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Rapidly rotating compact strange stars

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Abstract. We compute numerical, relativistic models of uniformly rotating strange stars for the recently proposed QCD-based equation of state (EOS) of strange quark matter (Dey et al. 1998). Static models based on this EOS are characterised by a larger surface redshift than strange stars within the MIT bag model. The frequencies of the fastest rotating configurations described by the Dey et al. (1998) model are much higher than those for neutron star models and for the simplest strange star MIT bag model. We determine a number of physical parameters for such stars and compare them with those obtained for neutron stars. We construct constant baryon mass equilibrium sequences, both normal and supramassive. We find the upper limits on the maximal masses and maximal frequencies of the rotating configurations. We show that the maximum mass limit does not coincide with the stability limit to quasi-radial perturbation. Just as for a neutron star model, a supramassive strange star, prior to its collapse to a black hole, increases its spin as it loses angular momentum. We find that for any given baryon mass, the maximal rotating configuration is not Keplerian. A normal and low mass supramassive strange star gaining angular momentum always slows down just before reaching the Keplerian limit. For a high-mass supramassive strange star sequence, the Keplerian configuration is the one with the lowest rotational frequency in the sequence. The value of T/W for rapidly rotating strange stars of any mass is significantly higher than that for ordinary neutron stars. For Keplerian configurations it increases as mass decreases. The results are robust for all linear self-bound equations of state.

Key words: dense matter – equation of state – stars: neutron – stars: peculiar

1. Introduction

The possibility of the existence of quark matter dates back to the early seventies. Bodmer (1971) remarked that matter consisting of deconfined up, down and strange quarks could be the absolute ground state of matter at zero pressure and temperature. If this is true then objects made of such matter so-called "strange stars" could exist (Witten 1984). Most of the previous calculations of the properties of strange stars (static models – see e.g. Alcock et al.,1986; Haensel et al., 1986, for rotating models, see references in Gourgoulhon et al. 1999) were done for an equation of state based on the MIT bag model, in which quark confinement and asymptotic freedom were postulated from the very beginning, and the deconfinement of quarks at high densities was not obvious.

Several authors have addressed the question of the existence of strange stars in the Galaxy. A binary merger of two strange stars (or of a strange star with a neutron star) might contaminate the entire Galaxy with strange matter seeds, thus precluding the formation of young neutron stars in supernovae (Madsen 1988; Caldwell & Friedman 1991). However, Alpar (1987) noted that glitching radio pulsars are not strange stars, but young neutron stars. Hence, it would appear, that strange stars are not present in binaries of the Hulse–Taylor type. By extension, it seems unlikely that strange stars are born in ordinary supernovae, as a fraction of supernovae must in fact occur in progenitor systems of Hulse-Taylor type binaries. However, strange stars could exist as millisecond pulsars and be formed from neutron stars in low-mass X-ray binaries through a phase transition (Kluźniak 1994; Cheng & Dai 1996).

Strange stars described by the simple MIT bag model with massless and non-interacting quarks have orbital frequencies in the marginally stable orbit which are higher than the lowest maximum frequency of kHz quasiperiodic oscillations observed in LMXBs (Bulik et al. 1999a,b) . However the corresponding frequencies for more sophisticated models (MIT bag model with massive strange quarks and lowest order QCD interactions, and/or rotation (Stergioulas et al. 1999; Zdunik et al. 2000a,b) do allow frequencies as low as 1 kHz, in agreement with observations.

Recently Dey et al. (1998) derived an EOS for strange matter which has asymptotic freedom built in and describes deconfined quarks at high density and confinement at zero density. Restoration of chiral quark masses at high density is incorporated in this model and using the model parameter for this restoration one can calculate the density dependence of the strong coupling constant (Ray et al. 2000). This model, with an appropriate choice of the EOS parameters, gives absolutely stable strange quark matter. This equation of state was used to calculate the structure of static strange stars and the mass-radius relations. Later, it was suggested by Li et al. (1999a) that the millisecond X-ray pulsar SAX1808.4-3658 is a strange star. This equation also allowed the explanation of the observed properties of other objects: an analysis of semi-empirical mass-radius relations in 4U 1728-34 (Li et al. 1999b) and 4U 1820-30 (Bombaci 1997), Her X-1 leads to the suggestion that these objects host strange stars (Dey et al. 1998). Two cases of this model have been used in these papers, which will be denoted as SS1 and SS2 equations of state. They both give a rather low value for the maximum gravitational mass $M_{max} = 1.33\,M_\odot$ for SS1 and $M_{max} = 1.44\,M_\odot$ for SS2. Very recently, the Dey model was used for calculating frequencies of marginally stable orbits around static strange stars and strange stars rotating with frequency 200 and 580 Hz (Datta et al. 2000). The authors conclude that very high QPO frequencies in the range of 1.9-3.1 kHz would imply existence of a non-magnetized strange star X-ray binary rather than a neutron star X-ray binary.

In this paper, we construct both normal and supramassive constant baryon mass equilibrium sequences in the frame of general relativity. To this end we use the well tested code for rapidly rotating compact objects, for description see Gourgoulhon et al. (1999). Calculations of equilibrium sequences of rapidly rotating compact stars are crucial to understand various astrophysical phenomena and objects, such as LMXBs, QPO and millisecond pulsars. In this paper we calculate the properties of rapidly rotating strange stars described by the models SS1 and SS2. We study which of these are characteristic for stars within the Dey et al. 1998 model and which are common for all models described by self-bound EOS. We compare the properties of compact rotating strange stars with those for neutron stars. We find the upper limits on observable astrophysical quantities, such as masses and frequencies of rotating stars. We locate the rotating maximum mass model and the maximum angular velocity model.

In Sect. 2 we outline briefly the equation of state used throughout this work and compare static model SS1 and SS2 stars with the MIT bag model strange stars. In Sect. 3 we describe the rotating configuration of the compact strange stars, and in Sect. 4 we discuss the results.

2. Equation of state and static strange star models

In the present paper we describe strange quark matter using the model presented by Dey et al. (1998). In this model quarks of the density dependent mass are confined at zero pressure and deconfined at high density. The quark interaction is described by an interquark vector potential originating from gluon exchange, and by a density dependent scalar potential which restores the chiral symmetry at high densities.

We start our calculation by noticing that the equations of state SS1 and SS2 can be very well approximated by a linear function $P(\rho)$- see Fig. 1:

$$P = a \cdot (\rho - \rho_0). \quad (1)$$

with $n(P)$ resulting from first law of thermodynamics $n(P) = n_0 \cdot \left[1 + \frac{1+a}{a} \frac{P}{\rho_0 c^2}\right]^{1/(1+a)}$. We find it very interesting that the

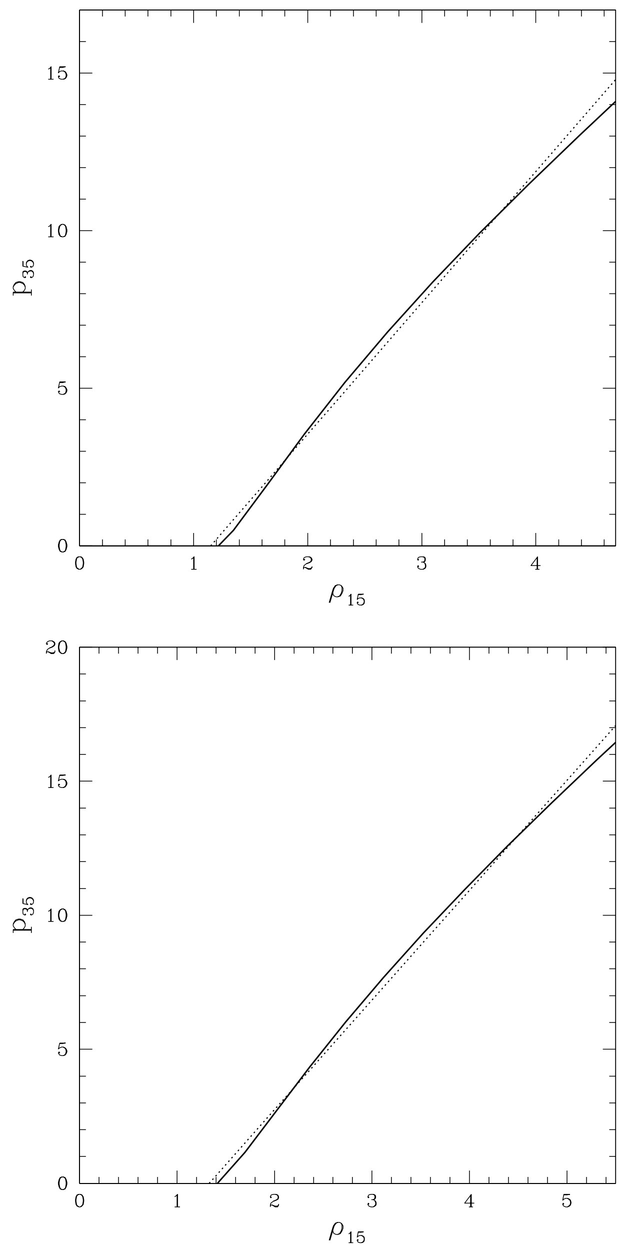

Fig. 1. Equations of state (the dependence of pressure in the units of 10^{35} dyne cm^{-2} on the density in the units of 10^{15} g cm^{-3}) considered in this work; top panel – SS1, bottom panel – SS2. The thick solid line is the tabulated equation of state, and the dotted line is the linear approximation.

equation of state based on rather complicated physics can have a simple form like that of Eq. (1). In general, the equation of the type (1) corresponds to self-bound matter at the density (mass-energy) ρ_0 at zero pressure and with a fixed sound velocity ($\sqrt{a}$). Thus equations of state SS1 and SS2 are physical realizations of linear equations of state considered previously, see e.g. Glendenning (1997). Such a parametrization is very convenient – we not only can calculate the stellar structure for stars described by equations of state SS1 and SS2 but also can use the scaling relations to extend the results to stars described by the EOS of the form given by Eq. (1). All stellar parameters are subject to the scaling relations with appropriate powers of ρ_0 for a fixed value of a (see e.g. Witten 1984; Zdunik 2000).

To calculate parameters a, ρ_0 and n_0 we use a least squares fit method taking into account the region of the densities, which is relevant to the interior of stable stellar configurations, i.e we neglect in the fitting the part of the EOS for densities larger than the central density in the last stable configuration (maximum mass in the non-rotating case). For the case of SS1 we obtained the values of $a = 0.463$, $\rho_0 = 1.15 \times 10^{15}$ g cm^{-3}, $n(P = 0) = n_0 = 0.725$ fm^{-3} and for the case SS2 the values are $a = 0.455$, $\rho_0 = 1.33 \times 10^{15}$ g cm^{-3}, $n_0 = 0.805$ fm^{-3}, see Fig. 1.

We have calculated the static stellar configurations using both the tabulated equation of state of Dey et al. (1998) and its linear approximation. The linear approximation agrees very well with calculations of stellar parameters of non-rotating configurations using the tabulated form of the EOS. The difference in the range of masses and radii is smaller than 2%, and the maximum mass point agrees within 0.2%.

We present the physical parameters for the maximum mass static strange stars described by Eqs. SS1, SS2 in the left columns of Table 1. The stars described by EOS SS1 and SS2 are very compact i.e. the gravitational redshifts z for the maximum mass configurations are much larger than those for SS within the MIT bag model (also larger than z for most models of neutron stars), for which z varies from 0.432 to 0.477 for a massive strange quark with $m_s = 250$ MeV and for massless quarks respectively. The maximal baryon mass in the case of static strange stars described by the SS1 and SS2 is relatively low (most of the neutron star models have a maximum baryon mass close to 2 $M_\odot$). In the case considered here the difference between the baryon mass and the gravitational mass is much higher (24% and 29%) than in the case of neutron stars (up to 10%) (for the MIT bag stars it is $\sim\ 20 - 34\%$). It should be however noted that in the case of strange stars we calculate the total baryon mass of the star using the nucleon mass and thus we include the binding energy of strange matter with respect to nuclear matter.

3. Rotating star configurations

We have calculated important properties of the uniformly rotating strange stars described by the SS1 and SS2 equations of state using the multi-domain spectral methods developed by Bonazzola et al. 1998. This method has been used previously for calculating rapidly rotating strange stars described by the MIT bag model (Gourgoulhon et al. 1999, Zdunik et al., 2000b). The multi-domain technique allows one to address the density discontinuity at the surface of self-bound stars even with a very high ρ_0.

We construct equilibrium sequences of rotating compact strange stars with constant baryon mass, i.e. the so-called evolutionary sequences (for example a pulsar keeps its baryon mass constant while slowing down; neglecting accretion a compact star keeps its rest mass constant during evolution). We identify normal and supramassive stars as done for neutron stars. A sequence is called normal if it terminates at the zero angular momentum limit with a static, spherically symmetric solution, and it is called a supramassive sequence if it does not. The boundary between these two sequences is the sequence with the maximum baryon mass of a static configuration. The angular momentum (and the central density) of a star changes monotonically along each sequence. Note that the rotational frequency f does not necessarily change monotonically along a sequence, since a star changes its shape and, consequently, moment of inertia with increasing angular momentum.

Table 1. Properties of the strange stars within the Dey model with maximal masses. The symbols are as follows: M and $M_{\rm bar}$ are gravitational and baryon masses respectively, R_{eq} is circumferential radius; $n_{\rm c}$ is the central baryon density; $\rho_{\rm c}$ is the central proper energy density divided by c^2, P is the rotation period; z^f_{eq}, z^b_{eq}, z_{pole} are the redshift for an emission at the equator and in the direction of rotation, the redshift for an emission at the equator and in the direction opposite to rotation and the redshift at the stellar pole respectively.

	static models		rotating models	
Model:	SS1	SS2	SS1	SS2
$M[M_\odot]$	1.435	1.323	2.05	1.88
$M_{\rm bar}[M_\odot]$	1.853	1.641	2.61	2.30
R_{eq}[km]	7.07	6.55	10.5	9.7
n_c[fm^{-3}]	2.35	2.64	1.75	1.77
$\rho_c[10^{15}$ g cm$^{-3}]$	4.68	5.6	3.16	3.64
P[ms]	–	–	0.39	0.36
z^f_{eq}	0.580	0.574	– 0.38	– 0.37
z^b_{eq}	0.580	0.574	3.63	3.50
z_{pole}	0.580	0.574	1.03	0.79

3.1. Equilibrium sequences

Stable solutions for rotating neutron stars have to satisfy four different constraints (Cook et al. 1994): the static constraint – for the normal evolutionary sequence of rotating stars, when angular momentum goes to zero the configuration should be identical to the one described by OV equations for the same baryon mass; the low mass constraint below which a neutron star cannot form, the mass-shed (Keplerian) constraint, and the constraint of stability to quasi-radial perturbations.

The first three constraints provide bounds on normal sequences stars (those are always stable to quasi-radial perturbations), while the two last limits provide bounds on the supramassive stars.

The mass-shed limit is reached when the velocity at the equator of a rotating star is equal to the velocity of an orbiting particle (a star becomes unstable when gravitational attraction is not sufficient to hold matter bound to the surface). For normal sequences of neutron stars and low mass supramassive neutron stars, Keplerian configurations are always these with the maximal rotational frequency. For high mass supramassive neutron stars the Keplerian configuration is the one with lowest rotational frequency in the sequence.

The fourth constraint is the requirement of stability to axisymmetric perturbations. For an evolutionary sequence parameterized by the central density, the model is (secularly) stable if $\left(\frac{\partial J}{\partial \rho_c}\right)_{M_{bar}} < 0$ (or $\left(\frac{\partial M}{\partial \rho_c}\right)_{M_{bar}} < 0$) and unstable otherwise (Friedman et al. 1986). The stability constraint imposes a limit which begins at the maximum mass static configuration and terminates at the Keplerian limit sequence near the maximum mass rotating configuration (see for example gravitational mass versus central density dependence at Fig. 1 in Cook et al. (1994). Normal sequences begin at the static limit and terminate at the mass-shed limit. Along such sequences angular momentum increases while central density decreases. Supramassive star sequences begin at the stability limit with a minimal angular momentum. As the angular momentum increases, the central density decreases until the configuration terminates at the Keplerian limit. The intersection of the mass-shed and the stability limits on the gravitational mass – central density plane give us the locations of the configuration rotating with maximum frequency. For neutron stars the maximum mass Keplerian configuration is not necessarily stable against axisymmetric perturbations. For some equations of state of neutron stars the maximum mass configuration is the same as the configuration with the maximum frequency.

3.2. Limits on gravitational mass

To find equilibrium sequences of compact strange stars we take only three of the above constraints into account: the low mass constraint is not relevant for self-bound matter. Other instabilities (for example to nonaxisymmetric perturbation) are not considered here since we cannot study them with our numerical code. Taking into account all constraints described above we found limits on masses and rotation frequencies for the SS2 model, shown as thick lines in Fig. 2. The mass shed limit is shown as a thick short-dashed line, the stability limit as long-dashed thick line, the fastest normal and low mass supramassive configurations as a dot-dashed line, and the "low rotational frequency" maximum mass configurations as a thick solid line. The thin solid lines in Fig. 2 correspond to normal and massive supramassive equilibrium evolutionary sequences and are labelled with their baryon mass. In Fig. 3 we show an evolutionary sequence with a baryon mass of $1.8 M_\odot$ as an example of the low mass supramassive stars (with baryon mass lower than 1.9 $M_\odot$). The marginally stable configurations with respect to quasi-radial perturbation is marked with an open circle. The angular momentum increases along each curve from $J = 0$ for static configurations ($J_{\rm min}$ for supramassive stars) to $J_{\rm max}$ for the Keplerian ones represented by filled circles for normal and low mass supramassive sequences. The evolutionary sequence with $M_{\rm bar} = 1.64$ separates normal stars from the supramassive ones.

The maximum mass configuration is found by considering a sequence of stars with a constant rotational frequency $f = \Omega/2\pi$ and parameterized by the central density. The maximum mass models for each sequence are represented by thick lines in Fig. 4. Chosen sequences are shown as thin solid lines.

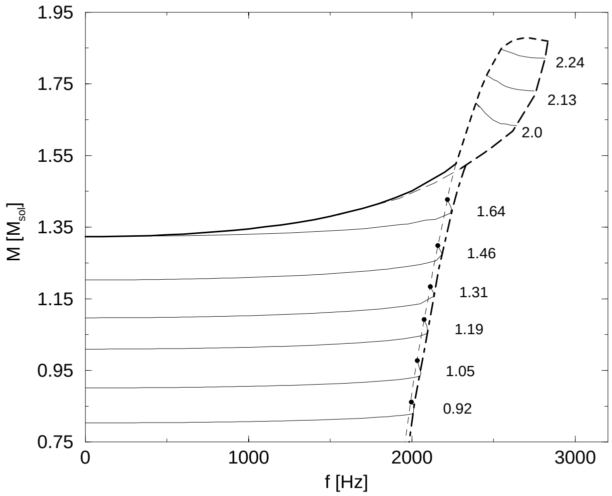

Fig. 2. Gravitational mass as a function of the rotation frequency for the SS2 model. Thin solid lines correspond to evolutionary sequences with fixed baryon mass labelled close to each line. The angular momentum increases along each curve from $J = 0$ for static configurations ($J_{\rm min}$ for supramassive stars) to $J_{\rm max}$ for the Keplerian ones represented by filled circles for normal and low massive supramassive stars. The thick lines correspond to the upper limits on gravitational mass and rotational frequency; the dashed line corresponds to the mass-shed limit, the long-dashed line is the quasi-radial stability limit and the dashed-dotted line shows the fastest rotating configurations.

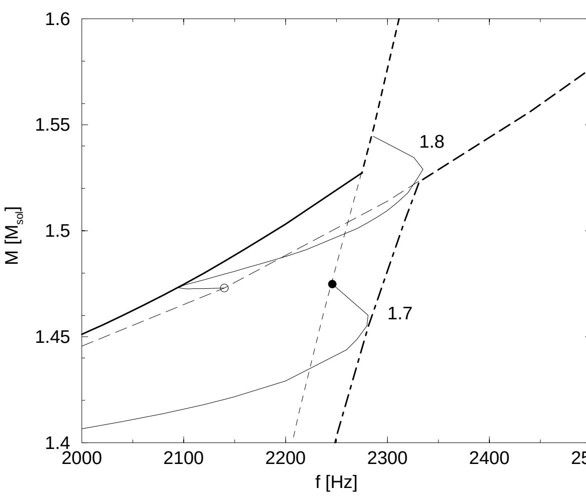

Fig. 3. Limits on gravitational mass and on the rotation frequency and evolutionary supramassive sequences with rest mass 1.7 and 1.8 $M_\odot$. All lines are as indicated in Fig. 2. The marginally stable configuration with respect to quasi-radial oscillation is shown as an open circle.

The Keplerian configurations are shown as a dashed line, while marginally stable configurations with respect to quasi- radial oscillation as long-dashed line. On one end of each sequence there is a Keplerian configuration (with the lowest central density in the sequence), and on the other end the last stable configuration with respect to axisymmetric perturbations (the densest object in the sequence). For $f < 2.27$ kHz the maximum mass configuration (shown as thick solid line) for each sequence is close to the marginally stable one. For fast rotating configurations,

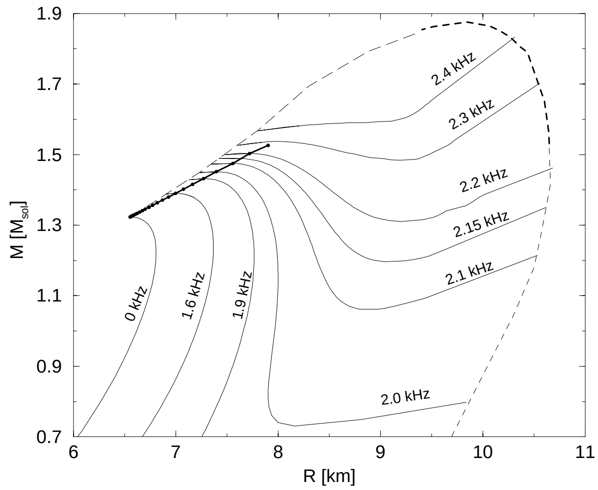

Fig. 4. Gravitational mass vs. radius for stars described by the SS2 model. The thin solid lines correspond to sequences of stars with constant rotational frequency $f = \Omega/2\pi$. The rotational frequency is labelled close to each line. The thick lines (solid and short-dashed) correspond to configurations with the maximal mass in each sequence. The dashed line corresponds to the mass-shed limit and the long dashed line is the quasi-radial stability limit. The intersection of the mass-shed and the stability limits gives the location of the configuration rotating with maximum frequency.

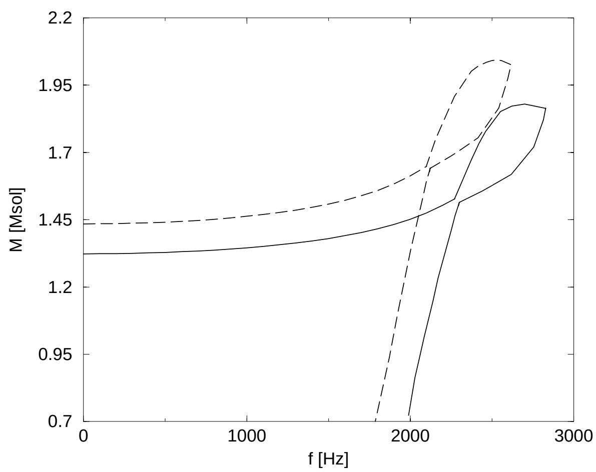

Fig. 5. Limits on gravitational mass and the rotation frequency for SS1 (dashed lines) and SS2 (solid lines).

$f > 2.27\,\mathrm{kHz}$ the maximum mass configurations are the Keplerian ones (for comparison see Fig. 1b in Gondek-Rosinska et al. 2000 for strange stars described by the MIT bag model).

The thick solid line and thick short dashed line in Fig. 2 correspond to the maximal gravitational mass of a rotating strange star described by the SS2 equation of state as a function of the rotational frequency. There are two cases as the rotational frequency increases, first for rotational frequencies lower than $2.27\,\mathrm{kHz}$ (in the case of the SS2 model) and second for $f > 2.27\,\mathrm{kHz}$. In the first case the maximal mass limit line goes very close to the stability limit (shown as thin long dashed line). For rotation frequencies lower than 1 kHz the stellar configuration is only slightly affected by rotation – the increase in the maximal mass is only a second order effect (where both the maximum mass line and the normal evolutionary sequence with maximum baryon mass are very close to each other) ie. $M_{\max}(f) - M^0_{max} \propto f^2$, where M^0_{max} is the static mass configuration. For higher frequencies $M_{\max}(f)$ increases faster. The maximum mass of rotating compact strange star in this case is approximately 15% greater than the maximum mass of a static stellar configuration. For $f > 2.27\,\mathrm{kHz}$ we see a strong increase in the maximum mass (thick dashed line). In this case the configurations with maximum mass are Keplerian. The maximum mass of a rotating compact strange star in this regime (the maximum allowed mass of rotating configurations) is 42% greater than the maximum mass of a static stellar configuration.

Note that the maximal mass configurations for a given rotational frequency are always supramassive i.e. no static configuration with such a mass can exist. It is worth noting that the central density of the rotating configurations is always lower than the central density of the maximum mass static star. This makes us confident that the rotating configurations found with the use of theapproximation of Eq. 1 are at least as accurate as the static configuration calculations.

3.3. *Limits on rotational frequency*

Let us now consider upper limits on rotational frequency of compact strange stars. In the case of high mass supramassive stars the maximum rotational frequency is determined by the condition of stability to axisymmetric perturbations, shown as a thick long dashed line in Fig. 2. Similarly to the supramassive neutron stars, the configurations to the left of this line are stable to radial collapse – they spin up loosing their angular momentum as discussed by Cook et al. (1994) in the case of supramassive neutron stars and by Gourgoulhon et al. (1999) in the case of supramassive MIT bag model strange stars.

In the case of normal and low mass supramassive stars, the maximum rotational frequency is slightly above the rotation frequency of the Keplerian configuration. The fastest configurations are shown as a thick dash-dotted line in Fig. 2 while the Keplerian one by filled circles. In Fig. 3 we present the details of the intermediate region of Fig. 2 for $2.0\,\mathrm{kHz} < f < 2.5\,\mathrm{kHz}$ to show low mass supramassive star sequences.

The rotational frequency decreases for large values of the angular momentum and the sequences turn back in Fig. 2 and Fig. 3 before reaching the Keplerian configuration. At this small part of an evolutionary sequence, configurations spin up by loosing the angular momentum (or slow down when obtaining the angular momentum). The mass-shed configurations are reached due to the increase of the equatorial radius relative to the deformation of the rotating star. The difference between the Keplerian frequency and the maximal rotation frequency for these evolutionary sequences SS2 model is of the order of 2%. Such a phenomenon was discussed by Zdunik et al. (2000b) in the case of normal sequences of MIT bag model strange stars. It is interesting to note that no such behavior was noticed for neutron stars. This feature is characteristic of stars described by a self-bound linear EOS.

The mass shed limit and the stability limit lines intersect twice: at frequency ≈ 2.25 kHz and at the frequency of ≈ 2.8 kHz. The intersection of the mass-shed limit (solid thick line in Fig. 2) and stability limit (solid dashed line in Fig. 2) determines the configuration rotating with maximum allowed frequency. Note that the maximum frequency occurs for only one extreme supramassive model and that this model is at the mass-shed limit and stability limit.

The configuration with the absolute maximum mass lies on the Keplerian limit line. Note that the maximum mass configuration is not rotating with the maximum allowed rotational frequency. For both SS1 an SS2 models the maximum mass rotating configuration is on the stable side of the mass-shed limit line.

In Fig. 5 we present the regions of the parameter space given by mass and rotation frequency where strange stars described by the SS1 (dashed line) and SS2 (solid line) equations of state can exist. The maximal mass and maximal frequency configurations lie on the Keplerian limit line. Details of the Keplerian configurations with the maximum mass are given in the right columns of Table 1. The maximum mass of rotating strange star given by SS1 and SS2 model is 42% larger (50% the equatorial radius) than in the case of a non-rotating star with maximum mass. For strange stars with massless and not interacting quarks these values are 44% and 54% respectively. The ratios of masses $M_{\rm max}^{\rm rot}/M_{\rm max}^{\rm stat}$ and equatorial radii $R_{\rm max}^{\rm rot}/R_{\rm max}^{\rm stat}$ do not depend on the parameter ρ_0, since static and rotating maximum mass configurations scale identically, and are functions only of a (sound velocity) in the case of stars described by Eq. (1).

For neutron stars the maximum mass which can be supported when the star is rotating uniformly increases by 14% to 21% depending on the equation of state while the radius increases by 30% to 39% (e.g. Cook et al. 1994; Datta et al. 1998) We note that a large increase (higher than in the case of neutron stars) of the maximum mass and corresponding equatorial radius in the case of strange stars of different EOS due to rotation is related to the fact that these EOS are self-bound with a very high density at the surface. They are much more compact than neutron stars and much higher rotation frequencies are required to deform them to reach Keplerian configurations.

3.4. T/W ratio for rotating strange stars

It has been shown by Gourgoulhon et al. (1999), Stergioulas et al. (1999) and Gondek-Rosińska et al. (2000) (where in addition to the treatment of first two papers, mass and interaction between quarks were taken into account) that T/W can be very high for strange stars described by the MIT bag model. In Fig. 6 we present the ratio of the rotational kinetic energy to the absolute value of the gravitational potential energy T/W for evolutionary sequences for SS2 model. Each sequence is labeled with its baryon mass. The thin dashed line corresponds to the sequence of configurations rotating with the same rotational frequency. Note that for a rotating strange star the value of T/W is significantly higher than that for an ordinary NS (eg. Cook et al. 1994). The large value of T/W results from a flat density profile combined with strong equatorial flattening of rapidly rotating strange stars. It increases as mass decreases for Keplerian configurations. This property is universal for all self-bound linear EOS. T/W does not depend on ρ_0 and its dependence on a is very weak. We can easily obtain a similar diagram for the simplest MIT model EOS of SS using the scaling relations $M_{\rm ss} = (a_{\rm D}\rho_{0,\rm ss}/a_{\rm ss}\rho_{0,\rm D})^{-1/2} M_{\rm D}$, $f_{\rm ss} = (a_{\rm D}\rho_{0,\rm ss}/a_{\rm ss}\rho_{0,\rm D})^{1/2} f_{\rm D}$ (Stergioulas et al 1999, for Keplerian configurations), where the subscript D denotes the EOS considered here. We check that the above scaling relations hold for any model in the evolutionary sequences. The difference in mass and rotational frequency between numerical results and rescaled ones are of the order of 2% and 4%. The value of T/W is quite large for all configurations close to Keplerian, so it is possible that the point of onset of secular instability to non-axisymmetric normal modes has already been passed. For mass-shed configurations T/W varies from 0.21 for maximum mass configuration to 0.27 for the low mass one (0.25 and 0.26 for gravitational mass 1.4 $M_\odot$ for SS1 and SS2 Keplerian models). This could be an indicator that rapidly rotating SS may constitute strong sources of gravitational waves (Gourgoulhon et al. 1999; Gondek-Rosińska et al. 2000; Gondek-Rosińska & Gourgoulhon 2000).

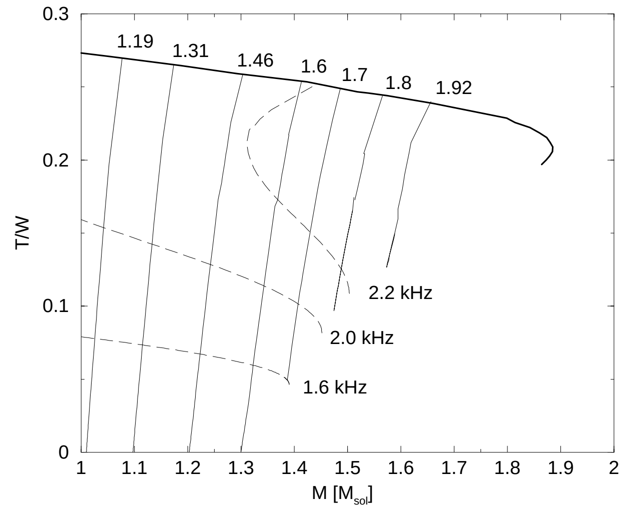

Fig. 6. The ratio of the rotational kinetic energy to the absolute value of the gravitational potential energy T/W for SS2 evolutionary sequences with fixed baryon mass labelled above each line in sollar mass units. The solid thick line corresponds to Keplerian configurations. The thin dashed line correspond to sequences of configuration rotating with the same rotational frequency. Models located to left and below of this line rotate with lower frequency.

4. Discussion

4.1. Summary of the properties of rotating compact strange stars

We have calculated numerical models of the uniformly rotating strange stars described by the SS1 and SS2 equations of state using the multi-domain spectral methods, which allows a treatment of the density discontinuity at the surface of self-bound stars with even very high ρ_0. The model used here describes the quark interactions self-consistently. The maximum mass of strange stars within this model is relatively low, and the stars are very compact. We find that the stars within the Dey model can rotate much faster than typical neutron stars, and also the MIT bag model strange stars. The maximum allowed rotational frequency is $2.6\,\mathrm{kHz}$ for SS1 and $2.8\,\mathrm{kHz}$ for SS2. The maximal mass of a rotating configuration is $2.05\,M_\odot$ for SS1 and $1.88\,M_\odot$ for SS2. The main physical reason for these high values of the rotational frequency and the low gravitational mass is that the parameter ρ_0 is quite large for this EOS.

Some properties of rotating strange stars described by the Dey EOS are universal and characteristic for all self-bound EOS. We find that: i) there are two cases for the maximal mass of a rotating configuration as the rotation frequency increases; first, for low rotation frequencies the increase in the maximal mass is only a second order effect ($M_{\max}(f)$ is very close to the line of limiting stability against a quasi-radial perturbation limit); second, for higher frequencies the maximal mass configurations are Keplerian; ii), the maximum mass of strange stars given by the MIT model and by SS1 and SS2 at the point of intersection with the line imposed by the Keplerian limit is approximately 15% greater than the maximum mass of a static configuration; iii), the maximum allowed mass is approximately 40% larger than the static maximum mass. This is much higher than for neutron stars; iv), we show that in contrast to normal neutron stars the maximal rotating frequency for both normal and supramassive stars is never the Keplerian one; v), we find that rotating strange stars have a very high ratio T/W. In the case of the Keplerian limit stars the ratio T/W increases with decreasing mass. Large values of T/W (higher than 0.2) imply that it is quite likely that the maximum rotational frequency can in fact be lower than found here.

4.2. Astrophysical aspects of the compact strange stars

The maximum frequency is very high – 2.6 kHz and 2.8 kHz for SS1 and SS2 models, respectively. It is important to remember that the maximum frequency occurs only for one extreme supramassive model and that this model is both on the mass-shed limit and the stability limit. But even for normal evolutionary sequences we reach very high frequencies – higher than 1.8 kHz and 2 kHz in the case of SS1 and SS2 respectively. The periods for stars rotating with maximal frequency can be shorter than half a millisecond, much shorter than the period $P = 1.56\ \mathrm{ms}$ of the fastest known millisecond pulsar PSR 1937 + 21.

The maximal masses for the SS1 and SS2 EOS are consistent with the observed masses of compact object. All observed pulsars have masses close to $1.4\,M_\odot$ and rotate with frequencies lower than the maximal frequencies for the SS1 and SS2 models. In the case of strange stars described by the SS1 and SS2 equations of state the maximum baryon mass are $1.85 M_\odot$ and $1.64 M_\odot$ (the difference between the baryon and gravitational mass is much greater than in the case of neutron stars). One can speculate that for high central densities in the core of a neutron star a phase transition to strange matter can take place. This can be accompanied by large energy release, and possibly a gamma-ray burst (Cheng & Dai 1996; Bombaci & Datta 2000). A rotating neutron star may become a strange star, conserving its total baryon mass and angular momentum. If the baryon mass of this star is lower than 1.84 $M_\odot$ it would become a normal sequence compact strange star. Otherwise, depending on its angular velocity, it can become a stable supramassive strange star and after it slows down finally a black hole. Just before transformation from a supramassive strange star to a black hole the star should accelarate (such phenomena was noticed by Cook et al. (1994) in the case of supramassive neutron stars). In Fig. 3 we show an evolutionary sequence with the baryon mass $1.8 M_\odot$ as an example of a low mass supramassive stars. This sequence begins at high J on the Keplerian limit, then losing angular momentum it reaches the maximum mass limit, and finally it spins up to reach the stability limit and collapse to a black hole. If a neutron star goes through a strange star stage and then ends up as a black hole, this may be an explanation as to why we do not observe pulsars with masses much higher than 1.4 $M_\odot$ (if pulsars are strange stars described by SS1 and SS2 equation of state.)

The masses of compact objects in LMXBs (inferred from kHz QPOs and assuming that the highest QPO frequency observed is related to a marginally stable orbit) are quite large, and extend even above $2\,M_\odot$. Such high mass neutron stars can still undergo a phase transition to form a supramassive compact strange star, which would consequently turn into a black hole. Note that the binary might be disrupted during the transition. To find such stars we would be looking for very fast millisecond pulsars, either single or in binaries.

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