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ASTRONOMY AND ASTROPHYSICS

Accurate masses of very low mass stars

IV. Improved mass-luminosity relations ⋆

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Abstract. We present improved visual and near-infrared empirical mass-luminosity relations for very low mass stars ($M<0.6\ M_\odot$). These relations make use of all stellar masses in this range known with better than 10% accuracy, most of which are new determinations with 0.2 to 5% accuracy from our own programme, presented in a companion paper.

As predicted by stellar structure models, the metallicity dispersion of the field populations induces a large scatter around the mean V band relation, while the infrared relations are much tighter. The agreement of the observed infrared mass-luminosity relations with the theoretical relations of Baraffe et al. (1998) and Siess et al. (2000) is impressive, while we find an increasingly significant discrepancy in the V band for decreasing masses. The theoretical mass-luminosity relation which is insufficiently steep, and has introduced significant errors in the local stellar mass functions derived from V band luminosity functions.

Key words: stars: binaries: general – stars: binaries: spectroscopic – stars: binaries: visual – stars: low-mass, brown dwarfs – techniques: radial velocities – stars: late-type

1. Introduction

The mass of a star is arguably its most basic characteristic, since most stellar properties have a very steep mass dependency. Yet, it can only be directly determined for some stars in multiple systems, and for most stars it has to be inferred from more directly observable parameters, the luminosity, the chemical composition, and the evolutionary status. An accurately calibrated Mass-Luminosity (hereafter M/L) relation, or in full generality a Mass-Composition-Age/Luminosity relation, is therefore an essential astrophysical tool (e.g. Andersen 1991, 1998). It is needed at many places to convert observable stellar light to the underlying mass, and perhaps most importantly, to derive stellar mass functions from more readily obtained luminosity functions. The latter in turn provide an essential diagnostic of star formation theories. They also represent a basic building block for galactic and stellar cluster dynamical models.

The M/L relation is fairly well constrained for solar-type and intermediate mass stars: a sizeable number of such stars in eclipsing systems have had their mass determined with better than 1% relative accuracy (Andersen 1991), and the theory of these stars approximately matches this excellent precision, when both evolutionary effects and metallicity are taken into account (Andersen 1998). For both smaller and more massive stars however, the M/L relation is significantly more uncertain, as theory and observations meet there with new difficulties.

Here we address the low-mass end of the HR diagram, below $0.6 M_\odot$, where stellar models face two major hurdles (Chabrier & Baraffe 2000, for a recent review):

- the onset of low temperature electron degeneracy in the stellar core (Chabrier & Baraffe 1997, 2000);
- a complex cold and high gravity stellar atmosphere, dominated by molecular and dust opacity (Allard et al. 1997; Jones & Tsuji 1997; Allard 1998).

Much progress has been made in the past few years, with the realization that an accurate atmosphere description *must* be used as an outer boundary condition to the stellar interior equations (Baraffe et al. 1995), and with an increased sophistication of the atmospheric models (Allard et al., in prep). State of the art models (Baraffe et al. 1998; Chabrier et al. 2000) now produce good to fair agreement with most observational colour-magnitude diagrams (Goldman et al. 1999, for an example). This lends considerable credence to their general reliability. Their description of some of the input physics however remains incomplete or approximate: some molecular opacity sources are still described by relatively crude approximations, or by line lists that remain incomplete (though vastly improved), the validity of the mixing-length approximation in the convective atmosphere is questionable, and atmospheric dust condensation and settling introduces new uncertainties at the lowest effective temperatures. The actual severity of these known shortcomings of the

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⋆ Based on observations made at the Observatoire de Haute Provence (CNRS), and at the CFH Telescope, operated by the NRCC, the CNRS and the University of Hawaii models is unclear, making an independent check of their M/L prediction most desirable.

Detached eclipsing M-dwarf binaries are rare, with only three known to date. Most mass determinations for Very Low Mass Stars (VLMS) are therefore instead obtained from visual and interferometric pairs, which until recently have not yielded comparable precisions. The current state of the art empirical M/L relation for M dwarfs (Henry & McCarthy 1993; Henry et al. 1999) as a result mostly rests on masses determined with 5-20% accuracy. The last two years have seen a dramatic evolution in this respect, with two groups breaking through the former $\sim$5% accuracy barrier. The first team used the 1 mas per measurement astrometric accuracy of the Fine Guidance Sensors (Benedict et al. 1999) on *HST* to determine system masses for three angulary resolved binaries with 2 to 10% accuracy (Franz et al. 1998; Torres et al. 1999; Benedict et al. 2000). Shortly thereafter our team demonstrated even better accuracies for masses of VLMS, of only 1-3% (Forveille et al. 1999; Delfosse et al. 1999b), by combining very accurate radial velocities with precise angular separations from adaptive optics. In a companion paper (Ségransan et al. 2000) we determine a dozen new or improved masses, with the same method and with accuracies that now range between 0.5 and 5%. The same paper also presents improved masses for two of the three known eclipsing M-dwarf systems, with 0.2% accuracy. Here we take advantage of this wealth of accurate new data, and reassess the VLMS M/L relation on a much firmer ground.

In Sect. 2 we briefly describe the sample of accurately determined very low stellar masses. We then discuss the resulting empirical M/L relation in Sect. 3, and compare it with theoretical models in Sect. 4.

2. Sample

2.1. Accurate masses for M dwarfs

We adopt a 10% mass accuracy cutoff for inclusion in our new M dwarf M/L relations, as a compromise between good statistics and the quality of the individual measurements. To our knowledge 32 M dwarfs fulfill this criterion. They can be divided into four broad categories:

1. Four systems have orbits of a quality that hasn't changed much since Henry & McCarthy (1993), but masses which are now somewhat better determined thanks to the availability of the Hipparcos parallaxes (Henry et al. 1999, and this paper): Gl 65, Gl 661, Gl 702, and Gl 860. One should note however that the Gl 661 masses derived by Martin et al. (1998) represent a $\sim 3\,\sigma$ correction to the Henry & McCarthy (1993) values. The new masses are much more consistent with the average M/L relation, and we believe that Henry & McCarthy (1993) had underestimated their standard errors for that particular pair. These four systems have long to very long periods, up to 90 years for Gl 702AB. Their inclusion is a testimony to the care and dedication of binary star observers over many decades, but the accuracy of most of these masses is unlikely to be significantly improved over the next few years.
2. The three eclipsing systems have extremely accurate masses, with relative precisions of 0.5% for CM Dra (Metcalfe et al. 1996) and 0.2% for YY Gem and GJ 2069A (Ségransan et al. 2000). Their parallaxes are unfortunately less precisely known, in part due to their slightly larger distances than those of the visual systems. Their luminosities are therefore more uncertain than their masses. Their flux ratios in the near-IR JHK bands have also not yet been determined.
3. The masses of Gl 473 (8%; Torres et al. 1999) and Gl 791.2 (2%; Benedict et al. 2000) result from the effort of the FGS astrometry team on HST (Benedict et al. 1999). We have chosen to temporarily exclude the measurement of the Gl 748 system mass by the same group (5%; Franz et al. 1998): a M/L relation has to be used to estimate individual masses for the two stars in that system. This would introduce an undesirable circular aspect to our discussion. We have on the other hand retained their Gl 473 determination, which formally has the same weakness. The two components of that system are sufficiently similar that this contributes negligible additional uncertainties.
4. Most of the masses in Table 3 result from our own programme: since 1995 we have been monitoring a sample of solar neighbourhood M dwarfs with high precision radial velocity and adaptive optics imaging observations (Delfosse et al. 1999c, for a complete presentation of the project). These observations have resulted in a new orbit for Gl 747, and in improved orbits and masses with 0.5 to 5% accuracy for Gl 234, Gl 644, Gl 831, Gl 866 (Ségransan et al. 2000), Gl 570B and Gl 623 (Ségransan et al. in prep). Some additionnal binaries, including discoveries from that programme (Delfosse et al. 1999c; Beuzit et al. in prep.) are nearing the time when their masses will be known with similar accuracies.

2.2. Photometry

Table 1 lists the basic properties of the selected systems: parallaxes, spectral types and integrated photometry. Table 2 lists the magnitude differences for the V, R, I, J, H and K photometric bands. The near-IR flux ratios were obtained either from litterature infrared speckle observations, or extracted from our adaptics images (Ségransan et al. 2000). For the three eclipsing systems the visible flux ratios were adopted from analyses of the light curves (Leung & Schneider 1978; Delfosse et al. 1999a; Lacy 1977). For the visual binaries, they were preferentially adopted from the FGS work of Henry et al. (1999), with the standard errors quoted in that article. When such measurements were unavailable, we relied instead on spectroscopic magnitude differences, using the relative areas of the ELODIE cross-correlation peaks as a proxy for the V band magnitude difference. The ELODIE cross-correlation has an effective bandpass centered close to the central wavelenghth of the Johnson V filter, but it is significantly broader. A "colour-tranformation" would

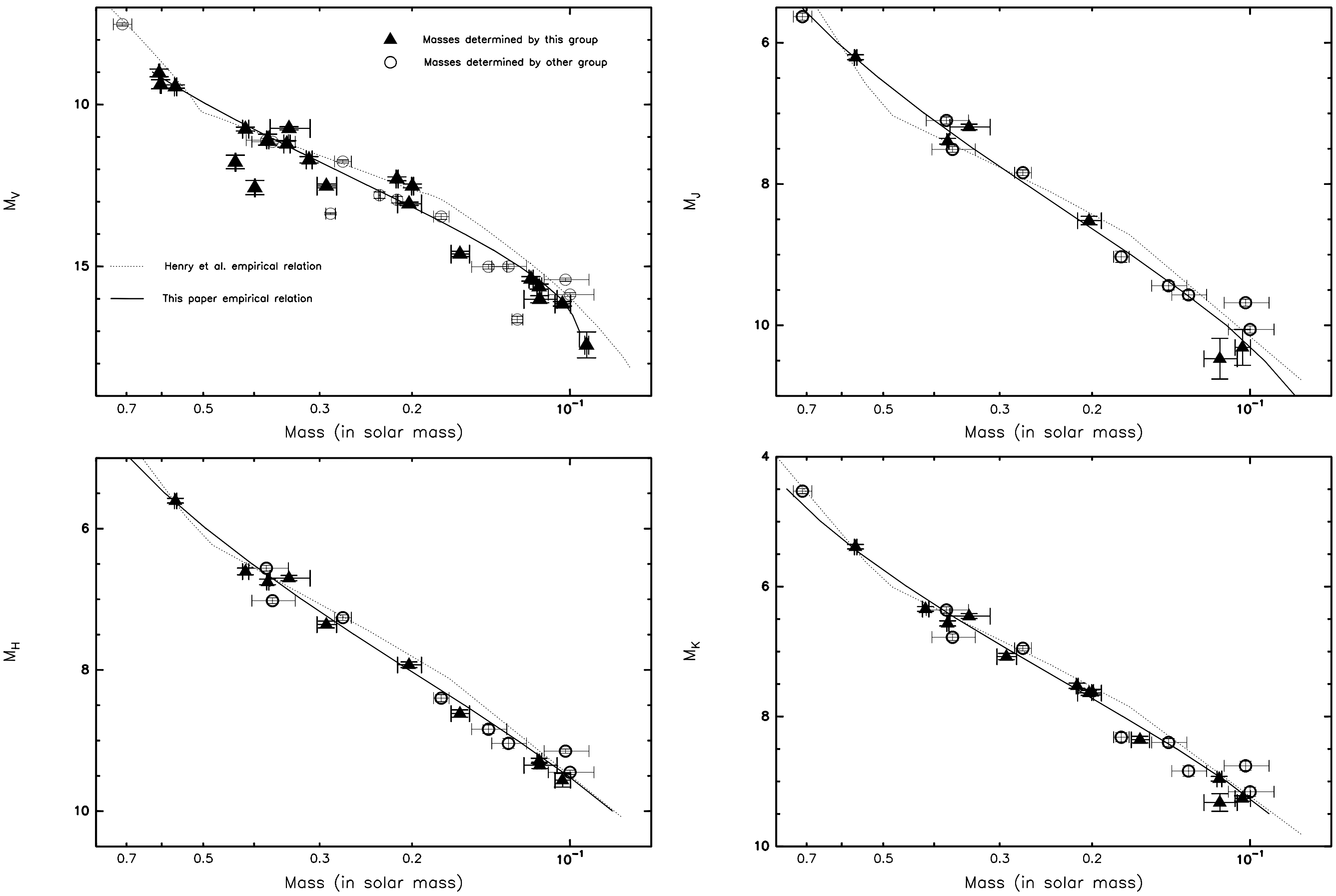

Fig. 1. V, J, H and K band M/L relations. The circles are data from Henry & McCarthy (1993), Torres et al. (1999), Henry et al. (1999), Benedict et al. (2000) and Metcalfe et al. (1996). The triangles represent our recent mesurements (Ségransan et al. 2000, in prep.; and Forveille et al. 1999). The masses and luminosities used in this figure are also listed in Table 3. The two curves represent the piecewise linear relation of Henry & McCarthy (1993; dotted line) and our polynomial fit (solid line).

thus in principle be needed to derive V band magnitude differences. A comparison with the direct measurements of Henry et al. (1999) for the sources in common shows maximum relative errors of ∼10% from neglecting this transformation: a 0.5 magnitude contrast is in error by at most 0.05 magnitude, and a 2 magnitudes one by at most 0.2 magnitude. We have therefore adopted the larger of 0.05 magnitude and 10% of the magnitude difference as a conservative estimate of the standard error for these spectroscopic magnitude differences.

3. Visible and infrared mass/Luminosity relations

The masses are listed in Table 3, with the individual absolute magnitudes derived from Table 1 and Table 2 for the four photometric bands (V, J, H and K) which have significant numbers of measurements. Fig. 1 shows the M/L relations for these 4 photometric bands. As can be seen immediately in Fig. 1, ∼20 stars define the V and K relations, while the J and H ones still have smaller numbers of stars. A number of systems still lack magnitude difference measurements in those two bands.

Fig. 2 presents the relation between stellar mass and the V-K colour index. This relation probably has too large an intrinsic dispersions to be generally useful, and is provided here mostly for illustration, and as a warning to potential users of similar relations.

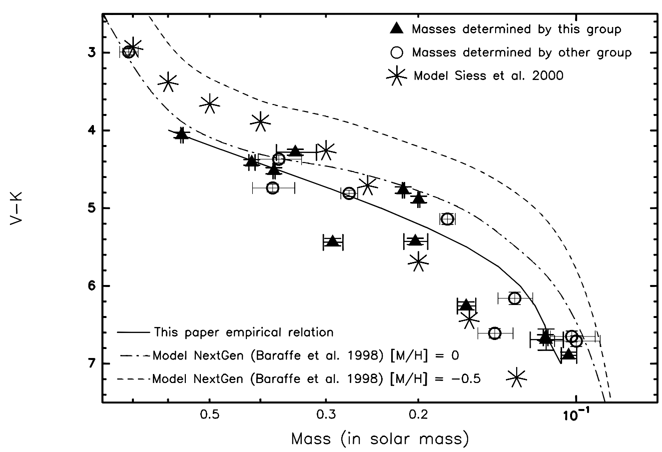

Fig. 2. mass-colour (V-K) relation for M dwarfs. The three curves are 5 Gyr theoretical isochrones from Baraffe et al. (1998) for two metallicities and our polynomial fit. The Siess et al. (2000) model are represented for 5 Gyr and solar metallicity with asterisks.

Table 1. Basic parameters for the M-dwarf systems with accurate masses. The parallaxes mostly originate from the Hipparcos catalog (ESA 1997), the Yale General Catalog of trigonometric Parallaxes (Van Altena et al. 1995), and from Ségransan et al. (2000). In that paper we derived optimal combinations of orbital and astrometric parallaxes, which often have a strong contribution from either an Hipparcos or a Yale catalog astrometric parallax. Some individual entries are additionally taken from Soderhjelm (1999), Forveille et al. (1999), and Benedict et al. (2000). All spectral types are from either Reid et al. (1995) or Hawley et al. (1997) and refer to the integrated light of the system. The photometry is taken from the extensive homogenized compilation of Leggett (1992), except for YY Gem, Gl 702AB and GJ2069A. The photometry of GJ2069A is from Weiss (1991). The optical photometry of YY Gem is from Kron et al. (1957) (RI), Eggen (1968) (UBV), Barnes et al. (1978) (UBVRI), and the infrared photometry from Johnson (1965), Glass (1975), and Veeder (1974). The optical photometry for Gl 702AB is from Bessel (1990) and the infrared photometry from Alonso et al. (1994). All photometry was converted to the Johnson-Cousins-CIT sytem adopted by Leggett (1992) using the colour transformations listed in that paper. Multiple measurements for the same band were averaged with equal weights.

Name	π (mas)	$\sigma(\pi)$ (mas)	Ref	Spectral type	B	V	R_c	I_c	J	H	K	L	L'
Gl 65AB	373.7	2.7	Yale95	M5.5V	13.87	12.00	10.37	8.31	6.24	5.67	5.33	5.00	
Gl 234AB	243.7	2.0	Ség00	M4.5V	12.80	11.08	9.77	8.06	6.40	5.78	5.49	5.33	
YY Gem	74.7	2.5	Yale95	M0V	10.50	9.07	8.10	7.09	6.01	5.35	5.18	5.16	
GJ 2069A	78.05	5.69	HIP	M3.5V		11.89	10.68	9.09					
Gl 473AB	227.9	4.6	Yale95	M5V	14.30	12.46	10.90	8.92	6.96	6.39	6.06		5.63
Gl 570B	169.8	0.9	For99	M1V	9.57	8.09	7.09	5.97	4.75	4.14	3.93	3.77	3.67
Gl 623AB	124.34	1.16	HIP	M2.5V	11.76	10.26	9.27	7.96	6.67	6.14	5.91		
CM Dra	69.2	2.5	Yale95	M4.5V	14.49	12.91	11.67	9.99	8.54	8.07	7.79		
Gl 644AB	154.8	0.6	Ség00	M3V	10.60	9.02	7.92	6.55	5.28	4.64	4.39	4.14	
Gl 661AB	156.66	1.37	Sod99	M3.5V	10.89	9.40	8.30	6.89	5.56	5.04	4.82	4.60	
Gl 702AB	195.7	0.9	Sod99	K0V	4.88	4.02	3.51	3.05	2.42	1.96	1.89		
Gl 747AB	120.2	0.2	Ség00	M3V	12.93	11.25	10.11	8.65	7.25	6.69	6.43		
Gl 791.2AB	112.9	0.3	Ben00	M4.5V	14.72	13.06	11.72	9.96	8.20	7.63	7.33		
Gl 831AB	117.5	2.0	Ség00	M4.5V	13.68	12.01	10.71	9.02	7.29	6.70	6.42		
Gl 860AB	247.5	1.5	HIP	M3V+M4V	11.25	9.59	8.40	6.91	5.56	4.97	4.71	4.48	
Gl 866ABC	293.6	0.9	Ség00	M5.5V	14.29	12.33	10.66	8.62	6.50	5.91	5.57	5.22	5.01

Fig. 1 shows the piecewise-linear relations adjusted by Henry & McCarthy (1993) to the J, H and K band data then available to them, and their piecewise-quadratic relation for the V band, with its Henry et al. (1999; V band) update for the lower masses. These relations provide a reasonable description of the new data, but they do show significant discrepancies, in particular around their breakpoints. Clearly the quality of the new masses warrants the use of higher order polynomials. We have found that the following fourth degree polynomials provide good descriptions of the data in Figs. 1 and 2:

$$\log(M/M_\odot) = 10^{-3}\times[0.3 + 1.87\times M_V + 7.6140\times {M_V}^2 \\ -1.6980\times {M_V}^3 + 0.060958\times {M_V}^4] \quad \text{for } M_V \in [9, 17]$$

$$\log(M/M_\odot) = 10^{-3}\times[1.6 + 6.01\times M_J + 14.888\times {M_J}^2 \\ -5.3557\times {M_J}^3 + 0.28518.10^{-4}\times {M_J}^4] \quad \text{for } M_J \in [5.5, 11]$$

$$\log(M/M_\odot) = 10^{-3}\times[1.4 + 4.76\times M_H + 10.641\times {M_H}^2 \\ -5.0320\times {M_H}^3 + 0.28396\times {M_H}^4] \quad \text{for } M_H \in [5, 10]$$

$$\log(M/M_\odot) = 10^{-3}\times[1.8 + 6.12\times M_K + 13.205\times {M_K}^2 \\ -6.2315\times {M_K}^3 + 0.37529\times {M_K}^4 \quad \text{for } M_K \in [4.5, 9.5]$$

$$\log(M/M_\odot) = 10^{-3}\times[7.4 + 17.61\times(V-K) \\ +33.216\times(V-K)^2 + 34.222\times(V-K)^3 \\ -27.1986\times(V-K)^4 + 4.94647\times(V-K)^5 \\ -0.27454\times(V-K)^6 \quad \text{for } V-K \in [4, 7]$$

One striking characteristic of Fig. 1 is the very different scatters in its V and K diagrams. The V plot displays considerable dispersion around a mean relation, and some of its best measurements are also some of the most discrepant. The K plot on the other hand shows a one to one relation between Mass and Luminosity, and its (mild) outliers are systems with larger than average errorbars. The J and H plots also have little dispersion, to the extent that this can be assessed from their smaller number of measurements. The V band scatter is much larger than the measurement errors, which on average are actually somewhat smaller for V than for K. This different behaviour of the visual and infrared bands, first seen so clearly here, is predicted by all theoretical models, as discussed for instance in the recent review by Chabrier & Baraffe (2000). It results from the metallicity dispersion of the solar neighbourhood populations, through the interplay of two physical mechanisms:

- a larger metallicity increases the atmospheric opacity in the visible range, which is dominated by TiO and VO molecular bands. For a given bolometric luminosity it therefore shifts the flux distribution towards the infrared;
- a larger metallicity decreases the bolometric luminosity for a given mass.

In the visible bands both effects work together, to decrease the visible luminosity of the more metal-rich stars at a given mass. In the near-infrared on the other hand, the redward shift of the flux distribution of the metal-rich stars counteracts their lower bolo-

Table 2. Magnitude differences for M-dwarf systems with accurate masses. Reference codes are: TYC for the Tycho catalogue HIP for the Hipparcos catalogue L77 for Lacy (1977), L78 for Leung & Schneider (1978), Hen93 for Henry & McCarthy (1993), C94 for Coppenbarger et al. (1994), B96 for Barbieri et al. (1996), Ben00 for Benedict et al. (2000), Tor99 for Torres et al. (1999) Hen99 for Henry et al. (1999), D99a for Delfosse et al. (1999a), For99 for Forveille et al. (1999), and D00 for the present paper. S stands for spectroscopic magnitude differences, infered from the relative line depths for double-lined spectroscopic binaries.

Name	ΔV	Ref	ΔR	Ref	ΔI	Ref	ΔJ	Ref	ΔH	Ref	ΔK	Ref
Gl 65AB	0.45±0.08	Hen99					0.38±0.03	Hen93	0.30±0.02	Hen93	0.40±0.07	Hen93
Gl 234AB	3.08±0.05	Hen99					1.79±0.30	C94	1.63±0.11	C94	1.62±0.02	D00
YY Gem	0.35±0.20	L78	0.34±0.19	L78	0.32±0.27	L78						
GJ 2069A	0.79±0.10	D99a										
Gl 473AB	−0.01±0.05	Hen99					0.13±0.04	Tor99	0.20±0.07	Tor99	0.44±0.09	Tor99
Gl 570BC	1.66±0.16	S					1.19±0.05	For99	1.15±0.05	For99	1.18±0.03	For99
Gl 623AB	5.28±0.10	B96					3.28±0.3	Hen93	2.65±0.03	Hen93	2.87±0.14	Hen93
CM Dra	0.14±0.03	S			0.15±0.01	L77						
Gl 644A-Bab	−0.08±0.05	S							−0.48±0.06	D00	−0.46±0.03	D00
Gl 644Ba-Bb	0.49±0.10	S										
Gl 661AB	0.05±0.05	TYC					0.41±0.01	Hen93	0.46±0.02	Hen93	0.42±0.07	Hen93
Gl 702AB	1.86±0.02	TYC					1.51±0.04	Hen93			0.74±0.03	Hen93
Gl 747AB	0.22±0.05	S									0.10±0.03	D00
Gl 791.2AB	3.27±0.10	Ben00										
Gl 831AB	2.10±0.06	Hen99							1.26±0.03	D00	1.28±0.02	D00
Gl 860AB	1.70±0.09	HIP					1.19±0.10	Hen93	1.14±0.05	Hen93	1.37±0.08	Hen93
Gl 866AC-B	0.40±0.10	Hen99							0.52±0.03	D00	0.54±0.03	D00
Gl 866AC	2.04±0.40	S										

metric luminosity. The models therefore predict that infrared absolute magnitudes are largely insensitive to metallicity, and our empirical M/L relations confirm this.

At visible wavelengths on the other hand, metallicity determinations now become a crucial limiting factor in accurate comparisons with stellar models, as has long been the case for more massive stars (e.g. Andersen 1991). Quantitative metallicity measurements of M dwarfs are unfortunately difficult in the optical range (e.g. Valenti et al. 1998), but near-IR spectroscopy offers better prospects (Allard, private communication). GJ 2069A and Gl 791.2 represent spectacular illustrations of the intrinsic dispersion of the V band M/L relation, as already discussed in their respective discovery paper (Delfosse et al. 1999a; Benedict et al. 2000): these four stars are underluminous by ∼2 magnitudes for their masses, compared to solar metallicity models and to other stars. We recently discovered an additional faint component (Beuzit et al., in prep.) in GJ 2069A, which, if anything, further slightly increases its distance from a solar metallicity M/L. This discrepancy is best explained if the Gl 791.2 and GJ2069 systems are metal-rich by ∼0.5 dex. Their near-IR absolute magnitudes should be much more consistent with the average relations, but have not yet been measured.

4. Comparison to theoretical models

Fig. 3 compares the empirical M/L data with the corresponding 5 Gyr theoretical isochrones (for which even the lowest-mass stars on the plot have settled on the main sequence) of Baraffe et al. (1998; hereafter BCAH) and Siess et al. (2000; hereafter SDF). For very low mass stars these two sets of models are up to now the only ones to use realistic model atmospheres as outer boundary conditions to the stellar interior equations. This has been found necessary for accurate results in this mass range (Chabrier et al. 1996). BCAH use atmospheres from Hauschildt et al. (1999), while SDF use the older Plez (1992) models. Besides this, and the different input physics that they use, the two sets of models differ by the technique used to compute observational quantities. SDF use the empirical bolometric correction tables of Kenyon & Hartmann (1995) to deduce absolute magnitudes in the various photometric bands from their theoretical bolometric luminosities and effective temperatures. BCAH on the other hand adopt a purely *ab initio* approach, and compute the absolute magnitudes from the stellar radii, the model atmosphere spectra, and the transmission profile of the photometric filters.

At the scale of Fig. 3 the BCAH and SDF models are nearly indistinguishable for the infrared bands, and both produce an impressive agreement there with the observational data. This is particulary striking for the K band, where many measurements define the M/L relation. The same is apparently true for J and H, where more data would nonetheless be welcome to confirm this behaviour.

In the V band (Fig. 3) on the other hand, neither of the two sets of models reproduces the observations perfectly. The BCAH models and the observations agree well above ∼0.5 $M_\odot$, though with significant dispersion, but they diverge somewhat for lower masses. Below 0.2 $M_\odot$, the solar metallicity models are systematically too luminous by ∼0.5 magnitude. As most of the bolometric flux of such stars emerges in the near-IR bands, where the agreement is excellent, the models necessarily provide a good account of the relation between mass and bolometric magnitude. Their overall description of the stars is therefore

Table 3. Masses and absolute magnitudes for the M dwarfs used in the M/L relation. The mass references are Met96 for Metcalfe et al. (1996), Hen99 for Henry et al. (1999), Hen93 for Henry & McCarthy (1993), Mar98 for Martin et al. (1998), Tor99 for Torres et al. (1999), Ben00 for Benedict et al. (2000), Seg00a for Ségransan et al. (2000), Seg00b for Ségransan et al. (in prep.) and For99 for Forveille et al. (1999). When relevant we have modified the masses from Henry & McCarthy (1993) and Henry et al. (1999) to reflect a more accurate parallax in Table 1 than was available to these authors. The individual absolute magnitudes are determined from the system magnitudes and parallaxes listed in Table 1, with the magnitude differences of Table 2.

Name		Mass (in $M_\odot$)	Ref	M_V	M_J	M_H	M_K
Gl 65	A	0.102 ± 0.010 (9.8%)	Hen99	15.41 ± 0.05	9.68 ± 0.05	9.15 ± 0.03	8.76 ± 0.07
	B	0.100 ± 0.010 (10.0%)	Hen99	15.87 ± 0.06	10.06 ± 0.05	9.45 ± 0.03	9.16 ± 0.07
Gl 234	A	0.2027 ± 0.0106 (5.2%)	Seg00a	13.07 ± 0.05	8.52 ± 0.06	7.93 ± 0.04	7.64 ± 0.04
	B	0.1034 ± 0.0035 (3.4%)	Seg00a	16.16 ± 0.07	10.31 ± 0.25	9.56 ± 0.10	9.26 ± 0.04
YY Gem	a	0.6028 ± 0.0014 (0.2%)	Seg00a	9.03 ± 0.12			
	b	0.6069 ± 0.0014 (0.2%)	Seg00a	9.38 ± 0.14			
GJ 2069A	a	0.4344 ± 0.0008 (0.2%)	Seg00a	11.78 ± 0.18			
	b	0.3987 ± 0.0007 (0.2%)	Seg00a	12.57 ± 0.19			
Gl 473	A	0.143 ± 0.011 (7.7%)	Tor99	15.01 ± 0.07	9.44 ±0.06	8.84 ± 0.06	8.40 ± 0.06
	B	0.131 ± 0.010 (7.6%)	Tor99	15.00 ± 0.07	9.57 ± 0.06	9.04 ± 0.07	8.84 ± 0.08
Gl 570	B	0.5656 ± 0.0029 (0.5%)	For99	9.45 ± 0.05	6.21 ± 0.03	5.61 ± 0.03	5.39 ± 0.03
	C	0.3770 ± 0.0018 (0.5%)	For99	11.09 ± 0.17	7.40 ± 0.04	6.76 ± 0.04	6.57 ± 0.04
Gl 623	A	0.3432 ± 0.0301 (8.8%)	Seg00b	10.74 ± 0.05	7.19 ± 0.04	6.70 ± 0.04	6.46 ± 0.04
	B	0.1142 ± 0.0083 (7.3%)	Seg00b	16.02 ± 0.11	10.47 ± 0.29	9.35 ± 0.05	9.33 ± 0.14
CM Dra	a	0.2307 ± 0.0010 (0.5%)	Met96	12.80 ± 0.1			
	b	0.2136 ± 0.0010 (0.5%)	Met96	12.94 ± 0.1			
Gl 644	A	0.4155 ± 0.0057 (1.4%)	Seg00a	10.76 ± 0.06		6.61 ± 0.05	6.35 ± 0.04
	Ba	0.3466 ± 0.0047 (1.3%)	Seg00a	11.22 ± 0.10			
	Bb	0.3143 ± 0.0040 (1.3%)	Seg00a	11.71 ± 0.10			
Gl 661	A	0.379 ± 0.035 (9.2%)	Mar98	11.10 ± 0.06	7.10 ± 0.05	6.56 ± 0.04	6.36 ± 0.05
	B	0.369 ± 0.035 (9.5%)	Mar98	11.15 ± 0.06	7.51 ± 0.04	7.02 ± 0.04	6.78 ± 0.05
Gl 702	B	0.713 ± 0.029 (4.1%)	Hen93	7.52 ± 0.05	5.63 ± 0.05		4.53 ± 0.04
Gl 747	A	0.2137 ± 0.0009 (0.4%)	Seg00a	12.30 ± 0.06			7.53 ± 0.04
	B	0.1997 ± 0.0008 (0.4%)	Seg00a	12.52 ± 0.06			7.63 ± 0.04
Gl 791.2	A	0.286 ± 0.006 (2.1%)	Ben00	13.37 ± 0.03			
	B	0.126 ± 0.003 (2.4%)	Ben00	16.64 ± 0.10			
Gl 831	A	0.2913 ± 0.0125 (4.3%)	Seg00a	12.52 ± 0.06		7.36 ± 0.05	7.08 ± 0.05
	B	0.1621 ± 0.0065 (4.0%)	Seg00a	14.62 ± 0.08		8.62 ± 0.05	8.36 ± 0.05
Gl 860	A	0.2711 ± 0.0100 (4.3%)	Hen99	11.76 ± 0.05	7.84 ± 0.04	7.26 ± 0.04	6.95 ± 0.04
	B	0.1762 ± 0.0066 (4.7%)	Hen99	13.46 ± 0.09	9.03 ± 0.08	8.40 ± 0.05	8.32 ± 0.07
Gl 866	A	0.1187 ± 0.0011 (0.9%)	Seg00a	15.39 ± 0.07			
	B	0.1145 ± 0.0012 (1.0%)	Seg00a	15.64 ± 0.08		9.29 ± 0.04	8.96 ± 0.04
	C	0.0930 ± 0.0008 (0.9%)	Seg00a	17.43 ± 0.40			

most likely correct, and the V band discrepancy probably points to a relatively localized problem in the models. The two leading explanations for this discrepancy (Baraffe & Chabrier, private communication) are either a V band opacity source that would be missing in the atmospheric models, or some low level problem in the physical description of the shallower atmospheric levels which emit the visible flux.

The SDF models by contrast are ∼0.5 mag too luminous in the V band above $0.3M_\odot$, produce an excellent agreement with the observations for $0.2M_\odot$, and look sub-luminous for $0.1M_\odot$. Here again, the excellent near-IR agreement with the observations indicates that these models nicely reproduce the relation between mass and bolometric magnitude, and the discrepancy rests in the V band bolometric correction. Indeed, the SDF models mostly target PMS stars and the Kenyon & Hartmann (1995) bolometric correction that they use applies for T Tauri stars, which have lower gravity than main sequence stars and hence somewhat different colours. The use of observational bolometric correction, which by definition are unaffected by missing opacity sources, on the other hand most likely explains why the SDF models agree better with the V band observations at $\sim 0.2M_\odot$.

The characteristics of very low mass stars are frequently derived from photometry in the redder CCD bands, R and especially I, where these objects are brighter than in the V band. The validity of the theoretical M/L relation for these red bands is thus of significant interest, but too few VLMSs with accurate masses have known luminosities in the R, I, or z filters to provide a fully empirical verification. One can note however that Baraffe et al. (1998) observe that below $T_{\rm eff} \sim 3700$ K the BCAH model $V-I$ and $V-R$ colours are too blue by 0.5 mag at a given luminosity, while colours which don't involve the V

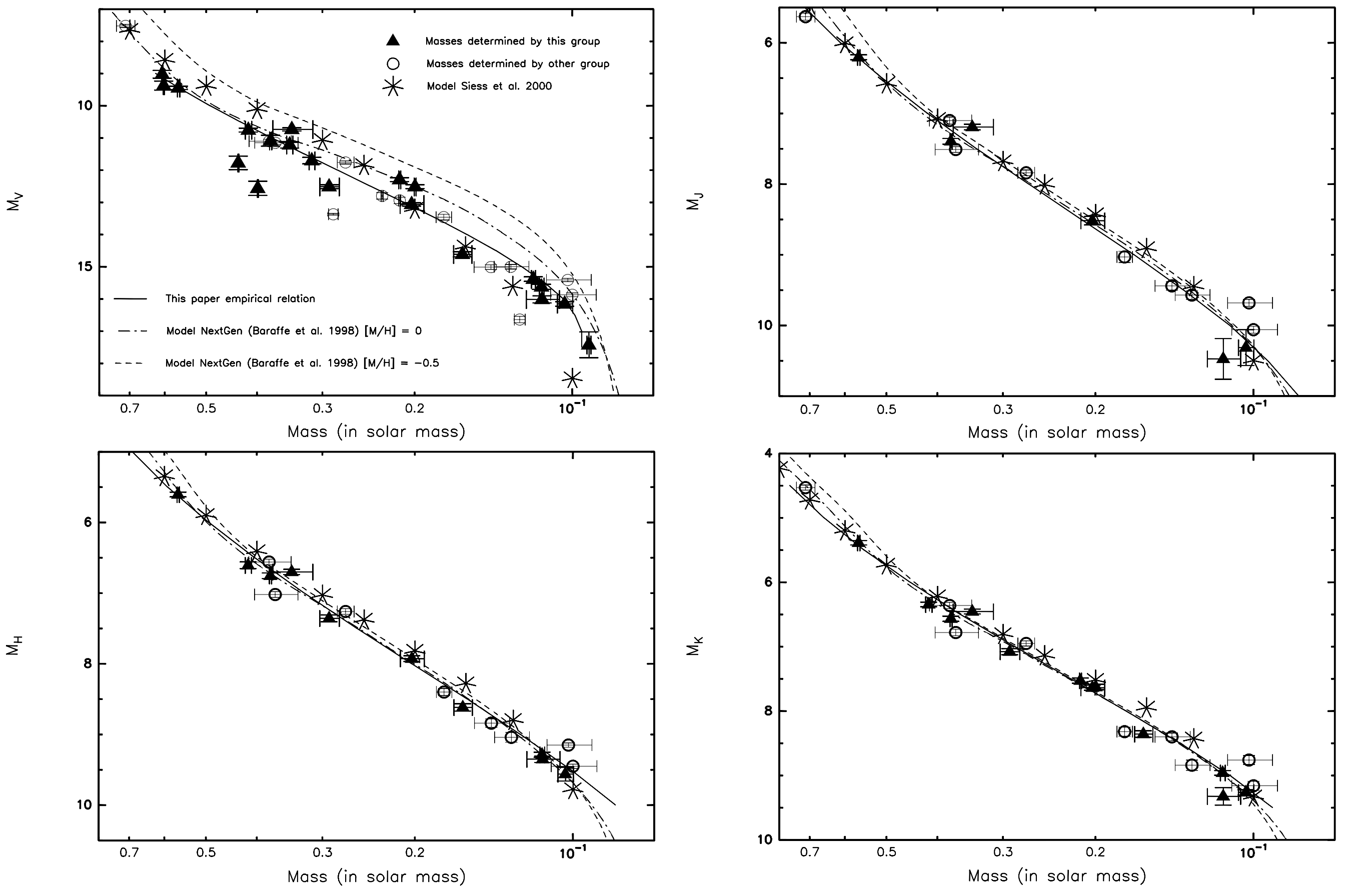

Fig. 3. comparison between V, J, H and K band M/L observational relation and theoritical ones. The three curves are 5 Gyr theoretical isochrones from Baraffe et al. (1998) for two metallicities and our polynomial fit. The asterisks represent 5 Gyr solar metallicity models from Siess et al. (2000).

band are much better predicted. Since the BCAH model M_V are also ∼0.5 mag too luminous for their mass, this suggests that the atmospheric models have a problem that is specific to the V band. The model M/L relations for the R, I and z bands are then probably more nearly correct. If valid this inference would suggest that the root of the problem rests in the V band opacity rather than in the physical description of the visible photosphere, which would probably affect a broader wavelength range.

5. Conclusions

Empirical masses of 0.2 to 10% accuracy validate the near-IR Mass/Luminosity relations predicted by the recent stellar models of Baraffe et al. (1998) and Siess et al. (2000), down to ∼0.1 $M_\odot$. They also point out some low level (∼0.5 mag) deficiencies of these models in the V band. Perhaps more importantly however, the V band M/L diagram represents direct evidence for an intrinsic dispersion around the mean M/L relation. This had previously remained hidden in the measurement noise, but there is, as theoreticians have kept telling us, no such thing as one single M/L relation for all M dwarfs. This is particularly true for the visible bands, while the dispersion in the near-IR JHK bands is much lower. Comparisons between measured masses and theoretical models will therefore increasingly depend on metallicity measurements for individual systems, which are not easily obtained.

The ∼0.5 magnitude discrepancy between observational and model masses derived from visible photometry has some consequences for mass functions determination. As mass cannot be determined for volume-limited samples, the mass function is always obtained from a luminosity function, by writing that

$$\frac{\mathrm{d}N}{\mathrm{d}M} = \frac{\mathrm{d}N}{\mathrm{d}L} \times \frac{\mathrm{d}L}{\mathrm{d}M}$$

and the slope of the M/L relation therefore plays a central role in its derivation. Below 0.5$M_\odot$ the $\mathrm{d}L/\mathrm{d}M$ slope of the empirical M/L relation is steeper than that of the BCAH models and shallower than for the SDF ones, by 10 to 20%. Their use will therefore respectively underestimate and overestimate the number of lower mass stars by this amount. Probably more seriously, the large dispersion around the V band M/L relation will introduce large Malmquist-like biases in the derived mass function, which would need an excellent characterization of this dispersion to be corrected. The infrared relations have both better agreement with the observations and much lower dispersion. We strongly recommend that they be used rather than the V band relations, whenever possible.

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Note added in proof: After the paper was accepted, Pavel Kroupa pointed out that we have omitted the Kroupa, Tout & Gilmore (1993, MNRAS 262, 545) relation in our comparison of the new data with published empirical mass-luminosity relations. The Kroupa et al. relation is in excellent agreement with the new data, except may be for the very end of main sequence ($M < 0.1 M_{\odot}$). We plan to show in a future paper the comparison with this empirical relation, which provides a significantly better description of the new data than the more commonly used Henry & McCarthy (1993) piece-wise linear relations and does not suffer from discontinuous derivatives. We are grateful to Pavel Kroupa for this comment.