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# Magnetic dynamo due to turbulent helicity fluctuations

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**Abstract.** Using the large-scale or long-time averaged magnetic field diffusion equation, we show that the inhomogeneous distribution of the turbulent helicity fluctuations (more correctly, the fluctuations of the $\alpha$-coefficient) gives rise to the large-scale enhancement of the mean magnetic field. This effect occurs even if the mean helicity is absent. This newly reported dynamo mechanism requires the differential rotation of an atmosphere and does not depend on the action of the Coriolis force. The estimations show that the $\alpha$-coefficients due to helicity fluctuations have the same (or greater) magnitude compared to the usual $\alpha$-coefficients, related to the action of the Coriolis force. However, this mechanism works in the regions of the convective zone where the inhomogeneity of the fluctuation distribution is most prominent. It is very anisotropic in character and is described by the $\alpha_{ij}$-tensor. The possible effect of this mechanism in the Sun is discussed.

**Key words:** magnetic fields – turbulence – Sun: magnetic fields – stars: magnetic fields

## 1. Introduction

It is commonly accepted that the $\alpha\omega$-dynamo provides the basis for an explanation of the 22-year cycle of solar activity (Parker 1970, 1979; Cowling 1981; Rädler 1990; Schmitt 1993; DeLuca & Gilman 1991; Weiss 1994; Rüdiger & Brandenburg 1995; Covas et al. 1997, 1998). Even old phenomenological theories of Babcock (1961) and Leighton (1964) partly revive accepting this mechanism (Zwaan 1996) and the conception that the dynamo region is located at the bottom of the convective zone where the buoyancy of a magnetic field is suppressed (Rosner & Weiss 1992). The problems of solar activity are similar to those of magnetic stars (Baliunas & Vaugham 1985).

There are some unresolved problems with the dynamo theory (Krause 1991; De Luca & Gilman 1991). Some of these difficulties arise from new data from helioseismology studies, concerning the revision of the angular velocity distribution inside the convective zone. Other, more conceptual difficulties are concerned with the existence of usual $\alpha$-effect as sufficiently effective mechanism to enhance the mean magnetic field in cosmic conditions.

### 1.1. Suppression of the $\alpha$-effect

The most difficult problem is the back-reaction of large magnetic fields on the structure of turbulent motion. Taking into account this back-reaction, Rüdiger & Kichatinov (1993), and Kichatinov et al. (1994) have shown numerically that the back-reaction decreases both the $\alpha$-coefficient and the turbulent diffusivity $\beta$. A more simple approximate approach of Gruzinov & Diamond (1994) demonstrates the drastic decrease which occurs in the $\alpha$-coefficient: $\alpha \approx \alpha_0/(1+R_m B_0^2/12\pi\rho u_0^2)$, where $\alpha_0$ is the value of the $\alpha$-coefficient calculated without any back-reaction, $R_m \approx \beta/\eta \approx u_0 R_0/\eta$ is the magnetic Reynolds number, $\eta$ is ohmic (molecular) diffusivity, $B_0$ is the mean magnetic field, $\rho$ is the density of a fluid, and $u_0, R_0$ and $\tau_0$ are the characteristic turbulent velocity, and the space and time scales of the correlation of the turbulent velocity field, respectively. Remembering that in cosmic conditions $R_m \sim 10^9 - 10^{14}$, one can conclude that the $\alpha$-coefficient is close to zero. Thus, according to Gruzinov & Diamond, the usual $\alpha$-effect does not occur in cosmic objects. More sophisticated numerical calculations of Rüdiger & Kichatinov (1993), and Kichatinov et al. (1994) do not show such large decreasing as a result of the back-reaction.

The disagreement between these authors is a result of the relation $\alpha = -\eta\langle \mathbf{b}\cdot\nabla\times\mathbf{b}\rangle/B_0^2$. This remarkable relation was derived and used by Gruzinov & Diamond. However, the most convincing derivation of this relation follows from the exact equation for fluctuating component of magnetic helicity, $H_m = \langle \mathbf{A}\cdot\mathbf{B}\rangle = \mathbf{A}_0\cdot\mathbf{B}_0 + \langle \mathbf{a}\cdot\mathbf{b}\rangle$ (see Seehafer 1994, 1996). In the following, we represent the stochastic values as a sum of the mean and fluctuating components, for example, the vector potential $\mathbf{A} = \mathbf{A}_0 + \mathbf{a}$, with $\langle\mathbf{A}\rangle = \mathbf{A}_0$ and $\langle\mathbf{a}\rangle = 0$. This exact relation is valid for infinite, homogeneous and stationary magnetized turbulence. It should be noted that both approaches mentioned use the linearized governing equations for the fluctuating fields $\mathbf{b}$ and $\mathbf{u}$ (here $\mathbf{u}$ is Eulerian turbulent velocity). The limits of the validity of these equations are unknown. It seems likely that they are very restrictive. We feel that such an appoach, in principle, does not describe the system when it is far from the initial state, particularly the stationary state. For this reason, the Gruzinov & Diamond (1994) formula for $\alpha$, mentioned above, is not internally consistent. The linearized approach can thus give us only some corrections to the initial value $\alpha_0$. Instead, their expression demonstrates the drastic decreasing of $\alpha$.

From the relation $\langle \mathbf{u} \times \mathbf{b} \rangle \equiv \alpha \mathbf{B}_0$, which is valid for homogeneous turbulence, we can use the estimation $\alpha \approx u_0 b_0 / B_0$, where $b_0^2 \equiv \langle b^2 \rangle$. The relationship $\alpha = -\eta \langle \mathbf{b} \cdot \nabla \times \mathbf{b} \rangle / B_0^2$ gives another, independent estimation $\alpha \approx \eta b_0^2 / B_0^2 l_0$, where $l_0$ is the characteristic scale of the magnetic fluctuations. The solution of these coupled relations is $\alpha \approx u_0 (u_0 l_0 / \eta) \gg u_0$ and $b_0 \approx B_0 (u_0 l_0 / \eta) \gg B_0$. Of course, these are maximal estimations because we did not take into account unknown angular dependencies of the vector equations considered.

The problem of the back-reaction of the magnetic field onto the turbulence itself has another side. Zeldovich (1957), for the case of two-dimensional turbulence, has shown the existence of very large magnetic fluctuations $b_{0\,\max}^2 \approx R_m B_0^2$, where $\mathbf{B}_0$ is the initial magnetic field. This estimation was accepted also for three-dimensional turbulence (see Vainshtein & Rosner 1991; Vainshtein & Cattaneo 1992; Gruzinov & Diamond 1994). According to this estimation, huge magnetic fluctuations first acquire equipartition with the turbulent kinetic energy and alter the character of turbulent motions. This occurs when the mean magnetic field is far below the equipartition level. Thus, the fluctuating field changes the turbulence, not the mean magnetic field (Vainshtein & Rosner 1991; Vainshtein & Cattaneo 1992).

The detailed consideration of this case has demonstrated that the estimation $b_0^2 \approx R_m B_0^2$ exists only at the moment $t_a$ of the maximum fluctuations of the vector potential and gives the relative level of magnetic fluctuations compared to the mean magnetic field. Moreover, this estimation changes drastically if one takes into account that the diffusion approximation, used by Zeldovich, gives the additional relation between all the involved values, namely $\beta \approx u_0 l_0 b_0 / B_0$. As a result, the estimation of magnetic fluctuations at the moment $t_a$ acquires the form: $b_0(t_a) \approx (u_0 l_0 / \eta) B_0(t_a)$. This is $(R_0/l_0)$ times smaller than the usually-accepted value.

For the spectra of turbulence with a slow decrease in the inertial interval (as in the Kolmogorov spectrum), $u_0 l_0 \sim \eta$ and $t_a \approx t_b$, where $t_b$ is the moment of the maximum of $b_0$. For this case one has $b_0 \approx B_0$, i.e. the level of fluctuations is near the level of the mean magnetic field. The estimation $b_0 \approx B_0$ is valid at the time $t_b$ both for 2D and 3D-turbulence.

For spectra containing a sharp decrease, have $l_0 \sim R_0$ and in the nonstationary case the relative energy of magnetic fluctuations can acquire very large values compared to the energy of the mean magnetic field. This occurs as a consequence of the different laws governing the decrease in these parts of the magnetic field. The mean magnetic field decreases according to the characteristic turbulent mixing time, and the fluctuations– according to the ohmic diffusivity time, i.e. more slowly.

Thus, we see that usual $\alpha$-effect is not suppressed enough to be excluded from consideration in cosmic objects.

### *1.2. The fluctuations of turbulent helicity*

There always exists the apprehension that some important physical process, which has a sufficiently large contribution, is not being included in the analyses. For this reason, we feel that it is important to pay special attention to the role of helicity fluctuations. Kraichnan (1976) was the first to consider the influence of helicity fluctuations on the value of turbulent diffusivity, $\beta$. He found that such fluctuations (in effect, the fluctuations of the $\alpha$-coefficient) lead to a decrease in turbulent diffusivity. Kraichnan did not excude the possibility of the negative diffusivity (see also Moffatt 1978).

A more detailed consideration of the $\alpha$-fluctuations is presented in the papers of Hoyng (1987a,b, 1988, 1996), Hoyng et al. (1994) and Hoyng & Schutgens (1995). In these papers, new effects of the dynamo theory were discovered, in particular, the excitation of higher eigenmodes of the oscillations. We note the very interesting discussion on the interpretation of various representations of the average and their relation to observable values (Hoyng 1987b).

Vishniac & Brandenburg (1997), using a numerical simulation of the magnetic field evolution in accretion discs have found that the fluctuations of the $\alpha$-coefficient itself, with the zero average $\alpha_0 = 0$, give rise to the enhancement of the mean magnetic field. Sokolov (1997) has presented some theoretical reasons to explain this effect. Silant'ev (1999) constructed a general mathematical theory which demonstrated that the effect of magnetic field enhancement occurs if the $\alpha$-fluctuations are distributed inhomogeneously and if differential rotation of the conducting fluid exists. It is interesting that the enhancement does not relate directly to the action of the Coriolis force.

Why does the turbulent helicity fluctuate, and what is the connection between these fluctuations and $\alpha$-fluctuations? We note that the turbulence itself consists of separate eddies which interact with each other, diminish in size and disappear due to viscous dissipation. Every eddy has its own angular momentum, corresponding to right-hand or left-hand rotation. This results in regions which contain fluid rotating in different directions. Due to their common origin and their interactions, there exists some space and time-correlation between these rotating eddies. This correlation exists even if rotations of opposite signs are present in equal amounts (the absence of the Coriolis force). For example, in many cases, a neighboring eddies will be rotating in the opposite directions.

We know (see Krause & Rädler 1980) that left-hand screw motions produce the $\alpha$-effect with $\alpha > 0$ and those of the right-hand screw give $\alpha < 0$. Let us present the $\alpha$-coefficient as a sum of the mean and fluctuating parts: $\alpha = \alpha_0 + \alpha'$. The previous consideration shows that there exists the nonvanishing two-point correlation function

$$A(1;2) \equiv \langle \alpha'(\mathbf{r}, t) \alpha'(\mathbf{r}', t') \rangle. \tag{1}$$

If the mean value $\alpha_0 = 0$ or it is sufficiently small, then only the correlator (1) is responsible for the large-scale enhancement of the magnetic field. The existence of the mean value $\alpha_0$ is provided by the action of the Coriolis force in the differentially rotating atmosphere. In contrast to $\alpha_0$, the fluctuations $\alpha'$ are not related to the effects of the Coriolis force, and the helicity of the velocity $H_0 \equiv \langle \mathbf{u} \cdot \nabla \times \mathbf{u} \rangle$ needed to construct the $\alpha'$–values intrinsically exists in the turbulent eddies (as explained above). It seems, for this reason, that the suppression of $\alpha'$-values by the large magnetic field differs from the suppression of the $\alpha_0$-coefficient. To suppress the turbulent eddies is more difficult because this requires the suppression of the angular moments of the eddies. For this reason, it is possible that the mean value of $A(1;1) \equiv \langle \alpha'(\mathbf{r},t)^2 \rangle = \alpha_1^2$ is larger than the value $\alpha_0^2$, and the new mechanism for magnetic enhancement is more effective than the usual $\alpha_0\omega$-mechanism.

The origin of the $\alpha$-effect is rather complex. The $\alpha$-coefficient itself has a statistical nature. Thus, the existence of the correlation function, Eq. (1), implies the existence of a second, large-scale or long-time averaging. In all the mentioned papers, the procedure of such a second average was used. In principle, it is possible to develop the theory using only one average. For such a theory, we need to know the dynamics of the evolution of the four-order velocity correlators to describe the value $A(1;2)$. This is too difficult in practice. However, the procedure of double averaging used previously makes it possible to study the effect of helicity fluctuations (as fluctuations of the $\alpha$-coefficient) in a fairly simple manner. At present, such an approach is the only one that can be used to study helicity fluctuations.

The goal of this paper is to show qualitatively and quantitatively the main features of the new mechanism to describe the large-scale amplification of magnetic fields – the amplification due to turbulent helicity fluctuations. This mechanism is averaged $\alpha^2\omega$-mechanism of the correlated fluctuations of turbulent helicity. It is not related to the existence of nonzero mean helicity and does not require the action of the Coriolis force. This mechanism gives rise to the independent tensor field $\alpha_{ij}(\mathbf{r},t)$ which can be used in the magnetic field induction equation. Some discussion of possible applications of this mechanism to solar magnetic activity problems also is given.

## 2. Basic equations

We begin our investigation from the known magnetic diffusion equation (see Cowling 1981):

$$\frac{\partial \mathbf{B}}{\partial t} - \eta\,\nabla^2\mathbf{B} = \nabla\times(\mathbf{U}_0\times\mathbf{B}) + \nabla\times\alpha\mathbf{B} + \nabla\times(\beta\nabla\times\mathbf{B}). \quad (2)$$

Here $\mathbf{U}_0(\mathbf{r},t)$ is the velocity of regular motion. The $\alpha$-coefficient and turbulent diffusivity, $\beta$, are stochastic functions and can be presented as the sums of mean and fluctuating parts:

$$\alpha(\mathbf{r},t) = \alpha_0 + \alpha'(\mathbf{r},t), \qquad \beta(\mathbf{r},t) = \beta_0 + \beta'(\mathbf{r},t). \quad (3)$$

Eq. (2) for the case $\mathbf{U}_0 = 0$, $\beta'(\mathbf{r},t) = 0$, $\alpha_0 = 0$ and an infinite isotropic medium was first considered by Kraichnan (1976). He found that the averaging of this equation gives rise to the same equation but with a renormalized turbulent diffusivity $\eta + \beta_0 + \beta$. The additional term $\beta$ was negative, i.e. the fluctuations of helicity decrease the total turbulent diffusivity. The consideration of Eq. (2) in the general case is presented in a prevoius paper (Silant'ev 1999). Here we give the main results of this investigation.

Substitution of Eq. (3) into the initial diffusion Eq. (2) leads to the basic stochastic equation:

$$\frac{\partial \mathbf{B}}{\partial t} - (\eta + \beta_0)\nabla^2\mathbf{B} - \alpha_0\nabla\times\mathbf{B} - \nabla\times(\mathbf{U}_0\times\mathbf{B}) =$$
$$\nabla\times\alpha'\,\mathbf{B} + \nabla\times(\beta'\,\nabla\times\mathbf{B}). \quad (4)$$

For simplicity we have assumed that $\alpha_0$ and $\beta_0$ are constant. The right-hand part of this equation, depending on the fluctuations $\alpha'(\mathbf{r},t)$ and $\beta'(\mathbf{r},t)$ is a stochastic function of the position $\mathbf{r}$ and time $t$. Using standard technique (see Dolginov & Silant'ev 1992; Silant'ev 1992) one can average this equation and obtain a new equation for the mean magnetic field $\mathbf{B}_0$:

$$\frac{\partial \mathbf{B}_0}{\partial t} - (\eta + \beta_0)\nabla^2\mathbf{B}_0 - \alpha_0\nabla\times\mathbf{B}_0 - \nabla\times(\mathbf{U}_0\times\mathbf{B}_0) =$$
$$\nabla\times\mathbf{F}_0, \quad (5)$$

$$F_{0i} = \alpha_{ij}(\mathbf{r},t)B_{0j}(\mathbf{r},t) + \beta_{ijk}(\mathbf{r},t)\nabla_k B_{0j}(\mathbf{r},t). \quad (6)$$

Eqs. (5) and (6) describe the turbulent magnetic dynamo in the diffusion approximation. The electromotive force $\mathbf{F}_0$ consists of two terms. The pseudo-tensors $\alpha_{ij}$ and $\beta_{ijk}$ generalize the pseudo-scalar $\alpha$-coefficient and the turbulent diffusivity $\beta$. They describe the influence of the helicity fluctuations $\alpha'(\mathbf{r},t)$ and the fluctuations of turbulent diffusivity $\beta'(\mathbf{r},t)$ on the evolution of the large-scale mean magnetic field $\mathbf{B}_0(\mathbf{r},t)$.

One can prove that the series of iterations of (5) using the Green function $G_{ij}^{(0)}(1;2)$ of the left-hand part of (5) converges rapidly. This occurs because the left-hand side mainly describes the process of turbulent diffusion and amplification of the mean magnetic field $\mathbf{B}_0$. We give the expressions for $\alpha_{ij}$ and $\beta_{ijk}$ as a result of the first iteration of (5):

$$\alpha_{ij}(\mathbf{r},t) \equiv \alpha_{ij}(1) =$$
$$\int d\mathbf{r}' \int_0^t dt' e_{jnm} G_{in}^{(0)}(\mathbf{r},t;\mathbf{r}',t')\langle \alpha'(\mathbf{r},t)\nabla'_m \alpha'(\mathbf{r}',t')\rangle. \quad (7)$$

Here, $e_{jnm}$ is the unit antisymmetric pseudo-tensor ($e_{xyz} = -e_{yxz} = -e_{xzy} = 1$, etc.). Below and in that which follows we shall use the convenient notations: $f(\mathbf{r}_n,t_n) \equiv f(n), d\mathbf{r}_n dt_n \equiv dn, d\mathbf{r}'dt' \equiv d2, \mathbf{R} = \mathbf{r}-\mathbf{r}', \tau = t-t'$ and so on. The formula for $\beta_{ijk}$ is more complex than that for $\alpha_{ij}$ and we use these symbolic notations:

$$\beta_{ijk}(1) = -\int d2\,[e_{jrn}G_{ir}^{(0)}(1;2)\langle\alpha'(1)(\nabla_n^{(2)}\alpha'(2)R_k)\rangle -$$
$$e_{its}(\nabla_t^{(1)}G_{sk}^{(0)}(1;2))\langle\beta'(1)(\nabla_j^{(2)}\beta'(2)\rangle +$$
$$e_{its}(\nabla_t^{(1)}G_{sj}^{(0)}(1;2))\langle\beta'(1)(\nabla_k^{(2)}\beta'(2)\rangle]. \quad (8)$$

In formulas (7) and (8) we have neglected the terms with the cross-correlator $\langle\alpha'(1)\rangle\beta'(2)\rangle$. This correlator vanishes when $\alpha_0 = 0$ (the value $\beta'(\mathbf{r},t)$ is an even function of the helicity $H_0$ and $\alpha'(\mathbf{r},t)$ is an odd one). It seems, that this correlator is small in all the cases because the turbulent diffusivity $\beta'(\mathbf{r},t)$ does not depend on the helicity if the degree of helicity is smaller than 50% (Silant'ev 1997a,b).

Very often, for simplicity, one uses the mean value of the diagonal elements:

$$\alpha(\mathbf{r},t) \equiv \frac{1}{3}\alpha_{ii}(\mathbf{r},t) =$$

$$e_{inm}\int d2\, G^{(0)}_{in}(1;2)\langle\alpha'(1)\nabla^{(2)}_m\alpha'(2)\rangle. \qquad (9)$$

Analogously, the value $\beta \equiv 1/6 e_{ijk}\beta_{ijk}$ has the sense of mean turbulent diffusivity due to the action of the helicity fluctuations $\alpha'(\mathbf{r},t)$ and the fluctuations of turbulent diffusivity $\beta'(\mathbf{r},t)$.

The case of isotropic turbulence with a nonzero mean helicity ($\alpha_0 \neq 0$) is the most simple. Our formulae give for this case:

$$\alpha = -\frac{2}{3}\int_0^\infty dp \int_0^t d\tau\, p^2 E_\alpha(p,\tau) g_1(p,\tau), \qquad (10)$$

$$\beta = -\frac{1}{3}\int_0^\infty dp \int_0^t d\tau\, [2\,p^2 E_\beta(p,\tau) g_0(p,\tau) +$$

$$E_\alpha(p,\tau) g_0(p,\tau) +\; pE_\alpha(p,\tau)\frac{\partial}{\partial p} g_0(p,\tau)]. \qquad (11)$$

Here, $E_\alpha(p,\tau)$ and $E_\beta(p,\tau)$ are the spectra of fluctuations:

$$\langle\alpha'(\mathbf{r},t)\alpha'(\mathbf{r},t+\tau)\rangle = \int_0^\infty dp E_\alpha(p,\tau), \qquad (12)$$

$$\langle\beta'(\mathbf{r},t)\beta'(\mathbf{r},t+\tau)\rangle = \int_0^\infty dp E_\beta(p,\tau). \qquad (13)$$

The functions $g_0(p,\tau)$ and $g_1(p,\tau)$ are the Fourier transforms of symmetric and antisymmetric parts of the Green tensor:

$$G^{(0)}_{nm}(\mathbf{p},\tau) = \delta_{nm} g_0(p,\tau) + i e_{nmk} p_k g_1(p,\tau), \qquad (14)$$

$$g_0(p,\tau) \equiv \int d\mathbf{R} g_0(R,\tau)\exp(-i\mathbf{p}\mathbf{R}) =$$

$$cosh(\alpha_0 p\tau)\exp(-\beta_0 p^2\tau),$$

$$g_1(p,\tau) = -\frac{1}{p} sinh(\alpha_0 p\tau)\exp(-\beta_0 p^2\tau). \qquad (15)$$

It should be noted that Moffatt (1978) obtained only the second term in Eq. (11). Eq. (10) demonstrates that the helicity fluctuations in a isotropic atmosphere increase the available $\alpha$-coefficient. In contrast with Eq. (10), Eq. (11) shows that an additional turbulent diffusivity term is negative.

Another situation takes place in an anisotropic and inhomogeneous turbulent atmosphere, a situation close to that found in the real convective zone. In such a medium, the large-scale enhancement of the mean magnetic field occurs even if the mean helicity is absent ($\alpha_0 = 0$). We now consider the form of $\alpha_{ij}$-tensor in the medium with the differential rotation.

## 3. $\alpha$-effect in an atmosphere with differential rotation

Let us consider the new dynamo mechanism in its "pure" form taking $\alpha_0 = 0$. The regular velocity in a rotating atmosphere has the form: $\mathbf{U}_0 = \omega(\rho,z)\mathbf{e}_z \times \mathbf{r} \equiv \mathbf{e}_\varphi \rho\omega(\rho,z)$. Here, $\omega(\rho,z)$ is the angular velocity of the rotation, $\rho$ is the distance of the point of the observation $\mathbf{r} = (\rho,\varphi,z)$ from the axis $OZ$ of rotation. In the cylindric frame of reference the integral equation for the Green tensor $G^{(0)}_{\alpha\beta}(\mathbf{r},t;\mathbf{r}',t')$ ($\alpha,\beta = \rho,\varphi,z$) has the form:

$$G^{(0)}_{\alpha\beta}(1;2) = g_{\alpha\beta}(1-2) -$$

$$\int d3\,\omega(3)\, g_{\alpha\gamma}(1-3)\frac{\partial}{\partial\varphi_3} G^{(0)}_{\gamma\beta}(3;2) +$$

$$\int d3\,\rho_3\, g_{\alpha\varphi}(1-3)[(\nabla^{(3)}_\rho\omega(3))G^{(0)}_{\rho\beta}(3;2) +$$

$$(\nabla^{(3)}_z\omega(3))G^{(0)}_{z\beta}(3;2)]. \qquad (16)$$

Here, the index $\gamma$ has all the values – $\rho,\varphi,z$, while the others are fixed. The Green tensor $g_{\alpha\beta}(1-2) \equiv g_{\alpha\beta}(\mathbf{R},\tau)$ is the usual diffusion Green function $g_{ij}(1-2) \equiv \delta_{ij} g_0(R,\tau)$ $(i,j = x,y,z)$, written in the cylindric frame of reference.

$$g_0(R,\tau) = (4\pi\beta_0\tau)^{-3/2}\exp(-R^2/4\beta_0\tau). \qquad (17)$$

Remember that the transition from a Cartesian frame to a cylindric one is described by the unitary matrix $U_{\beta k}(\varphi)$ according to the relations:

$$A_\beta = U_{\beta k}(\varphi)A_k, G_{\alpha\beta}(1;2) = U_{\alpha i}(\varphi_1)G_{ij}(1;2)U_{\beta j}(\varphi_2),$$

$$\alpha_{\gamma\beta}(\mathbf{r},t) = U_{\gamma i}(\varphi)\alpha_{ij}(\mathbf{r},t)U_{\beta j}(\varphi). \qquad (18)$$

The explicit form of the matrix $U_{\beta k}(\varphi)$ is the following:

$$U_{\beta k}(\varphi) = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}. \qquad (19)$$

According to (18) and (19) one has $g_{\alpha\beta}(1-2) = U_{\alpha\beta}(\varphi_1 - \varphi_2)$. It is easy to see that $G^{(0)}_{\alpha\beta}(1;2)$ depends on the difference $\psi = \varphi_1 - \varphi_2$.

We consider here the case when the statistical properties of the distribution of fluctuations have axial symmetry, i.e. the correlator $A(1;2) = \langle\alpha'(\mathbf{r},t)\alpha'(\mathbf{r}',t')\rangle$ does not change with the rotation of the points 1 and 2 as a whole. Such a correlator may be represented as $A(1;2) = A(z_1, z_2, \rho_1, |\mathbf{r}^{(1)}_\perp - \mathbf{r}^{(2)}_\perp|, \tau)$. Here the vector $\mathbf{r}^{(1)}_\perp(\rho,\varphi)$ determines the position of the projection of the radius-vector $\mathbf{r}_1$ in the plane $(xy)$.

We give the approximate solution of integral Eq. (16) in the form of the sum of the free term and the first iteration:

$$G^{(0)}_{\alpha\beta}(1;2) = U_{\alpha\beta}(\psi)g_0(R,\tau) - \frac{\partial}{\partial\psi}[U_{\alpha\beta}(\psi)\, f(1;2)] +$$

$$\begin{pmatrix} a\sin\psi - c\cos\psi & c\sin\psi - b\cos\psi & h\sin\psi - g\cos\psi \\ a\cos\psi + c\sin\psi & c\cos\psi + b\sin\psi & h\cos\psi + g\sin\psi \\ 0 & 0 & 0 \end{pmatrix}. \qquad (20)$$

Here we introduce the notations ($\psi=\varphi_1-\varphi_2, \psi'=\varphi_3-\varphi_2$):

$$f(1;2)=\int d3\,\omega(3)g_0(1-3)g_0(3-2), \quad (21)$$

$$a(1;2)=\int d3\,\rho_3(\nabla_\rho\omega(3))\cos^2\psi' g_0(1-3)g_0(3-2), \quad (22)$$

$$b(1;2)=\int d3\,\rho_3(\nabla_\rho\omega(3))\sin^2\psi' g_0(1-3)g_0(3-2), \quad (23)$$

$$c(1;2)=\int d3\,\rho_3(\nabla_\rho\omega(3))\sin\psi'\cos\psi' g_0(1-3)g_0(3-2), \quad (24)$$

$$g(1;2)=\int d3\,\rho_3(\nabla_\rho\omega(3))\sin\psi' g_0(1-3)g_0(3-2), \quad (25)$$

$$h(1;2)=\int d3\,\rho_3(\nabla_\rho\omega(3))\cos\psi' g_0(1-3)g_0(3-2). \quad (26)$$

The integration of (20) over the azimuthal angle $\varphi_1$ gives rise to an exact solution of Eq. (16) for the axisymmetric case when the components of $G^{(0)}_{\alpha\beta}(1;2)$ do not depend on the difference in the azimuthal angles $\psi=\varphi_1-\varphi_2$. Our approximation includes the latter case, which is usually used in dynamo models (see Hoyng & Schutgens 1995; Hoyng et al. 1994). However, the calculation of the tensor $\alpha_{ij}(\mathbf{r},t)$ according to Eq. (7) requires a knowledge of the Green tensor $G^{(0)}_{ij}(1;2)$ in its general form even if one considers the axisymmetric problems. Particularly, the substitution of the axisymmetric part of $G^{(0)}_{ij}(1;2)$ into (7) gives $\alpha_{\rho\rho}(\mathbf{r},t)=0$ and $\alpha_{\varphi\varphi}(\mathbf{r},t)\neq 0$. As we shall see below, the substitution of more general Eq. (20) gives rise to $\alpha_{\rho\rho}(\mathbf{r},t)\neq 0$. Moreover, these components are of the same order of magnitude.

It is useful to write here the general expressions for all the components of $\alpha_{\alpha\beta}(\mathbf{r},t)$:

$$\alpha_{\rho\rho}(\mathbf{r},t)=\int d2\{-\sin\psi G^{(0)}_{\rho\rho}(1;2)\nabla^{(2)}_z A(1;2)+$$
$$\cos\psi G^{(0)}_{\rho\varphi}(1;2)\nabla^{(2)}_z A(1;2)+$$
$$[\sin\psi\nabla^{(2)}_\rho A(1;2)-\cos\psi\nabla^{(2)}_\varphi A(1;2)]G^{(0)}_{\rho z}(1;2)\}, \quad (27)$$

$$\alpha_{\varphi\varphi}(\mathbf{r},t)=\int d2\{-\sin\psi G^{(0)}_{\varphi\varphi}(1;2)\nabla^{(2)}_z A(1;2)-$$
$$\cos\psi G^{(0)}_{\varphi\rho}(1;2)\nabla^{(2)}_z A(1;2)+$$
$$[\sin\psi\nabla^{(2)}_\varphi A(1;2)+\cos\psi\nabla^{(2)}_\rho A(1;2)]G^{(0)}_{\varphi z}(1;2)\}, \quad (28)$$

$$\alpha_{zz}(\mathbf{r},t)=\int d2[G^{(0)}_{z\rho}(1;2)\nabla^{(2)}_\varphi A(1;2)-$$
$$G^{(0)}_{z\varphi}(1;2)\nabla^{(2)}_\rho A(1;2)], \quad (29)$$

$$\alpha_{\rho\varphi}(\mathbf{r},t)=\int d2\{-\cos\psi G^{(0)}_{\rho\rho}(1;2)\nabla^{(2)}_z A(1;2)-$$
$$\sin\psi G^{(0)}_{\rho\varphi}(1;2)\nabla^{(2)}_z A(1;2)+$$
$$[\sin\psi\nabla^{(2)}_\varphi A(1;2)+\cos\psi\nabla^{(2)}_\rho A(1;2)]G^{(0)}_{\rho z}(1;2)\}, \quad (30)$$

$$\alpha_{\varphi\rho}(\mathbf{r},t)=\int d2\{\cos\psi G^{(0)}_{\varphi\varphi}(1;2)\nabla^{(2)}_z A(1;2)-$$
$$\sin\psi G^{(0)}_{\varphi\rho}(1;2)\nabla^{(2)}_z A(1;2)+$$
$$[-\cos\psi\nabla^{(2)}_\varphi A(1;2)+\sin\psi\nabla^{(2)}_\rho A(1;2)]G^{(0)}_{\varphi z}(1;2)\}, \quad (31)$$

$$\alpha_{\rho z}(\mathbf{r},t)=\int d2[G^{(0)}_{\rho\rho}(1;2)\nabla^{(2)}_\varphi A(1;2)-$$
$$G^{(0)}_{\rho\varphi}(1;2)\nabla^{(2)}_\rho A(1;2)], \quad (32)$$

$$\alpha_{\varphi z}(\mathbf{r},t)=\int d2[G^{(0)}_{\varphi\rho}(1;2)\nabla^{(2)}_\varphi A(1;2)-$$
$$G^{(0)}_{\varphi\varphi}(1;2)\nabla^{(2)}_\rho A(1;2)], \quad (33)$$

$$\alpha_{z\rho}(\mathbf{r},t)=\int d2\{\cos\psi G^{(0)}_{z\varphi}(1;2)\nabla^{(2)}_z A(1;2)-$$
$$\sin\psi G^{(0)}_{z\rho}(1;2)\nabla^{(2)}_z A(1;2)+$$
$$[-\cos\psi\nabla^{(2)}_\varphi A(1;2)+\sin\psi\nabla^{(2)}_\rho A(1;2)]G^{(0)}_{zz}(1;2)\}, \quad (34)$$

$$\alpha_{z\varphi}(\mathbf{r},t)=\int d2\{-\cos\psi G^{(0)}_{z\rho}(1;2)\nabla^{(2)}_z A(1;2)-$$
$$\sin\psi G^{(0)}_{z\varphi}(1;2)\nabla^{(2)}_z A(1;2)+$$
$$[\sin\psi\nabla^{(2)}_\varphi A(1;2)+\cos\psi\nabla^{(2)}_\rho A(1;2)]G^{(0)}_{zz}(1;2)\}. \quad (35)$$

These general formulae acquire a simpler form for various particular cases. In the case considered here, for $\omega=\omega(\rho,z)$, one has $G^{(0)}_{z\rho}(1;2)\equiv 0$ and $G^{(0)}_{z\varphi}(1;2)\equiv 0$ because both the rotation and the diffusion process do not create vertical magnetic force lines from horizontal ones. For this reason one has $\alpha_{zz}(\mathbf{r},t)=0$. If we have differential rotation in the plane $(xy)$, $\omega=\omega(\rho)$, then additionally $G^{(0)}_{\rho z}(1;2)=0$ and $G^{(0)}_{\varphi z}(1;2)=0$ take place because the vertical (along the $z$ - axis) magnetic fields do not transform to horizontal ones. Finally, remember that for the axisymmetric case with $\omega=\omega(\rho,z)$ only the components $G^{(0)}_{\rho\rho}(1;2)=G^{(0)}_{\varphi\varphi}(1;2)$, $G^{(0)}_{zz}(1;2)$, $G^{(0)}_{\varphi\rho}(1;2)$ and $G^{(0)}_{\varphi z}(1;2)$ are not equal to zero.

We present here explicitly only diagonal nonvanishing components of $\alpha_{\alpha\beta}(\mathbf{r},t)$:

$$\alpha_{\rho\rho}(\rho,z,t)=\int d2\{-f(1;2)\nabla^{(2)}_z A(1;2)-$$
$$[a(1;2)\sin^2\psi+b(1;2)\cos^2\psi]\nabla^{(2)}_z A(1;2)+$$
$$h(1;2)\sin\psi[\sin\psi\nabla^{(2)}_\rho A(1;2)-\cos\psi\nabla^{(2)}_\varphi A(1;2)]\}, \quad (36)$$

$$\alpha_{\varphi\varphi}(\rho,z,t)=\int d2\{-f(1;2)\nabla^{(2)}_z A(1;2)-$$

$$[a(1;2)\cos^2\psi + b(1;2)\sin^2\psi]\nabla_z^{(2)} A(1;2)+$$

$$h(1;2)\cos\psi[\cos\psi\nabla_\rho^{(2)} A(1;2) + \sin\psi\nabla_\varphi^{(2)} A(1;2)]\}, \qquad (37)$$

These expressions show that the components $\alpha_{\rho\rho}(\rho, z, t)$ and $\alpha_{\varphi\varphi}(\rho, z, t)$ are of the same order of magnitude. For this reason we present here the estimation of the mean value $\alpha_\perp = (\alpha_{\rho\rho} + \alpha_{\varphi\varphi})/2$ as:

$$\alpha_\perp \approx \omega\frac{\tau_\alpha^2\alpha_1^2}{L_z} + \frac{\tau_\alpha^2\alpha_1^2}{L_\rho}(\nabla_\rho\,\omega)l_{diff}(\tau_\alpha)+$$

$$\frac{\tau_\alpha^2\alpha_1^2}{L_z}(\nabla_z\,\omega)l_{diff}(\tau_\alpha). \qquad (38)$$

It should be remembered that this consideration is concerned with the rotating frame of reference where the permanent part of the angular velocity $\omega_0$ is excluded. Thus, our $\omega = \omega(\rho, z)$ denotes the remaining local rotations in this frame of reference. Only in such coordinate system can one consider that the mean magnetic field is locally homogeneous, which is required to calculate the $\alpha_{ij}(\mathbf{r}, t)$-tensor. As was mentioned, one usually considers axisymmetric problems where only the gradients of angular velocity play a role (the term with $f(1;2)$ disappears by azimuthal averaging). In the general case the local rotations also contribute.

In (38) the values $L_z$ and $L_\rho$ are characteristic scales for vertical and transverse variations of the correlator $A(1;2) = \langle\alpha'(\mathbf{r}, t)\alpha'(\mathbf{r}', t')\rangle$. The known expression $l_{diff}^2(\tau) \approx 6\beta_0\tau$ determines the length of turbulent diffusion mixing during the time $\tau$. The values $\alpha_1$ and $\tau_\alpha$ are the characteristic magnitude of the $\alpha'$-fluctuations and its life-time. By estimating $\alpha_1^2 \approx u_0^2$ and $\tau_\alpha \approx \tau_0$, we obtain a more detailed estimation:

$$\alpha_\perp \approx \xi_0^2\frac{R_0}{L_z}\omega R_0 + \xi_0^{5/2}\frac{R_0}{L_\rho}R_0^2\nabla_\rho\,\omega + \xi_0^{5/2}\frac{R_0}{L_z}R_0^2\nabla_z\,\omega, \qquad (39)$$

where $\xi_0 = u_0\tau_0/R_0 \equiv \tau_0/t_0$ is the turbulent Strouhal number, $t_0 = R_0/u_0$ is the turnover time of characteristic turbulent eddies. Usually one assumes $\xi_0 \approx 1$, but for long-lived eddies $\xi_0 \gg 1$.

The estimation of the usual $\alpha$-effect due to the action of the Coriolis force (see Weiss 1994) gives the value $\alpha_0 \approx R_0\omega$. Thus, Eq. (39) shows that new dynamo mechanism can give value $\alpha_\perp$ comparable with the $\alpha_0$-coefficient. However, this new dynamo is located in the regions where the inhomogeneity of the helicity fluctuations is most prominent. It seems that for the Sun, such regions occur in the upper and low layers of the convective zone.

## 4. Qualitative consideration of the new dynamo mechanism

We consider in this section some typical mechanisms corresponding to Eq. (28) for the component $\alpha_{\varphi\varphi}(\mathbf{r}, t)$. First, consider the contribution to $\alpha_{\varphi\varphi}(\mathbf{r}, t)$ from the second term in (28).

The initial homogeneous magnetic field $\langle\mathbf{B}\rangle \equiv \mathbf{B}_0$ is directed parallel to the azimuthal unit vector $\mathbf{e}_\varphi(1)$ at the point

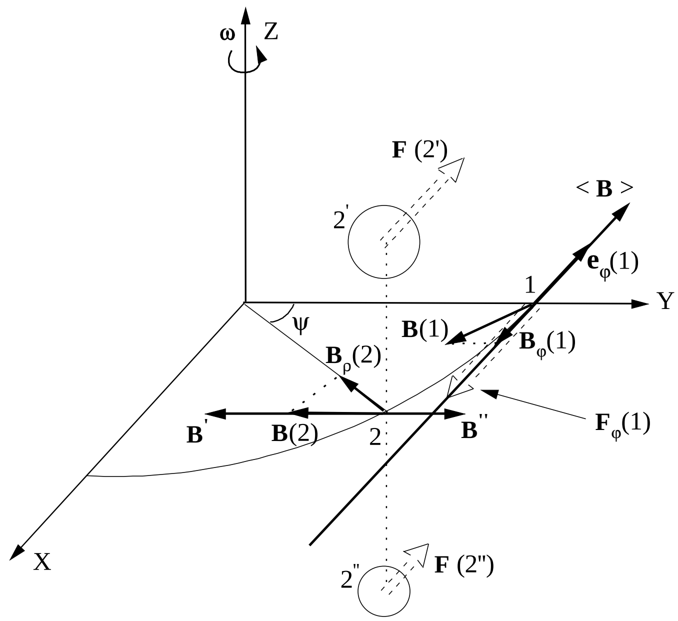


**Fig. 1.** The origin of the transversal $\alpha$-effect (the $\alpha_{\varphi\varphi}$-component) from the inhomogeneous distribution of the helicity fluctuations along the rotation axis $Z$ and the differential rotation in the perpendicular plane, $\omega = \omega(\rho)$. A detailed explanation is given in the text.

of the observation 1 (see Fig. 1). At the points $2'$ and $2''$, which equidistant from the plane $(xy)$, the field $\langle\mathbf{B}\rangle$ induces the electromotive forces $\mathbf{F}(2') = \alpha'(2')\langle\mathbf{B}\rangle$ and $\mathbf{F}(2'') = \alpha'(2'')\langle\mathbf{B}\rangle$. These electromotive forces give rise to electrical currents which create, at the point 2 on the plane $(xy)$, the magnetic fields $\mathbf{B}'$ and $\mathbf{B}''$ perpendicular to the field $\langle\mathbf{B}\rangle$. The resulting field $\mathbf{B}(2) = \mathbf{B}' + \mathbf{B}''$ has a radial component $B_\rho(2) = -\cos\psi|\mathbf{B}' + \mathbf{B}''|$. Due to radial differential rotation ($\omega = \omega(\rho)$), this component with a probability $\sim G_{\varphi\rho}^{(0)}(1;2)$ transforms to the azimuthal component at point 1 and there induces the azimuthal electromotive force $\mathbf{F}_\varphi(1) = -\cos\psi G_{\varphi\rho}^{(0)}(1;2)\alpha'(1)\nabla_z'\alpha'(2)\langle\mathbf{B}\rangle$, i.e. the $\alpha$-effect occurs. The sizes of circles in the figure correspond to the values of the function $\alpha'(\mathbf{r}, t)$ at the points $2'$ and $2''$. In Fig. 1 we represent the case $\nabla_\rho\,\omega > 0$.

It is seen from this qualitative picture that one requires inhomogeneity in the distribution of the $\alpha'$-fluctuations along the $z$-axis. The first term in (28), concerned with the azimuthal component $B_\varphi(2)$, can be considered in the same manner.

The last two terms in (28) may be considered in a simpler manner if one places the point of observation 1 on the $x$-axis, i.e. to take $\varphi_1 = 0$. The sum of these terms then may be presented as $G_{\varphi z}^{(0)}(1;2)\nabla_x^{(2)} A(1;2)$. Let us take the points $2'$ and $2''$ at the same distance from the circle $\rho = \rho_1$ (see Fig. 2). The mean homogeneous magnetic field $\langle\mathbf{B}\rangle$ directed along the unit vector $\mathbf{e}_\varphi(1)$ in this figure is parallel to the $y$-axis. The induced electromotive forces $\mathbf{F}(2') = \alpha'(2')\langle\mathbf{B}\rangle$ and $\mathbf{F}(2'') = \alpha'(2'')\langle\mathbf{B}\rangle$ create at the point 2 the resulting magnetic field $\mathbf{B}(2)$ directed along the $z$-axis. This field due to differential rotation $\omega = \omega(z)$ with the probability $\sim G_{\varphi z}^{(0)}(1;2)$ acquires the azimuthal component at the point 1. This component creates the azimuthal electromotive force $\mathbf{F}_\varphi(1) = G^{(0)}_{\varphi z}(1;2)\nabla^{(2)}_x A(1;2)\langle\mathbf{B}\rangle$. In Fig. 2 we assume $\nabla_z\,\omega > 0$.

In the same manner we can derive all the relations (27)-(35) for the components of the $\alpha_{\gamma\beta}$-tensor.

In all cases the induced fields $\mathbf{B}'$ and $\mathbf{B}''$ are perpendicular to the initial homogeneous field $\langle\mathbf{B}\rangle$, and to obtain some component parallel to $\langle\mathbf{B}\rangle$ we need differential rotation of the frozen field. We also need the inhomogeneous distribution of the helicity fluctuations, otherwise the fields $\mathbf{B}'$ and $\mathbf{B}''$ will have the same values and the resulting field $\mathbf{B}(2)$ does not appear.

According to Eq. (29), the $\alpha$-effect along the axis of rotation in a differentially rotating atmosphere cannot occur. If the mean homogeneous field $\langle\mathbf{B}\rangle$ is directed along the axis of rotation then the induced fields $\mathbf{B}'$ and $\mathbf{B}''$ are perpendicular to this axis. Both differential rotation and turbulent diffusion do not, statistically, transform magnetic force lines from the plane of rotation to the directions parallel to the axis of rotation. As a consequence, $\alpha_{zz}(\mathbf{r},t) \equiv 0$. For a vertical $\alpha$-effect one needs to have motions such as convection, which move matter from the horizontal plane into the vertical direction. In this case $G^{(0)}_{z\rho}$ and $G^{(0)}_{z\varphi}$ do not equal zero and $\alpha_{zz}(\mathbf{r},t) \neq 0$ can, in principle, exist.

It is seen from Eqs. (27)-(35) that the $\alpha$-tensor is not symmetric. The antisymmetric part of $\alpha_{ij}(\mathbf{r},t)$ has the meaning of some translatory velocity $\mathbf{g}$: $g_i = e_{ijk}\alpha_{jk}(\mathbf{r},t)/2$ (see Moffatt 1978). It seems that this velocity can play an important role as an additional source of dynamo waves (see Kichatinov 1993). This can be seen from consideration of the Green tensor of Eq. (5) for an infinite homogeneous medium:

$$G_{ij}(\mathbf{R},\tau) = \delta_{ij}G_0(\mathbf{R},\tau) + e_{isn}\alpha^{(s)}_{nj}\nabla_s G_1(\mathbf{R},\tau). \qquad (40)$$

Here, $\alpha^{(s)}_{nj}$ is the symmetric part of the tensor $\alpha_{ij}$. The explicit forms of $G_0(\mathbf{R},\tau)$ and $G_1(\mathbf{R},\tau)$ are given only for the case of $\alpha^{(s)}_{nj} = \delta_{nj}\alpha$, when the formulae acquire their simplest form:

$$G_0(\mathbf{R},\tau) = (4\pi\beta_0\tau)^{-3/2}\exp\{-[|\mathbf{R}-\mathbf{g}\tau|^2 - \alpha^2\tau^2]/4\beta_0\tau\}\times$$

$$\{cos\frac{\alpha|\mathbf{R}-\mathbf{g}\tau|}{2\beta_0} + \frac{\alpha\tau}{|\mathbf{R}-\mathbf{g}\tau|}\sin\frac{\alpha|\mathbf{R}-\mathbf{g}\tau|}{2\beta_0}\}, \qquad (41)$$

$$G_1(\mathbf{R},\tau) = \frac{1}{2\pi\alpha R}(4\pi\beta_0\tau)^{-1/2}\times$$

$$\exp\{-[|\mathbf{R}-\mathbf{g}\tau|^2 - \alpha^2\tau^2]/4\beta_0\tau\}sin\frac{\alpha|\mathbf{R}-\mathbf{g}\tau|}{2\beta_0}. \qquad (42)$$

These expressions describe wave propagation of magnetic field. If the antisymmetric part of the tensor $\alpha_{ij}(\mathbf{r},t)$ is equal to zero, then the wave character of the Green tensor disappears. Generally speaking, the antisymmetric components of $\alpha_{ij}(\mathbf{r},t)$ are not small compared with the symmetric ones.

## 5. Possible role of the new dynamo mechanism in the Sun

This new independent dynamo mechanism provides new possibilities for other dynamo models. Other models may use this

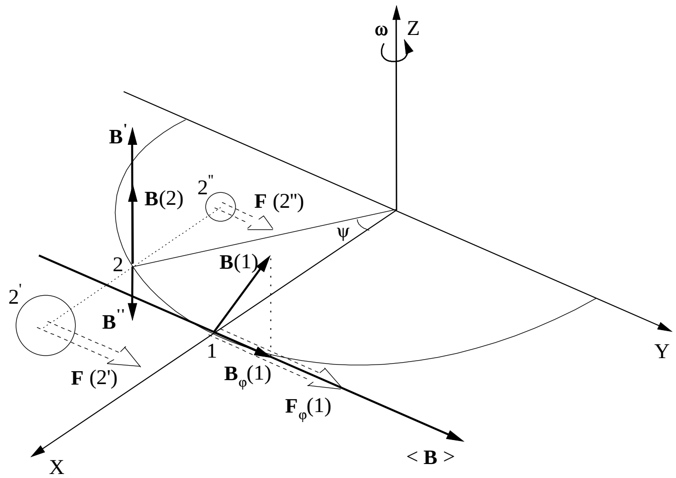


**Fig. 2.** The origin of the transversal $\alpha$-effect (the $\alpha_{\varphi\varphi}$-component) from the inhomogeneous distribution of the helicity fluctuations perpendicular to the rotation axis $Z$ and the differential rotation along the rotation axis, $\omega = \omega(z)$. A detailed explanation is given in the text.

mechanism to resolve some difficulties. We give here only the most important features of the new dynamo mechanism which can be applied to resolve some difficulties of existing dynamo models.

We first consider the basic features of these models (Rädler 1990). It is commonly accepted that the $\alpha_0(\mathbf{r},t)$-coefficient is positive in the northern hemisphere of the Sun and is proportional to $\cos\vartheta$, where $r, \vartheta$ and $\varphi$ are spherical coordinates of the point $\mathbf{r}$. For the existence of dynamo waves propagating towards the equator one needs the condition $\alpha\partial\omega/\partial r < 0$. The phase relation between the toroidal and radial magnetic components $B_\varphi B_r < 0$ demands the condition $\alpha > 0$. The back-reaction of the magnetic field (see Rüdiger & Kichatinov 1993) decreases the value $\alpha_0(\mathbf{r},t)$ (the $\alpha$-quenching phenomenon). This makes it difficult to obtain the needed 22-year period of dynamo waves.

The new mechanism does not depend on $\alpha_0(\mathbf{r},t)$-values. The fluctuations $\alpha'(\mathbf{r},t)$ are determined by the helical motions of turbulent eddies as they are formed by the action of all forces, including the magnetic back-reaction. Thus, the quenching phenomenon is automatically included in the observed picture of eddies with right-hand and left-hand helical motions. For this reason, the estimation $\langle\alpha'^2(\mathbf{r},t)\rangle \approx u_0^2$ seems to be rather natural. This estimation gives rise to the conclusion that the new mechanism is comparable or even more effective than the other models involving the action of the Coriolis force. The region of its action is restricted by those places where the inhomogeneity of the fluctuation distribution is most prominent.

It seems quite reasonable that such regions are the lower and upper boundaries of the convective zone. Every term for our $\alpha(\mathbf{r},t)$ depends on either $\nabla_z\omega(\rho,z)$ or $\nabla^{(2)}_z A(1;2)$, which have opposite signs in the northern and southern hemispheres. This implies that the signs of the $\alpha$-coefficient are opposite in these hemispheres, as needed for the solar dynamo mechanisms. Thus, it is sufficient to consider, say, only the northern hemisphere. It seems also quite natural that the gradient $\nabla^{(2)}A(1;2)$ has opposite directions in the upper and lower boundary layers of the convective zone. This property results in opposite signs of $\alpha(\mathbf{r},t)$ in these layers if the angular velocity $\omega(\rho,z)$ has the same direction of the gradient $\nabla\omega(\rho,z)$.

Using helioseismological data on the angular velocity distribution (see Weiss 1994; Rüdiger & Brandenburg 1995) and assuming that the number of helicity fluctuations increases towards the upper boundary of the convective zone, we estimate that the new mechanism (see Fig. 1) provides $\alpha(\mathbf{r},t) < 0$ beyond the overshoot layer of the convective zone. It seems that this is the highest magnitude the $\alpha$-coefficient acquires at the latitude $\vartheta \approx 45°$. This estimation follows from Eqs. (21)-(26) and the explicit formulae in Eqs. (36) and (37).

Of course, we do not know the explicit form of the fluctuation correlator $A(1;2) = \langle\alpha'(\mathbf{r},t)\alpha'(\mathbf{r}',t')\rangle$. For this reason our scenario cannot be wholly predictive.

The usual Coriolis force mechanism provides the $\alpha_0 > 0$ – values of the $\alpha$-coefficient in the northern semisphere of the Sun. This means that both mechanisms can annihilate one another beyond the overshoot layer.

Thus, in such a natural manner, the new independent dynamo mechanism allows us to move the region of magnetic field enhancement to the bottom of the convective zone, the so called overshoot layer. One supposes that the storage of the large magnetic fields generated there is situated in this layer, where the buoyancy of the magnetic field is suppressed (Rosner & Weiss 1992). This location of the solar magnetic dynamo is assumed now by many recent dynamo models (DeLuca & Gilman 1991; Schmitt 1993; Rüdiger & Brandenburg 1995).

One suggests that the overshoot layer is rather thin $\sim 0.02 - 0.14$ of the solar radius $R_\odot$, and is located at the distance $\approx 0.7R_\odot$ from the center of the Sun (see Rüdiger & Brandenburg 1995). It is natural to suppose that the number of helicity fluctuations increases towards the bottom of the overshoot layer. For this situation our formulae and helioseismology data give rise to the estimation $\alpha(\mathbf{r},t) > 0$ in the overshoot layer. Of course, at the equator, due to symmetry properties mentioned above, we have $\alpha(\mathbf{r},t) = 0$. Thus, the supposed behaviour of the helicity fluctuations gives rise to the positive total $\alpha$-coefficient in the overshoot layer where $\partial\omega/\partial r < 0$ exists. The dynamo wave from this region provides both the direction towards the equator and the true phase relation.

Due to small thickness of the overshoot layer, the turbulence therein shows large fluctuations (see DeLuca & Gilman 1991). In this situation, the dynamo mechanism due to turbulent helicity fluctuations can exceed in the magnitude the usual Coriolis force dynamo. Remembering that the helicity fluctuations decrease the turbulent diffusivity, we hope that the needed 22-year period of the dynamo wave is not crucial in our probable scenario.

Thus, the new mechanism provides avery natural physical explanation for the dynamos in the overshoot layer, as, for example, in the case of the dynamo of Rüdiger & Brandenburg (1995).

Another possible application of the helicity fluctuation mechanism may be the generation of poloidal fields near the polar region of the Sun. According to the scenario of Zwaan (1996) we need a source of such poloidal fields in this region. Our anisotropic mechanism (the $\alpha_{ij}$-tensor) can provide these poloidal fields which then sink to the bottom of the convective zone and there create the needed toroidal magnetic field due to differential rotation.

It seems that for the case of solar problems it is more natural to consider that the second averaging is the time averaging. Of course, one can also consider small scale, local action of the helicity fluctuation mechanism and assume the relatively small-scale spatial averaging.

Possible applications of the new dynamo mechanism concern two main features: i) this mechanism is independent of the usual mechanism due to the Coriolis force action, and ii) it is strongly anisotropic and is located at the bottom and the top of the convective zone. Thus, when investigating the magnetic activity of stars we have the opportunity to use the appropriate forms of the additional tensor $\alpha_{ij}(\mathbf{r},t)$ to explain the observed data.

## 6. Summary

Using the large-scale or long-time averaging procedures we have obtained the explicit formulae for the tensors $\alpha_{ij}(\mathbf{r},t)$ and $\beta_{ij}(\mathbf{r},t)$ which describe the influence of the helicity fluctuations on the enhancement and turbulent diffusion of the mean magnetic field.

We present a qualitative explanation of this new dynamo mechanism. It was demonstrated that this mechanism is independent of the existence of nonzero mean helicity which forms the basis of the usual dynamo mechanism. The new mechanism does not need the action of the Coriolis force and is very anisotropic in character.

The condition necessary for the new mechanism is the inhomogeneous distribution of helicity fluctuations in the differentially rotating atmosphere.

For the isotropic, homogeneous and mirror asymmetric turbulent medium the helicity fluctuations increase the existing $\alpha$-effect and decrease the turbulent diffusivity.

Our estimations show that this mechanism is comparable to or even more effective than the usual dynamo mechanism, but is concentrated near the boundaries of an atmosphere where inhomogeneity of fluctuations is most prominent.

The qualitative consideration of this mechanism in the Sun shows that it can diminish or even cancel the usual dynamo mechanism in the upper layers of the convective zone and displace the region of the dynamo action to the overshoot layer providing both the necessary phase condition between the radial and azimuthal magnetic components and the direction of the dynamo wave towards the equator.

## References

Babcock H.W., 1961, ApJ 133, 572

Baliunas S.L., Vaughan A.H., 1985, ARA&A 23, 379

Covas E., Tworkowski A., Brandenburg A., Tavakol R., 1997, A&A 317, 610

Covas E., Tavakol R., Tworkowski A., Brandenburg A., 1998, A&A 329, 350
Cowling T.G., 1981, ARA&A 19, 115
Dolginov A.Z., Silant'ev N.A., 1992, Geophys. Astrophys. Fluid Dyn. 63, 139
DeLuca E.E., Gilman P.A., 1991, In: Cox A.N., Livingston W.C., Matthews M.S. (eds.) Solar interior and atmosphere. The University of Arizona Press, Tucson, p. 275
Gruzinov A.V., Diamond P.H., 1994, Phys. Rev. Lett. 72, 1651
Hoyng P., 1987a, A&A 171, 348
Hoyng P., 1987b, A&A 171, 357
Hoyng P., 1988, ApJ 332, 857
Hoyng P., Schmitt D., Teuben L.J.W., 1994, A&A 289, 265
Hoyng P., Schutgens N.A.J., 1995, A&A 293, 777
Hoyng P., 1996, Solar Physics 169/2, 253
Kichatinov L.L., 1993, In: Krause F., Rädler K.-H, Rüdiger G., (eds.) The Cosmic Dynamo. IAU Symposium 157, Kluwer Academic Publ., Dordrecht, p. 13
Kichatinov L.L., Pipin V.V., Rüdiger G., 1994, Astron. Nachr. 315, 157
Kraichnan R.H., 1976, J. Fluid Mech. 75, 657
Krause F., Rädler K.-H., 1980, Mean-field magnetohydrodynamics and dynamo theory. Pergamon Press, Oxford
Krause F., 1991, In: Tuominen I., Moss D., Rüdiger G. (eds.) The Sun and Cool Stars activity, magnetism, dynamos. IAU Colloquium 130, Springer Verlag, Heidelberg, p. 3
Leighton R.B., 1964, ApJ 140, 1547
Moffatt H.K., 1978, Magnetic field generation in electrically conducting fluids. Cambridge Univ. Press, Cambridge
Parker E.N., 1970, ARA&A 8, 1
Parker E.N., 1979, Cosmical magnetic fields. Clarendon Press, Oxford
Rädler K.-H., 1990, In: Berthomieu G., Griber M. (eds.) Inside the Sun. IAU Colloquium 121, Kluwer Academic Publ., Dordrecht, p. 385
Rosner R., Weiss N.O., 1992, In: Harvey K.L. (ed.) The Solar Cycle. ASP Publ.
Rüdiger G., Kichatinov L.L., 1993, A&A 269, 581
Rüdiger G., Brandenburg A., 1995, A&A 296, 557
Schmitt D., 1993 In: Krause F., Rädler K.-H., Rüdiger G., (eds.) The Cosmic Dynamo. IAU Symposium 157, Kluwer Academic Publ., Dordrecht, p. 1
Seehafer N., 1994, Europhys. Lett. 27, 353
Seehafer N., 1996, Phys. Rev. E 53, 1283
Silant'ev N.A., 1992, JETP (Sov. Phys.) 74, 650
Silant'ev N.A., 1997a, JETP (Sov.Phys.) 85, 712
Silant'ev N. A., 1997b, Geophys. Astrophys. Fluid Dyn. 84, 99
Silant'ev N.A., 1999, JETP (Sov. Phys.) 89, 45
Sokolov D.D., 1997, AZh (Astron. Reports) 41, 68
Vainshtein S.I., Rosner R., 1991, ApJ 376, 199
Vainshtein S.I., Cattaneo F., 1992, ApJ 393, 165
Vishniac E.T., Brandenburg A., 1997, ApJ 475, 263
Weiss N.O., 1994, In: Proctor M.R.E., Gilbert A.D. (eds.) Lectures on solar and planetary dynamos. Cambridge Univ. Press, Cambridge, p. 59
Zeldovich Ya.B., 1957, JETP (Sov. Phys.) 4, 460
Zwaan C., 1996, Solar Physics 169/2, 265