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The transfer of comets from near-parabolic to short-period orbits: map approach

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Abstract. In the framework of the circular restricted three-body problem (the Sun-planet-comet), we generalize the map of Liu & Sun (1994) to study the transfer of comets from near-parabolic to Halley-family ones, under the perturbations of Jupiter. Numerical results show that the transfer is effective for comets on both the direct and retrograde orbits crossing the planet's orbit, and on the direct non-crossing orbits but with the perihelion distances close to the semi-major axis of the planet's orbit. The dependence of the transfer probability and of the average transfer time on the perihelion distances of the cometary orbits and on the planet mass are determined. The probabilities of comet transfer from the Oort Cloud are found to be 0.12 and 0.067 for comets on direct and retrograde orbits, respectively. This leads us to expect an average flux of 0.23 new Halley-family comets per year interior to Jupiter's orbit. The calculation shows that about 86% of the flux comes from comets on the Jupiter-crossing orbits.

Key words: diffusion – celestial mechanics, stellar dynamics – comets: general

1. Introduction

The capture of comets from the Oort Cloud (Oort 1950) is a subject with a long history and fruitful results (e.g. Newburn et al. 1991). Up to now there are nearly 940 comets with computed orbits recorded (end of 1997), of which about 80% are long-period (with period $P > 200$ yr) comets (Fernández 1999). Most of the discovered long-period (LP) comets have the perihelion distance q smaller than a few AU. Only 10 have perihelia beyond Jupiter's orbits ($q > 5.2$AU), the comet C/1991 R1 (McNaught-Russel) being the farthest with $q = 7$AU. The study of evolution of the LP comets subject to the perturbations of the outer planets led to the idea that the short-period (SP) comets could be the end products of the dynamical evolution of LP comets. The short-period (SP) comets are divided into two families: Halley-family comets (20yr$< P < 200$yr or $T < 2$, where T is the Tisserand parameter with respect to Jupiter) and Jupiter-family comets ($P < 20$yr or $T > 2$). Due to the flat near-ecliptic distribution, the Jupiter-family comets (JFCs) are believed to originate from a flat trans-Neptunian source (Duncan et al. 1995). As the Halley-family comets (HFCs) may come from the Oort Cloud, the question as to whether this mechanism is sufficient to maintain a steady-state population of the HFCs is still open (Fernández 1999).

One of the major difficulties in the statistical study of the comet dynamics is that one needs to integrate orbits of millions of comets over the lifetime of the Solar System, which is beyond the capacity of existing computers. Consequently, Monte Carlo simulations have been used (see, e.g. Valtonen et al. 1998). In recent years the map approach has been used widely in the study of long-term evolution of celestial bodies. It approximates the differential equation system with a map system, with the main properties of the original system being kept. The use of a map instead of integrating the original system saves much computing time, and it is easier to handle both analytically and numerically (for a review, see Froeschlé 1998).

For the study of comets captured from the Oort Cloud, Petrosky & Broucke (1988) and Liu & Sun (1994) have derived two-dimensional area-preserving maps, approximating the model of the restricted three-body problem. Recently, for the evolution of comets in high-eccentricity planet-crossing orbits, Malyshkin & Tremaine (1999) obtained a two-dimensional map similar to the Keplerian map obtained by Petrosky & Broucke (1988), and studied the role of resonance sticking on the evolution of cometary orbits. They have also compared the results obtained by the map with those by direct integrations of the cometary orbits. A good correspondence between both proved the map to be very helpful in the study of comet dynamics.

In this paper we study the transfer of comets from the Oort Cloud to the HFCs. The arrangement of the paper is as follows. In Sect. 2, we present the map model adopted for this work. In Sect. 3 the map is used to study the transfer of comets from near-parabolic orbits to short-period ones. The dependence of transfer probability and of the average capture time on the perihelion distance of cometary orbits and on the mass of the planet is

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computed in this section. Conclusions of the paper are given in the final section.

2. Map model

Liu & Sun (1994) derived a map which describes the evolution of comets on near-parabolic orbits in the framework of the planar circular restricted three-body problem (the Sun-planet-comet). In our work, we generalize this map in order to study the motion of comets on both planet-crossing and non-crossing orbits.

Let us take the distance between the Sun and Jupiter (i.e., 5.2AU) as the unit of length, the total mass of the Sun and Jupiter as the unit of mass, and the period of Jupiter's circular motion as 2π units of time. Denote the mass of planet as μ (μ_J for Jupiter). In an inertial coordinate system with the origin at the mass center of the Sun and Jupiter, the planar motion of a comet located at (x, y) has the following form:

$$\begin{aligned}&\ddot{x} = (1-\mu)(x_S - x)/r_{13}^3 + \mu(x_J - x)/r_{23}^3\\&\ddot{y} = (1-\mu)(y_S - y)/r_{13}^3 + \mu(y_J - y)/r_{23}^3\\&r_{13}^2 = (x - x_S)^2 + (y - y_S)^2\\&r_{23}^2 = (x - x_J)^2 + (y - y_J)^2,\end{aligned} \quad (1)$$

where (x_S, y_S) and (x_J, y_J) are the positions of the Sun and Jupiter, respectively. They can be expressed as follows,

$$\begin{aligned}&x_S = -\mu\cos(t - t_0), \qquad x_J = (1-\mu)\cos(t - t_0),\\&y_S = -\mu\sin(t - t_0), \qquad y_J = (1-\mu)\sin(t - t_0),\end{aligned} \quad (2)$$

where t_0 is the initial time. Expanding the right hand members of Eq. (1) to $O(\mu)$, one has

$$\begin{aligned}\ddot{x} &= -x/r^3 + \mu F(x, y, t, t_0) + O(\mu^2),\\\ddot{y} &= -y/r^3 + \mu G(x, y, t, t_0) + O(\mu^2).\end{aligned} \quad (3)$$

where $r = (x^2 + y^2)^{1/2}$, and

$$\begin{aligned}F(x, y, t, t_0) &= \tfrac{x - \cos(t-t_0)}{r^3}\\&+ \tfrac{3x[x\cos(t-t_0) + y\sin(t-t_0)]}{r^5}\\&+ \tfrac{\cos(t-t_0) - x}{\{[x-\cos(t-t_0)]^2 + [y-\sin(t-t_0)]^2\}^{3/2}}\\G(x, y, t, t_0) &= \tfrac{y - \sin(t-t_0)}{r^3}\\&+ \tfrac{3y[x\cos(t-t_0) + y\sin(t-t_0)]}{r^5}\\&+ \tfrac{\sin(t-t_0) - y}{\{[x-\cos(t-t_0)]^2 + [y-\sin(t-t_0)]^2\}^{3/2}}\end{aligned} \quad (4)$$

The initial epoch is chosen such that the comet passes through the perihelion at $t = 0$, and the initial phase angle of Jupiter motion is g. Thus $t_0 = -g$ in the equations from (1) to (4).

For LP comets, the orbital energy K is the parameter that suffers the greatest variation during a passage in the planetary region. The (double of) orbital energy of a comet in the unperturbed motion is

$$K = (\dot{x}^2 + \dot{y}^2) - 2/r = -1/a, \quad (5)$$

where a is the semi-major axis of the comet orbit. For the near-parabolic comets, we approximate the orbits by $r = q\sec^2(f/2)$, where q and f are the perihelion distances and the true anomalies of the orbits, respectively. According to the definition (5), the variation of K can be expressed as a function of time, with the help of Eq. (3) and the above parabolic-orbit approximation:

$$dK/dt = 2\mu[\dot{x}(t)F(x(t), y(t), t) + \dot{y}(t)G(x(t), y(t), t)]. \quad (6)$$

Thus the change of comet energy during one perihelion passage can be obtained by integration the above Eq. (6):

$$\Delta K \equiv 2\mu\psi(g, q) = 2\mu\int_{-\infty}^{+\infty}[\dot{x}(t)F(t) + \dot{y}(t)G(t)]dt \quad (7)$$

with g and q being parameters. The integral limits are set as infinite due to the parabolic-orbit approximation. The change of q during each perihelion passage is of order μ, and its effect on ΔK is of order μ^2, therefore it can be treated as a constant during the evolution of the orbital energy of the comet (Petrosky & Broucke 1988; Liu & Sun 1994). For a given parameter q, ψ is a 2π-periodic function of g, and it is anti-symmetric with respect to π (Liu & Sun 1994).

The period of time between the two successive perihelion passages is $\Delta t = 2\pi/(-K)^{3/2}$, and the change of the phase angle g of Jupiter during this period is $\Delta g = \Delta t$. Combining the expression (7), one obtains a two-dimensional area-preserving map:

$$\begin{aligned}&K_{n+1} = K_n + 2\mu\psi(g_n, q)\\&g_{n+1} = g_n + 2\pi/(-K_{n+1})^{\frac{3}{2}} \pmod{2\pi}.\end{aligned} \quad (8)$$

where the subscript n numbers the perihelion passages. This map can be applied to both the direct and retrograde comet motions.

Unlike in Liu & Sun (1994), where the function ψ in (8) is expanded into Fourier series in g, we calculate the integral (7) by numerical quadrature. This method has two advantages: 1) it avoids the problem of series truncation, as the convergence of the series is very slow when q is close to one. 2) it can be generalized to $q < 1$, which is important in the transfer of comets. While the Fourier expansion diverges in these situations, the numerical integration of (7) is still valid.

In practice, we compute the integral $\int_{-\infty}^{\infty}(\dot{x}F + \dot{y}G)dt$ in (7) as $\int_{-20}^{20}(\dot{x}F + \dot{y}G)dt$. The discarded part $|t| > 20$ corresponds to the situation where the comet is more than 100 AU away from Jupiter, and thus the perturbations by Jupiter can be safely neglected. Considering the periodic properties of $\psi(g)$, we calculate the values of ψ on 2×10^4 values of g uniformly spaced in the interval $[0, 2\pi]$, and obtain ψ for other values of g by linear interpolation. Fig. 1 shows the graphs of ψ as the function of g for two different q. As one can see, ψ is continuous for $q > 1$; while for $q < 1$, there are two values of g where ψ is discontinuous due to the comet-Jupiter collisions. The widths of these two interpolation intervals with the two collision values of g are $10^{-4}\pi$, which corresponds to the length of the same order of magnitude as the equator radius of Jupiter. If a comet falls in one of these collision intervals, we assume that it collides with Jupiter and do not continue its evolution.

Fig. 1 can be compared to Malyshkin & Tremaine (1999), where the function ψ is interpolated (called the Kicked func-

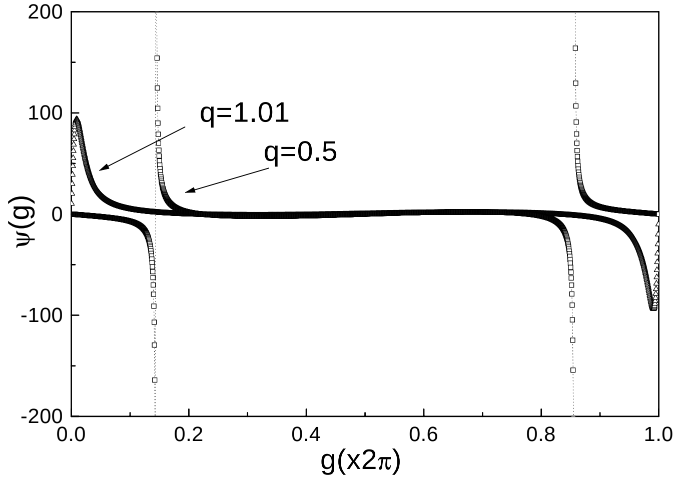

Fig. 1. The function $\psi(g)$ for $q = 1.01$ and $q = 0.5$. Each curve is plotted at 2000 discrete values of g uniformly distributed in $[0, 2\pi]$.

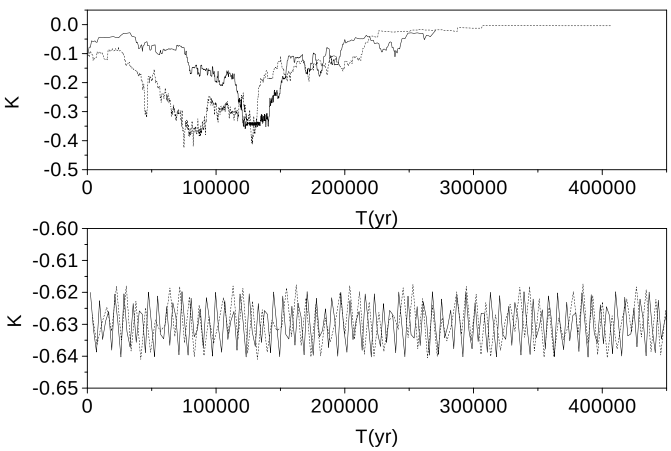

Fig. 2. Evolution of the energy of a comet by the integration of the original system (1) (dash lines) and the use of the map (8) (solid lines) with the same initial values: $(K = -0.64, g = 0.4\pi)$ for the regular orbit (lower window) and $(K = -0.10, g = 0)$ for the escape orbit (upper window).

tion) from discrete values obtained by integration of the original Hamiltonian system. The maps in their work and in this paper are nearly equivalent, and should have almost the same computational efficiency.

To test the validity and the computational speed of the map (8), we calculate the orbits with the same initial configurations by the map (8) and by integration of the original system (1). The Runge-Kutta-Fehlberg 7(8) method is used in our integration. Fig. 2 shows that the final qualitative results (escape, remain or transfer) of the evolution of individual orbits in the original system (1) is preserved by the map (8). However, for the orbit presented in the lower window of Fig. 2, the use of map (8) takes only 1 minute for the orbit evolving up to 10^7 passages, which takes 30 minutes to integrate the original system (1) up to 10^5 passages. Thus the use of map (8) is in this case some 3000 times faster than the integration of the differential equation system (1).

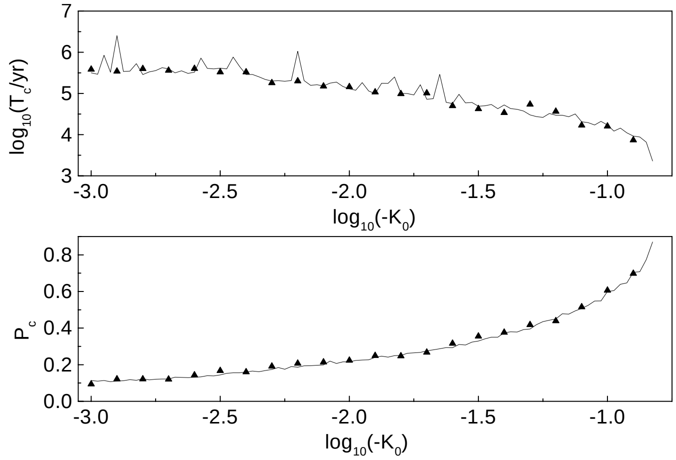

Fig. 3. Variations of the transfer probability (lower window) and of the average transfer time (upper window) with the initial energy, obtained by integration of the original system (1) (triangles) and by the use of the map (8) (solid lines), respectively. We set $q = 0.8$.

The statistical validity of the map (8) is also verified. We take 10^3 comets with the same initial energy and calculate their orbits up to 10^7 years. Then determine the transfer probability (the average ratio of comets whose final energy $K < -0.152$ to 10^3) and the average transfer time (number of passages preceeding the transfer). Fig. 3 shows the variations of transfer probability and averaged transfer time for different initial energies and $q = 0.8$. As one can see, the results obtained by the integration of the original system (1) coincide with those obtained by the map (8) quite well.

Since the use of parabolic orbits to approximate the orbits of the comets, the map is not valid when the comets are in orbits with very small eccentricities. According to our numerical experiments, the minimum eccentricity that the map (8) is still valid is ~ 0.3. Since the LP comets we studied in the paper all have large eccentricities (at least 0.7) because of their large semi-major axes and small perihelion distances, this limitation does not affect our results.

3. Numerical results

Fig. 4 shows a phase diagram of map (8), which has both regular and chaotic orbits. The islands correspond to the locations where a comet is in the exterior mean motion resonance with Jupiter. For example, the first resonance visible at $K = -0.32$ corresponds to a $n : n_J = 2 : 11$ resonance. The sticking effect of these resonances were studied by Malyshkin & Tremaine (1999). The chaoticity of orbits in the phase space with $-0.3 < K < 0$ comes from the fact that, according to the map (8), for the small $|K|$, the phase angles g of two consecutive perihelion passages are almost independent.

As we intend to study the possibility of orbital transfer of comets with near zero energy, we concentrate on cases when a comet obtains enough (negative) energy from the planet to change its orbit from a LP one to a SP one. The relation between the energy K and the period P in our units is $K = -(11.86$ yr

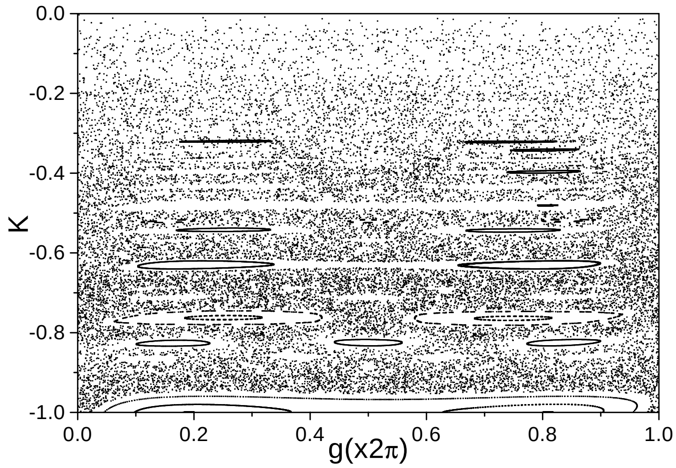

Fig. 4. Phase diagram of the map (8) with $\mu = \mu_J$ and $q = 1.1$.

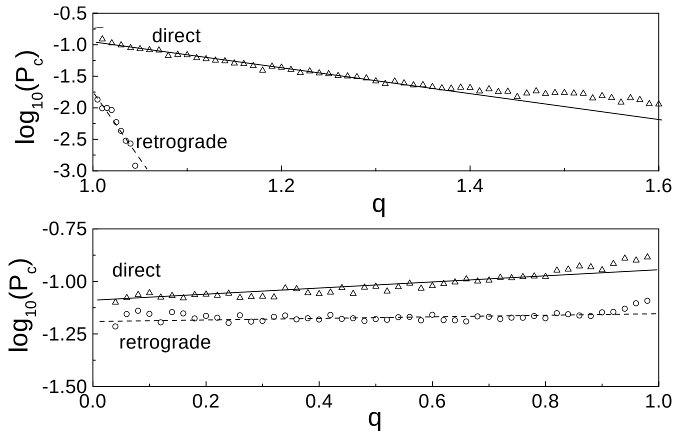

Fig. 5. Dependence of the transfer probability P_c on q with $\mu = \mu_J$. The fitting lines have the form of $\log_{10} P_c = -0.94 + 0.2(q-1)$, $\log_{10} P_c = -1.16 + 0.025(q-1)$ for $q < 1$ and $\log_{10} P_c = -0.95 - 2(q-1)$, $\log_{10} P_c = -1.8 - 22(q-1)$ for $q > 1$, respectively. The unit of q is 5.2AU. Note the different scales between the two windows.

$/P)^{\frac{2}{3}}$. The boundary of period between LP and SP comet is 200 year, which corresponds to the energy of about $K_f = -0.152$. Since the Oort Cloud is supposed to be located at $10^4 - 10^5$ AU, we study comets with the initial energy of -0.001.

In all of the following experiments, we take sets of 10^4 orbits each with initial energy $K = -0.001$ uniformly distributed in $g \in [0, 2\pi]$, and compute the evolution of each orbit one by one with the map (8), until it either escapes ($K > 0$) or is transferred to $K < -0.152$. Then we make the statistics for these 10^4 orbits. Note that for comet orbits with $q < 1$, there is also the possibility of collision with Jupiter, though it is very small ($0.07\% - 0.2\%$ according to our calculations). Three quantities related the transfer dynamics of the comets are determined: the transfer probability (P_c), the average transfer time (n_f) and the average energy change of transferred comets per passage (σ). P_c is defined as the ratio of transferred orbits (with final energy $K < K_f$) to the total number of the initial orbits, i.e. 10^4. For the transferred comets, n_f is defined as the average number of passages needed for accomplishing the transfer, and σ is defined as the root-mean-square average of the energy change per perihelion passage. We study the dependence of these quantities on the perihelion distance of the comet orbit (which is assumed constant during the energy evolution) and the planet mass.

3.1. Dependence on perihelion distance

In this subsection we fix $\mu = \mu_J = 1/1047.355$ and study the transfer dynamics with the perihelion distance q varying from 0.2 to 1.6. Fig. 5 shows the variation of P_c with the perihelion distance q. We note that, for $q < 1$, P_c has values ranging from 0.09 to 0.13 for comets on direct orbits and it is about 0.07 for comets on retrograde orbits. In both cases the variation of P_c with q is small. In contrast, for comets with perihelion distances $q > 1$, the variation of P_c with q is large. P_c decreases from 0.11 to 0.01 as q increases from 1 to 1.6 for direct motion, and it decreases drastically for retrograde motions ($P_c \sim 10^{-3}$ at $q = 1.05$). Thus the transfer of comets with $q > 1$ on retrograde orbits should be negligible. For comets on direct motion with $q > 1$, only those with $q \sim 1$ can be transferred efficiently.

According to the fitting lines in Fig. 5, the variations of P_c obey exponential laws with the form of

$$\begin{array}{ll} P_d^{<1} \simeq 0.11 \times 10^{0.2(q-1)} & (q < 1) \\ P_d^{>1} \simeq 0.11 \times 10^{-2(q-1)} & (q > 1) \\ P_r^{<1} \simeq 0.069 \times 10^{0.025(q-1)} & (q < 1) \\ P_r^{>1} \simeq 0.016 \times 10^{-22(q-1)} & (q > 1). \end{array} \quad (9)$$

where the subscripts "d" denote for comets in direct orbits and "r" for those on retrograde orbits, respectively. Assuming these exponential laws, we define the integral of transfer probability:

$$I_d^{<1} = \int_0^1 P_d^{<1}(q)dq, \quad I_d^{>1} = \int_1^\infty P_d^{>1}(q)dq \quad (10)$$

and similarly for $I_r^{<1}$ and $I_r^{>1}$. The calculation gives,

$$\begin{array}{lll} I_d^{<1} = 0.092, & I_d^{>1} = 0.024, & I_d^{<1} + I_d^{>1} \approx 0.12, \\ I_r^{<1} = 0.067, & I_r^{>1} = 0.0003, & I_r^{<1} + I_r^{>1} \approx 0.067. \end{array} \quad (11)$$

Thus the flux of transferred comets from direct orbits is ≈ 1.8 times as large as that from retrograde orbits, this may be the reason why most of the observed HFCs are on the direct orbits (Fernández 1999). Moreover, the flux of transferred comets from Jupiter-crossing orbits is about $(I_d^s + I_r^s)/(I_d^g + I_r^g) \approx 6.7$ times as large as that from non-crossing orbits. This phenomenon is noticed for the first time, as far as we know.

The variation of the average transfer time n_f for the transferred orbits with q is shown in Fig. 6. We find that the variation of n_f is smaller for $q < 1$ than that for $q > 1$. For $q < 1$, n_f varies with q very slowly and equals to $10 - 25$ passages for the direct motion and to about 60 passages for the retrograde motion. For $q > 1$, the number of passages needed for transfer from direct motion increase from 10 to 3000 as q increases from

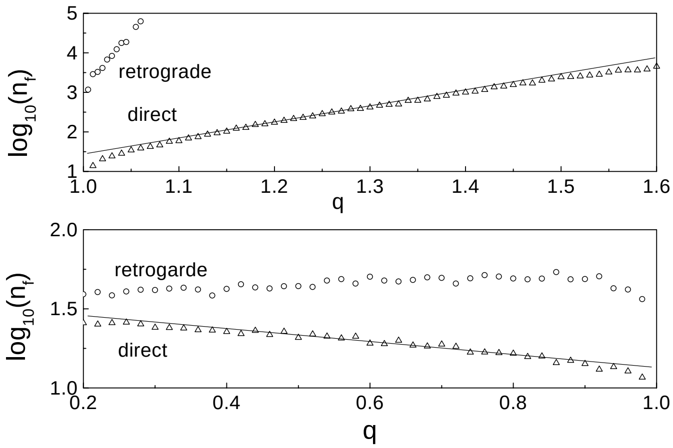

Fig. 6. Dependence of the averaged transfer time n_f on q, with $\mu = \mu_J$. The fitting lines have the form of $\log_{10} n_f = 1.13 - 0.4(q-1)$ and $\log_{10} n_f = 1.55 - 3.95(q-1)$ for $q < 1$ and $q > 1$, respectively.

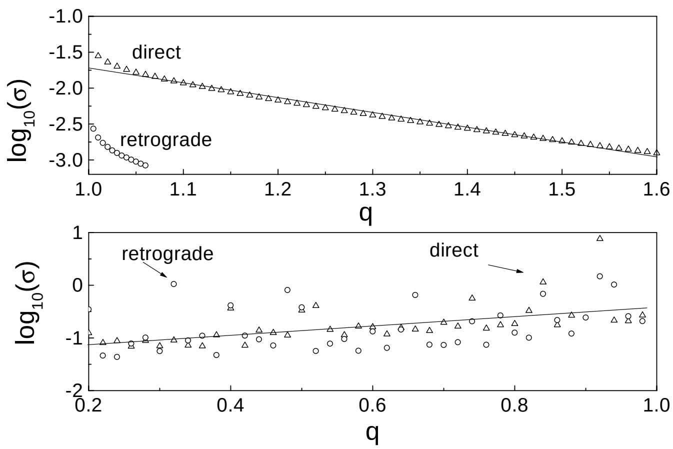

Fig. 7. Dependence of the average energy change σ on q when $\mu = \mu_J$. The fitting lines have the form $\log_{10}\sigma = -0.55 + 0.8(q-1)$ and $\log_{10}\sigma = -1.7 - 2.0(q-1)$ for $q < 1$ and $q > 1$, respectively.

1 to 1.6. For comets in direct motion, the dependence of n_f on q can be roughly fitted with the exponential laws:

$$\begin{aligned} n_f^{<1} &\simeq 13 \times 10^{-0.4(q-1)} \quad (q < 1), \\ n_f^{>1} &\simeq 35 \times 10^{4(q-1)} \quad\;\; (q > 1). \end{aligned} \tag{12}$$

To explain the above exponential laws, we calculate the average energy change per passage σ for the transferred comets. The dependence of σ on q is shown in Fig. 7. We notice that the values of σ for $q < 1$ are large, sometimes even $\sigma \sim 1$. This is because the encounters for $q < 1$ can be much closer than those with $q > 1$. From the graph of ψ (Fig. 1), one can also see that the energy exchange of the comet with $q < 1$ is much larger than those with $q > 1$. For the direct motions σ varies with q exponentially with the form

$$\begin{aligned} \sigma^{<1} &\simeq 0.28 \times 10^{0.8(q-1)} \quad\; (q < 1) \\ \sigma^{>1} &\simeq 0.023 \times 10^{-2(q-1)} \quad (q > 1). \end{aligned} \tag{13}$$

The exponential dependence of σ on q gives an explanation of the exponential forms of Eqs. (9) and (12). In fact, according

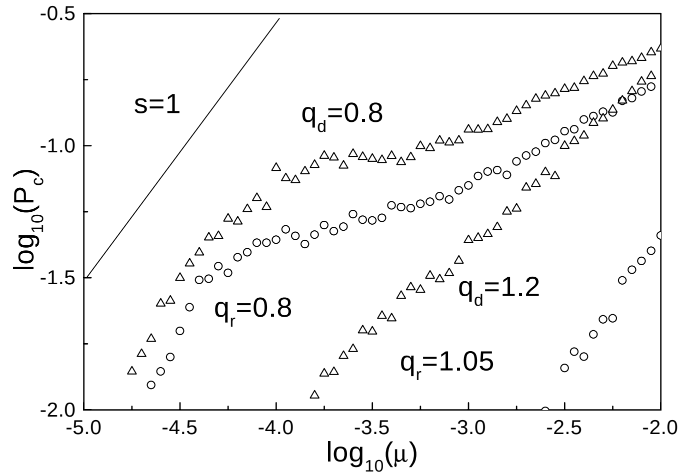

Fig. 8. Dependence of transfer probability P_c on μ with different q. The straight line in the up-left corner has a slope $s = 1$. q_d stands for the perihelion distance of comets in the direct motion, q_r stands for that in the retrograde motion.

to Fig. 7, the typical values of σ is about $10^{-3} - 10^{-2}$ for $q > 1$ in the direct motions, which is much less than the energy decrement required for a comet with near zero initial energy to reach $K_f = -0.152$. Thus the evolution of energy for $q > 1$ obeys the diffusion approximation. According to the diffusion approximation, the transfer probability and average transfer time for a comet with initial energy near zero to reach energy K_f obey (Fernández & Gallardo 1994)

$$P_c \simeq 0.5\sigma/(-K_f), \qquad n_f \simeq (K_f/\sigma)^2. \tag{14}$$

Thus the exponential laws in Eqs. (9) and (12) for $q > 1$ can be deduced from the Eq. (13). However, for $q < 1$, due to the large average energy change per passage, which is of the same order as $|K_f|$, the condition of the diffusion approximation is no more fulfilled.

3.2. Dependence on the planet mass

To see the influence of the planet mass on the transfer of comets, we investigate the evolution of above 10^4 orbits with μ varying from 10^{-2} to 10^{-5}. The variations of the transfer probability P_c with μ are shown in Fig. 8. One can see that the probabilities for $q < 1$ are larger than those for $q > 1$ as long as $\mu < 10^{-2}$. This is the case for our Solar System, where the largest perturbation for comet motions comes from Jupiter ($\mu_J \sim 10^{-3}$).

As μ increases, we find approximately that $P_c \sim \mu$ in the cases of (a) $\mu < 10^{-4}$ for $q < 1$ and (b) $\mu < 10^{-2}$ for $q > 1$. However, in the case of $q < 1$ with $\mu > 10^{-4}$, this relationship does not hold (Fig. 8). This is caused by the differences in the average energy change σ. To demonstrate this we show the dependence of σ on μ (Fig. 9), where we find $\sigma \sim \mu$ in all the studied cases. This is expected since Eq. (7) shows that the change of energy is proportional to μ.

With the relation of $\sigma \sim \mu$, the variation of P_c on μ can be explained. For the situations (a) and (b) listed above, the average energy changes are small so that the diffusion approximation

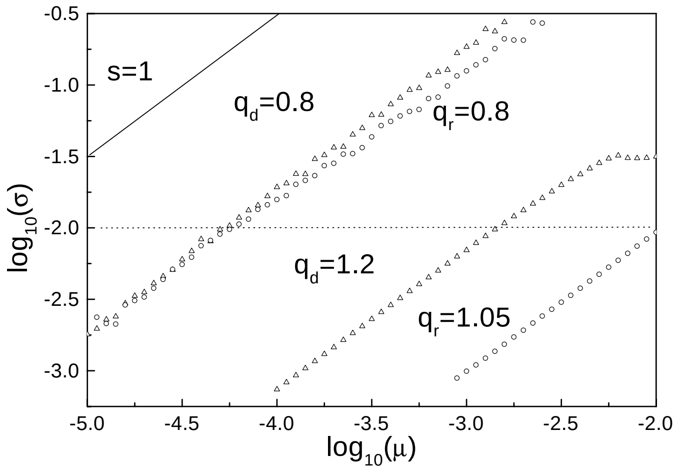

Fig. 9. Dependence of the average energy change σ on μ with different q. The straight line in the up-left corner has a slope $s = 1$. The dotted line shows the critical value σ_c below which the diffusion approximation holds.

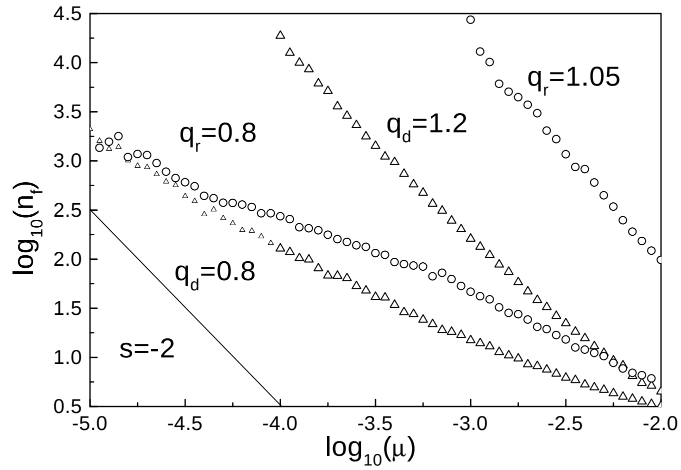

Fig. 10. Dependence of the averaged transfer time n_f on μ with different q. The straight line in the down-left corner has the slop $s = -2$.

holds, with $P_c \sim \mu \sim \sigma$, which obeys the relation (14). For the other situations, the diffusion approximation is not fulfilled due to the large energy change, thus the relation (14) does not hold since $P_c \sim \mu$ does not hold.

Finally we present the dependence of the average transfer time n_f on μ (Fig. 10). For the situations (a) and (b) listed above but with $\mu < 10^{-4.5}$ for $q < 1$, one can identify roughly a power-law with the form of $n_f \sim \mu^{-2}$, which is consistent with the relation (14). In the other cases, when conditions of the diffusion approximation is not fulfilled, the relation $n_f \sim \mu^{-2}$ is no more obeyed.

In the above qualitative discussion of this subsection, when the mass of planet μ is changed, the energy boundary between LP comets and HFCs is fixed as $k_f = -0.152$. However, when the only perturbing planet is changed (e.g. Saturn, Uranus or Neptune), K_f should be also changed due to the different orbital periods of the planets. We calculate the transfer probabilities in the cases of different planet pertubations as in the last subsection and Fig. 5. The results are showed in Table 1. As we can see, only Saturn contribute by a non-negligible amount to the transferred comets while the contributions of Uranus and Neptune are negligible.

Table 1. Integral transfer probability for different planet perturbations

planet	K_f	$I_d^{<1}$	$I_r^{<1}$	$I_d^{>1}$	$I_{\rm sum}$
Jupiter	–0.152	0.0920	0.0672	0.0244	0.184
Saturn	–0.279	0.0457	0.0296	0.0045	0.080
Uranus	–0.560	0.0005	0.0029	0	0.003
Neptune	–0.875	0.0010	0.0021	0	0.003

Note: $K_{\rm f} = -(P{\rm yr}/200{\rm yr})^{\frac{2}{3}}$, $I_{\rm sum} = I_d^{>1} + I_r^{<1} + I_d^{>1}$.

4. Conclusions

In this paper, we studied the dynamical transfer of comets from near-parabolic orbits to short period ($< 200{\rm yr}$) orbits, under the perturbation of a large planet. Numerical results showed that due to the difference of average energy change per perihelion passage, the transfer of comets on planet-crossing orbits (with $q < 1$) is more efficient than the transfer of comets in non-crossing orbits ($q > 1$). The comets with $q < 1$ are easier to be transferred in both direct and retrograde motions. For the comets with $q > 1$, only those on direct orbits with q near 1 have a higher probability of transfer. According to our calculations, the flux of transferred comets with $q < 1$ is about 4 times as large as that of $q > 1$ on direct orbits. It is difficult to confirm this conclusion at the moment by observations since the small amount of observed SP comets.

The size of the HFCs comets population has been estimated by several authors (e.g. Fernández & Gallardo 1994). Even though the number of actually known HFCs is only somewhat more than 20, the observations are strongly biased towards small perihelion distances and bright absolute magnitudes. From the observed frequency of apparitions of new comets and making allowance for missed comets, the influx rate of new comets interior to Jupiter's orbit brighter than absolute magnitude 11 is about 0.5 yr^{-1}AU^{-1} (Fernández & Ip 1991). This gives about 2.6 comets/year interior the Jupiter orbit. According to our results, the integral transfer propability is 0.12 for comets in direct motions and 0.067 for those in retrograde motions. Provided a mean value of transfer probability 0.09, the flux of HFCs transfered from the new comets flux is thus 0.23 comets per year in the regions $q < 5.2$ AU. Supposing the physical lifetime of a typical HFC as $2000 \sim 10000$ year (e.g. Levison & Duncan 1994), the steady-state population of HFCs interior Jupiter's orbits is $460 \sim 2300$. This can be compared with the steady-state population of 300 HFCs with $q < 2$AU according to the estimate of Fernández & Gallardo (1994).

Among HFCs the retrograde comets represent less than 20% of the total population. We expect retrograde comet captures (in practice) only from $q < 1$, and also their P_c is generally lower than in direct orbits. According to our resluts, the retrograde comets should be 37%. This is somewhat overestimated.

We think this is mainly due to the plannar approximation of the model. The actual ratio also depends on the width of the capture region in inclination which we have not studied here. However, from previous studies (e.g. Valtonen et al. 1992) we know that the capture probability as a function of inclination follows reasonably well the inclination distribution of Halley type comets.

The transfer of comets is a subject of orbital energy evolution under the perturbations of planets. In the case of Jupiter perturbations, the evolution of the orbital energy for a comet with $q > 1$ obeys the diffusion approximation, i.e., the relation (14) holds, due to the small average energy exchange per passage as compared with the total transferred energy -0.152. However for the comets with $q < 1$, due to the large average energy change, the evolution of the energy may not obey the diffusion approximation. This is related to the problem of the validity of the diffusion approximation on the comet evolution, which will be addressed in more detail in our future works.

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