Letter to the Editor

Entropy & equation of state (EOS) for hot bare strange stars

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Abstract. Compactness of some stars is explained if they are strange stars (SS) as shown by Dey et al. (1998) (D98) and Li et al. (1999a). One of these compact star candidates is the SAX J1808.4−3658 (SAX in short) believed to be an important link in the genesis of radio pulsars. SS have also been suggested for bursting X-ray pulsars (GRO J1744−28, Cheng et al. 1998), from quasi-periodic oscillations (QPO) of X-ray binaries (4U 1728−34, Li et al. 1999b) and from peculiarity of properties of radio pulsars (PSR 0943+10, Xu et al.1999; Kapoor et al. 2000). We now extend the calculation to include high temperatures upto $T = 70$ MeV $\sim 8 \times 10^{11}$ ^{o}K and find that the nature of the mass (M) and radius (R), derived from astrophysical data, is still retained. The entropy is calculated and matches onto that calculated from hadronic models thus supporting the idea that the quark-hadron transition may be continuous.

Key words: dense matter – elementary particles – equation of state – stars: fundamental parameters

1. Introduction

A calculation for cold strange stars (D98) enabled us to draw conclusions about chiral symmetry restoration in QCD when the EOS was used to get SS fitting definite mass-radius (MR) relations (D98; Li et al. 1999a; Li et al. 1999b; Ray et al. 2000b). The empirical MR relations were derived from astrophysical observations like luminosity variation and some properties of quasi-periodic oscillations from compact stars. The calculations are compared to these stars which emit X-rays, generated presumably due to accretion from their binary partner.

During the genesis of these stars higher T may be encountered and in this paper we deal with a generalized case of an object at a uniform T. We show at upto $T = 70$ MeV a self sustained system can be supported by the parameter set eos1 (D98) (called SS1 in Li et al. 1999a).

The changes in star masses are shown in the range of T mentioned above. The conclusions of D98 and Li et al. (1999a, 1999b) are still valid for the finite T cases. Secondly, parameters of the single particle potential at finite T are tabulated. In particular the entropy is studied and this can be compared with hadronic models. The comparison shows that the entropy may indeed be continuous supporting the idea of a continuous phase transition between hadrons and quarks.

2. Strange stars at finite temperatures

The interesting point made in D98 is that starting from an empirical form for the density dependent masses of the up (u), down (d) and strange (s) q-s given below one can constrain the parametric form of this mass from recent astronomical data[1]:

$$M_i = m_i + M_Q \operatorname{sech}(\nu \frac{\rho_B}{\rho_0}), \quad i = u, d, s. \tag{1}$$

Restoration of chiral quark masses at high density is incorporated in this model. Using the model parameter (ν) for this restoration one can calculate the density dependence of the strong coupling constant (Ray et al. 2000b). In other words the masses of stars in units of solar mass, ($M/M_\odot$), found as a function of the star radius R, calculated using the above Eq. 1, - produces constraints which enable us to restrict the parameter ν. At high ρ_B the q- mass M_i falls from its constituent value M_Q to its current one m_i. The parameter M_Q is taken to be 310 MeV to match up with constituent q- masses assuming the known fact that the hadrons have very little potential energy. The results are not very sensitive in so far as changing M_Q to 320 MeV changes the maximum mass of the star from 1.43735 $M_\odot$ to 1.43738 $M_\odot$ and the corresponding radius changes from 7.0553 kms to 7.0558 kms.

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[1] $\rho_B = (\rho_u + \rho_d + \rho_s)/3$ is the baryon number density, $\rho_0 = 0.17$ fm^{-3} is the normal nuclear matter density, and ν is a numerical parameter. The current q masses m_i used in the following are 4, 7 and 150 MeV for u, d and s respectively.
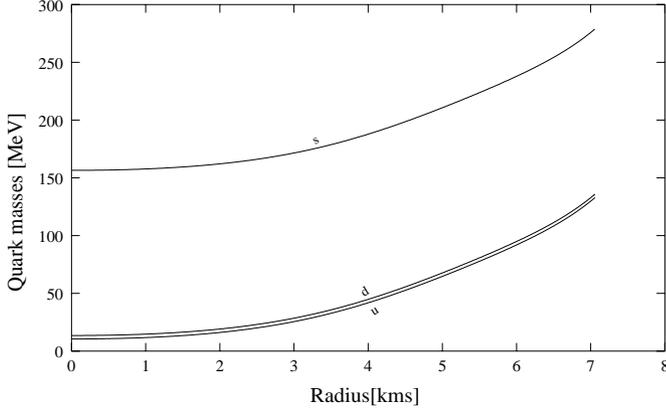
Fig. 1. The smooth restoration of chiral symmetry inside the star for each of the u, d and s flavours (note that this is for the zero temperature result).

It is interesting to plot the up (u), down (d) and strange (s) q-masses at various radii in a star. This is done with strong coupling constant $\alpha_0 = 0.2$, chiral symmetry restoration parameter $\nu = 1/3$ and the QCD scale parameter $\Lambda = 100$ already discussed in D98 & Li et al. (1999a, 1999b). Fig. 1 shows that the quarks do not have the constituent masses as in zero density hadrons nor do they have the current masses of the bag model. Upto a radius about 2 kms the quarks have their chiral current mass but in the major portion of the star their masses are substantially higher. At the surface the strange q- mass is about 278 MeV and the u, d q- masses ~ 130 MeV.

We use the large N_c (colour) approximation of 't Hooft (1974) for quarks, where quark loops are suppressed by $1/N_c$ and the calculation can be performed at the tree level with a mean field derived from a qq interaction (Witten 1979). This was done for baryons (Dey et al. 1986; Dey et al. 1991; Ray et al. 2000a) and extended to dense systems like stars (D98). Following is the Hamiltonian, with a two-body potential V_{ij}:

$$H = \sum_i (\alpha_i \cdot \mathbf{p}_i + \beta_i M_i) + \sum_{i<j} \frac{\lambda(i) \cdot \lambda(j)}{4} V_{ij}. \qquad (2)$$

The vector potential in Eq. 2 between quarks originate from gluon exchanges, and the λ-s are the color SU(3) matrices for the two interacting quarks. In the absence of an accurate evaluation of the potential (e.g. from large N_c planar diagrams) we borrow it from meson phenomenology, namely the Richardson potential (Richardson 1979). The potential reproduces heavy as well as light meson spectra (Crater et al. 1984). It has been well tested for baryons in Fock calculations (Dey et al. 1986; Dey et al. 1991). Recently, using the Vlasov approach with the Richardson potential, Bonasera (1999) finds a transition from nuclear to quark matter at densities 5 times ρ_0.

We had to calculate the potential energy (PE) contribution in two steps: the single particle potential, $U_i(k)$, for momentum k, is first calculated and this is subsequently integrated to get the PE. The $U_i(k)$ is needed for doing finite T calculation.

The $U_i(k)$ is parametrized as sum of exponentials in k (i.e., $U_i(k) = -\exp(a_0 + a_1 k + a_2 k^2)$ for a given flavour), where the
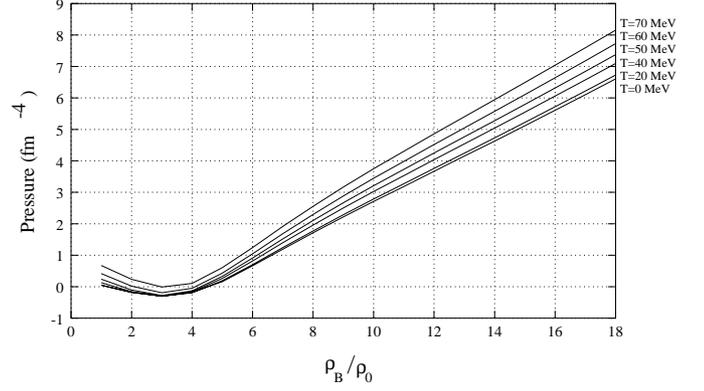

Fig. 2. The equation of states for different T. It is to be noted that beyond $T = 70$ MeV, the zero of the pressure cannot be attained.

Table 1. Parameters of $U_i(k) = -\exp(a_{0i} + a_{1i}k_i + a_{2i}k_i^2)$

i	a_0	a_1	a_2
u	−2.3524	0.10162	−0.09832
d	−2.3486	0.10281	−0.10003
s	−2.3109	0.12042	−0.11023

parameters for a given set, reported in this paper, with $\nu = 1/3$, $\Lambda = 100$ MeV, $\alpha = 0.2$ and a typical density $10\rho_0$ are given in Table 1.

Finite temperature $T(= 1/\beta)$ can be incorporated in the system through the Fermi function:

$$\mathrm{FM}(k, T) = \frac{1}{\exp[\beta(\epsilon - \epsilon_F)] + 1} \qquad (3)$$

with the flavour dependent single particle energy

$$\epsilon_i = \sqrt{k^2 + M_i(\rho)^2} + U_i(k). \qquad (4)$$

Now we evaluate

$$\frac{\gamma}{2\pi^2} \int_0^\infty \phi(\epsilon) k^2 \mathrm{FM}(k, T) \mathrm{d}k$$

For $\phi(\epsilon) = 1$ we get the number density and for $\phi(\epsilon) = \epsilon$, the energy density. $\gamma = 6$ is the spin-colour degeneracy.

Even at very high T which is around 70 MeV, the chemical potential is very large, of the order several hundred MeV. The entropy is calculated as follows:

$$s(T) = -\frac{3}{\pi^2} \int_0^\infty k^2 [\mathrm{FM}(k, T) \ln(\mathrm{FM}(k, T))$$
$$+ (1 - \mathrm{FM}(k, T)) \ln(1 - \mathrm{FM}(k, T))] \mathrm{d}k \qquad (5)$$

The pressure (P) is calculated from the free energy

$$f = \epsilon - Ts$$

as follows:

$$P = \sum_i \rho_i \frac{\partial f_i}{\partial \rho_i} - f_i \qquad (6)$$

We find $P = 0$ points only upto $T = 70$ MeV on plotting P as a function of ρ at different T (Fig. 2).
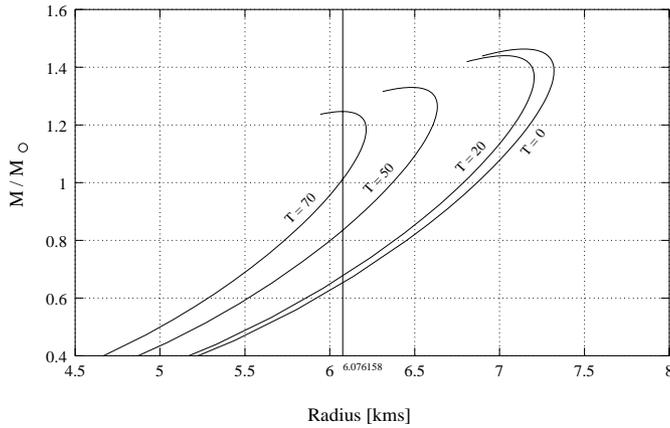
Fig. 3. MR curves for $T = 0$, 20, 50 & 70 MeV. Core densities for $T \neq 0$ are 16.5, 19.5 and 22 ρ_0. We have drawn a vertical line at $R_{\max}$ for $T = 70$ MeV to show that the mass of star increases as T increases, for fixed radius.

Fig. 4. The entropy per quark at different T(20, 50 & 70 MeV) as a function of the star radius.

3. Results and discussions

With this EOS at finite T, the TOV equation is solved to find out the MR relationship of the strange star at finite T. Here we find that the stellar mass and radius decrease with increasing T and the central energy density goes upto 22 times the normal saturation density of nuclear matter. No star can be formed beyond $T = 70$ MeV since the star surface must have pressure zero for SS. The MR curves for the stars at the three values of T are given in Fig. 3 showing that the radii and the masses decrease slightly with T. The conclusions of D98 and Li et al. (1999a & 1999b) remain unchanged so far as compactness of the stars is concerned.

We thank the anonymous referee for pointing out to us that the decrease in the maximum mass and the corresponding radius $R_{\max}$ of the star at finite T is somewhat surprising. We offer explanations of this fact as follows:

(1) We show in the Fig. 3 that the mass of the star with a given radius increases with T. This is clear from the vertical line which connects the maximum mass for $T = 70$ MeV with $R_{\max} \equiv 6.076$ km to the star mass for $T = 0$ for the same radius. This is in accord with the usual expectation that the star mass should increase with T.

(2) With increasing T, the size of any self sustaining system decreases due to the restriction placed on the energy balance by the increased thermal energy. The shrinking of a self-sustained system with increasing T is also seen for the Skyrmion (Dey and Eisenberg 1994). Interestingly the latter is also an example of the success of the large N_c phenomenon. Note that P in our system (Fig. 2), or the Skyrmion, is calculated self consistently whereas in the earlier literature on strange stars at finite T (Kettner et al. 1995) employing bag model, the variation of the bag pressure with T is neglected. It is pointed out in (Chmaj and Słomiński 1989) at low P the bag pressure dominates and as P grows the results are close to the free relativistic gas limit. With an unchanged bag parameter therefore bag model calculations find an almost unchanged mass and radius for the stars at finite T.
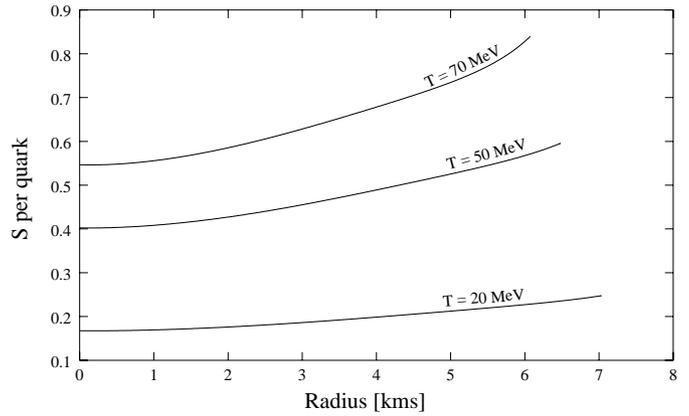
(3) As thermal energy increases the binding energy per baryon decreases. This supports the argument given in (1) and (2) above that as T decreases the masses corresponding to the same radius will decrease.

The entropy S increases with T and decreases with density. Comparable to our entropy is that calculated by Das, Tripathi and Cugnon (1986) (DTC) for interacting hadrons. The results of DTC checks with experiment. The minimum T considered by DTC is 10 MeV and S/A is a little less than 1 at a density 2.5 times ρ_0.

In Fig. 4 we have plotted S/A as a function of the star radius for $T = 20$, 50 and 70 MeV. It is interesting to see here that entropy is maximum at the surface showing that the surface is more disordered than the core. The extrapolated entropies from the plots of DTC at 5 ρ_0 agree with our S/A at the stellar surface for all the three cases.

Recently Glendenning (Glendenning 2000) has argued that the SAX could be explained as a neutron star rather than bare SS, not with any of the existing known EOS, but with one based only on well-accepted principles and having a core density about 26 ρ_0. Of course, such high density cores imply hybrid strange stars, subject to Glendenning's assumption that such stars can exist with matching EOS for two phases. There is the further constraint that if the most compact hybrid star has a given mass, all lighter stars must be larger. It was found in Li et al. (1999b) that the star 4U 1728−34 may have a mass less than that of SAX and yet have a radius less than $R_{\rm SAX}$. Another serious difference is that in our model the strange quark matter EOS derived, using the formalism of large N_c approximation, indeed shows a bound state in the sense of having minimum at about 4.8 ρ_0 whereas in Glendenning (2000) one of the assumptions is that strange matter has no bound state.

In summary, we find that beyond $T = 70$ MeV, the EOS has no zero pressure point. A self-bound star cannot exist in our model at higher T. The entropies at $T = 20$, 50 and 70 MeV (intermediate values are quite obvious) match onto hadronic entropies at corresponding T, suggesting the possibility of smooth phase transition between the hadronic and the quark states.

References

Bonasera A. 1999, Phys. Rev. C60, 212.

Cheng K. S., Dai Z. G., Wai D. M. & Lu T. 1998, Sci 280, 407.

Chmaj T. & Słomiński W. 1989, Phys. Rev. D40, 165.

Crater H. W., & van Alstine P. 1984, Phys. Rev. Lett. 53, 1527.

Das C., Tripathi R. K. & Cugnon J. 1986, Phys. Rev. Lett. 56, 1663.

Dey J., Dey M., & LeTourneux J. 1986, Phys. Rev. D34, 2104.

Dey J., Dey M., Mukhopadhyaya G., & Samanta B. C. 1991a, Can. J. Phys. 69, 749.

Dey J. & Eisenberg J. M. 1994, Phys. Lett. B334, 290.

Dey J., Tomio L., Dey M., & Frederico T. 1991b, Phys. Rev. C44, 2181.

Dey M., Bombaci I., Dey J., Ray S. & Samanta B.C. 1998, Phys. Lett. B438, 123; Addendum 1999, B447, 352; Erratum 1999, B467, 303; 1999, Indian J. Phys. 73B, 377.

Glendenning N. 2000, Phys. Rev. Lett. 85, 1150.

Kapoor R. C. & Shukre C. S. 2000, "Are radio pulsars strange stars?" astro-ph/0011386.

Kettner Ch., Weber F., Weigel M. K. & Glendenning N. K. 1995, Phys. Rev. D51, 1440.

Li X., Bombaci I., Dey M., Dey J. & van den Heuvel E. P. J. 1999a, Phys. Rev. Lett. 83, 3776.

Li X., Ray S., Dey J., Dey M. & Bombaci I. 1999b, ApJ 527, L51.

Ray K., Dey J. & Dey M. 2000a, Mod. Phys. Lett. A15, 683.

Ray S., Dey J. & Dey M. 2000b, Mod. Phys. Lett. A15, 1301.

Richardson J. L. 1979, Phys. Lett. B82, 272.

't Hooft G. 1974, Nucl. Phys. B72, 461; 1974, B75, 461.

Witten E. 1979, Nucl. Phys B160, 57.

Xu R. X., Qiao G. J. & Zhang B. 1999, ApJ 522, L109.