Astron. Astrophys. 317, 104–107 (1997)

ASTRONOMY AND ASTROPHYSICS

Research Note

Apsidal motion in the eclipsing binary OX Cassiopeiae

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Received 17 November 1995 / Accepted 14 April 1996

Abstract. We report several new reliable times of minimum of the eccentric eclipsing binary OX Cas (P = 2.48934 days, e = 0.04, V = 9.92 mag, B2V + B2V). The O-C diagram is analyzed using all precise timings found in the literature and improved values for the elements of the apsidal motion and the internal structure constant are computed. We have found a short period of apsidal motion U = 37.3 ± 0.7 yr and the internal structure constant $\bar{k}_{2,obs}$ = 0.0060.

Key words: stars: binaries: eclipsing - stars: individual: OX Cas - stars: fundamental parameters

1. Introduction

Eclipsing binaries are excellent laboratories for studying a wide variety of processes in stellar astrophysics. Their usefulness extends far beyond their textbook role in the determination of stellar masses and radii. The study of apsidal motion in detached eclipsing binary systems with eccentric orbit is known as an important source of information for the stellar internal structure as well as the possibility of verification of the general relativity. The suitable objects for this research were recently collected by Giménez (1994).

The detached eclipsing binary OX Cassiopeiae (BD +60° 0169 = BV 4 = GSC 4030 0784; α(2000) = 1 h 9 min 0 s, δ(2000) = +61° 28.2', V_{max} = 9.92 mag; Sp. B2V + B2V) is a relatively well investigated, rather bright early-type binary with known eccentric orbit (e = 0.04) and a period about 2.5 days. It was discovered to be a variable star photographically by Reim (1957), who obtained also the first photographic light-curve and determined an orbital period of 1.2 days. The next photographic observations were obtained by Geyer (1958) and Filin (1962). Frazier & Hall (1975) made a first photoelectric study of this system and found that the orbital period was actually twice the earlier period. The light-curve of Frazier & Hall was analyzed by Mardirossian et al. (1980) by means of WINK computer model (Wood 1972). They predicted an apsidal motion period shorter than 10 yr. Busch (1978) carried out a new analysis of the period including new photographic times of minimum from Sonneberg and Hartha Observatories. The only spectroscopic investigation found in the literature was announced by Schiller and Milone (1987), who obtained the spectroscopic orbit, from which they determined a preliminary mass ratio of 0.88. The apsidal motion in OX Cas was also detected by Khaliullin et al. (1987) using the two photoelectric ligt-curves obtained in two different epochs. Their resulting rate of the apsidal motion 9.1°/yr leads to the value of interal structure constatnt k_2, which is smaller than the theoretical prediction. The possible membership of OX Cas in the open cluster NGC 381 was excluded by Crinklaw & Talbert (1988). Crinklaw & Etzel (1989) made a new $uvby$, Hβ photometry of the system and derived photometric elements and absolute parameters of the orbit. The combination of the photometric elements and the preliminary spectroscopic orbit of Schiller and Milone (1987) provided determination of the masses and radii for the two components of OX Cas. These values (m_1 = 7.2 ± 0.5 M$\odot$, m_2 = 6.3 ± 0.5 M$\odot$, R_1 = 4.69 ± 0.3 R$\odot$, R_2 = 4.22 ± 0.3 R$\odot$) are consistent with early B type stars within the main-sequence band.

This paper is a continuation of a series of the author's publications on the apsidal motion analysis in eccentric eclipsing binaries (Wolf & Diethelm 1993; Wolf 1994, 1995; Wolf & Šarounová 1995).

2. Observations of minimum light

To enlarge the collection of times of minima, all reliable times of minimum light were gathered in the literature and new observations of the system were carried out. Our new CCD observations were performed at two observatories, the Ondřejov Observatory, Czech Republic and R. Szafraniec Observatory, Metzerlen, Switzerland.

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Table 1. New times of minimum of OX Cas

JD Hel.- 2400000	Error (10^{-4}d)	Min. type	N	Observatory
49906.4416	12	II	75	Ondřejov
49947.4934	7	I	73	Ondřejov
49987.318[a]	50	I	20	Metzerlen
50013.4826	16	II	24	Metzerlen

[a] published also in BBSAG Bull. No. 110

At the Ondřejov a 65cm reflecting telescope with a CCD-camera SBIG ST-6 in the primary focus was used in July and August 1995. The measurements were done using the standard Johnson R and V filters with 60 s exposure time. The later observations were perfomed with a 35cm Cassegrain telescope with the same type of CCD-camera, without filter and 10 s integration time. The stars GSC 4030 0550 (V = 9.9 mag) and GSC 4030 0732 = SAO 11 600 (V = 9.3 mag) on the same frame as OX Cas served as a comparison and check stars. During the observations no variations in the brightness of these stars exceeding the possible error of measurements (σ = 0.01 mag) were detected. The data were reduced using software developed at Ondřejov Observatory by P. Pravec and M. Velen. The new times of primary and secondary minimum and their error were determined using the Kwee-van Woerden (1956) method. These times of minimum are presented in Table 1. In this table, N stands for the number of observations used in the calculations of minimum time.

3. Apsidal motion analysis

The apsidal motion of OX Cas was studied by means of an O-C diagram analysis. We have collected the all reliable times of minimum light gathered from the literature. All times presented in Table 2 were taken into consideration. The photoelectric measurements obtained by Khaliullin et al. (1987) were recalculated and the new times of minimum are also given in this table. The epochs were calculated using the linear light elements given by Crinklaw & Etzel (1989):

Pri. Min. = HJD 24 46733.7912 + 2.489329 · E.

We employed the following data reduction procedure. All photoelectric times of minimum were used in our computation, with a weight of 10. The photoelectric time obtained by Busch (1994) as well as some our less precise measurements were weighted with a weight of 5. The earlier visual and photographic times of minimum were not used because the large scatter of these data. The measurement of Diethelm (1988) at JD 24 47368 were excluded. A total 21 times of minimum light were used in our analysis, with 6 secondary eclipses among them.

A suitable numerical method for the apsidal motion analysis was described by Giménez & García-Pelayo (1983). This method is a weighted least squares iterative procedure, including terms in the eccentricity up to the fifth order. Due to the smaller value of eccentricity we used only the terms up to the third order in our calculation. The derived relation for the prediction of the times of minimum T_j, used for minimization by the least-squares method, is

Table 2. Precise times of minimum of OX Cas used for apsidal motion analysis.

JD Hel.- 24 00000	Epoch	Weight	Reference
41244.7396	-2205.0	10	Frazier & Hall (1975)
41269.6322	-2195.0	10	Frazier & Hall (1975)
41355.5483	-2160.5	10	Frazier & Hall (1975)
46074.1168[a]	-265.0	10	Khaliullin et al. (1987)
46075.2946[a]	-264.5	10	Khaliullin et al. (1987)
46299.344	-174.5	5	Diethelm (1985)
46733.7908	0.0	10	Crinklaw & Etzel (1989)
47064.8712	133.0	10	Crinklaw & Etzel (1989)
47119.6357	155.0	10	Crinklaw & Etzel (1989)
47871.4160[b]	457.0	10	Agerer (1990)
48085.4915	543.0	10	Agerer (1991)
48187.5544[b]	584.0	10	Agerer (1991)
48232.3646[b]	602.0	10	Agerer (1991)
48588.328	745.0	5	Diethelm (1992)
49005.2820	912.5	10	Diethelm (1993)
49290.324	1027.0	5	Busch (1994)
49636.3297[b]	1166.0	10	Agerer & Hübscher (1995)
49906.4416	1274.5	10	this paper
49947.4934	1291.0	10	this paper
49987.318	1307.0	5	this paper
50013.4826	1317.5	10	this paper

[a] recalculated original data
[b] mean value of the B and V measurements

$$
\begin{aligned}
T_j &= T_0 + P_s E + (j-1)\frac{P_a}{2} + (2j-3)\frac{eP_a}{2\pi}\Big[2+\cot^2 i \\
&- \frac{e^2\cot^2 i}{4}\{3-(2+\csc^2 i)\csc^2 i\}\Big]\cos\omega \\
&+ \frac{e^2 P_a}{4\pi}\left[\frac{3}{2}+(2+\csc^2 i)\cot^2 i\right]\sin 2\omega \\
&- (2j-3)\frac{e^3 P_a}{8\pi}\Big[\frac{4}{3}+(2+\csc^2 i)\cot^2 i\csc^2 i \\
&+ 3\cot^2 i\Big]\cos 3\omega, \qquad (1)
\end{aligned}
$$

where $\omega = \dot\omega E + \omega_0$ and j = 1 and 2 are taken for primary and secondary minima, respectively. In this equation P_a denotes the anomalistic period, P_s the sidereal period, e represents the eccentricity, i the orbital inclination and $\dot\omega$ is the rate of periastron advance in degrees per cycle. The zero epoch is given by T_0 and the corresponding position of the periastron is represented by ω_0. There are five independent variables $(T_0, P_s, e, \dot\omega, \omega_0)$ in this procedure. The relation between the two different periods, P_s and P_a, is given by

$$P_s = P_a\,(1-\dot\omega/360°)$$

and the period of apsidal motion is

$$U = 360° P_a/\dot\omega.$$

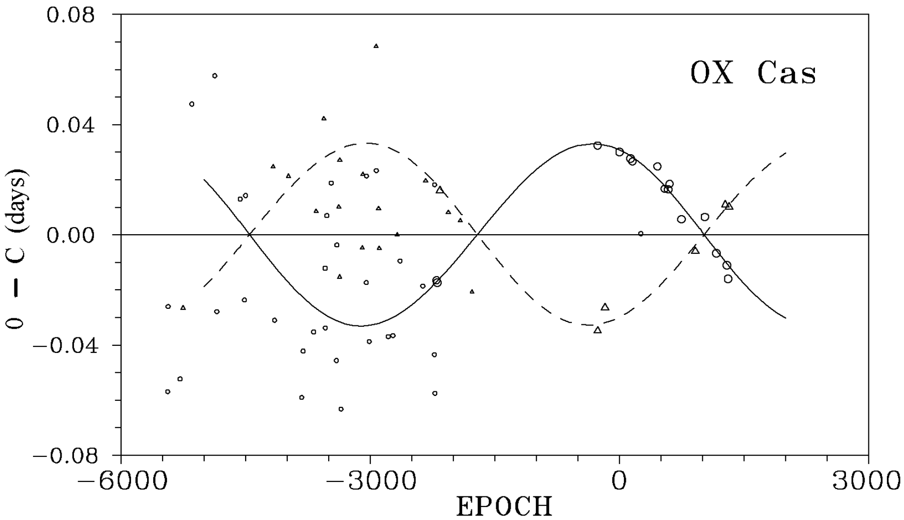

Fig. 1. $O-C$ residuals for the times of minimum of OX Cas with respect to the linear part of the apsidal motion equation. The continuous and dashed curves represent predictions for primary and secondary eclipses, respectively. The individual primary and secondary minima are denoted by circles and triangles, respectively. Larger symbols correspond to the photoelectric measurements which were taken into calculations.

Table 3. Apsidal motion elements of OX Cas

T_0	=	2446733.7607 ± 0.0008
P_s	=	2.4893445 ± 0.0000002 d
P_a	=	2.4897997 ± 0.0000002 d
e	=	0.0415 ± 0.0005
$\dot\omega$	=	$(0.06580 \pm 0.0012)^\circ$ cycle^{-1}
	=	$(9.65 \pm 0.08)^\circ$ yr^{-1}
ω_0	=	$202.8 \pm 0.2^\circ$
U	=	$5471\ P_a$ = 37.3 ± 0.7 yr

Adopting the orbital inclination, derived from the light curve solution by Crinklaw & Etzel (1989), $i = 84.15^\circ$, the mean apsidal motion elements given in Table 3 can be computed. Their standard errors are indicated. The relatively high internal accuracy of these results, however, is decreased in next calculations by poorly determined fractional radii of the components.

The $O-C$ residuals for all times of minimum with respect to the linear part of the apsidal motion equation are shown in Fig. 1. The non-linear predictions, corresponding to the fitted parameters, are plotted as continuous and dashed curves for primary and secondary eclipses, respectively. The original photographic measurements of Busch (1978) are also plotted for the O-C difference within ± 0.08 days. It is evident, that determination of the apsidal motion elements using only the photographic and visual data set was impossible.

4. Internal structure constant

The empirical method to study the internal structure of stars is given by the analysis of apsidal motion in an eccentric binary system. The period of rotation of the periastron in eccentric eclipsing binaries does not allow us to derive the individual internal structure constant (ISC) of the component stars. The observational average value $\bar{k}_{2,obs}$ of the ISC is given by the known formula:

$$\bar{k}_{2,obs} = \frac{1}{c_{21} + c_{22}} \frac{P_a}{U} = \frac{1}{c_{21} + c_{22}} \frac{\dot\omega}{360^\circ}, \tag{2}$$

where c_{21} and c_{22} are functions of the orbital eccentricity, fractional radii, masses of the components and the ratio between actual rotational velocity of stars and the Keplerian velocity, which corresponds to synchronization with the average orbital velocity. In accordance with Kopal (1978) we assumed, that the rotation of the component stars are synchronized with the orbital velocity at periastron, where the tidal forces are at maximum, so that

$$\frac{\omega_i}{\omega_K} = \sqrt{\frac{1+e}{(1-e)^3}}. \tag{3}$$

Comparing the photometric elements and absolute parameters and their errors derived independently (Mardirossian et al. 1980, Khaliullin et al. 1987, Crinklaw & Etzel 1989), we adopted the following values for the fractional radii and masses of the components computed by the last mentioned authors:

$$m_1 = 7.2\,\mathrm{M}\odot,\ m_2 = 6.3\,\mathrm{M}\odot,$$

$$r_1 = 0.2560,\ r_2 = 0.2304.$$

Using values of the eccentricity and sidereal period given in this paper, we could substract from $\dot\omega$ a relativistic correction $\dot\omega_{rel}$ (Giménez 1985)

$$\dot\omega_{rel} = 5.45 \times 10^{-4} \frac{1}{1-e^2} \left(\frac{m_1 + m_2}{P_s}\right)^{2/3}, \tag{4}$$

which is $\dot\omega_{rel} = 0.00169$ degrees per cycle or 2.6 percent of the total apsidal motion rate. Finally, the mean internal structure constant has value of $\bar{k}_{2,obs} = 0.00595 \pm 0.00025$.
As the system of OX Cas consists of two almost identical components, the value $\bar{k}_{2,obs}$ of the internal structure constant is close to both values k_{21} and k_{22}.

5. Conclusions

We derived new precise apsidal motion elements and the value of the internal structure constant by means of O-C diagram analysis using the current data set. Our resulting orbital eccentricity and period of apsidal motion is slightly larger than the previous results obtained by Khaliullin et al. (1987). The value of internal structure constant is in a good agreement with the theoretical value k_2 derived by Jeffery (1984), which lies between 0.0055 and 0.0060. From the current O-C diagram no firm conclusion concerning the presence of a third body, as was annonced by Crinklaw & Etzel (1989), can be done. There is no significant variations on the O-C diagram which could be caused by a light-time effect. Approximately 30 percent of the apsidal motion period is well-covered by precise photoelectric observations, so that more high-accuracy timings of this eclipsing system are necessary in the future to enlarge the time span for better analysis of the apsidal motion and to confirm the parameters given in this paper. Nevertheless, we can conclude that OX Cas belongs to the early type eclipsing binaries with the shortest known period of the apsidal motion, like in U Oph (21.3 yr, Kämper 1986), GL Car (25.2 yr, Giménez & Clausen 1986) or DR Vul (36.6 yr, Wolf & Diethelm, 1993).

We plan to continue our photometric study of OX Cas and will attempt to obtain more precise photoelectric timings of primary and secondary eclipses in the next future.

Acknowledgements. This work has been supported in part by the Grant Agency of the Czech Republic, grant No. 205-95-1498 and by the ESO C&EE Programme, grant No. A-02-069. MW would like to thank the staff of the Astronomical Institute, University Basel, for the use of their facilities and for their hospitality during the stay in October 1995. This research has made use of the SIMBAD database, operated at CDS, Strasbourg, France.

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