

Evolutionary scenario for metal-poor pulsating stars

I. Type II Cepheids

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Abstract. Pulsating convective models and evolutionary tracks are used to constrain mass, luminosity, and effective temperature of metal-poor (Z in the range of 0.0001 to 0.001) low-mass ($0.52 M_{\odot}$ – $0.80 M_{\odot}$) fundamental pulsators connected with the horizontal branch and asymptotic branch phase.

On this basis the fundamental period $P(F)$ and the absolute blue magnitude B of the pulsators are calculated. We show that, by scaling the blue magnitude of the pulsator to the value $B(3.83)$ of the zero age horizontal branch at the mean effective temperature of the RR Lyrae strip ($\log T_e = 3.83$), the distribution of theoretical fundamental pulsators in the $B(3.83) - B$ vs $P(F)$ plane is independent of metallicity. The limits of this distribution, which depend on the adopted edges of the instability strip, conform well with data of known type II Cepheids and show that these variables are fundamental pulsators with mass in the range of $0.59 M_{\odot}$ to $0.52 M_{\odot}$.

Moreover, since for fundamental pulsating models there is a linear correlation of the blue magnitude with period and blue amplitude $A(B)$, we are able to predict that the period-luminosity-blue amplitude relation for type II Cepheids with $P \leq 15$ days is $B(3.83) - B = -0.12 + 1.39 \log P + 0.57 A(B)$.

The comparison with the theoretical evolutionary tracks yields that the mass of type II Cepheids decreases with increasing the period and that the brightest variables with $P \geq 10$ days should likely belong to the very rapid phases of the $0.52 M_{\odot}$ and $0.53 M_{\odot}$ evolution.

The agreement between observations and calculated expectations turns out to be almost satisfactory, suggesting that the pulsational and evolutionary scenario is capable to match the properties of metal-poor pulsating stars.

Key words: stars: Population II – stars: evolution – stars: oscillations – II Cepheids

1. Introduction

Pulsating stars are quite a common phenomenon in old stellar populations. The majority of them belongs to the class of RR Lyrae stars, named after a field variable star discovered in 1899 by Fleming. According to the morphology of the light curve, RR Lyrae stars are classified into two groups: type *ab* and type *c*. The former show asymmetric light curves, period P in the range of ~ 0.4 to ~ 0.8 days, and large blue amplitude $A(B)$ which increases for shorter periods; the latter show symmetric light curves, $P \sim 0.2$ – 0.4 days and almost constant small amplitudes. Such a distinctive behavior in the P - $A(B)$ plane (the well-known “Bailey diagram”) allows a firm separation between RR*ab* and RR*c* stars, first correctly interpreted by Schwarzschild (1941) as fundamental (F) and first overtone (H) pulsators respectively.

When they are members of globular clusters, RR Lyrae stars are located in a well-defined zone of the horizontal branch (HB), at $(B - V)$ colors in the range of ~ 0.2 to ~ 0.4 and at roughly constant visual magnitude. As for their intrinsic luminosity, it is generally assumed that (at fixed helium content) it depends on the metal abundance Z . For this reason, even though the value of the metallicity coefficient is still under debate (see Sandage & Cacciari 1990; Carney et al. 1992; Castellani & De Santis 1994) and it is now clear that age and evolutionary status of the variable also may play a not-negligible role (see e.g. Lee et al. 1990; Caputo et al. 1993; Caputo & Degl’Innocenti 1995), these variables are the main distance estimator for old stellar populations.

Together with RR Lyrae, other variables are observed in globular clusters and other metal-poor stellar systems (the reader should refer to the reviews by Harris 1985; Wallerstein 1990; Whitelock et al. 1991 for discussion of their pulsational characteristics). In the present investigation we focus our attention on radial pulsators generally referred to as type II Cepheids (CII). Several investigations have been recently devoted to these variables (e.g. Sandage, Diethelm & Tammann 1994, hereafter SDT;

Nemec, Nemec & Lutz 1994, hereafter NNL) and here we only need to recall their main distinguishing features.

Type II Cepheids are found mainly in globular clusters with blue HB and moderate to extreme metal deficiencies. They are intrinsically brighter than RR Lyrae stars, with period in the range of ~ 1 day to ~ 26 days and blue amplitude in the range of ~ 0.5 to ~ 1.8 . In the $P - A(B)$ plane no similarity with the Bailey diagram can be found and consequently no safe separation between fundamental and first overtone pulsators (if any) can be made. When their apparent mean magnitude is scaled to the RR Lyrae level, CII arrange along a strip, with the less luminous stars at lower periods and the more luminous stars (up to ~ 3 mag above the RR Lyrae level) at larger periods. As for their evolutionary status, it is widely accepted that CII are produced by post-HB evolving stars which, starting from the higher temperatures of the zero age horizontal branch (ZAHB), move towards the asymptotic branch (AGB) or leave the AGB on rapid blueward loops (see Gingold 1985 for discussion and references).

After this brief outline it is evident that CII appear to obey a distinctive period-luminosity ($P - L$) relation which should be useful in establishing the distance to the parent stellar cluster. Moreover, since they are significantly brighter than RR Lyraes, they could be detectable in almost all the galaxies of the Local Group.

There are several ways of approaching the $P - L$ relation for these variables. In general, their absolute magnitude is differentially evaluated by setting the RR Lyrae stars at a given luminosity (e.g., see NNL). One can also adopt a fixed mass and then predict the period-luminosity relation as given from pulsation theory (e.g., see McNamara 1995), even if the most reliable approach should include the mass range as suggested from stellar evolution theory. The literature offers only sporadic examples of the last method (e.g., see SDT). The reason is that, while the mass and luminosity range of RR Lyraes has been matter of accurate computations since the 70's, only in very recent times a homogeneous set of pulsational and evolutionary models for lower and larger masses has become available.

The theoretical background is shortly discussed in Sect. 2 and the predicted period-luminosity-amplitude relation is given in Sect. 3. The comparison with observed data is presented in Sect. 4, while in Sect. 5 the mass of individual variables are derived. The main results are summarized in Sect. 6.

2. Theoretical background

2.1. The pulsational scenario

The pulsational frame is based on the Bono & Stellingwerf (1994) investigation and we refer to it for all questions concerning numerical and physical assumptions.

That paper contains an extensive grid of non-linear, non-local and time-dependent convective pulsating models, both for F- and H-mode, with fixed helium abundance ($Y = 0.30$) and mass ($M = 0.65$) and different luminosities ($1.51 \leq \log L \leq 1.81$) and effective temperatures ($5700 \leq Te \leq 8000$). The

survey yields useful information on the location and internal structure of the instability strip [i.e. on the blue and red edges for fundamental and first overtone mode), as well as on the correlation of the light curve bolometric amplitude with the luminosity and temperature of the model.

The dependence of these parameters on mass and helium abundance has been matter of further analyses (Bono et al. 1995; Bono 1996; Marconi 1994,1996; Staiano 1994) and the final result is a reference set of pulsating models with a wide range of mass ($0.58-0.75 M_{\odot}$), luminosity ($1.51 \leq \log L \leq 2.2$), and helium abundance ($0.20 \leq Y \leq 0.30$).

From fundamental models with $Y = 0.24$, $M \leq 0.75$, and $\log L \geq 1.7$ we derive that the pulsation equation is

$$\log P(F) = 11.627 + 0.824 \log L - 0.582 \log M - 3.505 \log Te \quad (1)$$

and that the red edge (RE) of the instability strip, namely the lowest effective temperature for pulsation, depends linearly on luminosity and mass as

$$\log Te(RE) = 3.949 - 0.093 \log L + 0.085 \log M. \quad (2)$$

As for the blue fundamental edge (BE), the available models with $\log L \leq 1.7$ suggest $\Delta \log Te = BE - RE \sim 0.075$, while from those with larger luminosity one has

$$\Delta \log Te = 0.075 + 0.057(\log L - 1.7). \quad (3)$$

Concerning the occurrence of H-pulsators, the new calculations (see also Bono, Castellani & Stellingwerf 1995) confirm an early suggestion (Tuggle & Iben 1972) that above a luminosity level, which depends on the helium content and mass of the model, only the fundamental mode is stable.

With $Y=0.24$ such a limit can be approximated as

$$\log L = 2.48 + 2.02 \log M. \quad (4)$$

Moreover, stellar atmosphere models (Kurucz 1992) are used to get the static blue magnitude B and the amplitude $A(B)$ of the light curve for both F- and H-pulsators.

The results of particular relevance to the present study are:

1. At fixed mass and luminosity the blue amplitude of F-pulsators regularly increases from the red edge (where $A(B) \sim 0$) towards the blue edge (where $A(B) \sim 2.2$) for fundamental pulsation. On the contrary, the blue amplitude of H-pulsators reaches a maximum near the centre of the first overtone instability strip (for details, see Bono, Caputo & Stellingwerf 1995, hereafter BCS).

2. At fixed mass, the blue amplitude of F-pulsators at constant period increases from lower to higher luminosities. At fixed luminosity, the blue amplitude of F-pulsators at constant period increases from larger to smaller mass. This is shown in Fig. 1 for F-models with $M = 0.65$, $\log L = 1.72, 1.81, 1.91$ (solid line) and with $M = 0.75$, $\log L = 1.72$ (dashed line).

3. The blue magnitude B of the pulsators depends on the physical parameters of the model (mass, luminosity, effective temperature) and on the adopted metallicity, as well. However,

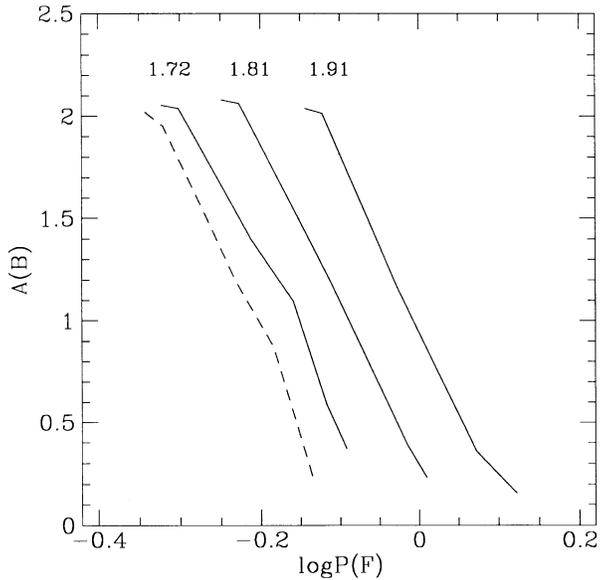


Fig. 1. Blue amplitude versus period for fundamental pulsators with $Y = 0.24$, $M = 0.65$, $\log L = 1.72, 1.81, 1.91$ (solid line) and $M = 0.75$, $\log L = 1.72$ (dashed line)

with Z in the range of 0.0001 to 0.001, the blue magnitude of F-pulsators with a given mass and metallicity is linearly correlated with period and blue amplitude.

In particular, with $M = 0.65$ we derive

$$dB = -2.87d \log P(F) - 0.57dA(B). \quad (5)$$

2.2. The evolutionary scenario

As a first point, let us remark that our chief interest are metal-poor low-mass pulsators connected to the HB and AGB evolutionary phase. For this reason the adopted evolutionary framework takes into account computations with Z in the range of 0.0001 to 0.001.

The models used in the present analysis are taken from the work by Castellani, Chieffi & Pulone (1991, hereafter CCP) and by Dorman, Rood & O'Connell (1993, hereafter DRO). Specifically, we use the CCP canonical (solar-scaled chemical compositions) models with $Z = 0.0001$ and 0.0004, and the DRO oxygen-enhanced models with $[\text{Fe}/\text{H}] = -2.26$, $[\text{O}/\text{Fe}] = 0.50$ ($Z \sim 0.0002$) and $[\text{Fe}/\text{H}] = -1.48$, $[\text{O}/\text{Fe}] = 0.60$ ($Z \sim 0.001$).

The computations adopt an original helium abundance $Y \sim 0.23$ and refer to ages of ~ 15 Gyr. We note that all the models evolving within the instability strip are taken into account, irrespectively of the evolutionary timescale.

3. Predicted P-L-A(B) relation

The procedure for predicting the observational parameters of F-pulsators is in principle quite simple. By adopting that the edges for fundamental pulsation follow Eqs.(2) and (3), we derive the intersection of each evolutionary track with the appro-

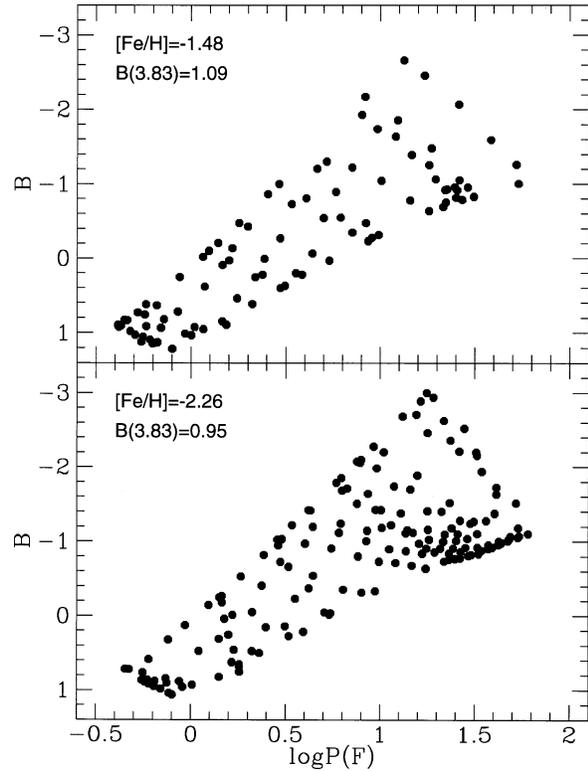


Fig. 2. The predicted distribution of low-mass F-pulsators in the B vs $P(F)$ plane, as derived from DRO models

priate instability strip. This provides the range of luminosity and effective temperature of F-pulsators with a given mass and then, through Eq. (1), the predicted range of period.

Furthermore, through stellar atmosphere models (Kurucz 1992), the blue absolute magnitude B of each model within the instability strip is derived, yielding the predicted distribution of F-pulsators in the B vs $P(F)$ plane. As shown in Fig. 2, where the results from DRO models are presented, there is a slight dependence on the metallicity. However, if the magnitude of pulsators is scaled to the magnitude $B(3.83)$ of the ZAHB model at the mean effective temperature of the RR Lyrae strip ($\log T_{\text{eff}} = 3.83$, which corresponds to $\log P(F) \sim -0.35$), then the predicted distribution of F-pulsators in the $B(3.83) - B$ vs $P(F)$ plane turns out to be independent of metallicity and of the ratio between oxygen and heavy elements, as well. This is shown in Fig. 3a which refers to the DRO models with $[\text{Fe}/\text{H}] = -2.26$, $[\text{O}/\text{Fe}] = 0.50$ (dots) and $[\text{Fe}/\text{H}] = -1.48$, $[\text{O}/\text{Fe}] = 0.60$ (triangles), and in Fig. 3b, which refers to the CCP models with $Z=0.0001$ (dots) and $Z = 0.0004$ (triangles).

The upper and lower envelope of the theoretical distribution, which are given from the adopted edges of the instability strip, are drawn with solid lines. The equation of the upper envelope is

$$B(3.83) - B = +1.18 + 2.30 \log P(F), \quad (6)$$

while for the lower envelope we obtain

$$B(3.83) - B = -0.12 + 1.39 \log P(F), \quad (7)$$

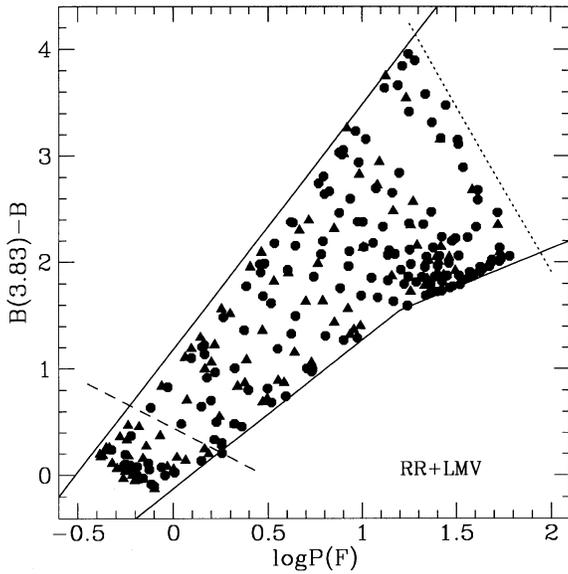


Fig. 3a. The calculated difference in blue magnitude between the ZAHB model at $\log Te = 3.83$ ($\log P(F) \sim -0.34$) and low-mass fundamental pulsators against the fundamental period. Dots and triangles refer to DRO models with $[\text{Fe}/\text{H}] = -2.26$, $[\text{O}/\text{Fe}] = 0.50$ and $[\text{Fe}/\text{H}] = -1.48$, $[\text{O}/\text{Fe}] = 0.60$, respectively. Solid lines depict the envelopes of the distribution, as given from the intersection of the evolutionary tracks with the edges of the fundamental instability strip. The dashed line is the limit for the occurrence of first overtone pulsators, while the dotted line is the limit as given from the $0.52 M_{\odot}$ evolution (see text)

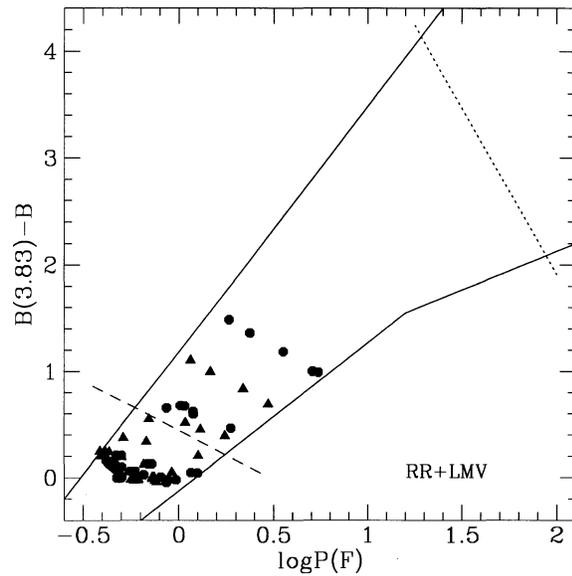


Fig. 3b. As in Fig. 3a, but for CCP models with $Z=0.0001$ (dots) and $Z = 0.0004$ (triangles)

with $P(F) \leq 15$ days, and

$$B(3.83) - B = +0.68 + 0.72 \log P(F), \quad (8)$$

for longer period.

Moreover, through Eq. (4) we derive that metal-poor stars with $M \leq 0.59$ pulsate only in the fundamental mode. For these only-fundamental low-mass variables (hereafter LMV) the lower limit for the period (dashed line) is given by the equation

$$\log P(F) = +0.47 - 1.08[B(3.83) - B]. \quad (9)$$

Finally, since the evolutionary track with mass smaller than $0.52 M_{\odot}$ fails to feed the instability strip (“AGB – manque” stars in DRO), we derive that the upper limit for the period of LMV (dotted line) is given by the equation

$$\log P(F) = 2.61 - 0.32[B(3.83) - B]. \quad (10)$$

4. Fitting observations

Figure 4 shows the comparison between the theoretical limits presented in Figs. 3a–3b and data for known type II Cepheids. The measured mean magnitude $-\langle B \rangle$ of the variable (data from NNL) is scaled to the observed lower envelope $-B(RR)le$ of RR Lyrae stars, read at $\log P = -0.34$. The triangles refer

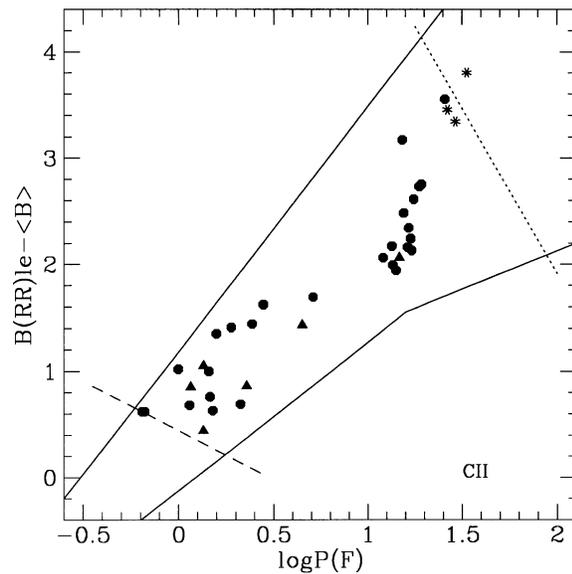


Fig. 4. Observed difference in blue magnitude between the RR Lyrae lower envelope, read at $\log P(F) = -0.34$, and known type II Cepheids in globular clusters (dots) and ω Cen (triangles) in comparison with the predicted limits of Fig. 2. The asterisks refer to RV Tau type stars

to stars in ω Cen, while the asterisks refer to variables with RV Tau characteristics (see NNL).

Bearing in mind that the envelope lines drawn in the figure represent the limits for all the possible LMV, the predictions conform with observed data, with the following points worth of notice.

1) Almost all the data are at the right side of the predicted limit for H-pulsators (dashed line). This confirms that likely none of known CII is a first overtone pulsator (see also McNamara 1995).

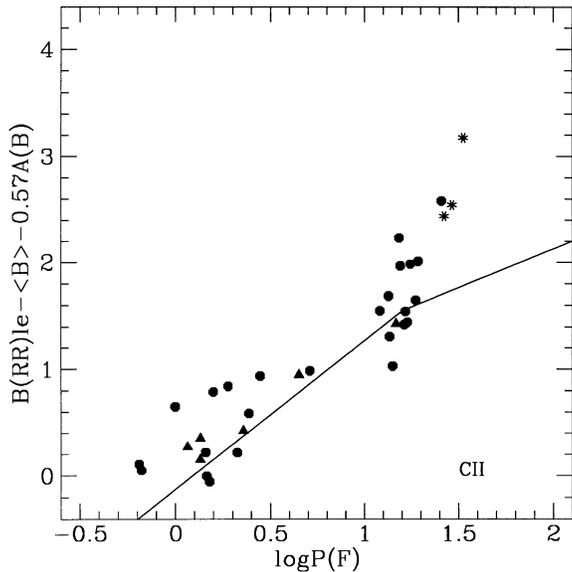


Fig. 5. As in Fig. 4, but for amplitude corrected magnitude. The solid line is the lower envelope of Figs. 3a–3b

2) All the known CII are well at the left side of the “structural” limit of $0.52 M_{\odot}$ (dotted line), except the RV Tau type stars and V42 in M5.

3) By excluding the RV Tau type variables and the deviant stars V154 in M3 and V42 in M5, the equation of the ridge line through the data with $P \leq 15$ days (not drawn)

$$B(RR)le - \langle B \rangle = 0.67 + 1.37 \log P \quad (11)$$

has a slope in agreement with the predicted lower envelope, but a zero point brighter by ~ 0.8 mag. Since the observed amplitude are generally larger than ~ 0.80 , whereas the theoretical pulsators at the lower envelope should have $A(B) \sim 0$, this is not surprising at the light of Eq. (5).

4) By assuming that Eq. (5) holds for all the masses, we derive that the CII $P \leq 15$ days (i.e. excluding the RV Tau type stars, V42 in M5, and likely V154 in M3) turn out to be fitted with the predicted lower envelope. This is shown in Fig. 5 where the observed difference in blue magnitude is corrected for the blue amplitude. Considering that the observed blue magnitudes and amplitudes are subject to a mean error of ± 0.05 mag and ± 0.10 mag, respectively, and that our calculated expectations adopt the static magnitude whereas the measurements refer to the mean magnitude, the comparison between predictions and observed data suggests that the current evolutionary and pulsational scenario can explain not only the properties of RR Lyrae stars (see Bonom et al. 1994), but also type II Cepheids, at least in a general sense.

5. Masses of type II Cepheids

Figures 6 to 8 show the known CII in individual globular clusters in comparison with DRO evolutionary tracks for the various masses. We note that stars with $M \leq 0.59$ traverse the instability

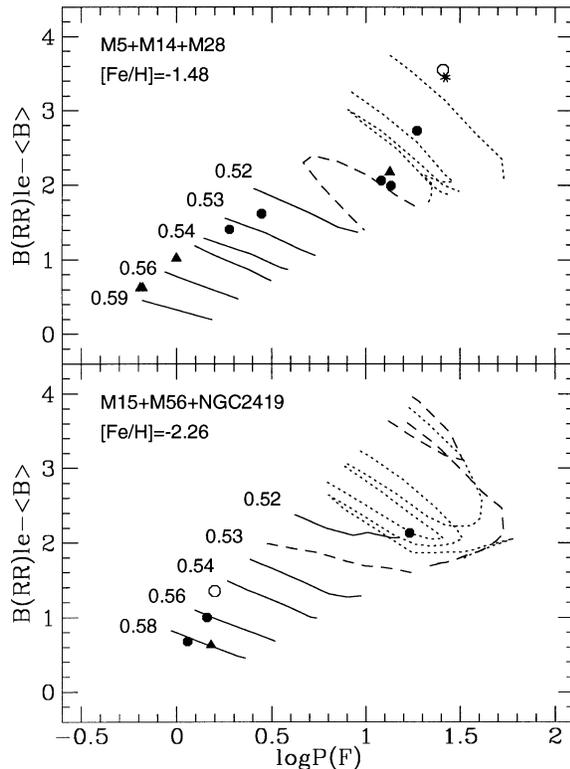


Fig. 6. (*lower panel*)- Type II Cepheids in M15 (dots), M56 (triangle), and NGC 2419 (circle) in comparison with the DRO evolutionary tracks with $[\text{Fe}/\text{H}] = -2.26$ and $[\text{O}/\text{Fe}] = 0.50$. (*upper panel*)- Type II Cepheids in M14 (dots), M28 (triangles) and M5 (circle) in comparison with the DRO evolutionary tracks with $[\text{Fe}/\text{H}] = -1.48$ and $[\text{O}/\text{Fe}] = 0.60$. The asterisk refers to the RV Tau star in M5. Dotted and dashed lines refer to the faster pulsating phases of the $0.52 M_{\odot}$ and $0.53 M_{\odot}$ evolution, respectively

strip on timescales of ~ 1 – 2 million years. However, while for the larger masses we have only one crossing, models with $M = 0.52$ and 0.53 show that the instability strip may be crossed two or three times, as a consequence of thermal pulses (see DRO). In this case we have a “slow” crossing, typically lasting ~ 1.5 million years (solid line), and a series of “fast” ones lasting ~ 0.7 million years (dotted line with $M = 0.52$ and dashed line with $M = 0.53$).

For M15, M56, and NGC 2419 ($[\text{Fe}/\text{H}] \sim -2.1$) we show the DRO models with $[\text{Fe}/\text{H}] = -2.26$ and $[\text{O}/\text{Fe}] = 0.50$ ($Z \sim 0.0002$), while for M5, M14, and M28 ($[\text{Fe}/\text{H}] \sim -1.4$) those with $[\text{Fe}/\text{H}] = -1.48$ and $[\text{O}/\text{Fe}] = 0.60$ ($Z \sim 0.001$). For the remaining clusters ($[\text{Fe}/\text{H}] \sim -1.7$) the comparison refers to both the sets of evolutionary models. The figures are self-explanatory and show that the mass of type II Cepheids varies from $\sim 0.59 M_{\odot}$ to $\sim 0.52 M_{\odot}$, decreasing with increasing the period. This seems to disagree with the hypothesis that variables with $P \geq 10$ days are more massive than the shorter period ones (Mc Namara 1995).

Looking at the figures, one may notice that the number of variables on the brighter, faster phases of the ~ 0.52 – $0.53 M_{\odot}$ evolution increases with increasing the adopted metallicity. By

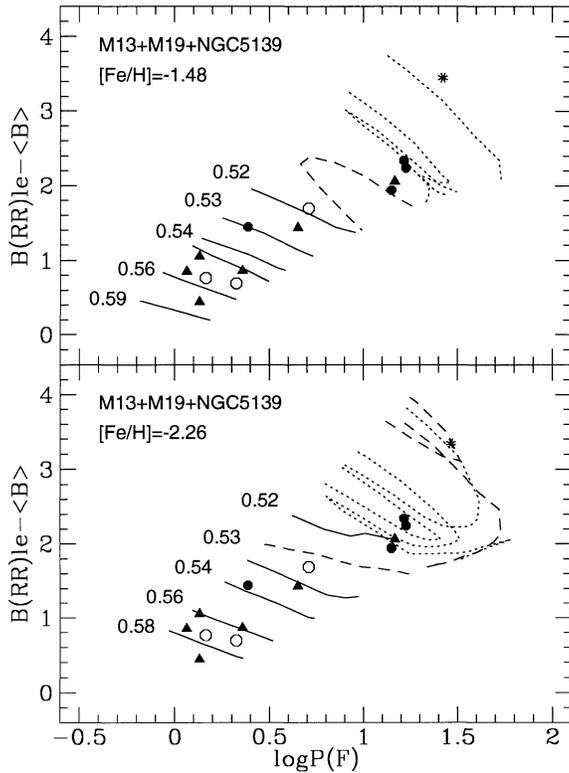


Fig. 7. Type II Cepheids in M19 (dots), M13 (circles), and ω Cen (triangles) in comparison with the DRO evolutionary tracks with $[\text{Fe}/\text{H}] = -2.26$ and $[\text{Fe}/\text{H}] = -1.48$. The asterisk refers to the RV Tau star in ω Cen. Lines as in Fig. 6

excluding the three RV Tau type stars, at $[\text{Fe}/\text{H}] = -2.26$ and $[\text{O}/\text{Fe}] = 0.50$ ($Z \sim 0.0002$) we have 6 “fast” and 13 “slow” variables with $M \sim 0.52\text{--}0.53$, whereas at $[\text{Fe}/\text{H}] = -1.48$ and $[\text{O}/\text{Fe}] = 0.60$ ($Z \sim 0.001$) the relative numbers are 14 and 5. On the other hand, the ratio of the timescales gives the probability of finding stars on the fast and slow crossings ($p \sim 1/3$ and $q \sim 2/3$, respectively) and one may easily evaluate the expected distribution of N stars along the two phases. With $N = 19$ we derive that the average number of “fast” variables is 6, with a standard deviation $\sigma = 2$. Since the overall metallicity of our sample of globular clusters is likely lower than $Z \sim 0.001$, we could conclude that the evolutionary timescales are not discordant with the observed relative numbers of the variables.

6. Conclusion

Convective pulsating models and evolutionary computations of metal-poor ($Z \leq 0.001$) low-mass ($M \leq 0.59$) stars in HB and AGB phase appear successful in matching the general properties of type II Cepheids with $P \leq 15$ days.

We show that it is likely that all the CII are fundamental pulsators and that variables with $P \leq 15$ days (i.e. excluding the RV Tau type stars, V42 in M5, and likely V145 in M3) are in agreement with the predicted period-luminosity-amplitude relation

$$B(3.83) - B = -0.12 + 1.39 \log P(F) + 0.57A(B). \quad (12)$$

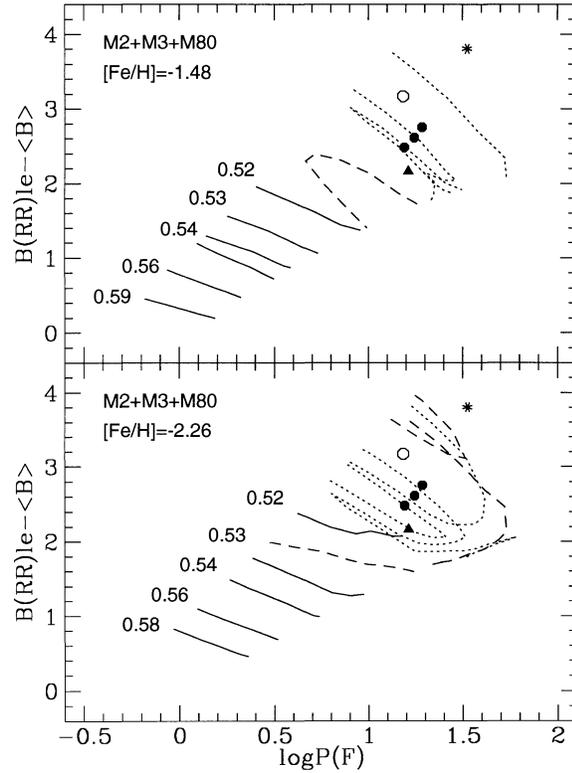


Fig. 8. As in Fig. 7, but for type II Cepheids in M2 (dots), M80 (triangles), and M3 (circle). The asterisk refers to the RV Tau star in M2

As for the mass of the variables, we show that it generally decreases with increasing the period and that the brighter variables with longer period should belong to the faster phases of the $0.52 M_{\odot}$ and $0.53 M_{\odot}$ evolution. With a total number of 19 variables with $M \sim 0.52\text{--}0.53$, the observed number of the “fast” ones varies from 6 at $Z \sim 0.0002$ to 14 at $Z \sim 0.001$, while the number predicted from evolutionary timescales is $\sim 6 \pm 2$ (standard deviation). Since it is likely that the overall metallicity of globular clusters with observed CII is lower than $Z \sim 0.001$, we could conclude that theoretical timescales are not in disagreement with observations.

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