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Dynamo driven by weak plasma turbulence

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Abstract. The dynamo mechanism driven by weak plasma turbulence, called the p-ω dynamo, is studied. Under the assumption of the uniform rotation of a celestial body, a dynamo equation is derived from the magnetic induction equation based on mean field electrodynamics. It is well known that differential rotation is indispensable for an α-ω dynamo to be operative. In contrast to the α-ω dynamo model, no matter if the rotation angular velocity is uniform or not, no matter in the absence or the presence of the convection, the p-ω dynamo is operative. The coupling between uniform rotation and plasma turbulence waves can also produce source terms in the dynamo equation which lead to the generation of a poloidal and toroidal field. The existence of differential rotation will evidently enhance the action of the p-ω dynamo. We present a complete uniform-rotational solution of the dynamo equation here, and dynamo solutions for several types of differential rotation are given. Finally, we discuss the physical nature of these dynamo solutions in Sect. 5.

Key words: magnetic fields – turbulence – plasmas – MHD

1. Introduction

As more magnetic fields are detected on celestial bodies, the problem of the magnetic field origin becomes more important than ever. The initial investigation of magnetic field origin was called the laminar flow dynamo theory. Cowling (1934) proved that a laminar-flow axisymmetric system could not generate and maintain a magnetic field, so the initial theory cannot be applied to such bodies. But most celestial bodies are axisymmetric, which presents a difficulty to explain the origin of the magnetic field. Parker (1955a, 1955b) put forward an idea of the turbulent fluid dynamos, in which the rising blobs of charged fluid, under the action of the Coriolis' force, will yield sets of screw-like vortices generating and maintaining a magnetic field. This is the so-called α-effect. Since then, the dynamo theory has been further developed by many scientists, proposing such as kinematic dynamos (Steenbeck & Krause 1966, Roberts & Stix 1971; Yoshimura 1975 & Stix 1976a, 1976b, 1983) and the magnetohydrodynamic dynamo in which the Lorentz force is considered (Malkus & Proctor 1975; Krause & Radler 1980; Jones & Soward 1990), etc. The study of dynamos has been extended from the earth dynamo and the solar dynamo to the others such as galactic and intergalactic dynamos (Deinzer et al. 1990; Brandenburg et al. 1990; Donner & Brandenburg 1990; Elstner et al. 1990, 1992), etc.

In the cosmos, many celestial bodies have no differential rotation, and moreover, many have no convective interior. In such cases, the α-effect does not apply, but such a star can have a strong magnetic field, e.g. stars near the final stage of evolution. The plasma turbulence dynamo model proposed by Li & Song (1981) possibly can explain the magnetic origin of this class of celestial bodies. The substantial difference between plasma turbulence and fluid turbulence is that the plasma turbulence wave has a characteristic frequency $\omega \neq 0$ and propagates in a certain direction. Obviously, in inhomogeneous media, the wave amplitude and the wave vector are inhomogeneous, which leads to a source term in the dynamo equation such as to generate and maintain the magnetic field of a celestial body. It was pointed out by Li & Song (1981) that, even if there are neither differential rotation nor convection, the weak plasma turbulent wave can only result in a poloidal field. Their model cannot provide a toroidal field. By considering the effect of uniform rotation on the plasma turbulence dynamo, Tong et al. (1985) showed that the coupling between uniform rotation and plasma turbulence leads to a new source in the dynamo equation; so as to generate not only the poloidal field but also a toroidal field. Because they only considered a special class of celestial bodies with very small angular velocity ω, their dynamo solution is not complete.

In this paper, we will consider a more general condition and then obtain more general solutions. In Sect. 3, the large rotational speed is considered, so the advection term $(\boldsymbol{V}_0 \cdot \nabla)\boldsymbol{B}_0$, neglected in Tong et al. (1985), is now added into the dynamo

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equation and a uniform-rotational solution of the dynamo equation is obtained. Dynamo solutions of several types of differential rotation are given in Sect. 4. We discuss the physical nature of these dynamo solutions in Sect. 5. We call our dynamo the "p-ω dynamo".

2. Derivation of the dynamo equation

Following Tong et al. (1985), we separate the magnetic induction equation

$$\frac{\partial B_i}{\partial t} - \eta\nabla^2 B_i = -V_j\frac{\partial B_i}{\partial x_j} + B_j\frac{\partial V_i}{\partial x_j} - B_i\frac{\partial V_j}{\partial x_j} \tag{1}$$

into mean and fluctuating components. Let

$$B_i = B_{0i} + b_i \qquad V_i = V_{0i} + v_i \tag{2}$$

where B_{0i} and V_{0i} represent the mean components, b_i and v_i represent the fluctuating components. By ignoring the change of average velocity in comparison with the fluctuating field, we can obtain the mean field equation from Eq. (1)

$$\left(\frac{\partial}{\partial t} - \eta\nabla^2 + V_0\cdot\nabla\right)B_{0i} = -\left\langle v_j\frac{\partial B_i}{\partial x_j}\right\rangle + \left\langle b_j\frac{\partial v_i}{\partial x_j}\right\rangle - \left\langle b_i\frac{\partial v_j}{\partial x_j}\right\rangle \tag{3}$$

and the fluctuation equation(only to first-order)

$$\left(\frac{\partial}{\partial t} - \eta\nabla^2\right)b_i = -V_{0j}\frac{\partial b_i}{\partial x_j} - v_j\frac{\partial B_{0i}}{\partial x_j} + B_{0j}\frac{\partial v_i}{\partial x_j} + b_i\frac{\partial V_{0i}}{\partial x_j} - B_{0i}\frac{\partial v_j}{\partial x_j} - b_i\frac{\partial V_{0j}}{\partial x_j} \tag{4}$$

Using the Fourier transformation with the expression

$$(LM)_K = \int L_{K_1}M_{K_2}dK_1dK_2\delta(K - K_1 - K_2)$$
$$dK \equiv d\boldsymbol{K}d\omega \tag{5}$$

from Eq. (4), we obtain

$$b_i = \frac{B_{0j}(iK_j)v_{iK} - B_{0i}(iK_j)v_{jK}}{\eta K^2 - i(\omega - v_{0j}K_j)} \tag{6}$$

where B_{0i} and v_{0i} are slowly varying fields of large scale. Let the first approximation be $B_{0i,K} = B_{0i}\delta(K)$, where $K = (\boldsymbol{K}, \boldsymbol{\omega})$ and $\delta(K)$ is the Dirac function. Suppose the expanded terms

$$A_i \propto A_{i,K}\exp(\mathrm{i}\boldsymbol{K}\cdot\boldsymbol{r} - \mathrm{i}\omega t)$$

where K is a slowly varying function of the coordinates. Only the first-order approximation is adopted .

By substituting Eq. (6) into Eq. (3) and making use of Eq. (5), and after some complex manipulation, we obtain

$$\left(\frac{\partial}{\partial t} - \eta\nabla^2 + V_0\cdot\nabla\right) = \frac{1}{\omega_K - V_{0j}K_j} \times\frac{\partial}{\partial x_j}\left\{B_{0i}K_l\left[\left(\delta_{ij} - \frac{K_iK_j}{K^2}\right)|U^t|^2 + \frac{K_iK_j}{K^2}|U^l|^2\right]\right\} - \frac{1}{\omega_K - V_{0j}K_j}\frac{\partial}{\partial x_j}\left(K_jB_{0i}|U^l|^2\right) \tag{7}$$

where $|U_T|^2$ is the square of the turbulent wave amplitude $|U_T|^2 = \int|v_k|^2dk$.

To simplify Eq. (7) we assume that the energy of transverse turbulence is nearly equal to the longitudinal one, i.e., $|U_T^t|^2 \approx |U_T^l|^2$, and the following expressions are approximately valid:

$$\frac{\partial}{\partial x_1}\left(K_1|U_T|^2\right) \approx \frac{\partial}{\partial x_2}\left(K_2|U_T|^2\right) \approx \frac{\partial}{\partial x_3}\left(K_3|U_T|^2\right)$$
$$K_j|U_T^l|^2 = \Phi(x_j)$$

Omitting the non-source terms on the right-hand side of Eq. (7), the dynamo equation (7) becomes

$$\frac{\partial\boldsymbol{B}_0}{\partial t} = \eta\nabla^2\boldsymbol{B}_0 - (\boldsymbol{V}_0\cdot\nabla)\boldsymbol{B}_0 + p\frac{\boldsymbol{B}_0}{1 - \beta\omega r\sin\theta} \tag{8}$$

where p is given by

$$p = -\frac{2}{\omega_k}\frac{\partial}{\partial z}\left(k_z|U_T|^2\right), \quad b = k_\phi/\omega_k \tag{9}$$

This is the parameter that reflects the divergence of the momentum density of plasma turbulence. We will discuss its physical interpretation.

By means of the cylindrical polar coordinates (R, ϕ, z), Eq. (8) becomes

$$\frac{\partial\boldsymbol{B}_0}{\partial t} = \eta\nabla^2\boldsymbol{B}_0 - (\boldsymbol{V}_0\cdot\nabla)\boldsymbol{B}_0 + p\frac{\boldsymbol{B}_0}{1 - \beta V_0} \tag{10}$$

where $\boldsymbol{B}_0$ is the mean magnetic field, $\boldsymbol{V}_0 = \omega\times\boldsymbol{R}$ the mean fluid velocity, ω the angular velocity of the rotation, k_ϕ the ϕ-component of wave number, ω_k the dispersion frequency of the turbulent plasma wave. Equation (10) shows that the generation of the mean magnetic field $\boldsymbol{B}_0$ depends upon the effect of the coupling between weak plasma turbulence p and the rotation ω.

Using the separation of variables as follows,

$$\boldsymbol{B}(t, \boldsymbol{r}) = A(t)\boldsymbol{B}(\boldsymbol{r}) = A_0\mathrm{e}^{\lambda t}\boldsymbol{B}(\boldsymbol{r})$$

Eq. (10) becomes

$$\nabla^2\boldsymbol{B}(\boldsymbol{r}) - \frac{1}{\eta}(\boldsymbol{V}\cdot\nabla)\boldsymbol{B}(\boldsymbol{r}) - \frac{\lambda}{\eta}\boldsymbol{B}(\boldsymbol{r}) + p\frac{\boldsymbol{B}(\boldsymbol{r})}{\eta(1 - \beta\boldsymbol{V})} = 0 \tag{11}$$

Assume axisymmetry ($\frac{\partial}{\partial\phi} = 0, V_R = V_z = 0$). The three components of dynamo equation can be written as

$$\left(\nabla^2 - R^{-2}\right)B_\phi(R, z) - \frac{\lambda}{\eta}B_\phi(R, z) - \frac{1}{\eta}\omega(R, z)B_R(R, z) + p\frac{B_\phi(R, z)}{\eta[1 - \beta\omega(R, z)R]} = 0 \tag{12}$$

$$\left(\nabla^2 - R^{-2}\right)B_R(R, z) - \frac{\lambda}{\eta}B_R(R, z) + \frac{1}{\eta}\omega(R, z)B_\phi(R, z) + p\frac{B_R(R, z)}{\eta[1 - \beta\omega(R, z)R]} = 0 \tag{13}$$

$$\nabla^2B_z(R, z) - \frac{\lambda}{\eta}B_z(R, z) + p\frac{B_z(R, z)}{\eta[1 - \beta\omega(R, z)R]} = 0 \tag{14}$$

Equation (12) is the toroidal component, Eq. (13) and Eq. (14) together comprise the poloidal component.

3. Solutions of the p-ω dynamo equation in the case of uniform rotation

($\omega = \omega_0 =$constant)

Now we shall consider two different cases as follows.

a) The case of $\beta\omega_0 R_0 \ll 1$:
We can adopt the approximation of $1/(1-\beta\omega_0 R) \approx 1$, in which case Eqs. (12), (13) and (14) become

$$\left(\nabla^2 - R^{-2}\right) B_\phi(R,z) + \Gamma^2 B_\phi(R,z) - \Omega_0 B_R(R,z) = 0 \quad (15)$$

$$\left(\nabla^2 - R^{-2}\right) B_R(R,z) + \Gamma^2 B_R(R,z) + \Omega_0 B_\phi(R,z) = 0 \quad (16)$$

$$\nabla^2 B_z(R,z) + \Gamma^2 B_z(R,z) = 0 \quad (17)$$

where R_0 is the radius of the celestial body, $\Gamma^2 = (p-\lambda)/\eta$, $\Omega_0 = \omega_0/\eta$.

To solve the above equations, we may take the solution obtained previously (Tong et al. 1985)

$$B_R(R,z) = A_m J_1(nR)\cos(mz)$$

as the first approximation and substitute it into Eq. (15). Then Eq. (16) reduces to

$$\left(\nabla^2 - R^{-2}\right) B_\varphi(R,z) + \Gamma^2 B_\varphi(R,z) = A_m \Omega_0 J_1(nR)\cos(mz)$$

or

$$\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial B_\phi}{\partial R}\right) - \frac{B_\phi}{R^2} + \Gamma^2 B_\phi + \frac{\partial^2 B_\phi}{\partial z^2} = A_m \Omega_0 J_1(nR)\cos(mz) \quad (18)$$

First we can get a special solution of Eq. (18):

$$\overline{B_\phi}(R,z) = A_m \Omega_0 J_1(nR)\left[\frac{z}{2m}\sin(mz)\right] \quad (19)$$

This is only a solution of first approximation. After substituting Eq. (19) into Eq. (16), Eq. (12) becomes

$$\left(\nabla^2 - R^{-2}\right) B_R(R,z) + \Gamma^2 B_R(R,z) = -A_m \Omega_0^2 J_1(nR)\left[\frac{z}{2m}\sin(mz)\right] \quad (20)$$

which has a solution of second approximation

$$\overline{B_R}(R,z) = -A_m \Omega_0^2 J_1(nR) \times \left[\frac{1}{8m^2}\left(\frac{z}{m}\sin(mz) - z^2\cos(mz)\right)\right]. \quad (21)$$

Then substituting Eq. (17) into Eq. (11) again, we have

$$\left(\nabla^2 - R^{-2}\right) B_\phi(R,z) + \Gamma^2 B_\phi(R,z) = -A_m \Omega_0^3 J_1(nR)\left[\frac{1}{8m^2}\left(\frac{z}{m}\sin(mz) - z^2\cos(mz)\right)\right] \quad (22)$$

which has a solution of second approximation

$$\overline{B_\phi}(R,z) = -A_m \Omega_0^3 J_1(nR)\left[\frac{1}{16m^4}\left(\frac{z}{2m}\sin(mz) - 2z^2\cos(mz) - \frac{1}{3}mz^3\sin(mz)\right)\right] \quad (23)$$

Under the non-rotational condition ($\Omega_0 \equiv 0$), Eq. (18) can be written as

$$\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial B_\phi}{\partial R}\right) - \frac{B_\phi}{R^2} + \Gamma^2 B_\phi + \frac{\partial^2 B_\phi}{\partial z^2} = 0$$

which has a general solution of the form

$$\sum_{m_1} A_{m_1}\left(J_1(n_1 R) + D_{m_1} N_1(n_1 R)\right)\cos(m_1 z).$$

Here $J_i(x)$ and $N_i(x)$ are the Bessel functions of the first and second kind, respectively, and

$$n^2 = \Gamma^2 - m^2, \quad n_1^2 = \Gamma^2 - m_1^2.$$

The integral constant A_{m_1} of the solution of the toroidal component has been proved (Li & Song 1981) to be

$$A_{m_1} \equiv 0.$$

Thus the solution of Eq. (15) is given by

$$B_\phi(R,z) = -A_m \Omega_0^3 J_1(nR)\left[\frac{1}{16m^4}\left(\frac{z}{2m}\sin(mz) - 2z^2\cos(mz) - \frac{1}{3}mz^3\sin(mz)\right)\right] \quad (24)$$

and the solution of Eq. (16) by

$$B_R(R,z) = \sum_m A_m\left[J_1(nR) + D_m N_1(nR)\right]\cos(mz) - A_m \Omega_0^2 J_1(nR)\left[\frac{1}{8m^2}\left(\frac{z}{m}\sin(mz) - z^2\cos(mz)\right)\right] \quad (25)$$

where the first term of the right hand side is the non-rotational solution and the second term is produced by the rotation effect.

From Eq. (17), the z-component of the dynamo equation is not affected by the rotation, and its solution is

$$B_z(R,z) = \sum_{m_2} A_{m_2}\left[J_0(n_2 R) + D_{m_2} N_0(n_2 R)\right]\cos(m_2 z) \quad (26)$$

where $n_2^2 = \Gamma^2 - m_2^2$.

In this case, there is an amplifying poloidal component to maintain the magnetic field given by

$$\begin{aligned}\boldsymbol{B}_p(\boldsymbol{r},t) &= \mathrm{e}^{\lambda t}\left\{\boldsymbol{e}_R B_R(R,z) + \boldsymbol{e}_z B_z(R,z)\right\} \\ &= \mathrm{e}^{\lambda t}\left\{\boldsymbol{e}_R\left[\sum_m A_m\left[J_1(nR) + D_m N_1(nR)\right]\cos(mz) - A_m\Omega_0^2 J_1(nR)\left[\frac{1}{8m^2}\left(\frac{z}{m}\sin(mz) - z^2\cos(mz)\right)\right]\right.\right. \\ &\quad \left.\left. + \boldsymbol{e}_z\left[\sum_{m_2} A_{m_2} J_0(n_2 R) + D_{m_2} N_0(n_2 R)\right]\cos(m_2 z)\right]\right\} \end{aligned} \quad (27)$$

and toroidal components given by

$$\begin{aligned}\boldsymbol{B}_\phi(\boldsymbol{r},t) &= \mathrm{e}^{\lambda t}\boldsymbol{e}_\phi B_\phi(R,z) \\ &= -\mathrm{e}^{\lambda t}\boldsymbol{e}_\phi A_m \Omega_0^3 J_1(nR)\left[\frac{1}{16m^4}\left(\frac{z}{2m}\sin(mz) - 2z^2\cos(mz) - \frac{1}{3}mz^3\sin(mz)\right)\right]\end{aligned} \quad (28)$$

b) The case of $\beta\omega_0 R_0 < 1$:

We can adopt the approximation of $1/(1-\beta\omega_0 R) \approx 1 + \beta\omega_0 R$, Eqs. (8), (9) and (10) become

$$\left(\nabla^2 - R^{-2}\right) B_\phi(R,z) + \Gamma^2 B_\phi(R,z) - \Omega_0 B_R(R,z) + a\Omega_0 R B_\phi(R,z) = 0 \tag{29}$$

$$\left(\nabla^2 - R^{-2}\right) B_R(R,z) + \Gamma^2 B_R(R,z) + \Omega_0 B_\phi(R,z) + a\Omega_0 R B_\phi(R,z) = 0 \tag{30}$$

$$\nabla^2 B_z(R,z) + \Gamma^2 B_z(R,z) + a\Omega_0 R B_z(R,z) = 0 \tag{31}$$

where $a = pb$.

Under our assumption that $a\Omega_0 R$ is a small quantity, the influence of the last terms on the left hand of the above equations may be neglected, thus the special solutions of Eq. (29) and (30) are approximately equal to the expressions of Eq. (21) and (23). Solutions of the following equations

$$\left(\nabla^2 - R^{-2}\right) B_\phi(R,z) + \Gamma^2 B_\phi(R,z) + a\Omega_0 R B_\phi(R,z) = 0$$
$$\left(\nabla^2 - R^{-2}\right) B_R(R,z) + \Gamma^2 B_R(R,z) + a\Omega_0 R B_\phi(R,z) = 0$$
$$\nabla^2 B_z(R,z) + \Gamma^2 B_z(R,z) + a\Omega_0 R B_z(R,z) = 0$$

were already obtained (Tong et al. 1985). Therefore, the solutions of Eqs. (29), (30) and (31) are

$$B_\phi(R,z) = y_1 \cos(m_1 z) - A_m \Omega_0^3 J_1(nR)\left[\frac{1}{16m^4} \times\left(\frac{z}{2m}\sin(mz) - 2z^2\cos(mz) - \frac{1}{3}mz^3\sin(mz)\right)\right] \tag{32}$$

$$B_R(R,z) = \sum_m \left\{A_m\left[J_1(nR) + D_m N_1(nR)\right] + y\right\}\cos(mz) - A_m\Omega_0^2 J_1(nR)\left[\frac{1}{8m^2}\left(\frac{z}{m}\sin(mz) - z^2\cos(mz)\right)\right] \tag{33}$$

$$B_z(R,z) = \sum_{m_2} A_{m_2}\left[J_0(n_2 R) + D_{m_2} N_0(n_2 R) + y_2\right]\cos(m_2 z) \tag{34}$$

where

$$y = \frac{\pi}{2} n^2 a\Omega_0\left[J_1(nR)\int R^2 J_1(nR) N_1(nR) dR - N_1(nR)\int R^2 J_1^2(nR) dR\right] \tag{35}$$

$$y_1 = \frac{\pi}{2} n_1^2 a\Omega_0\left[J_1(n_1 R)\int R^2 J_1(n_1 R) N_1(n_1 R) dR - N_1(nR)\int R^2 J_1^2(n_1 R) dR\right] \tag{36}$$

$$y_2 = \frac{\pi}{2} n_2^2 a\Omega_0\left[J_0(n_2 R)\int R^2 J_0(n_2 R) N_0(n_2 R) dR - N_0(n_2 R)\int R^2 J_0^2(n_2 R) dR\right] \tag{37}$$

So the poloidal field is given by

$$\begin{aligned}\boldsymbol{B}_p(\boldsymbol{r},t) &= \mathrm{e}^{\lambda t}\left\{\boldsymbol{e}_R B_R(R,z) + \boldsymbol{e}_z B_z(R,z)\right\}\\ &= \mathrm{e}^{\lambda t}\Big\{\boldsymbol{e}_R\Big[\sum_m \left\{A_m\left[J_1(nR) + D_m N_1(nR)\right] + y\right\}\cos(mz)\\ &\quad - A_m\Omega_0^2 J_1(nR)\left[\frac{1}{8m^2}\left(\frac{z}{m}\sin(mz) - z^2\cos(mz)\right)\right]\\ &\quad + \boldsymbol{e}_z\Big\{\sum_{m_2} A_{m_2}\left[J_0(n_2 R) + D_{m_2} N_0(n_2 R)\right] + y_2\Big\}\\ &\quad \times\cos(m_2 z)\Big]\Big\}\end{aligned} \tag{38}$$

and the toloidal component by

$$\begin{aligned}B_p(\boldsymbol{r},t) &= \mathrm{e}^{\lambda t}\boldsymbol{e}_\phi B_\phi(R,z)\\ &= \mathrm{e}^{\lambda t}\boldsymbol{e}_\phi\Big\{y_1\cos(m_1 z) - A_m\Omega_0^3 J_1(nR)\\ &\quad\times\Big[\frac{1}{16m^4}\Big(\frac{z}{2m}\sin(mz) - 2z^2\cos(mz)\\ &\quad - \frac{1}{3}mz^3\sin(mz)\Big)\Big]\Big\}\end{aligned} \tag{39}$$

4. Solutions of the p-ω dynamo equation in the case of differential rotation

($\omega = \omega(R,z)$)

From observations of the sun, we know that the solar rotational angular velocity decreases as the latitude on the sun increases. Following Parker (1979), non-uniform rotation with equatorial symmetry is assumed to take the form

$$\omega(z) = \omega_0 + \omega_m\left(\frac{z}{R_0}\right)^m \tag{40}$$

According to the specific details of the variation of rotation velocity with latitude, we may adopt terms of different order in Eq. (40).

(1) Assuming that the variation of rotational angular velocity with latitude is small and linear, Eq. (40) takes its simplest form

$$\omega(z) = \omega_0 + \omega_1\frac{|z|}{R_0} \tag{41}$$

a) For the case of $\beta\omega R_0 \ll 1$, adopting the approximation $1/(1-\beta\omega R_0) \approx 1$, Eqs. (12), (13) and (14) become

$$\left(\nabla^2 - R^{-2}\right) B_\phi(R,z) + \Gamma^2 B_\phi(R,z) - \Omega_0 B_R(R,z) - \Omega_1\frac{|z|}{R_0} B_R(R,z) = 0 \tag{42}$$

$$\left(\nabla^2 - R^{-2}\right) B_R(R,z) + \Gamma^2 B_R(R,z) + \Omega_0 B_\phi(R,z) + \Omega_1\frac{|z|}{R_0} B_\phi(R,z) = 0 \tag{43}$$

$$\nabla^2 B_z(R,z) + \Gamma^2 B_z(R,z) = 0 \tag{44}$$

where $\Gamma^2 = (p - \lambda)/\eta$, $\Omega_0 = \omega_0/\eta$, $\Omega_1 = \omega_1/\eta$, and R_0 is the radius of the celestial body. Using the method stated in the last section, we can get the solutions of Eqs. (42), (43) and (44)

$$B_\phi(R,z) = -A_m J_1(nR)\Big\{\Big[\frac{\Omega_0^3}{16m^4}\Big(\frac{z}{2m}\sin(mz) -2z^2\cos(mz) - \frac{1}{3}mz^3\sin(mz)\Big)\Big] + \frac{\Omega_0^2\Omega_1|z|}{32m^5R_0}\Big[\frac{3}{2}m^2z^3\sin(mz) - \frac{1}{3}mz^2\cos(mz) -\frac{21}{2}z\sin(mz) - \frac{21}{2m}\cos(mz)\Big]\Big\} \quad (45)$$

$$B_R(R,z) = \sum_m A_m\left[J_1(nR) + D_mN_1(nR)\right]\cos(mz) -A_mJ_1(nR)\left[\frac{\Omega_0^2}{8m^2}\left(\frac{z}{m}\sin(mz) - z^2\cos(mz)\right)\right] +A_mJ_1(nR)\frac{\Omega_0\Omega_1|z|}{4m^3R_0}\Big[-\frac{mz^2}{2}\cos(mz) +z\sin(mz) - \frac{1}{m}\cos(mz)\Big] \quad (46)$$

$$B_z(R,z) = \sum_{m_2} A_{m_2}\left[J_0(n_2R) + D_{m_2}N_0(n_2R)\right]\cos(m_2z) \quad (47)$$

where $J_i(x)$ and $N_i(x)$ are Bessel functions of first and second kind with imaginary argument, and $n^2 = \Gamma^2 - m^2$, $n_1^2 = \Gamma^2 - m_1^2$, $n_2^2 = \Gamma^2 - m_2^2$.

In this case, there are amplifying poloidal component and toroidal components to maintain the magnetic field

$$\boldsymbol{B}_p(\boldsymbol{r},t) = \mathrm{e}^{\lambda t}\left\{\boldsymbol{e}_RB_R(R,z) + \boldsymbol{e}_zB_z(R,z)\right\} = \mathrm{e}^{\lambda t}\Big\{\boldsymbol{e}_R\Big\{\sum_m A_m\left[J_1(nR) + D_mN_1(nR)\right]\cos(mz) -A_m\Omega_0^2J_1(nR)\left[\frac{1}{8m^2}\left(\frac{z}{m}\sin(mz) - z^2\cos(mz)\right)\right] +A_mJ_1(nR)\frac{\Omega_0\Omega_1|z|}{4m^3R_0}\Big[-\frac{1}{2}mz^2\cos(mz) +z\sin(mz) - \frac{1}{m}\cos(mz)\Big]\Big\} +\boldsymbol{e}_z\Big[\sum_{m_2}A_{m_2}\left[J_0(n_2R) + D_{m_2}N_0(n_2R)\right]\cos(m_2z)\Big]\Big\} \quad (48)$$

$$\boldsymbol{B}_\phi(\boldsymbol{r},t) = \mathrm{e}^{\lambda t}\boldsymbol{e}_\phi B_\phi(R,z) = -\mathrm{e}^{\lambda t}\boldsymbol{e}_\phi A_mJ_1(nR)\Big[\frac{\Omega_0^3}{16m^4}\Big(\frac{z}{2m}\sin(mz) -2z^2\cos(mz) - \frac{1}{3}mz^3\sin(mz)\Big) +\frac{\Omega_0^2\Omega_1|z|}{32m^5R_0}\Big[\frac{3}{2}m^2z^3\sin(mz) - \frac{1}{3}mz^2\cos(mz) -\frac{21}{2}z\sin(mz) - \frac{21}{2m}\cos(mz)\Big]\Big] \quad (49)$$

b) For the case of $\beta\omega_0R_0 < 1$, adopting the approximation of $1/(1 - \beta\omega_0R) \approx 1 + \beta\omega_0R$, and then substituting them into Eqs. (12), (13) and (14) neglecting the second-order small quantity $\beta\Omega_1$, we obtain

$$\left(\nabla^2 - R^{-2}\right)B_\phi(R,z) + \Gamma^2B_\phi(R,z) + a\Omega_0RB_\phi(R,z) = \left(\Omega_0 + \Omega_1\frac{|z|}{R_0}\right)B_R(R,z) \quad (50)$$

$$\left(\nabla^2 - R^{-2}\right)B_R(R,z) + \Gamma^2B_R(R,z) + a\Omega_0RB_R(R,z) = -\left(\Omega_0 + \Omega_1\frac{|z|}{R_0}\right)B_\phi(R,z) \quad (51)$$

$$\nabla^2B_z(R,z) + \Gamma^2B_z(R,z) + a\Omega_0RB_z(R,z) = 0 \quad (52)$$

where $a = pb$

Due to our assumption that $\beta\Omega_0R$ is a small quantity, the influence of the third terms on the left hand of Eq. (50) and (51) may be neglected when we solve their special solutions. The approximate solutions of Eqs. (50), (51) and (52) can finally be obtained

$$B_\phi(R,z)_\phi = -y_1\Omega_0\left[\frac{1}{8m_1^2}\left(\frac{z}{m_1}\sin(m_1z) - z^2\cos(m_1z)\right)\right] -A_mJ_1(nR)\Big\{\Big[\frac{\Omega_0^3}{16m^4}\Big(\frac{z}{2m}\sin(mz) -2z^2\cos(mz) - \frac{1}{3}mz^3\sin(mz)\Big)\Big] +\frac{\Omega_0^2\Omega_1|z|}{32m^5R_0}\Big[\frac{3}{2}m^2z^3\sin(mz) - \frac{1}{3}mz^2\cos(mz) -\frac{21}{2}z\sin(mz) - \frac{21}{2m}\cos(mz)\Big]\Big\} \quad (53)$$

$$B_R(R,z)_R = \sum_m A_m\left\{\left[J_1(nR) + D_mN_1(nR)\right] + y\right\}\cos(mz) -A_mJ_1(nR)\left[\frac{\Omega_0^2}{8m^2}\left(\frac{z}{m}\sin(mz) - z^2\cos(mz)\right)\right] -A_mJ_1(nR)\frac{\Omega_0\Omega_1|z|}{4m^3R_0}\Big[\frac{1}{2}mz^2\cos(mz) -z\sin(mz) - \frac{1}{m}\cos(mz)\Big] \quad (54)$$

$$B_z(R,z) = \sum_{m_2}A_{m_2}\left\{\left[J_0(n_2R) + D_{m_2}N_0(n_2R)\right] + y_2\right\} \times\cos(m_2z) \quad (55)$$

where y, y_1 and y_2 are defined by Eqs. (35), (36) and (37).

Therefore, we get the amplifying poloidal and toroidal fields to maintain the magnetic field.

$$\boldsymbol{B}_p(\boldsymbol{r},t) = \mathrm{e}^{\lambda t}\left\{\boldsymbol{e}_RB_R(R,z) + \boldsymbol{e}_zB_z(R,z)\right\} = \mathrm{e}^{\lambda t}\Big\{\boldsymbol{e}_R\Big\{\sum_m A_m\left\{\left[J_1(nR) + D_mN_1(nR)\right] + y\right\}\cos(mz)$$

$$-A_m J_1(nR)\left[\frac{\Omega_0^2}{8m^2}\left(\frac{z}{m}\sin(mz)-z^2\cos(mz)\right)\right]\Bigg\}$$
$$-A_m J_1(nR)\frac{\Omega_0\Omega_1|z|}{4m^3R_0}$$
$$\times\left[\frac{1}{2}mz^2\cos(mz)-z\sin(mz)-\frac{1}{m}\cos(mz)\right]$$
$$+\boldsymbol{e}_z\sum_{m_2}A_{m_2}\Big\{\left[J_0(n_2R)+D_{m_2}N_0(n_2R)\right]+y_2\Big\}\cos(m_2z)\Bigg\} \quad (56)$$

$$\boldsymbol{B}_\phi(\boldsymbol{r},t)=\mathrm{e}^{\lambda t}\boldsymbol{e}_\phi B_\phi(R,z)$$
$$=\mathrm{e}^{\lambda t}\boldsymbol{e}_\phi\Bigg\{-y_1\Omega_0\left[\frac{1}{8m_1^2}\left(\frac{z}{m_1}\sin(m_1z)-z^2\cos(m_1z)\right)\right]$$
$$-A_m J_1(nR)\Bigg\{\Bigg[\frac{\Omega_0^3}{16m^4}\Big(\frac{z}{2m}\sin(mz)$$
$$-2z^2\cos(mz)-\frac{1}{3}mz^3\sin(mz)\Big)\Bigg]$$
$$+\frac{\Omega_0^2\Omega_1}{32m^5R_0}\left[\frac{3}{2}m^2z^4\sin(mz)-\frac{1}{3}mz^3\cos(mz)\right.$$
$$\left.-\frac{21}{2}z^2\sin(mz)-\frac{21}{2}\frac{z}{m}\cos(mz)\right]\Bigg\}\Bigg\} \quad (57)$$

(2) For the sun and solar-like stars, we suppose that the variation of the rotational angular velocity with latitude is proportional to z^2/R_0^2. Eq. (40) gives

$$\omega(z)=\omega_0+\omega_2\left(\frac{z}{R_0}\right)^2 \quad (58)$$

a) For the case of $\beta\omega R_0 \ll 1$, adopting the approximation $1/(1-\beta\omega R_0)\approx 1$, Eqs. (15), (16) and (17) become

$$\left(\nabla^2-R^{-2}\right)B_\phi(R,z)+\Gamma^2B_\phi(R,z)$$
$$-\Omega_0B_R(R,z)-\Omega_2\left(\frac{z}{R_0}\right)^2B_R(R,z)=0 \quad (59)$$
$$\left(\nabla^2-R^{-2}\right)B_R(R,z)+\Gamma^2B_R(R,z)$$
$$+\Omega_0B_\phi(R,z)+\Omega_2\left(\frac{z}{R_0}\right)^2B_\phi(R,z)=0 \quad (60)$$
$$\nabla^2B_z(R,z)+\Gamma^2B_z(R,z)=0 \quad (61)$$

where $\Omega_2=\omega_2/\eta$.

Similarly, we can obtain the solutions of Eqs. (59), (60) and (61)

$$\boldsymbol{B}_\phi(\boldsymbol{r},t)=\mathrm{e}^{\lambda t}\Bigg\{\boldsymbol{e}_R\Bigg\{\sum_m A_m\left[J_1(nR)+D_mN_1(nR)\right]\cos(mz)$$
$$+A_mJ_1(nR)\left[\frac{\Omega_0^2}{8m^2}\left(\frac{z}{m}\sin(mz)-z^2\cos(mz)\right)\right.$$
$$+\frac{\Omega_0\Omega_2}{R_0^2}\frac{1}{32m^4}\Bigg(-\frac{7}{3}m^2z^4\cos(mz)$$
$$\left.+\frac{17}{3}mz^3\sin(mz)+\frac{21}{2}z^2\cos(mz)+\frac{5}{2}z\sin(mz)\Bigg)\right]\Bigg\}$$
$$+\boldsymbol{e}_z\sum_{m_2}A_{m_2}\left[J_0(n_2R)+D_{m_2}N_0(n_2R)\right]\cos(m_2z)\Bigg\} \quad (62)$$

$$\boldsymbol{B}_\phi(\boldsymbol{r},t)=\mathrm{e}^{\lambda t}\boldsymbol{e}_\phi\Bigg\{A_mJ_1(n_1R)\Bigg[\Omega_0\frac{z}{2m}\sin(mz)+\frac{\Omega_2}{R_0^2}\frac{1}{4m^2}$$
$$\times\left(\frac{mz^3}{6}\sin(mz)+z^2\cos(mz)-\frac{z}{m}\sin(mz)\right)\Bigg]\Bigg\} \quad (63)$$

b) In the case of $\beta\omega_0R_0 < 1$, adopting the approximation of $1/(1-\beta\omega_0R)\approx 1+\beta\omega_0R$, Eqs. (15), (16) and (17) become

$$\left(\nabla^2-R^{-2}\right)B_\phi(R,z)+\Gamma^2B_\phi(R,z)+a\Omega_0RB_\phi(R,z)$$
$$=\left(\Omega_0+\Omega_2\frac{z^2}{R_0^2}\right)B_R(R,z) \quad (64)$$
$$\left(\nabla^2-R^{-2}\right)B_R(R,z)+\Gamma^2B_R(R,z)+a\Omega_0RB_R(R,z)$$
$$=-\left(\Omega_0+\Omega_2\frac{z^2}{R_0^2}\right)B_\phi(R,z) \quad (65)$$
$$\nabla^2B_z(R,z)+\Gamma^2B_z(R,z)+a\Omega_0RB_z(R,z)=0 \quad (66)$$

Therefore, the amplifying poloidal and toroidal fields are

$$\boldsymbol{B}_p(\boldsymbol{r},t)=\mathrm{e}^{\lambda t}\Bigg\{\boldsymbol{e}_R\Bigg\{\sum_m A_m\left[J_1(nR)+D_mN_1(nR)\right]+y\Bigg\}$$
$$\times\cos(mz)-y_1\Omega_0\frac{z}{2m_1}\sin(m_1z)$$
$$-A_mJ_1(nR)\left[\frac{\Omega_0^2}{8m^2}\left(\frac{z}{m}\sin(mz)-z^2\cos(mz)\right)\right.$$
$$+\frac{\Omega_0\Omega_2}{R_0^2}\frac{1}{32m^4}\Bigg(-\frac{7}{3}m^2z^4\cos(mz)+\frac{17}{3}mz^3\sin(mz)$$
$$\left.+\frac{21}{2}z^2\cos(mz)+\frac{5}{2}z\sin(mz)\Bigg)\right]\Bigg\}$$
$$+\boldsymbol{e}_z\Bigg\{\sum_{m_2}A_{m_2}\left[J_0(n_2R)+D_{m_2}N_0(n_2R)\right]+y_2\Bigg\}$$
$$\times\cos(m_2z)\Bigg\} \quad (67)$$

$$\boldsymbol{B}_\phi(\boldsymbol{r},t)=\mathrm{e}^{\lambda t}\boldsymbol{e}_\phi\Bigg\{A_mJ_1(n_1R)\Bigg[\Omega_0\frac{z}{2m}\sin(mz)$$
$$+\frac{\Omega_2}{R_0^2}\frac{1}{4m^2}\Bigg(\frac{mz^3}{6}\sin(mz)+z^2\cos(mz)$$
$$-\frac{z}{m}\sin(mz)\Bigg)+y_1\cos(mz)\Bigg]\Bigg\} \quad (68)$$

5. Discussion

As a system with many degrees of freedom, plasma is highly unstable. Many turbulent waves can be excited by the equipartition of energy over each possible degree of freedom. Since the celestial bodies with magnetic fields are largely composed of turbulent plasma, which is easy to excite the inertial Alfvén wave and thereby derive a p-ω dynamo. We believe that where the α-ω dynamo is operative, there the p-ω dynamo is also operative. Therefore, the p-ω dynamo is much more general than other dynamos.

It is well-known that differential rotation is indispensable for the α-ω dynamo action to occur (Robert 1972). But in contrast to the α-ω dynamo, no matter if the rotation angular speed ω be uniform or not, the p-ω dynamo is always operative. Both rotational cases lead to the generation of the poloidal and toroidal fields. In fact, the p-ω dynamo does not depend on the so-called α-effect, which is the result of a coupling between rotation and convection of turbulent fluid. The fluid-turbulence dynamo may be operative in the absence of differential rotation, in view of the fact that the α-effect can generate a toroidal current (and hence the poloidal field) from the toroidal field, as for instance the α^2-dynamo. Nevertheless, the α-effect vanishes if there is no rotation, and hence there is no fluid-turbulence dynamo. It must be emphasized, however, that the plasma- turbulence dynamo is operative even if there is no rotation. Li & Song (1981) have proved that if the body does not rotate, i.e., $\omega = 0$, the plasma turbulence wave can only generate a poloidal field, and the dynamo equation may be written as

$$\frac{\partial \boldsymbol{B}}{\partial t} = \eta \nabla^2 \boldsymbol{B} + p\boldsymbol{B}$$

Here we can give a physical interpretation of the p parameter. To this end, following Whitham (1974), the momentum density of plasma turbulence can be expressed as

$$\boldsymbol{P}_T = \frac{E_T}{\boldsymbol{C}_k}$$

where E_T is the kinetic energy density and $c_K = \frac{\omega_k}{\boldsymbol{k}}$ the phase velocity of plasma turbulent wave. Note that

$$E_T \sim |\boldsymbol{U}_T|^2 = \int \left|\boldsymbol{v}_k^2\right| d\boldsymbol{k}$$

with the integral over $k = |\boldsymbol{k}|$ running from 0 to ∞, and where $\left|v_k^2\right| d\boldsymbol{k}$ is the contribution to the kinetic energy density from the wave-number range $(k, k + dk)$. Obviously, $\boldsymbol{P}_T$ is a vector in the $\boldsymbol{k}$ directon given by

$$P_T \sim \left(\frac{k_x|U_T^2|}{\omega_k}, \frac{k_y|U_T^2|}{\omega_k}, \frac{k_z|U_T^2|}{\omega_k}\right) .$$

Because we restrict the plasma turbulence to the case of

$$\frac{\partial}{\partial x_1}\left(K_1 \left|U_T\right|^2\right) \approx \frac{\partial}{\partial x_2}\left(K_2 \left|U_T\right|^2\right) \approx \frac{\partial}{\partial x_3}\left(K_3 \left|U_T\right|^2\right)$$

the parameter p may itself be expressed as

$$\nabla \cdot \boldsymbol{P}_T \sim \left[\frac{\partial}{\partial x}\left(\frac{k_x|U_T^2|}{\omega_k}\right) + \frac{\partial}{\partial y}\left(\frac{k_y|U_T^2|}{\omega_k}\right) + \frac{\partial}{\partial z}\left(\frac{k_z|U_T^2|}{\omega_k}\right)\right]$$
$$\sim \frac{1}{\omega_k}\frac{\partial}{\partial z}\left(k_z \left|U_T\right|^2\right)$$
$$p \sim \nabla \cdot \boldsymbol{P}_T$$

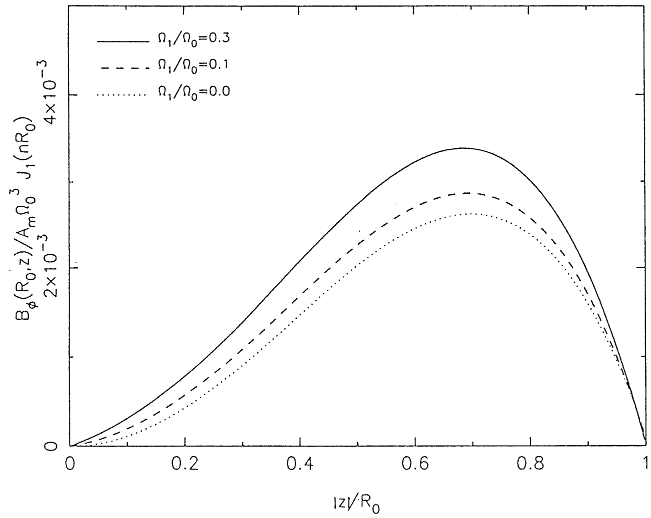

Fig. 1. Effect of differential rotation on the toroidal magnetic field at the surface of a celestial body, as compared with uniform rotation. See text for explanation

Therefore, the parameter p is a measure of the divergence of the momentum density of plasma turbulence. It is well-known that the plasma turbulent wave always has some non-zero characteristic frequency $\omega_k \neq 0$, the momentum density can at least be propagated in a certain direction, so the parameter p is always non-zero. In fact, as Moffatt (1978) pointed out, some diffusion is essential to generate an α-effect. Fluid turbulence (or a random wave field) that lacks reflectional symmetry can arise in a natural way only in a rotating system in which there is necessarily a preferred direction (the direction of the rotation vector), so α can be non-zero only if the turbulence lacks reflectional symmetry. As for the plasma turbulence, p is always non-zero in all directions, which is very different from the fluid turbulence. Hence, a non-rotating celestial body, plasma turbulence still generates a magnetic field.

However, in a rotating system, even if the rotation angular velocity is so small that the third terms in the right-hands of Eqs. (12) and (13) may be neglected, Tong et al. (1985) showed that both toroidal and poloidal fields are generated by the plasma turbulence. It is worth note that the coupling between uniform rotation and plasma turbulence can generate a toroidal field, so that the magnetic field can be maintained for a long time.

Moffatt (1978) pointed out that the effect of differential rotation is to generate a toroidal magnetic field from an initially axisymmetric poloidal magnetic field. Thus it is anticipated that the differential rotation will enhance the toroidal magnetic field, and hence enhance the action of the turbulent plasma dynamo. To illustrate the effect of differential rotation described in Sect. 4, Eq. (45) may be rewritten as

$$B_\phi(R, z)/A_m J_1(nR) = -\left\{\left[\frac{1}{16m^4}\left(\frac{z}{2m}\sin(mz)\right.\right.\right.$$
$$\left.\left. -2z^2\cos(mz) - \frac{1}{3}mz^3\sin(mz)\right)\right]$$

$$+\frac{\Omega_1|z|}{32m^5}\left[\frac{3}{2}m^2z^3\sin(mz)-\frac{1}{3}mz^2\cos(mz)\right.$$
$$\left.\left.-\frac{21}{2}z\sin(mz)-\frac{21}{2}\frac{1}{m}\cos(mz)\right]\right\} \quad (69)$$

where R and z are measured in units of R_0, and the variables representing angular velocity have been non-dimensionalized with respect to Ω_0, i.e., $R_0 \equiv 1$, $\Omega_0 \equiv 1$.

Note that $\boldsymbol{B}_\varphi$ vanishes at $|z| = 1$, so the parameter m is to be determined from the boundary condition at $|z| = 1$:

$$\left[(-9\Omega_1+4)m^3+(63\Omega_1-6)\right]\sin(m)$$
$$+\left[(2\Omega_1+24)m^2+63\Omega_1\right]\cos(m)=0 \quad (70)$$

From Eq. (70) we have $m = 1.7083$ for $\Omega_1 = 0$ (uniform rotation), $m = 1.7853$ for $\Omega_1 = 0.1$, $m = 1.8722$ for $\Omega_1 = 0.3$.

At the surface of the celestial body ($R = R_0 = 1$), the toroidal magnetic field can be calculated from Eq. (69). The z-profile of $B_\varphi/A_m J_1(n)$ is plotted in Fig. 1. The solid curve represents $B_\varphi/A_m J_1(n)$ for $m = 1.8722$ and $\Omega_1 = 0.3$, the dashed line for $m = 1.7853$ and $\Omega_1 = 0.1$, the dotted line for $m = 1.7083$ and $\Omega_1 = 0$, namely uniform rotation. Figure 1 shows that the greater the differential rotation, the greater the toroidal magnetic field. Moreover, the maximum of $B_\varphi/A_m J_1(n)$ lies at $z = 0.7$. The toroidal magnetic field is concentrated towards large z, which agrees with the observations of magnetic field at the solar surface that sunspots appear and disappear mainly within $\pm 35°$ of the equatorial plane. At the beginning of each cycle, new active regions appear at high latitudes corresponding to about $z = 0.7$. Therefore it is clear that the differential rotation enhances the action of the p-ω dynamo.

As a note of caution, the above solutions of the p-ω dynamo equation are limited to the conditions $\beta V \ll 1$ and $\beta V < 1$ ($V/V_\mathrm{A} < 1$). This raises the question of whether these solutions can apply directly to astrophysical circumstances. It is natural for us to consider the solar circumstance first. As is well known, at the upper convection region of the sun, the density is about 10^{-7} g cm^{-3} and the magnetic intensity around the granulations is several hundred Gauss so the velocity of the Alfvén wave is about 50 km s^{-1}. If the rotational speed on the surface of the Sun is taken as 2 km s^{-1}, the restrictive condition $V/V_\mathrm{A} \approx 2/50 < 1$ is satisfied. In the region from the surface of the sun downward to a certain depth, the restrictive condition is kept. Therefore, the solar dynamo may be included in our theory in this paper (the detailed research will be made later). In accordance with our estimation for solar-like stars from F-type to K-type, the condition $V/V_\mathrm{A} < 1$ is satisfied also.

It is obvious that the condition $V/V_\mathrm{A} < 1$ is satisfied in upper envelopes of CP stars with stronger magnetic fields and smaller densities than solar-like stars. Since CP stars have no convection zone but a convective core, the α-ω dynamo has to operate in the core. So far no physical mechanism can transport a core-generated field to the stellar surface within the age for CP stars to enter the main sequence (Moss 1989). This is a problem for the dynamo theory of CP stars. We think that the p-ω dynamo is the key to settle this problem, because it can operate in the envelopes of CP stars where no convection exists.

As a final example, consider the supergiant ν Cep (HD 207260) of spectral type A2Ia with strong magnetic field of about 2000 Gauss (Scholz & Gerth 1981) and mean density of about 10^{-4} of the solar mean density (Kenneth 1992). Its Alfvén speed is then calculated to be 150 km s^{-1}, if we suppose that the density of its outer envelope is 10^{-4} of the mean density and its rotational velocity less than 50 km s^{-1}, hence the condition $V/V_\mathrm{A} < 1$ is satisfied. Evidently the dynamo solution given here is of sufficient importance to apply to this astrophysical circumstance.

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