Astron. Astrophys. 317, 919–924 (1997)

ASTRONOMY AND ASTROPHYSICS

Seismic sounding of the solar core: purging the corruption from the Sun's magnetic activity

W.A. Dziembowski[1,2] **and Philip R. Goode**[2,1]

[1] N. Copernicus Astronomical Center, Polish Academy of Sciences, Bartycka 18, PL-00-716 Warszawa, Poland
[2] Physics Department, New Jersey Institute of Technology, 323 Martin Luther King Boulevard, Newark, NJ 07102, USA

Received 25 March 1996 / Accepted 11 June 1996

Abstract. Probing the structure and rotation of the solar core is one of the greatest challenges to helioseismology. We show that the seismic information in the observed low degree solar oscillations which probe the core is severely contaminated. This contamination arises from the Sun's near surface magnetic activity. The effect on the oscillation frequencies varies with the solar cycle–vanishing at solar minimum and growing with increasing surface activity. We demonstrate that this contamination can be quantified and removed after determining the fine structure of the entire oscillation spectrum.

Key words: Sun: oscillations – Sun: interior – Sun: activity

1. Introduction

The solar core, especially the inner core, is where we expect to see the imprint of solar evolution in the hydrogen abundance profile and the angular momentum distribution. Techniques of helioseismology enable one to sound this region. In helioseismology, one does not directly probe the chemical composition, instead one investigates the sound speed, the behavior of which in the inner core is affected by the mean molecular weight. The angular velocity of rotation is directly probed from the fine structure by measuring rotational splitting in the spectrum of solar oscillations. However, the contribution to rotational splitting arising from the core is very small– comparable to the errors in the splittings.

The Sun's p-modes are the only tool we have for seismic investigations. This tool is not ideal because only a small fraction of the observed modes exhibit any sensitivity to core properties. The ones that do are of low degree(l). However, even these modes are primarily sensitive to the Sun's envelope structure which includes its near surface, where the Sun is more complicated and still quite poorly understood. The primary problem here is the effect of vigorous convection both on the mean structure and the adiabaticity of the individual modes of vibration. Inversions of p-mode data to determine the Sun's internal structure have treated the near surface complications as though they possess spherical symmetry. This method used is not applicable to removing the effect of the near surface magnetic field because the perturbation it causes depends on latitude.

In this work, we demonstrate that the magnetic perturbation is significant. We then show how to account for it through the use of the fine structure in the spectrum of solar oscillations. More specifically through the even-a coefficients of Duvall, Harvey, Pomerantz(1986),

$$\nu_{nlm} - \nu_{nl0} = L \sum_{i=1}^{N} a_{i,nl} P_i(\tfrac{m}{L}), \qquad (1)$$

where ν the cyclic frequency of an individual (nlm)-mode, $L = \sqrt{l(l+1)}$, P is a Legendre polynomial, and N is the order of the Legendre expansion provided by the observers. The (nlm) are the radial order, angular degree and azimuthal order of the oscillation, respectively. The fine structure in each (nl) multiplet is labelled by m. The odd a-coefficients arise from the linear effect of rotation. While the even-a's arise from asphericities, like those from magnetic and centrifugal forces.

Before discussing the effect of the surface magnetic field on the oscillations which sound the core, one needs to have a target precision for the frequencies which probe the core.

2. Accuracy needed in p-mode frequencies to precisely sound the core

The Sun's p-mode frequencies are the most accurately known solar parameters. For over a 1000 modes, the precision is better than 3×10^{-5}. For comparison, the precision in the determination of the Sun's mass is 6×10^{-4} and that for its radius 10^{-4}. For a typical 3 mHz p-mode, this precision translates to a 0.1 μHz uncertainty in mode frequency. We shall see that this is the kind of accuracy required in low-l frequencies to test stellar evolution theory. Many low-l modes have quoted errors which are smaller (Elsworth et al. 1994).

Admissible modifications in the input to a standard solar model lead to rather subtle changes in the core's sound speed at the level of a fraction of a percent. The kind of modifications

Send offprint requests to: W.A. Dziembowski

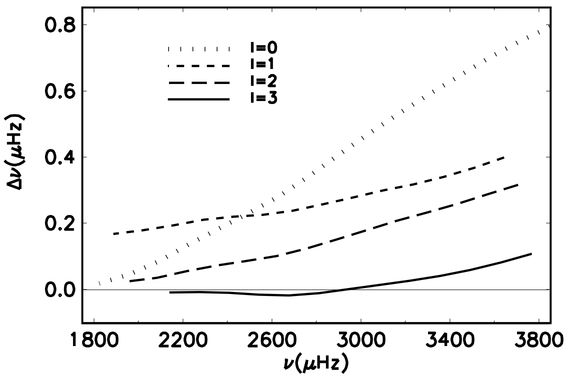

Fig. 1. Frequency shift caused by a 1% increase in $u = \frac{p}{\rho}$ throughout the inner core($r < 0.1R_{\odot}$) is shown as a function of mode frequency for modes with l between 0 and 3.

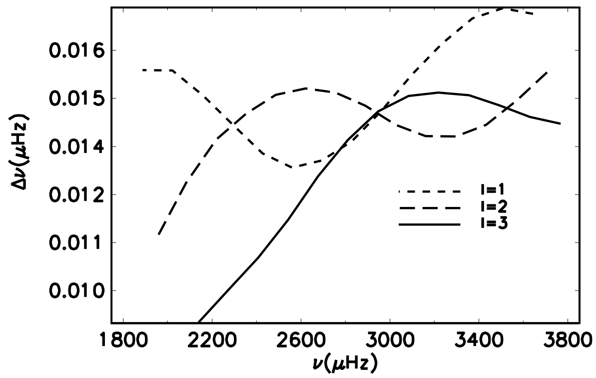

Fig. 2. Increase in the fine structure spacing caused by a 40% increase in rotation throughout the core($r < 0.25R_{\odot}$) is shown as a function of mode frequency for modes ranging from l=1 to 3.

we have in mind, like changing the solar age or certain nuclear reaction cross sections, leave an unmistakable signature only in the inner core ($r < 0.1R_{\odot}$; Dziembowski et al. 1994). To detect such effects, one would need to reach better than a 1% precision in the helioseismic determination of the square of the speed of sound. This latter quantity is the one that is typically chosen to be the primary seismic probe. We use $u = p/\rho$, the square of the isothermal speed of sound, where p is the gas pressure and ρ is the local density.

We calculated the frequency change caused by a 1% increase in u in the inner core($r < 0.1R_{\odot}$). The results are shown in Fig. 1 for the lowest degree oscillations, l=0-3, which are the ones detected in whole disk observations. In the figure, we focus on the whole disk modes found by Elsworth et al. (1994). We will fix out attention on these modes throughout this paper.

It may seem surprising in Fig. 1 that at low frequencies the l=1 modes are more sensitive to the inner core structure than the l=0 modes. This happens because the kernel for sound speed in the inner core goes through zero for low frequency l=0 modes, whilst remaining positive for the l=1 modes. One should recall that in the asymptotic approximation, the kernel is proportional to the energy density and, therefore it is positive definite. However, that approximation fails badly in the inner core.

The probing of the core is sensitive to the differences in $\Delta\nu$ between modes of different l at similar frequencies. Except for the highest frequency modes, the differences are less than 0.5 μHz. Still, this picture sends a rather optimistic message about the prospects for probing the inner core because these differences are large compared to the observers' errors. However, we have to convince ourselves that what we measure as a frequency difference actually reflects a core structure at variance with the solar model rather than an effect arising from near the solar surface. We shall see that the near surface effects, which vary with the solar cycle, cause frequency shifts of up to 0.3 μHz.

Probing the rotation of the solar core is more problematic. Indeed, conflicting announcements have been made concerning core rotation. The results from the IRIS network, Fossat et al. (1995) indicate that the core ($r < 0.25R_{\odot}$) rotates some 40% faster than the envelope. Whereas, results from the BISON network, Elsworth et al. (1995) point to the core rotating more slowly than the envelope.

Precision requirements for measuring just the mean value of rotation in the core are indeed quite high. In Fig. 2, we show the incremental increase in rotational splitting, as a function of frequency for the lowest l modes, that would result from the aforementioned 40% spin-up in core rotation. For spherical rotation, the splittings themselves are the Zeeman-like uniform spacing between adjacent m-peaks in the fine structure of individual (nl)-mulitplets. In whole disk observations, peaks having $(l + m)$-odd are not seen, and, therefore, the observed separation between neighboring peaks is twice the rotational splitting. The rotational splitting is close to 0.45 μHz. Fossat et al. (1995) quote an aggregated splitting for l=1 of $0.459{\pm}0.010\mu$Hz and $0.450{\pm}0.013$ for l=2. Elsworth, et al. (1995) quote errors which are somewhat larger. From Fig. 2, it is clear that the errors in the aggregated splittings are comparable to the signal that would be caused by the 40% change in rotation in the core. We will see that the effect of the near surface magnetic field on the frequency splittings can be as much as an order of magnitude larger.

3. The near surface magnetic perturbation determined from BBSO data

Libbrecht and Woodard (1990) extracted the a-coefficients of Eq. (1) for l=5-60 from their observations at the Big Bear Solar Observatory. They noted that the coefficients changed significantly between the activity minimum in 1986 and 1988 when the Sun was in the rising phase of solar activity. They also found a corresponding sizeable shift in the centroid frequencies (m=0). They argued that the source of these changes must reside very near the solar surface because the size of the effect scaled with mode inertia. They also noted that the source of the perturbation is strongest in the active latitudes. We (Dziembowski and Goode 1991) used these data in a search for the signature of a strong, internal magnetic field. We found the signature of a buried quadrupole toroidal field near the base of the convection zone. We emphasized that in the 1988 data the dominant contributor to the frequencies, by far, was the near surface magnetic field. Our analysis allowed us to localize the source of this effect to within some 1 Mm of the base of the photosphere. A similar conclusion was reached by Goldreich et al. (1991) from their analysis of the centroid shifts.

The data of Libbrecht and Woodard went through several preliminary iterations of which they generously provided us the results. The BBSO data we use in this paper are from their final iteration. The four sets of observations were made in the middle of 1986, 1988, 1989 and 1990, respectively. Each of these data sets spans l from 5 to 140 and covers a three month period. From these data, we find no evidence for an internal magnetic field. Of the aforementioned conclusions arising from use of earlier releases of the BBSO data, only this one is changed.

Our purpose here is to derive information about the near surface perturbation, and use it to evaluate its effect on low-l mode frequencies. To that end, we use Eq. (7) of Dziembowski and Goode (1991), where we drop the term due to internal magnetic field. This equation may be written in the following form,

$$a_{2k,nl} = (a_{2k,nl})_{\rm rot} + P_{2k}(0)\frac{\gamma_k(\nu_{nl})}{LI_{nl}}, \qquad (2)$$

where the $(a_{2k,nl})_{\rm rot}$ describe the contribution of the second order effect of rotation to the fine structure and next term describes the near surface perturbation. The I's are the mode inertia and L is again $\sqrt{l(l+1)}$, and the $\gamma_k(\nu)$ are what we want. We assume a common normalization of the eigenfunctions fixing their amplitude at the base of the photosphere. The contribution of the second order effect of rotation is calculated using the seismically determined rotation rate in the interior(which is determined from the odd-a coefficients). For the p-modes, our ignorance of the rotation in the core is irrelevent here because the second order effect of rotation is dominated by the distortion it causes which arises mostly in the outer layers. Furthermore, its overall effect is small. Only for the lowest-l values and for a_2 does the second order effect of rotation play any role, and even there it is within the errors.

The form of the near surface contribution to even-a coefficients given in Eq. (2) is easy to justify. At this point, we do not specify the perturbation, but we assume that it is located well above the lower turning point of the modes considered. For the set of modes we consider, the lower turning point is located below $0.95R_\odot$. Therefore, the radial eigenfunctions are l-independent. Second, we assume that the perturbing force may be expanded in Legendre polynomials, $P_{2k}(\cos\theta)$. The expansion starts with k=1 because the spherically symmetric part does not contribute to the a-coefficients, and we truncate it at k=3 because we have data up through a_6. With these assumptions, the variational expression for frequency perturbation (nlm)-mode (e.g. Eq. (2) of Dziembowski and Goode 1991) leads to

$$\Delta\nu_{nlm} = \sum_{k=1}^{3} \frac{Q_{klm}}{I_{nl}}\gamma_k(\nu). \qquad (3)$$

The γ_k-coefficients are ν-dependent because the radial eigenfunctions are strong functions of the mode frequency. The angular integral defining the term Q is given by

$$Q_{klm} = \int |Y^l_m|^2 P_{2k}(\cos\theta) d(\cos\theta) d\phi. \qquad (4)$$

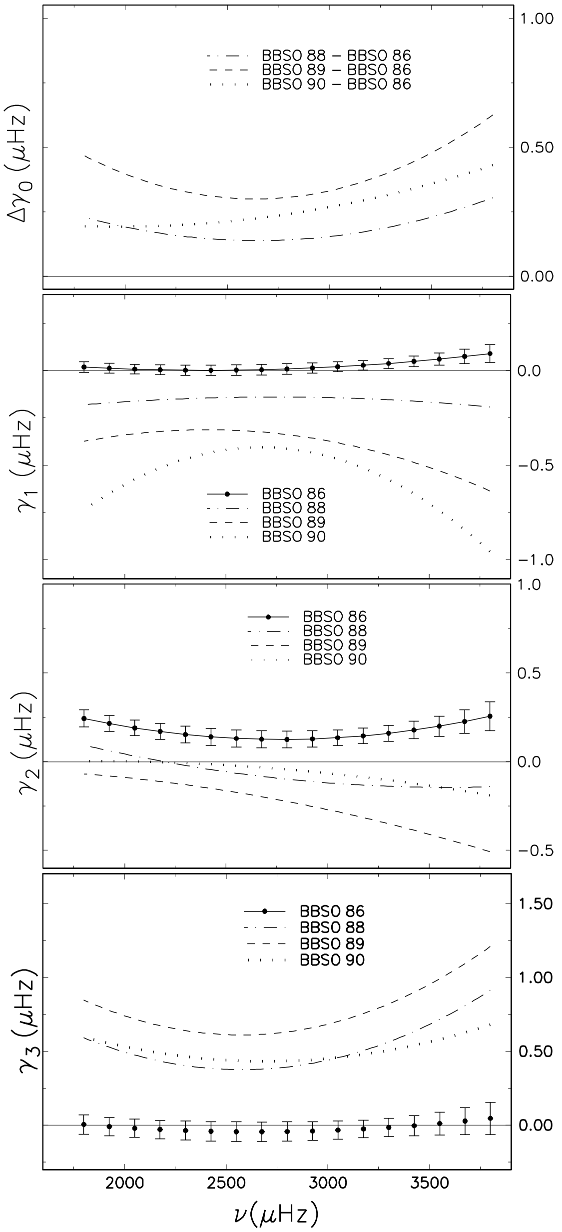

Fig. 3. From top panel to bottom $\Delta\gamma_0$, γ_1, γ_2 and γ_3 are shown as a function of mode frequency. $\Delta\gamma_0$ is calculated with respect to the 1986 BBSO data. The other γ's are calculated for 1986, 1988, 1989 and 1990 BBSO data. The normalization of the radial displacement amplitude at the base of the photosphere was chosen to 3×10^4. With such a normalization, the mode inertia in Eq. (2) is of order unity for modes of frequency near 3 mHz.

The l-dependence of the perturbation enters through this coefficient and I. Modes with higher l's are trapped in the outer layers and naturally have lower inertia. The m-dependence arises only from the Q-coefficients which are polynomials of order $2k$ in m. In the limit of $l \gg k$, we have

$$Q_{klm} \approx P_{2k}(0)P_{2k}(\tfrac{m}{L}), \qquad (5)$$

where

$$P_{2k}(0) = (-1)^k \frac{(2k-1)!!}{(2k)!!}. \tag{6}$$

In this limit, $P_{2k}(\cos\theta)$ contributes only to a_{2k} coefficient, Goode and Kuhn(1990). This approximation is applicable to the set of modes we consider. It doesn't work perfectly for the lowest l-values in the BBSO data, however, its accuracy is well within the quoted errors for those modes. The approximation given by Eq. (5) implies that the centroid shifts have the same form as a spherically symmetric perturbation. Therefore, if the structural inversion uses only modes for which this approximation is valid, then the near surface perturbation does not corrupt the inversions. However, for probing the core, we need low l modes for which Eq. (5) is invalid.

Our objective is to find the functions, $\gamma_k(\nu)$. To this end, we discretize each $\gamma_k(\nu)$ function by a three term Legendre expansion of argument $2(\nu-\bar{\nu})/(\nu_{max}-\nu_{min})$ where ν_{max} and ν_{min} are the maximum and minimum frequencies in the data and $\bar{\nu} = (\nu_{max}+\nu_{min})/2$. We determine the coefficients by a least squares fitting. The resulting γ's are shown in Fig. 3. In the uppermost panel, we show $\Delta\gamma_0$ describing the centroid shift relative to 1986. The quantity γ_0–the spherically symmetric part of the magnetic perturbation–cannot be observationally determined. We emphasize that only the γ_2 coefficient differs significantly from zero in the 1986 data, and that this term corresponds to a quadrupole toroidal field geometry for the near surface perturbation.

One may compare the γ's for 1986 and 1988 in Fig. 3 to their counterparts in Fig. 1 of Dziembowski and Goode (1991). In doing so, we stress that the γ's defined in the latter paper incorporated the $P_{2k}(0)$ factor. Beyond this, there are only slight differences between the two sets of results.

The changes in frequency are linked to the activity cycle. Woodard et al. (1991) have observed that the frequency changes are correlated with the surface magnetic activity on a timescale of months. That leaves no room for doubt that the frequencies reflect magnetic changes occuring in the outer layers. Precisely how this occurs is unclear. Whether it is due to a changing fibril structure or whether there is a role for a subsurface velocity field remains to be seen. Henceforth, we denote the near surface perturbation varying with activity as NSPA. We make this distinction not only to emphasize the fact that we are studying this effect, but also that the NSPA is different from any other near surface effect, like that due to vigorous convection. The ν-dependence of γ results from the fact that the radial eigenfunctions in the outer layers vary significantly with frequency. The fact that the frequency dependence in all the γ's is as weak as it is in Fig. 3 argues for a localization of the perturbing agent very close to the photosphere where the radial eigenfunctions have been uniformly normalized.

4. Frequency shift in low degree modes induced by the NSPA

Having determined γ, we can easily evaluate the corresponding frequency shift for low degree modes using Eq. (3). But now we cannot use Eq. (5), instead we use the exact expression for Q. For the moment, we ignore γ_0 which would lead to l-independent frequency shifts. Such shifts do not affect the results of inversions for either structure or rotation. Fig. 4 shows the frequency shifts for the l=1-3 modes for selected m-values. In addition to the centroid shifts, we show shifts for m-values of the modes visible in whole disk observations. The size of each shift, $\Delta\nu$, should be compared to the observational errors and to the effect of the artificial 1% sound speed perturbation shown in Fig. 1. The errors quoted by Elsworth et al. (1994) for modes in the vicinity of 3 mHz are typically 0.04, 0.05, 0.08, 0.15 μHz for l=0, 1, 2 and 3, respectively. From Fig. 4, we see that the shifts in the 1986 observations were within the errors, while in 1988 the shift is comparable to the errors. In 1989 and 1990, years of high activity, the plotted shifts due the NSPA exceed the observational errors by factors as large as three. A comparison with Fig. 1 forces the conclusion that structural inversions may be significantly corrupted if one does not account for the NSPA. That is, one might well mistakenly interpret the NSPA as a perturbation of the speed of sound in the inner core. The largest shifts are observed at the highest frequencies which reflects the decrease of inertia with increasing frequency. Note from Fig. 1, that the high frequency modes are the most important ones for probing the innermost part of the Sun.

Care must be taken in extracting centroid frequencies from whole disk measurements. The NSPA has a strong m-dependence. In particular, for l=1 the sign of the shift is opposite for m=0 and 1. For l=2 and 3, there is an ambiguity because we see contributions from two different m-values and the contributions have opposite signs in the whole disk observations. How the various m-components contribute to measured frequencies depends on their amplitudes. If energy equipartition is assumed, then the amplitude ratios may be calculated (Christensen-Dalsgaard and Gough, 1982; Christensen-Dalsgaard, 1989). In detail, the result depends on what type of helioseismic observations are made. However, modes of higher m always have larger predicted amplitudes.

In practice, NSPA only corrupts the probing of the core. Information about the envelope relies mostly on high-l modes in which the perturbation behaves as though it were spherically symmetric. For the core, we rely on low-l modes. In this case, the centroids are shifted in an l-dependent way. Furthermore, much of our low-l information comes from whole disk observations for which peaks are seen only if $l+m$ is even, whereas higher l data come from resolved disk observations from which all peaks, in principle, can be identified in a multiplet. For the former kind of observations, extraction of the centroid frequency is not straightforward.

4.1. *Comparsion with direct observations of centroid shifts in whole disk observations*

Frequency shifts in whole disk observations have been measured by Anguera Gubau et al. (1992) and by Elsworth et al. (1994). We can compare the results of their direct observations with our inferences from the BBSO data using the 1986.5 data as a reference. In Table 1, we give the average frequency shift

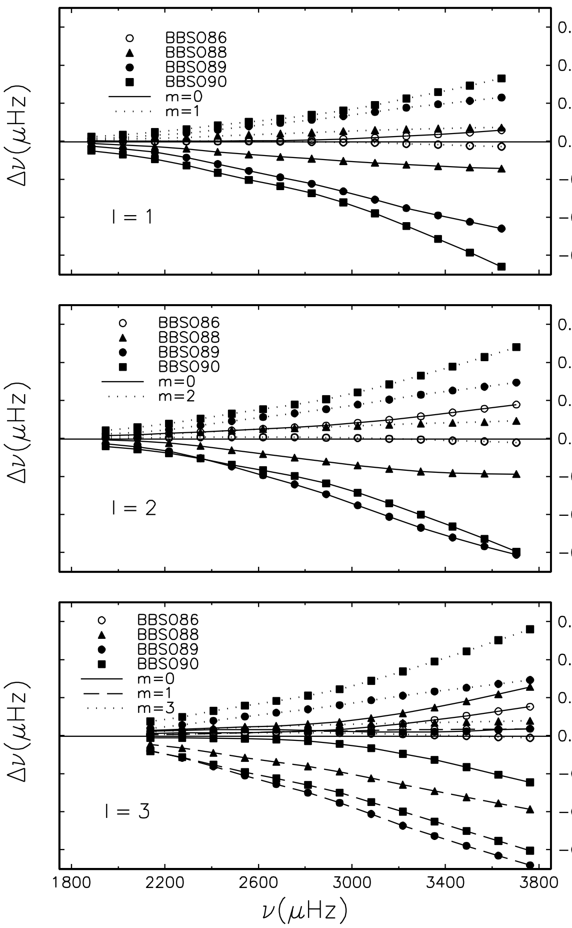

Fig. 4. The frequency shift, $\Delta\nu$, is shown as a function of mode frequency for l from 1 to 3. The shifts are for the centroid(m=0) and the modes visible in whole disk observations–those with $(l + m)$ being even.

Table 1. Mean frequency shifts relative to 1986.5 from BISON data

Year	l=0	l=1	l=2
1988.5	-0.03	0.18	0.29
1989.5	0.21	0.21	0.52
1990.5	0.26	0.28	0.32

relative to 1986.5 from the BISON data for 1988.5, 1989.5 and 1990.5. In Table 2, we show the predicted average frequency shift for the same BISON modes, but with result being calculated from the BBSO data. Here we make use of $\Delta\gamma_0$, as shown in Fig. 3. The averages are from modes in the middle of the 5 min band of oscillations. For l=0-1, the averages are over modes ranging in n from 17 to 24 and for l=2 the averages are over n=16-23.

From Anguera Gubau et al. (1992) the average shifts of frequencies between periods of high activity and low activity are 0.17, 0.46 and 0.23 μHz for l=0, 1 and 2, respectively. The

Table 2. Mean frequency shifts relative to 1986.5 calculated from BBSO data

Year	l=0	l=1, m=1	l=2, m=0	l=2, m=2
1988.5	0.145	0.187	0.039	0.175
1989.5	0.310	0.408	0.102	0.392
1990.5	0.244	0.357	0.062	0.361

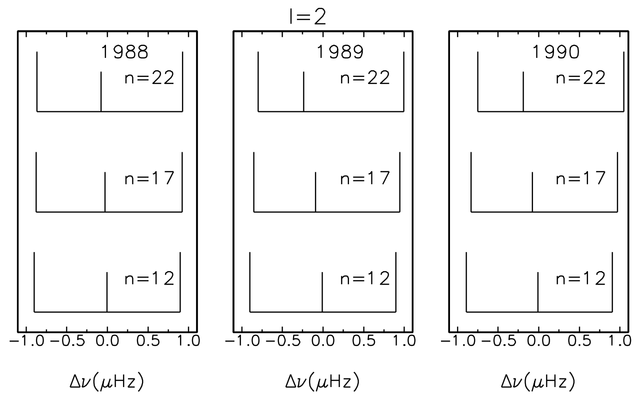

Fig. 5. The $(l + m)$ even fine structure peaks which are those detected in whole disk observations are shown for l=2 for n=12, 17 and 22 as a function of frequency. The non-uniform spacing arises from the NSPA. Modes with higher n are more affected by the NSPA.

corresponding averages of Elsworth et al. (1994) are 0.19, 0.19 and 0.31 μHz.

There is a very large spread in the shifts among individual modes with respect to n for all low-l data both from Anguera Gubau et al.(1992) and Elsworth et al. (1994). With caveat in mind, we conclude that there is an overall agreement between our calculated values and those from whole disk observations. Certainly, there is no evidence that the frequency shifts measured are due to anything but the NSPA. A similar conclusion was reached by Elsworth et al. (1994).

4.2. Effect of the NSPA *on fine structure*

In Figs. 5 and 6, we show the effect on the fine structure arising from the NSPA for l=2 and 3. Except for the lowest order modes, we see that in years of higher activity the NSPA visibily distorts the fine structure pattern from the Zeeman-like uniform spacing predicted by the linear effect of rotation. We note that the symmetric departure from uniform spacing arising from the latitudinal dependence of the rotation rate in the convective envelope is small enough that it could not be noticed in the figures. For these figures, we assumed a constant splitting due to rotation of 0.45 μHz, and added the calculated effect of centrifugal distortion.

This latter effect is well-approximated by

$$\Delta\nu = 1.2 \times 10^{-5} \frac{L^2 - 3m^2}{4L^2 - 3} \nu \ \mu\text{Hz}. \quad (7)$$

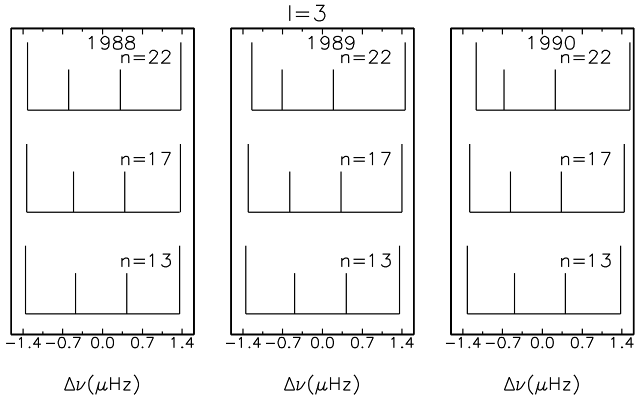

Fig. 6. The $(l+m)$ even fine structure peaks which are those detected in whole disk observations are shown for l=3 for n=13, 17 and 22 as a function of frequency. The non-uniform spacing arises from the NSPA. Modes with higher n are more affected by the NSPA.

One can easily calculate that the size of the effect never exceeds 0.01 μHz, and is below the current accuracy in frequency for individual modes, but it is of the size of the errors in the splittings measured by Fossat et al. (1993).

This visible departure from uniformity would not effect the determination of the internal rotation if we could rely only on measurements of separation between fine structure components with the same value of $|m|$. That is, the frequency differences defined by $\nu_{n,l,m} - \nu_{n,l,-m}$. However, for low degree modes from whole disk observations one does not rely on individual peaks, but rather the fit is to the whole multiplet, and therefore the NSPA must be taken into account.

5. Remarks

We have shown that the NSPA, which varys with the magnetic cycle, corrupts the low degree mode spectrum impeding a precise seismic sounding of the Sun's core. In years of low activity, the effect is well within the observers' quoted errors. One could evade the problem by discarding data from years of high activity. However, such an approach would not take advantage of data from long timebase observational programs like BISON, IRIS, GONG or SOHO. Another way to evade is to keep only low frequency modes which are much less affected by the NSPA, but it is the high frequency modes, as Figs. 1 and 2 show, which contain most of the information about the inner core. We must realize that there is abundant information about the NSPA in the high-l modes, and the NSPA may be precisely calculated from these modes. The requisite information base is vast and is contained in the even-a coefficients. This information may be straightforwardly converted into corrections to low-l mode frequencies.

Acknowledgements. We are grateful to Ken Libbrecht and Martin Woodard for providing us their fine data over the years. We are also grateful to Yvonne Elsworth for the numbers in Table 2. This work was supported by AFOSR-95-0070, NSF-INT-93-14820, NSF-AST-96-14803, and by KBN-2P304-013-07.

References

Anguera Gubau, M., Pallé, P.L., Perez Hernández, F., Régulo, C. & Roca Cortés, T. 1992, A&A, 55, 363.
Christensen-Dalsgaard, J. & Gough, D.O. 1982, MNRAS, 198, 141.
Christensen-Dalsgaard, J. 1989, MNRAS, 238, 977.
Duvall, T.L., Jr., Harvey, J.W., Pomerantz, M.A. 1986, Nat, 321, 400.
Dziembowski, W.A. & Goode, P.R. 1991, ApJ. 376, 782.
Dziembowski, W.A.,Goode, P.R., Pamyatnykh, A.A., & Sienkiewicz, R. 1994, ApJ., 432, 417.
Elsworth, Y., Howe, R., Isaak, G.R., et al., 1994, ApJ, 434, 801.
Elsworth, Y., Howe, R., Isaak, G.R., et al., 1995, Nat, 376, 669.
Goldreich, P., Murray, N., Willette, G. & Kumar, P. 1991, ApJ, 370, 752.
Goode, P.R. & Kuhn, J.R. 1990, ApJ, 356, 310.
Libbrecht, K.G. & Woodard, M.F. 1990, Nat, 345, 779.
Woodard, M.F. & Libbrecht, K.G. 1991, ApJ, 374, L61.
Woodard, M.F., Kuhn, J.R., Murray, N. & Libbrecht, K.G., 1991, ApJ, 373, L81.

This article was processed by the author using Springer-Verlag LaTeX A&A style file *L-AA* version 3.