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Asymptotic analysis of linear non-adiabatic non-radial oscillation of the solar p-modes

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Abstract. The non-adiabatic frequencies of a standard solar model are discussed in this paper. The physics describing the radiative losses is included in the non-adiabatic effects. An asymptotic approximation to the non-adiabatic non-radial p-modes of the Sun is developed. An exact solution of non-adiabatic oscillations, which results in an eigenfrequency equation with the model-dependent surface phase shift, is assumed in the outmost layers, where the asymptotic description is invalid. For the lower degree modes, the phase shift is a function of frequency alone. Compared with an adiabatic approximation, the non-adiabatic approximation of the p-mode frequencies fills more of the gap between theory and observation.

Key words: solar oscillation

1. Introduction

The global solar p-modes have been observed in the past decade or so, thousands of solar oscillation frequencies are measured with a relative accuracy as high as 10^{-5} (Libbrecht, Woodard and Kaufman 1990), and the frequencies of the modes have been successfully modeled at the one percent level as adiabatic oscillations of a standard solar model (Deubner and Gough 1984; Christensen-Dalsgaard, Gough and Toomre 1985; Libbrecht 1988a; Voronstov and Zharkov 1989). The comparison of the solar p-mode frequencies with the theoretical frequencies computed for the solar models reveals some significant discrepancies which are taken to be largely the result of non-modeled details in the structure of the solar atmosphere and the non-adiabatic character of the oscillation modes at the solar surface (Christensen-Dalsgaard 1988; Christensen-Dalsgaard, D*ä*ppen and Lebreton 1988; Cox, Guzik and Raby 1990; Guenther et al.1992).

It is obvious that the solar p-mode surface brightness oscillations are the result of non-adiabatic effects which can be very easily measured (Libbrecht 1990). The solar oscillations are not the adiabatic disturbances because the pulsations disturb the local equilibrium and cause the stellar material to exchange heat with the flow of energy through the star by radiation or convection. In the present paper we neglect the effect of the pulsations on convection and only the perturbations in the radiative field.

Ando & Osaki (1975) and Christensen-Dalsgaard & Frandsen (1983) carried out the first non-adiabatic oscillation calculations for the Sun. Ando & Osaki came to the conclusion that most of the observed p-modes are overstable; in contrast, Christensen-Dalsgaard & Frandsen, in whose paper the effects of departures of thermal equilibrium in the solar model were also included, found all the modes to be stable. More recently, Guzik & Cox (1993) and Cox, Guzik & Kidman (1989) calculated the non-adiabatic p-mode oscillation frequencies for a modern standard solar model. They gave an explanation for the radiative losses by a diffusion approximation, but they did not discuss the stability of the modes. By including the helium diffusion in their standard solar model and by making adjustments to the opacities near the base of the convection zone and near the surface, Guzik and Cox (1993) were able to reduce the discrepancy between the non-adiabatic frequencies of their model and the Sun to $\pm\, 2\mu$Hz.

In Sect. 2, the equations of the eigenfrequencies are obtained by matching an asymptotic solution for the interior region with the exact solution in the exterior region, with an approximation of the phase shift. In Sect. 3, the frequency-dependence of the phase shift β_{ad} is used to compare the p-mode frequencies of the standard solar model with the non-adiabatic frequencies and with the observed frequencies. And finally the results are summarized.

2. The equations for the eigenfrequencies

The non-adiabatic effects associated with the radiative losses and convective transfer add small imaginary parts to the solar p-mode eigenfrequencies, which corresponds to the fact that the eigenfrequencies and eigenfunctions are complex in the mathematical description of the linear non-adiabatic oscillations. The non-adiabatic non-radial oscillation equations are more difficult to solve numerically as compared with the adiabatic equations, so we have focused attention on the nature of the non-adiabatic effects on the frequencies rather than on the size. For simplicity, by considering a surface radiative envelope and employing the Cowling approximation, the differential equations of the solar oscillation can be written as

$$\frac{d}{dr}(r^2\Delta r) = (\frac{\ell(\ell+1)}{\sigma_{ad}^2} - 1)\frac{r^2}{c^2}\zeta + \frac{g}{c^2}r^2\delta r + \Lambda r^2, \quad (1)$$

$$\frac{d\zeta}{dr} = (\sigma_{ad}^2 - N^2)\delta r + \frac{N^2}{g}\zeta + g\Lambda, \quad (2)$$

with

$$\Lambda = \frac{(\Gamma_3 - 1)}{i\sigma_{ad}c^2}[\delta S - \frac{\partial \delta L}{\partial m}]$$

where $\zeta = P'/\rho$ and σ_{ad} is the angular frequency of the adiabatic oscillation. We take the non-adiabatic effects into account in the dissipative terms in Λ.

We introduce $f_1 = \exp[\int -\frac{g}{c^2}dr]$ and $f_2 = \exp[\int -\frac{N^2}{g}dr]$. Assuming $\sigma_{ad}^2 \gg \frac{\ell(\ell+1)c^2}{r^2}$ and eliminating ζ, we obtain

$$\frac{d^2y}{d\tau^2} + [\sigma_{ad}^2 - v^2]y + d = 0, \quad (3)$$

where

$$y = c^{-1/2}hr^2\delta r,$$

$$v^2 = N^2 + c^2(\frac{1}{h}\frac{d^2h}{dr^2} - \frac{1}{c^{1/2}}\frac{d^2c^{1/2}}{dr^2}),$$

$$d = (\frac{f_1}{f_2}c)^{1/2}\frac{d}{dr}(f_2c^2\Lambda) + (\rho c)^{1/2}g\Lambda,$$

$$h = \frac{c}{r}(\frac{f_2}{f_1})^{1/2}$$

and 'acoustic depth', $\tau = \int \frac{dr}{c}$. The 'acoustic potential' v^2 is identical to that introduced by Vorontsov and Zharkov (1989), and has a general increase to the surface, reaching the value $v(0) \approx c/2H$ at the temperature minimum $\tau = 0$, where H is the scale height in the isothermal atmosphere. This value of H corresponds to the acoustic cut off frequency $\sigma_{ad} = c/2H$.

A proper evaluation of the inhomogeneous term d in Eq. (3) requires integration of the thermal equation simutaneously with Eq. (3). However, for simplicity, we treat the effect of the radiative losses as the exponential behaviour, so that d can be expressed as a constant times y. Therefore, to take non-adiabatic effects into account we modify Eq. (3) and write it as a linear Schrödinger-type equation

$$\frac{d^2y}{d\tau} + [\sigma^2 - v^2]y = 0, \quad (4)$$

where the complex frequency, $\sigma = \sigma_{ad} - i\sigma_i$, is the angular frequency of the non-adiabatic oscillation and σ_i is the imaginary part of the eigenfrequency due to the radiative losses. Although σ_i is closely related to the quantity d, it is difficult to obtain the functional relations between d and σ_i (Ge Dunren 1965).

Using the method of the phase functions known in quantum mechanics (Babikov 1976), we obtain the solution of Eq. (4) on the form

$$y = A(\tau)\cos[\sigma\tau - \pi\alpha(\tau) - \frac{\pi}{4}] \quad (5)$$

and

$$\alpha(\tau) = \alpha_{ad}(\tau) - i\sigma_i(\tau - \tau_0) - \Delta\alpha, \quad (6)$$

where $\alpha_{ad}(\tau)$ is the phase of the adiabatic approximation for $\sigma_i = 0$, τ_0 is the 'acoustic depth' of the radiative envelope and $\Delta\alpha$ is given by

$$\tan\Delta\alpha = \frac{i\sigma_i \tan(\sigma_{ad}\tau - \pi\alpha_{ad})}{\sigma_{ad} + \sigma\tan(\sigma_{ad}\tau - \pi\alpha_{ad})}, \quad (7)$$

with the additional requirement that the amplitude function $A(\tau)$ should satisfy the condition

$$\frac{dy}{d\tau} = -\sigma A(\tau)\sin[\sigma\tau - \frac{\pi}{4} - \pi\alpha(\tau)]. \quad (8)$$

In addition, the phase function $\alpha(\tau)$ is determined only by the structure of the outer solar regions. This provides us with a very convenient means of testing the models of the solar envelope independently without any recourse to the models of the solar interior. Then, the phase function is determined by the first-order nonlinear differential equation

$$\frac{d\pi\alpha}{d\tau} = \frac{v}{\sigma}\cos^2[\sigma\tau - \frac{\pi}{4} - \pi\alpha(\tau)]. \quad (9)$$

At the surface the value of $\alpha(0) = \alpha_s$ is determined by the corresponding boundary condition and may be written as

$$-\frac{\pi}{4} - \pi\alpha(0) = \arctan[-(\frac{v(0)}{\sigma^2} - 1)^{1/2}] \quad (10)$$

Finally, the asymptotic description of p-modes is developed by matching the asymptotic solution of the oscillation equation in the interior with the exact (non-asymptotic) solutions near the surface (Vorontsov and Zharkov 1989), and this leads to the eigenfrequency equation with the boundary condition Eq. (10)

$$F(\omega) = \pi\frac{n + \alpha(\sigma)}{\sigma} \quad (11)$$

and

$$\omega = \frac{\sqrt{\ell(\ell+1)}}{\sigma_{ad}},$$

where ℓ is the degree, n is the radial order of a p-mode and

$$F(\omega) = \int_{r_1}^{R}(\frac{r^2}{c^2} - \frac{\ell(\ell+1)}{\sigma_{ad}^2})^{1/2}\frac{dr}{r}$$

where r_1 is the inner turning points corresponding to $\sigma_{ad}^2 = \frac{\ell(\ell+1)c^2}{r^2}$. The function $F(\omega)$ is determined by the distribution of the velocity of sound in the solar interior.

3. The effects of radiative losses

The frequency-dependence of the phase shift is given by the function

$$\beta = -\sigma^2 \frac{d}{d\sigma}(\frac{\alpha}{\sigma}), \tag{12}$$

In addition, β can be written as

$$\beta \approx \beta_{ad} + \sigma_{ad}^2 \frac{d}{d\sigma_{ad}}(\frac{\Delta\alpha}{\sigma_{ad}}), \tag{13}$$

where the function β_{ad} corresponds to the adiabatic oscillations and the function β is obtained from the full non-adiabatic treatment.

For the low and intermediate-degree ℓ, the differentiation of the asymptotic Eq. (13) with respect to frequency gives

$$\frac{d\sigma}{dn} \approx \frac{\sigma}{n+\beta}, \tag{14}$$

The determined results of β_{ad} from the observed frequencies with $5 \leq \ell \leq 20$ (Libbrecht and Zirin 1986) and with $30 \leq \ell \leq 60$ (Libbrecht 1990) are shown in Fig. 1, denoted by the small circles and the small x, respectively. The determined result of β_{ad} from the theoretical eigenfrequencies of the non-radial adiabatic oscillations for the JCD standard model (Christensen-Dalsgaard 1989) is shown in Fig. 1 by the solid line.

Fig. 1 shows the significant difference between the results of β coming from the observed frequencies and the theorectial adiabatic frequencies of the same modes of oscillations for the standard solar model. This leads to the conclusion that the main source of discrepancies in the frequencies of the p-mode oscillations is due to the structure of the outer solar regions rather than the solar interior. In addition, the values of β predicted by the standard solar model are too low. It is clearly seen in Fig. 1 that the maximum discrepancy between the frequency dependence of the phase shift determined from the observation and that from the standard model is $\Delta\beta \approx 0.15$.

The main problem in all our attempts to make the standard model in agreement with the observed frequency of a p-mode oscillation is to modify β. We have taken a first step that the nature of the non-adiabatic effects rather than the size of the non-adiabatic effects on the frequencies is considered. The modification of the non-adiabatic approximation is obtained as follows:

$$\Delta\beta = \beta - \beta_{ad} = \sigma_{ad}^2 \frac{d}{d\sigma_{ad}}(\frac{\Delta\alpha}{\sigma_{ad}}), \tag{15}$$

where $\Delta\beta$ is determined by theory and related to the quantity d. The non-adiabatic effects of the perturbation of the radiative fields may significantly reduce the difference between observation and theory.

It is interesting to compare the improvement in the agreement of oscillation frequencies by the solar model with the variations in β. From Eq. (15), it is known that σ_i will be non-zero and that this will induce a correction to β.

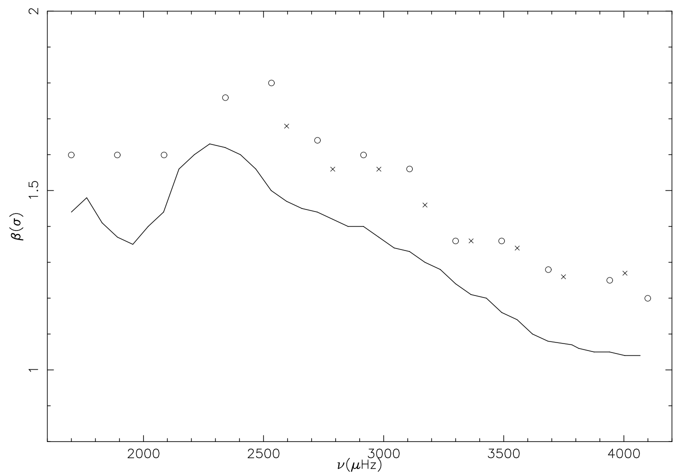

Fig. 1. The frequency dependence of the phase shift obtained from the observation and from the theoretical adiabatic eigenfrequencies of the JCD standard solar model.

The contribution to the imaginary part of the mode eigenfrequency σ_i due to the radiative losses is given by Goldreich & Kumar (1991)

$$\sigma_i \sim \frac{L_\odot}{M_\alpha c_t^2}(\frac{\sigma}{\sigma_{ac}})^2, \tag{16}$$

where $L_\odot$ is the solar luminosity, c_t is the sound speed at the photosphere, $\sigma_{ac} \sim g/c_t$ is the acoustic cutoff frequency and M_α is the mass of mode α. Eq. (16) provides an estimate of the order of magnitude for σ_i.

For the solar 5-minute oscillations, it takes place in the very outer surface layer where $\sigma\tau \sim \sigma\frac{\delta r}{c}$ is large considering variation in sound speed very slowly with r, here δr is a depth below the surface, then from Eq. (7) and using by the fact that the imaginary part of the eigenfrequency, σ_i,is much smaller than the real one, we may obtain

$$\tan\Delta\alpha \sim \frac{\sigma_i}{\sigma_{ad}} \tag{17}$$

From the combination of Eqs. (15),(16) and (17), we have a simple form of $\Delta\beta$ when $\nu \leq 3$mHz, i.e.

$$\Delta\beta \sim \frac{\sigma_i}{\sigma_{ad}} \tag{18}$$

If the convective energy transport is neglected, the non-adiabatic effects associated with the radiative losses act as a source for the line widths, and therefore the observation of the line widths of the solar p-modes can be used to provide the estimate of the radative loss σ_i. The observations by Libbrecht (1988b) establish that $\sigma_i \sim 10^{-3}(\frac{\sigma}{\sigma_0})^{4.2}\sigma_0$(mHz) at $\sigma = 2$ mHz, and in which we use σ_0 as the unite of σ and define $\sigma_0 = 1$mHz. As a result, for $\nu \leq 3$mHz we obtain from Eq. (18)

$$\Delta\beta \sim 10^{-3}(\frac{\sigma}{\sigma_0})^3 \tag{19}$$

Eq. (19) only gives rough estimates of non-adiabatic effects on the solar p-mode oscillations. For example, if $\nu = 2.0$mHz, we may obtain $\Delta\beta \sim 0.001$ from Eq. (19). The result gives rise to some improvement in the agreement between the normal adiabatic models and the observations. However, the cases are very limited."

4. Discussion

It is clear that the discrepancy between the observation and adiabatic theory is reduced for the non-adiabatic determination of β but has not eliminated.

The discrepancy between the frequency dependence of the phase shift determined from observation and that from theory opens a new way for the diagnostics of the outer solar regions. We have built the theory of the non-adiabatic non-radial oscillation which includes the non-adiabatic terms associated with the radiative losses. But further and extensive studies are still required, including, for example, the physical effects associated with the turbulent convective heat and momentum flux.

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