Astron. Astrophys. 318, 275–288 (1997)

ASTRONOMY AND ASTROPHYSICS

Spin and orbital angular momentum exchange in binary star systems

R. Keppens

Kiepenheuer-Institut für Sonnenphysik, Schöneckstr. 6, D-79104 Freiburg, Germany

Received 2 December 1995 / Accepted 22 June 1996

Abstract. We present a comprehensive model for studying the angular momentum (AM) evolution in binary star systems, taking into account: (i) evolutionary effects of both component stars on the Pre-Main Sequence (PMS), on the Main Sequence (MS) and during the (initial) ascent onto the giant branch; (ii) spin-orbital AM exchange through 'tidal' interactions; and (iii) AM loss from one or both component stars due to stellar winds. This allows us to assess whether, when and how the synchronization of spin and orbital rotation rates, and the circularization of eccentric orbits, is achieved within a composite system of two evolving stars.

We develop the formalism for spin and orbital AM exchange in binary systems such that 'standard' (and sometimes rivaling) theories of tidal interactions and stellar winds can easily be incorporated and compared, in so far as they lead to qualitative differences in the overall AM evolution.

When using our model for a binary system of solar-type stars, we use a 2-component model for each star (as in MacGregor & Brenner 1991), with possibly differentially rotating core and envelope zones. These two zones are coupled through visco-magnetic mechanisms. The model calculations presented illustrate how the combined effects of structural evolution, tidal interactions, stellar winds, and the visco-magnetic coupling mechanisms lead to rich scenarios for the AM evolution. We concentrate in this paper on the model and its potential for gaining new insights in the physical effects that play a role in the binary AM balance. It is pointed out how it can be used for a direct interpretation of many observational results, but this is postponed to a forthcoming paper (Keppens et al. 1996).

Key words: stars: binaries: close – stars: evolution – stars: rotation

1. Introduction

In this paper, we focus on the angular momentum (AM) evolution within binary star systems, and use the structural evolution of individual stars as a necessary ingredient. It is our intention to present a model for the evolution of the AM balance within binary star systems, so that the different effects that play a role in this evolution can be identified and compared. For a recent review on binary star topics, we refer to Shore (1994). The kind of binary star systems to which we pay particular attention are well-separated systems of solar-type stars, although it should be clear that this restriction is not inherent to the model itself. Rather, it is inspired by the fact that most of the effects playing a role in the spin AM evolution for single solar-type stars are grossly understood (see, e.g., Keppens, et al. 1995; and references therein), so that we can concentrate more on how spin and orbital AM is exchanged within the system.

Therefore, we develop a formalism which is physically transparent, but at the same time is sufficiently sophisticated to allow for a self-consistent study of the combined effects of stellar structural evolution, mutually exerted tidal interactions, and AM loss induced by stellar winds. We address how binary systems get synchronized and circularized, the role played by phases of rapid structural evolution (PMS and ascent onto the giant branch), and touch upon the evolution of Chromospherically Active (hereafter, CA) binary star systems (with a wealth of observations, see Strassmeier et al. 1993). As this is an ambitious endeavour, we choose to postpone a direct confrontation with observational results to a later paper (Keppens et al. 1996). Here, we discuss the mathematical and physical assumptions in detail, and illustrate how the model is used and how its results must be interpreted physically. At the same time, we gain new insights in the mechanisms regulating the AM in binary star systems, and make qualitative statements about its predictions.

Of course, we are not the first to investigate the AM evolution of binary star systems, taking into account evolutionary effects. Of direct relevance to our study are the results of Zahn & Bouchet (1989), who studied tidal effects on the PMS phase in late-type binaries; and those of Habets & Zwaan (1989) concerning evolutionary effects on tidal interactions from the MS up to the red giant branch (taking into account magnetic braking). The main advantadge of our model with respect to these and other earlier investigations lies in its ability to incorporate all relevant effects at once, and thus perform a self-consistent cal-

Send offprint requests to: R. Keppens, FOM institute for Plasma-Physics Rijnhuizen, P.O. Box 1207, 3430 BE Nieuwegein, The Netherlands

culation all the way from early PMS phases to the giant branch. Moreover, we can compare different theories for tidal and other effects (e.g. winds) in a straightforward manner.

In Sect. 2, we explain the underlying assumptions entering the model. We make the link with standard tidal theories, and results pertaining to single stars. In Sect. 3 we discuss the model qualitatively, and show how it is used from a complete discussion of two illustrative cases. We consider binary systems of identical solar-type stars, and discover intruiging aspects of binary AM evolution. They represent two physically distinct cases where tidal effects play dominant and secondary roles, respectively. A discussion of these results is given in Sect. 4.

2. Equations for AM evolution

2.1. Preliminaries: the 2-body problem

At the lowest level of approximation (Newtonian approach), the two components are mass points M_1 and M_2, revolving in elliptical orbits about their combined center of mass (CM). Acknowledging that the motion is planar, the constant magnitude of the orbital AM J_{orb} with respect to the CM is given by

$$J_{orb} = \frac{M_1 M_2}{M_1 + M_2} \Omega_{orb} a^2 \sqrt{1 - e^2}, \tag{1}$$

where Ω_{orb} denotes the orbital revolution rate, and a is the semi-major axis of the elliptic orbit with eccentricity $0 \leq e < 1$. Of course, Kepler's third law immediately links the orbital revolution period with the semi-major axis.

2.2. Allowing for mutual tidal torques

In order to address the exchange of spin and orbital AM in real binary systems, one must allow for the mutual torques exerted on the envelope regions of the component stars, as a result of the gravitational disturbance induced by the mere presence of a component star. To keep our formalism physically transparent, we follow a Darwinistic approach and note that, qualitatively speaking, as a result of a disturbing mass point M_2, the spinning star M_1 will be deformed from an oblate spheroid (the oblateness due to its own spin) to an ellipsoid (assuming that its spin axis is perpendicular to the orbital plane; see however Stawikowski & Glebocki 1994, for some peculiar observational facts). As a result of some a priori unspecified viscous mechanism acting in the envelope region of M_1, this ellipsoid will be misaligned with the vector $\boldsymbol{d}_{12}$ connecting the center of mass of M_1 with the mass point M_2, and its misalignment is denoted by the angle α_1. When we further assume that neither the deformation of M_1, nor this misalignment angle α_1 change during one orbital revolution P_{orb} (an admittedly crude assumption, especially for very eccentric orbits), it is straightforward to show that the mass point M_2 exerts a torque on the envelope of the star M_1 whose magnitude with respect to CM is given by

$$S_{12} = \frac{3}{2} \frac{G M_1 M_2}{M_1 + M_2} \frac{1}{d_{12}^3} (B^{(1)} - A^{(1)}) \sin(2\alpha_1). \tag{2}$$

In this expression, G is the gravitational constant, and the 'ellipsoidal' deformation of M_1 due to its own spin and the presence of the disturbing mass point M_2 is quantitatively incorporated in the difference between the equatorial principal moments of inertia $A^{(1)}$ and $B^{(1)}$ (we envision, for simplicity, that equatorial and orbital planes coincide). To calculate the net effect over one orbital revolution, one must substitute for an orbit-averaged factor $< 1/d_{12}^3 >$, since our assumptions guarantee that all other factors in S_{12} are constants over one orbit. At the same level of qualitative approximation, we recognize that the deformation as measured by the difference $B^{(1)} - A^{(1)}$ relates ultimately to the disturbing tidal potential raised by the mass point M_2 at mean distance a on the surface of M_1, so that we expect a proportionality $B^{(1)} - A^{(1)} \propto M_2/a^3$. As the distortion will most likely affect the envelope region of M_1 only, and in keeping with dimensions, we therefore suggest to use

$$B^{(1)} - A^{(1)} = \frac{M_{1,env} R_1^2}{5} \left[\frac{M_2 R_1^3}{M_1 a^3} \right], \tag{3}$$

in which R_1 is the radius of M_1, and $M_{1,env}$ is the mass of the envelope zone of M_1 only. The term between brackets ensures the desired proportionality, in a dimensionless fashion. The choice $M_{1,env} R_1^2/5$ is inspired from the expressions for the principal moments of inertia of a homogeneous ellipsoid of mass M and semi-major axes (a, b, c) where the difference is $[B - A] = \frac{M}{5}(a^2 - b^2)$. An expression for the misalignment angle α_1 can be taken in accord with more standard theories of tidal interaction, namely

$$\alpha_1 = \frac{R_1^3}{G M_{1,env} \tau_{1,V}} (\Omega_{orb} - \Omega_{1,env}). \tag{4}$$

Here, $\Omega_{1,env}$ denotes the spin rate of the envelope region of M_1. In this approach, the angle α_1 dissappears when orbit and spin are exactly synchronized, so that the torque S_{12} vanishes. The timescale $\tau_{1,V}$ plays a crucial role in the time evolution of the binary system: it mimics the time within which 'viscous effects' within the envelope zone of the star M_1 are operative, causing the tidal 'lag'. However, to bring our formalism in line with more sophisticated treatments of tidal interactions in binary systems, we will assign this timescale $\tau_{1,V}$ a much broader meaning.

Clearly, if this were the only torque operative on the envelope zone of M_1, we could derive a synchronization timescale $\tau_{1,sync}$ (as e.g., in Zahn 1977; Tassoul & Tassoul 1992) defined from

$$\frac{1}{-(\Omega_{1,env} - \Omega_{orb})} \frac{d\Omega_{1,env}}{dt} = \frac{1}{\tau_{1,sync}}. \tag{5}$$

To this end, we assume rigid rotation of the envelope of M_1 so that its spin AM is $J_{1,env} = I_{1,env} \Omega_{1,env}$, with $I_{1,env}$ an appropriate moment of inertia for this envelope (obviously, in terms of the afore-mentioned equatorial principal moments of inertia $A^{(1)}$ and $B^{(1)}$, this $I_{1,env}$ represents some suitable mean thereoff), and for long timescales $\tau_{1,V}$ or small differences $\Omega_{orb} - \Omega_{1,env}$, we get

$$\tau_{1,sync} \simeq \frac{5}{3} \frac{I_{1,env}}{M_1 R_1^2} \frac{M_1}{M_2} \frac{M_1 + M_2}{M_2} \left(\frac{a}{R_1} \right)^6 \tau_{1,V}. \tag{6}$$

Hence, the synchronization timescale τ_{sync} has the essential dependence $\tau_{sync} \propto (a/R)^6 \tau_V$, and it is this realization that makes it possible to implement a more realistic treatment of the problem of AM exchange in binary systems into our naive formalism presented so far.

2.3. Allowing for secular changes in the orbit

Indeed, with our *ad hoc* description of the deformation of M_1 due to the presence of the component star M_2, we circumvent the tedious but physically more appropriate decomposition of the gravitational field external to each component star in terms of spherical harmonics and locally corotating Fourier components (see Zahn 1977; and references therein). In such a description, the tidal problem accounts fully for the fact that the true distortion of M_1 (primary star) is produced by the gravity field of the component star M_2 (secondary star), which modifies the stars (M_1) own gravitational field. This, in turn, submits M_2 to additional accelerations in directions parallel and perpendicular to the line joining their centers in its revolution about M_1. The situation is similar viewed from M_2's perspective as a primary star. One can then use classical principles of perturbation theory, to derive the secular changes in the orbital elements. Through these secular changes in the orbital elements, one essentially tracks the redistribution of energy and AM within the binary system (see e.g. Collins 1989). It turns out that the factors multiplying the radial and tangential components of the additional accelerations in the equation governing the secular change in the orbital eccentricity e are odd, respectively even, functions of the mean anomaly. In that respect, it is noteworthy that our naive picture presented above, through the assumption of constant deformations and misalignment angles $\alpha_{1,2}$ during one orbit, essentially makes the extra accelerations dependent on the separation distance d_{12} only, so that both are even functions of the mean anomaly. Therefore, when averages over one orbital revolution for the secular change in orbital eccentricity are desired, only the tangential accelerations related to the torques ($S_{12} + S_{21}$) enter our formalism. Hence, we ignore the fact that the radial accelerations may produce a net effect over one orbit, although, in reality, they need not be an even function of the mean anomaly. Formally, our approach states that the mass point M_2 in its revolution about M_1, and in the spherical polar coordinates of the primary, varies its distance as d_{12}, but its polar angle in the orbital plane ϕ_{12} remains fixed at α_1, which is fixed at the polar angle $\theta_{12} = \pi/2$ (no torques perpendicular to the orbital plane), while subject to the non-spherically symmetric potential of M_1 as given by

$$\begin{aligned}\Phi_1(d_{12},\theta_{12},\phi_{12}) = & -\tfrac{GM_1}{d_{12}} \\ & +\tfrac{G}{d_{12}^3}\tfrac{(A^{(1)}-B^{(1)})}{4} P_2^2(\cos\theta_{12})\cos(2\phi_{12}) \\ & +\tfrac{G}{d_{12}^3}\left(C^{(1)} - \tfrac{A^{(1)}+B^{(1)}}{2}\right) P_0^2(\cos\theta_{12}).\end{aligned} \tag{7}$$

Here, $C^{(1)}$ is the third (polar) principal moment of inertia for the 'ellipsoidal' M_1 and the function P_l^m is the associate Legendre function of the first kind of order m and degree l. Of course, only the second term in Φ_1 as written above is important for our purposes. Completely analogous formulae apply when the role of M_1 and M_2 are interchanged. The formal Eq. (7) serves to illustrate how our approach to treat the tidal interaction between the two component stars deviates from the standard tidal theories. Through our assumption of a deformation as measured by $(A^{(1)}, B^{(1)}, C^{(1)})$, a misalignment angle α_1, and $\theta_{12} = \pi/2$, which are taken as constant for a whole orbital revolution, the secular change of the eccentricity after averaging over one orbit, worked out up to fourth order in e (as will be used in the calculations), is

$$\begin{aligned}-\frac{1}{e}\frac{de}{dt} = & \tfrac{15}{4}(1+\tfrac{7}{2}e^2)\tfrac{G}{\Omega_{orb}a^5} \\ & \times \left[(B^{(1)}-A^{(1)})\sin(2\alpha_1)\right. \\ & \left.+(B^{(2)}-A^{(2)})\sin(2\alpha_2)\right].\end{aligned} \tag{8}$$

This yields an approximate expression for a circularization timescale $\tau_{circ}^{-1} \equiv \frac{-1}{e}\frac{de}{dt}$ (as e.g., in Zahn 1977; Tassoul & Tassoul 1992) as (take $M_1 = M_2$)

$$\tau_{circ} \simeq \frac{1}{3}\tau_V \left(\frac{a}{R}\right)^8 \frac{\Omega_{orb}}{\Omega_{orb}-\Omega_{env}}. \tag{9}$$

2.4. Orbital stability and τ_{circ}

In the form presented so far, and essentially through the adopted dependence of the misalignment angles α_i on the difference $(\Omega_{orb}-\Omega_{i,env})$ we get that the circularization timescale as given by (9) contains the term $(\Omega_{orb}-\Omega_{env})^{-1}$, expressing that for synchronous systems, no torques remain in our formalism, and the circularization time becomes infinite.

However, as long as the orbit is (even slightly) eccentric, the tidal interactions are differential during one orbit, and eventually tend to drive the system in the lowest energy state consistent with the AM conservation: a circular orbit with synchronized spins. We therefore need to correct the equation for the evolution of the orbital eccentricity. To this end, we rely on established tidal theory to incorporate a more appropriate stability condition (separating positive and negative de/dt). Quoting Zahn (1977): "a circular orbit is stable only if the ratio of the rotational to the orbital velocities satisfies the inequality $\Omega_{env}/\Omega_{orb} < 18/11$, a result first established by Darwin". Within our approximative approach, we therefore incorporate this factor $18/11$ in the dependence of the misalignment angles α on rotation rates, but only in the equation governing the orbital eccentricity. This does not influence the synchronization timescale as given by (6) significantly, and achieves that circularization (through tidal torques) takes place on a timescale (set $\Omega_{orb} = \Omega_{env}$ as synchronization is reached first) $\tau_{circ} \simeq \frac{11}{21}\tau_V \left(\frac{a}{R}\right)^8$.

2.5. Standard tidal theories and the parameter τ_V

Using expressions (6) and (the corrected) (9) for the synchronization and circularization timescales, we now address how we can, at least qualitatively, introduce the essential aspects of more standard theories of tidal interaction. As the timescale τ_V is the only unspecified parameter so far, we explain how exactly this parameter can be used to model any kind of tidal effect that is believed to play a role in the AM balance of the binary system.

As mentioned before, a realistic treatment of tides must make appropriate decompositions of the external gravitational fields, in locally corotating Fourier components. Then, one incorporates the fact that the deformation of each component is not merely a static phenomenon, and that *both* an equilibrium tide and a dynamical tide are raised on each star (Zahn 1977; Zahn 1989).

The *equilibrium tide* is the hydrostatic adjustment of the structure of the star to the perturbing force exerted by the companion. Our formalism resembles this equilibrium tide. As a lag of this equilibrium tide occurs when a dissipative mechanism is believed to operate over the whole envelope zone of the star, the timescale τ_V must be a friction timescale derived from envelope stellar parameters. According to Zahn (1977, 1989), for stars possessing a convective envelope, turbulent dissipation in this envelope is the most effective viscous mechanism. Following Zahn (1977), we get a typical friction time from $\tau_V \sim (MR^2/L)^{1/3}$, with L the stellar luminosity. This makes the circularization and synchronization timescales mentioned above, for stars with convective envelopes, mainly dependent on the eight and sixth power of a/R, respectively. Other timescales τ_V derived from (envelope) stellar parameters alone, may be inserted to mimic other dissipative mechanisms retarding the equilibrium tide.

The non-static component of the tide raised on the star may also affect the AM evolution. Indeed, due to the elastic properties of the star, the dynamical component of the perturbance may lock into resonance with free modes of oscillation of the star. It is clear that our formalism neglected this possibility, as we took out any time dependence of the perturbing potential felt by each component during one orbit, except for the mutual distance. This is what underlies expression (7). Of course, we can model a *dynamical tide* by introducing an artificial dependence of the timescale τ_V on the parameters appropriate for such resonance locking. To recover a Zahn-like description, we proceed as follows. Evidently, since the resonance locking effect is no longer solely a property of the stellar envelope alone, but requires the revolution of the component star at mean distance a, one may expect $\tau_V \propto aP_{orb}$. The proportionality must be expressible in terms of stellar parameters which indicate how the excited oscillations suffer damping within the star, thereby transfering orbital and spin AM. In any case, the synchronization timescale for such tidal interaction then has the essential dependence $\tau_{sync} \propto (a/R)^{17/2}$, and the circularization time becomes $\tau_{circ} \propto (a/R)^{21/2}$. As pointed out by Zahn (1977), in stars with convective envelopes, the mechanism of the equilibrium tide is much more effective in redistributing spin and orbital AM, through its weaker dependence on the ratio a/R. The dynamical tide has been advocated to operate effectively in binary systems containing a star with a radiative envelope. The excited oscillations would thereby suffer from radiative cooling in the stellar outer region.

More recently, Tassoul (1987, 1988; see Tassoul & Tassoul 1992 and references therein) has proposed yet another mechanism which may redistribute spin and orbital AM even more efficiently than any of the afore-mentioned tides. He notices that the mere deformation of each star in the system (hence the non-vanishing of the difference $[B^{(1)} - A^{(1)}]$) necessarily induces large-scale meridional flow patterns in each component star. In this *hydrodynamical spin-down* mechanism, an Ekman-type suction layer of thickness δ/R develops near the stellar surface, and it is this layer that regulates the overall AM transport. Clearly, the timescale τ_V must be taken inversely proportionate to the Ekman-layer thickness, and as this mechanism is viewed as a spin-down problem in the reference frame rotating with the orbital angular velocity Ω_{orb}, it is necessary to take $\tau_V \propto [P_{orb}(\delta/R)]^{-1}$. Following Tassoul & Tassoul (1992), the essential dependence of the Ekman-layer thickness is $\delta/R \propto (a/R)^{3/8}$ (a dependence of this thickness on mean distance a is to be expected), and we find $\tau_{sync} \propto (a/R)^{33/8}$ and $\tau_{circ} \propto (a/R)^{49/8}$. For stars with a convective envelope, such dependences on the ratio a/R would make the hydrodynamical spin-down mechanism even more effective than the equilibrium tide. The same statement applies also to stars with radiative envelopes. On the other hand, Rieutord (1992) showed that the large-scale flows driven by Ekman pumping in the spin-up/down of a tidally distorted star do not efficiently reduce the synchronization timescale. It is argued in Tassoul & Tassoul (1992) how the analysis of Rieutord (1992) involves a mixup of the relevant spin-down timescale, and the much longer viscous decay time of residual motions in the distorted star.

We note that the physical mechanism underlying all afore-mentioned tidal interactions can be traced to viscous forces operative in the whole or in a part of the stellar envelope. Especially in stars with convective envelopes, both Zahn (1989) and Tassoul (in Tassoul & Tassoul 1992) agree that the increased macroscopic viscosity due to turbulence in such envelopes plays an essential role. In this respect, the sophistication inherent in either description must ultimately rely on a parametrization for turbulent viscosity, which is poorly understood. In the formalism presented so far, this dependence on dissipative effects is thereby conveniently incorporated in the parameter τ_V.

2.6. Allowing for structural evolution

Aside from the controversy between current tidal theories regarding the large-scale mechanism responsible for the spin and orbital AM exchange (for instance in stars with convective envelopes: is it the hydrostatic adjustment in combination with a viscous lag, or is it the hydrodynamical spin-down achieved by the development of a free-boundary viscous layer?), there is general agreement that the true AM evolution in binary systems must be influenced by the structural evolution of each component star separately. Especially during the PMS, it is known that for single stars, the structural evolutionary effects dominate the spin AM evolution (Keppens et al. 1995). These evolutionary effects include, for instance, the gradual development of a radiative core and a convective envelope in solar-type stars, together

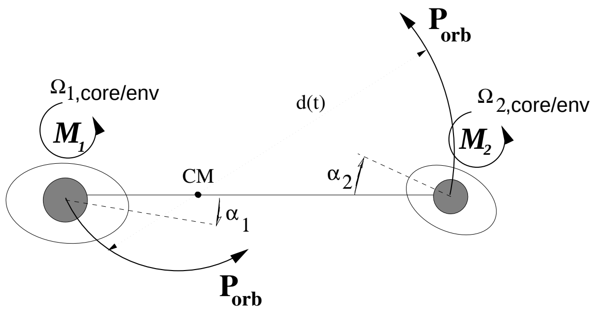

Fig. 1. A schematical representation of the ideas underlying the mathematical approach to the AM evolution in the binary system. The tidal effects over one orbit are modeled as deformations and misalignment angles α that are constant over the orbit. The link with standard tidal theories is made through the viscous timescales $\tau_{i,V}$, which in turn determines the magnitude of the angles α_i. Structural evolution of the component stars and resulting changes in the orbital elements are treated self-consistently.

with overall size changes. Additionally, for stars in isolation possessing convective envelopes, the AM evolution on the MS seems regulated by the action of stellar winds, braking the envelope region of the star all along the MS by carrying away mass and AM. Such stellar winds must still be responsible for AM loss within binary star systems containing components with convective envelopes (a discussion is found in van 't Veer & Maceroni 1992). This motivates a study of CA binary star systems, to deduce observationally how effective these winds are in carrying away AM, and to learn indirectly about the dynamo activity in the component stars. At the final stages of the MS evolution of the (initially) more massive of the two component stars, dramatic influences of its structural evolution on the binary AM evolution must occur: the expansion to a giant star may eventually cause Roche-lobe overflow, with additional mass and AM exchange in at present poorly understood accretion processes.

It is our opinion that a treatment of AM exchange in binary systems should try to incorporate (some of) these evolutionary effects for the component stars, in unison with the tidal effects that they induce on each other. Allowing for the PMS and MS evolutionary effects of each star that are at present best understood, we will adopt a 'two-component' model for each star separately (MacGregor & Brenner 1991), to quantify its spin evolution. In this approach, each star in the system is viewed, at any instant of time, as consisting of an envelope and a core region, structurally evolving during PMS and MS phases as if the star were in isolation, so that standard evolutionary calculations for stars may serve as known input to the AM evolution in the binary system. The envelope and core region of each star ($i = 1, 2$ of known mass M_i) thereby change their respective moment of inertia, and when we assume that each region rotates rigidly so that the spin angular momenta are given by $J_{env}^{(i)} = I_{env}^{(i)}\Omega_{i,env}$ and $J_{core}^{(i)} = I_{core}^{(i)}\Omega_{i,core}$, we may write

$$\frac{dJ_{core}^{(i)}}{dt} = -\frac{\Delta J^{(i)}}{\tau_c^{(i)}} + \frac{2}{3}\Omega_{i,env}R_{i,core}^2\frac{dM_{i,core}}{dt},$$

$$\frac{dJ_{env}^{(i)}}{dt} = +\frac{\Delta J^{(i)}}{\tau_c^{(i)}} - \frac{2}{3}\Omega_{i,env}R_{i,core}^2\frac{dM_{i,core}}{dt} - \frac{J_{env}^{(i)}}{\tau_W^{(i)}} + S_{ij}. \tag{10}$$

Hereby is

$$\Delta J^{(i)} = \frac{J_{core}^{(i)}I_{env}^{(i)} - J_{env}^{(i)}I_{core}^{(i)}}{I_{core}^{(i)} + I_{env}^{(i)}},$$

the amount of AM exchanged between core and envelope region in a typical 'coupling timescale' $\tau_c^{(i)}$, and $\tau_W^{(i)}$ is an appropriate timescale for AM loss induced by a stellar wind. These equations thereby incorporate the overall structural evolution of the star through the (prescribed) evolving moments of inertia, and the (also known) mass reappointment between core and envelope region (the second terms on the RHS). More details pertaining to Eq. (10) can be found in Keppens et al. (1995). Supplementing these equations for the spin AM evolution with the equations governing the orbital AM, namely

$$\frac{dJ_{orb}}{dt} = -S_{12} - S_{21}, \tag{11}$$

and Eq. (8) giving the time evolution of the orbital eccentricity, we can then resort to a numerical integration of the combined system of Eq. (8), Eqns. (10), and Eq. (11). For consistency, we must write the tidal torque terms up to fourth order in the eccentricity. Thereby must one specify the initial semi-major axis and eccentricity to get the initial condition for Eq. (11) from Eq. (1). At each time, we can then calculate a new orbital revolution and semi-major axis for the binary system from the evolved set (J_{orb}, e), using expression (1) and Kepler's third law. Initial conditions for the system (10) are derived from the known structural evolution, further assuming that in the early PMS phase, the core and envelope rotation rate are equal to a typical PMS rotation rate for the type of star under consideration. Fig. 1 is a schematical representation of the ideas underlying the equations discussed, where we suggestively assume M_1 somewhat larger than M_2.

3. Two illustrative calculations

The set of six equations described in the previous section allows one, in principle, to perform a detailed study of their parametric dependence. Given a combination of initial rotation rates and orbital parameters at a certain age, we can easily evolve the system in time. However, many different timescales enter these equations: the timescale(s) determining the mass exchange between core and envelope zones and similar evolutionary timescales, the synchronization timescale(s) and circularization timescale, the coupling timescale(s), and the wind braking timescale(s). Choices must be made regarding their dependence on the (evolving) model parameters in agreement with the physical processes that they represent. We briefly comment on these processes in what follows.

- *The evolutionary timescale* τ_{ev}*:* This timescale is reasonably understood, as long as it can be taken from standard evolutionary tracks. We use evolutionary tracks kindly provided by Vandenberg, mimicing the changing internal structure of late-type stars, all the way from fully convective early-PMS stellar objects, to their initial ascent onto the giant branch. These tracks provide us with the quasi-static evolution of *non-rotating, single* stars of fixed mass. Hence, they restrict our study to moderately rotating components in sufficiently separated binary systems. Indeed, when the individual (rapid) rotation, or the presence of a (close) binary component starts to influence significantly the stars own evolution, our approach must be abandoned.
- *The synchronization* (τ_{sync}) *and circularization* (τ_{circ}) *timescales:* We already pointed out that the actual dependence of the synchronization and circularization timescales on the physical parameters of the binary star system is currently debated. Table 1 lists their dependences on the binary parameters according to Zahn (1977), Tassoul (from Tassoul & Tassoul 1992), and our formalism, for stars with a convective envelope. The standard descriptions invoke a viewpoint where one star (M_1) is denoted as primary, to which all stellar parameters in Table 1 refer to. In particular, $k^2 \equiv I/MR^2$ is the squared fractional gyration radius of the primary (where I represents the moment of inertia of the whole star), and k_2 its so-called apsidal motion constant, which incorporates the effect of the internal stratification. It is a measure of the density concentration: an upper limit for k_2 is $3/2$, which is exact for a homogeneous 'star', but typical values are of the order of $O(10^{-2})$ (for a definition see Kopal 1969; historical values may be found from Motz 1952, where a value of $k_{2,\odot}$ = 0.00599 for the sun is quoted). In a more recent paper, Zahn (1989) has provided updated expressions for both timescales, bringing this effect of the internal stratification in line with our current understanding of stellar convection zones. Therefore, in the updated expressions, k_2 is replaced with a similar quantity directly calculable from the internal structure of the primary. The effect of the secondary (M_2) then enters through the mass ratio $q \equiv M_2/M_1$. However, to compare the circularization timescales, we simply took q = 1.
- *The coupling timescale* τ_c*:* This timescale characterizes the transport of AM from core to envelope regions, through visco-magnetic coupling (see, among others, Tassoul & Tassoul 1989; Charbonneau & MacGregor 1993). In the present-day sun, the radiative core and convective envelope zone are more-or-less in uniform rotation, suggesting efficient coupling (see Goode et al. 1991; Schou et al. 1995). However, it is unclear whether the coupling between core and envelope region for a single solar-type star is, at all times during its evolution, as efficient as seems to be the case in the present-day sun. A certain amount of differential rotation between core and envelope zone, especially on the PMS, may well be required to explain the observational results pertaining to the observed rotation rates of single (solar-type) stars of different ages (see also Keppens et al. 1995). We therefore argue to use a finite value for this coupling timescale (formally, τ_c = 0 would ensure solid body rotation throughout the evolution), small enough to ensure solid body rotation at the solar age, but large enough to allow for a certain amount of differential rotation at the Zero Age Main Sequence (ZAMS).
- *The wind braking timescale* τ_w*:* For single, solar-type stars, the wind braking timescale dominates the AM evolution beyond the ZAMS. Thereby, the dynamo operative in the convection zone or overshoot layer of the star plays an important role, as it links the stars rotation rate with the mean generated fieldstrength, which in turn determines the amount of AM lost. For moderate rotation rates, a linear dynamo relation is certainly appropriate, but for fast-rotating stars (say, for rotation rates faster than 20 times solar rotation $\Omega_\odot \approx 3 \times 10^{-6} s^{-1}$), one may argue for a saturation of the dynamo-generated fieldstrength (e.g., Stauffer et al. 1994). How the presence of a (close) component star modifies the efficiency of the wind braking torque for a solar-type star is simply unknown. Hence, to get reasonable prescriptions for the wind braking timescale from known results for single stars, we should avoid 'rapid' rotators and 'close' binary systems. However, the restrictions imposed by the qualifications 'rapid' and 'close' are not severe, as we may always calculate the torque exerted by the stellar wind as if it was a spherically symmetric outflow, so that the so-calculated torque will yield an upper limit for the actual wind driven AM loss.

In view of the many uncertainties involved, it seems appropriate at this point, to postpone an observationally guided parametric study to a following paper, and focus here on a complete physical description of a few illustrative cases. We emphasize *in what manner* the variety of physical effects that enter our description *may* influence the evolutionary scenarios. The particular choices we make for the different timescales involved will be reasonable estimates, and arguably nothing more than that. In particular, we consider a symmetric binary system where both components are of solar mass, so that the evolutionary timescales are identical for both component stars. For the synchronization and circularization timescales, we simply took $\tau_V = (MR^2/L)^{1/3}$ (a Zahn-type prescription)[1]. For the coupling timescale, we adopt a constant τ_c = 10 Million years. The wind timescale is taken from a spherically symmetric MHD stellar wind model, originally presented as an exact solution of the ideal MHD equations for quantifying the solar wind by Weber & Davis (WD, 1967). To calculate this timescale for other than solar rotation rates, we

[1] Unfortunately, the evolutionary tracks available to us at present do not provide us with (an equivalent of) the apsidal motion constants k_2. The use of $\tau_V = (MR^2/L)^{1/3}$ means that if we take $I_{env}/I \approx 0.11$, $k^2 \approx 0.072$, as is the case for the present-day sun (Fig. 3.1), set q = 1 and assume $k_{2,\odot}$ = 0.00599, our τ_{sync} is smaller than the Zahn-value for the sun as a primary star by a factor of 76, but still about a factor of 52 larger than the corresponding Tassoul value (for $a/R_\odot$ = 15). Likewise, for the circularization timescale τ_{circ}, our value for a present-day sun is a factor of 10 below the Zahn-value, but about 148 times larger than a Tassoul-value.

Table 1. Synchronization and circularization (for q = 1) timescales (in years) for a binary system of solar-type stars, according to Tassoul (from Tassoul & Tassoul 1992), Zahn (1977) and the formalism presented here. Through our parameter τ_V, we may mimic different types of tidal interactions. See text for a discussion.

τ (yr)	Tassoul	Zahn	
τ_{sync}	$\frac{14.4\times10^{-10/4}}{q(1+q)^{\frac{3}{8}}} \frac{\left(\frac{M}{M_\odot}\right)^{-\frac{1}{8}}\left(\frac{R}{R_\odot}\right)^{\frac{9}{8}}}{\left(\frac{L}{L_\odot}\right)^{\frac{1}{4}}} \left(\frac{a}{R}\right)^{\frac{33}{8}}$	$\frac{0.0718k^2}{k_2q^2} \frac{\left(\frac{M}{M_\odot}\right)^{\frac{1}{3}}\left(\frac{R}{R_\odot}\right)^{\frac{2}{3}}}{\left(\frac{L}{L_\odot}\right)^{\frac{1}{3}}} \left(\frac{a}{R}\right)^6$	$\frac{5k^2I_{env}(1+q)}{3Iq^2}\tau_V \left(\frac{a}{R}\right)^6$
τ_{circ}	$\frac{14.4\times10^{-10/4}}{k^2(2)^{\frac{11}{8}}} \frac{\left(\frac{M}{M_\odot}\right)^{-\frac{1}{8}}\left(\frac{R}{R_\odot}\right)^{\frac{9}{8}}}{\left(\frac{L}{L_\odot}\right)^{\frac{1}{4}}} \left(\frac{a}{R}\right)^{\frac{49}{8}}$	$\frac{0.0273}{k_2(2)} \frac{\left(\frac{M}{M_\odot}\right)^{\frac{1}{3}}\left(\frac{R}{R_\odot}\right)^{\frac{2}{3}}}{\left(\frac{L}{L_\odot}\right)^{\frac{1}{3}}} \left(\frac{a}{R}\right)^8$	$\frac{11}{21}\tau_V \left(\frac{a}{R}\right)^8$

use a linear dynamo-relation between the mean coronal field-strength and the envelope rotation rate up to $20\Omega_\odot$ (which will never be exceeded in the cases presented). The evolutionary effects, especially the change in stellar radius, have to be incorporated to determine this timescale, as they induce significant changes in the magnetic (through flux conservation $\propto R^2$) and centrifugal ($\propto R$) parameters that enter the WD wind model. For a complete description how this can be done, we refer to Keppens et al. (1995; Appendix A). It is worthwhile mentioning that the combination of a coupling timescale τ_c of the order of 10 Myr, and the WD wind timescale as found from a linear dynamo relation saturated at $20\Omega_\odot$, was found to yield satisfactory agreement between observed and calculated rotational velocity distributions of single, solar-type stars at different ages.

3.1. Case I: short (5 day) period

We start the evolution at an age of 2 Million years: for a star of solar mass, the star radius R_* is then about $2R_\odot$, and a radiative core has developed up to approximately $0.2R_*$, but its associated moment of inertia is still two orders of magnitude smaller than that of the overlying contracting convective envelope. A plausible assumption at this initial age is to take both regions corotating, since the star was fully convective prior to about 2 Million years, and therefore likely in a state of rigid rotation. We consider a 'slow' rotation with, for each component star, the equatorial rotational velocity $V_{eq}(t = 2Myr)$ = 5 kms^{-1}, so that the actual initial rotation rates $\Omega_{1,2/env,core}$ are $\approx 1.25\Omega_\odot$. This may seem like a small value compared to typical rotation rates for single stars of the same age (15 kms^{-1} for a $1M_\odot$ T Tauri star, Bouvier 1991) but may not be an unacceptable value when binary systems indeed form out of direct fragmentation of a collapsing protostellar dense molecular cloud (Boss 1995). The conservation of AM in such a binary star formation process will store most of the AM of the parent cloud in the orbital AM of the system, while the collapse of a cloud without fragmentation necessarily bundles all the AM in the spin AM of the so-formed single star. Guided by the observational results on PMS binary systems of about the same age (as in Mathieu et al. 1989), it seems not unreasonable to start our calculation with a 5 day orbital rotation period and a low value of the eccentricity e = 0.05. This choice ensures that spin and orbital rotation rates ($\Omega_{orb} \approx 4.8\Omega_\odot$) are asynchronous by about a factor of 4. The semi-major axis is then initially about $15.5R_\odot$, and both stars fit comfortably within their critical Roche radius (about $5.9R_\odot$), so effects of mass exchange can be excluded, and evolutionary scenarios for single stars should be adequate to model the evolutionary effects. However, as the critical Alfvén radius appearing in the WD wind solution for the present day sun is at about $36.5R_\odot$, while the slow magnetosonic point for the sun lies at approximately $9.2R_\odot$, the calculation of the wind braking torque from a WD wind model may be poor. In any case, we follow the AM evolution of the binary system according to the equations and underlying assumptions, up to an age of 11.7 billion years. Presented in Fig. 3.1 are the accompanying evolutionary changes experienced by each component. These are drawn in the typical format we will adopt throughout this paper: changes of physical parameters shown as a function of time (in years), but with a logarithmic timescale along the PMS, and a linear timescale on the MS. This because the time of ZAMS (for a star of solar mass at approximately 4.6×10^7 years) 'separates' an era of fast structural evolution (PMS) from the slow evolutionary effects on the MS. Shown in Fig. 3.1 is the evolution of the moments of inertia of core (I_{core}) and envelope (I_{env}) zone (in 10^{55} cgs units), and the change in stellar radius R_* (in units of $R_\odot$). We clearly identify (i) the PMS contraction phase, (ii) the gradual development of a significant radiative core (extending to about $0.7R_*$ along the MS) during this phase, and (iii) the ascent onto the giant branch (at $\sim$ 11 billion years) with the overall expansion of the stellar envelope and its accompanying increase in moment of inertia. Other evolving stellar parameters (core radius, mass exchange between core and envelope, stellar luminosity) are likewise available from the evolutionary tracks (from D. Vandenberg).

Fig. 3 shows the resulting evolution of rotation rates and Fig. 4 the changing orbital parameters. We also present the evolution of the relevant timescales in Fig. 5 (as $log_{10}(\tau)$, with τ in years). To aid in the interpretation of the results, we have, at all times, identified the largest spin-up ($\uparrow$) or spin-down ($\downarrow$) torques in the set of Eqns. (10) and added this information onto

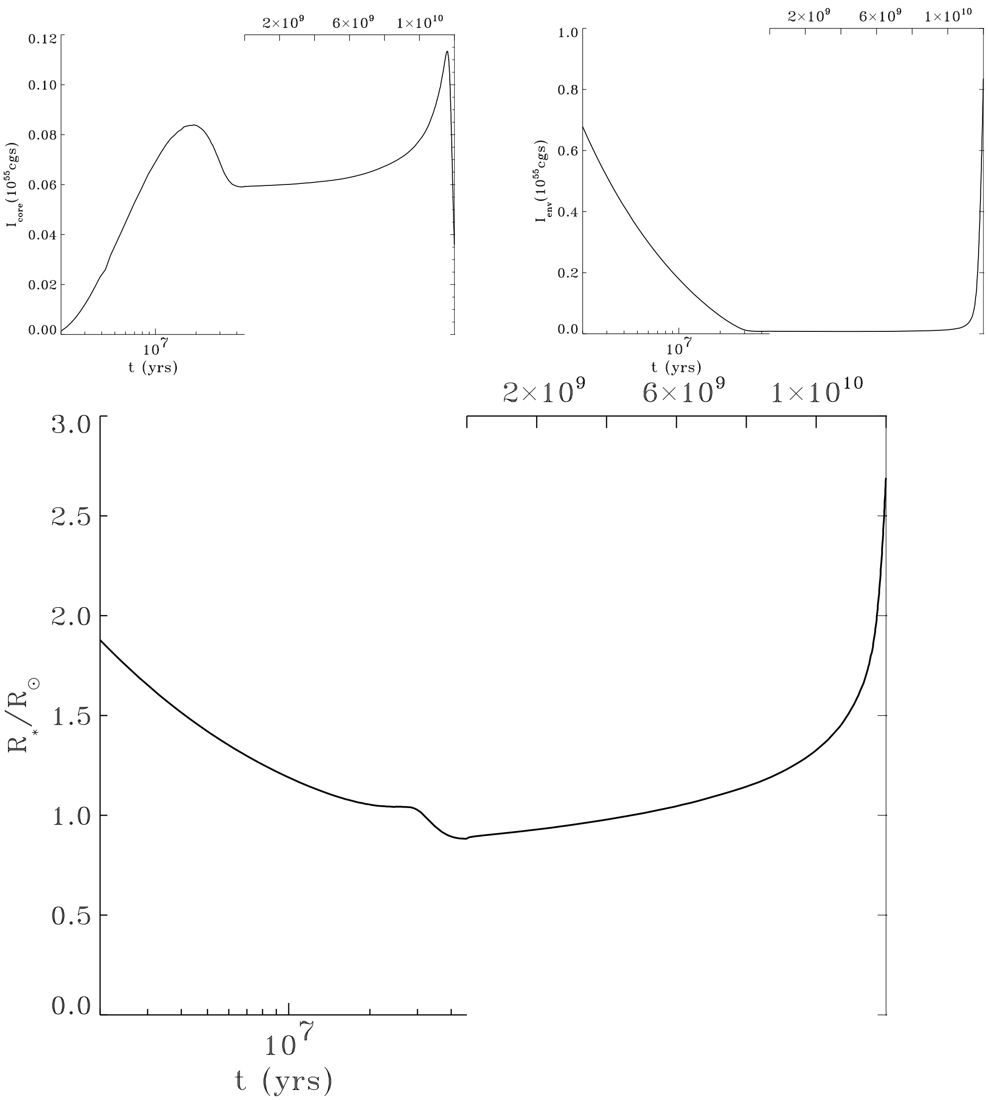

Fig. 2. Time evolution of the moment of inertia for the radiative core (top left, in 10^{55} cgs units), of the moment of inertia for the convective envelope (top right), and of the stellar radius (in $R_\odot$ units) for a star of solar mass. Note the logarithmic timescale (in years) on the PMS, and the linear timescale on the MS. Data from evolutionary tracks provided by D. Vandenberg.

Fig. 3. For instance, the indication $\uparrow \tau_{me}$ denotes that during that time, the effect of mass exchange between core and envelope represents the largest (in absolute magnitude) spin-up ($\uparrow$) torque for the stellar region whose rotation rate is plotted as a solid line. A vertical dotted line marks a time where the largest torque term changes from spin-up to spin-down, or vice-versa. In conjunction with the changes in the respective moments of inertia (Fig. 3.1), we are then able to pin down which effect forces the rotation rates to evolve in the manner calculated. For the eccentricity and semi-major axis, vertical dotted lines in Fig. 4 mark times where the angle α changes sign (with the factor 18/11 in the eccentricity evolution!). For the calculation of the evolutionary timescale τ_{ev} in Fig. 5, we simply took the shortest logarithmic time derivative among core mass, stellar radius, and moment of inertia of core and envelope.

As can be seen from the evolution of the timescales (Fig. 5), during the PMS, evolutionary and synchronization effects are most important, and coupling and wind effects can be ignored. Therefore, we witness in Fig. 3 how the envelope rotation rates (of both components) try to achieve synchronization with the orbital rotation rate (dash-dotted line), as the tidal torque is the largest torque operative on the envelope region for most of the PMS evolution. However, due to evolutionary effects, the binary system is unable to achieve perfect synchronization on the PMS. Indeed, the envelope rotation spins up rapidly in an initial phase mainly because of the efficient tidal torque, but also due to the overall contraction. As a result, the envelope rotation 'overshoots' the orbital rotation rate, and the tidal torque then acts to brake the stars rotation. For the envelope zone, the mass exchange term represents an additional braking torque, which is even dominant during a limited time (mainly due to its dependence on $\Omega_{env} R^2_{core}$ in this case). As a result, the envelope rotation rate never exceeds $5.5\Omega_\odot$ during the PMS. Meanwhile, primarily the mass reappointment from the envelope to the core causes a spin-up of the core rotation rate to approximately $13.5\Omega_\odot$ right before ZAMS.

Then, a lot happens on the transition from PMS to MS. The structural changes diminish and take place on a longer timescale,

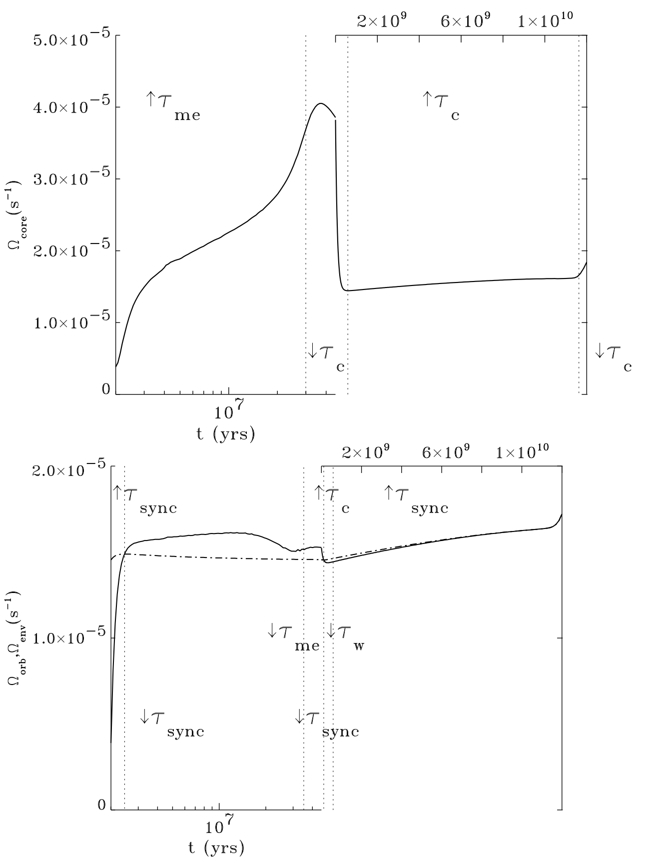

Fig. 3. The calculated evolution of the rotation rates (in s^{-1}) for a symmetric binary star system of solar mass stars, starting at 2 Myr from a mildly eccentric (e = 0.05), relatively close (P_{orb} = 5 days) asynchronous system (initial isorotating core and envelope zones corresponding to an equatorial velocity of 5 kms^{-1}). Top: the core rotation rate(s). Bottom: the envelope rotation rate(s) (solid line) and the orbital rotation rate (dash-dotted line). Indicated on the figures are the largest (in absolute magnitude) torques operative at each time within the system of Eqns. (10), to aid in the interpretation (see text). Vertical dashed lines denote a change in this torque from spin-up (↑) to spin-down (↓) or vice-versa.

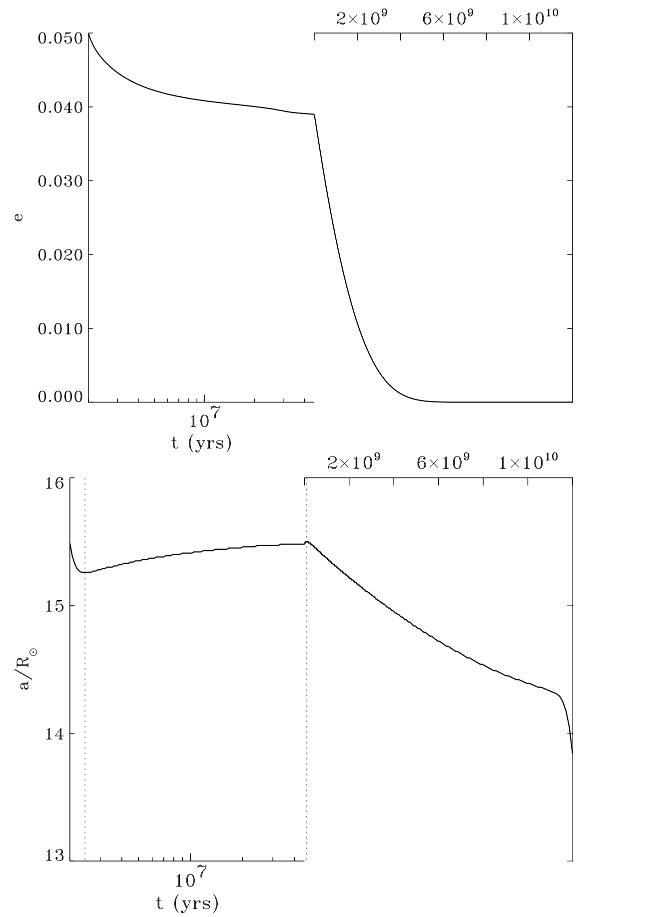

Fig. 4. For the same system as in Fig. 3, the evolution of the orbital eccentricity (top) and semi-major axis a (bottom, in units of $R_{\odot}$). Vertical dotted lines denote changes in sign of the misalignment angles α. See text for details.

the stellar wind starts to play a role, and especially the coupling between core and envelope begins to gain in efficiency (remember that τ_c = 10 million years must elapse before this effect is of any importance). The coupling mechanisms tend to force the component stars into uniform rotation, and act as a sink for AM for the core, and as a source of AM for the envelope. Thus, the envelope experiences a slight rotational increase before the ZAMS, while the core rotation rate drops to about its surface value short after the ZAMS.

From then on, all is dominated by the combined action of stellar winds and tidal effects. The wind brakes the stellar envelope, whose rotation rate again dips below the orbital rotation rate. As the synchronization timescale remains the shortest timescale involved, we essentially deal with a synchronous system all along the MS. But a noticable asynchronism remains possible during a considerable time along the MS, as the braking effect of the wind may cause the spin rotation rate to 'undershoot' the orbital rotation rate during a short time where the system is almost perfectly synchronized (this effect is also discussed by Habets & Zwaan 1989). Along the MS, we witness how the AM loss from both stellar winds essentially carries away AM out of the orbit from the synchronized system, and both components close in on each other. Note that although we can call the system synchronized from, say, 2 billion years onwards, it is the negligible amount of asynchronism, in combination with the short synchronization timescale, that enforces the torque terms S_{ij} to remain the largest torques involved during the MS. This keeps the system synchronized. Thereby is the core rotation rate identical to the envelope rotation rate due to the short coupling time.

Note how phases where spin AM is converted to orbital AM, and where orbital AM is converted to spin AM, can be identified from the change (essentially, its slope) of the semi-major axis a in Fig. 4. Remember that a is closely related (Kepler's law) with the variation of Ω_{orb}, from which we can derive estimates for the change of the orbital period P_{orb}: a linear interpolation from $\Omega_{orb}(t)$ between 3 and 30 Million years on the PMS yields

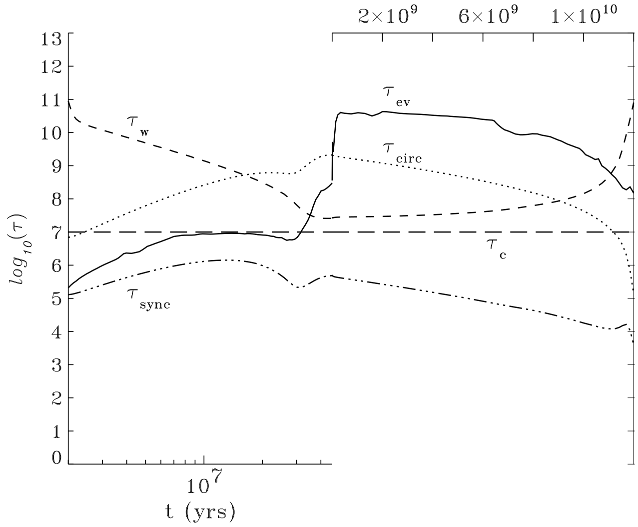

Fig. 5. For the system from Fig. 3 and 4, the evolution of the timescales involved in the AM balance. Timescales τ in $\log_{10}$, with τ in years. Evolutionary timescale: solid line; wind timescale: short dashes; coupling timescale: long dashes; synchronization timescale: dash-dotted; circularization timescale: dotted.

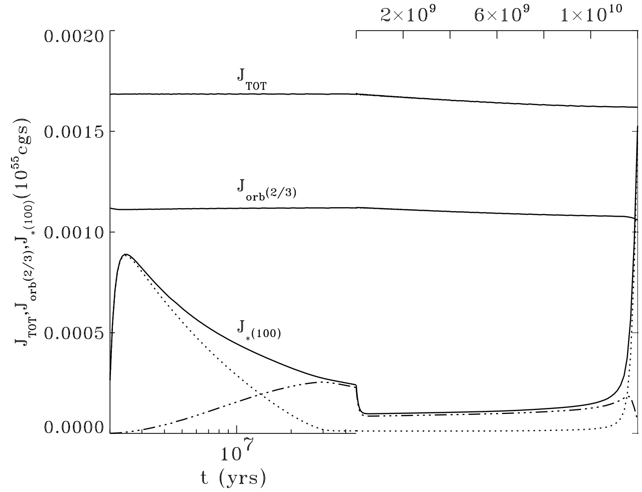

Fig. 6. The AM balance during the evolution of the initial 5 day period binary star system. Shown is the total AM J_{TOT}, the (scaled by a factor 2/3) orbital AM J_{orb}, and the angular momentum in each component star J_* (multiplied by 100 to fit on the figure). The latter consists of the added contributions from core (dash-dotted) and envelope regions (dotted).

a value for $d\ln P_{orb}/dt$ = +6.8 × $10^{-10}yr^{-1}$, and on the MS, using values at 2 and 8 billion years, we get $d\ln P_{orb}/dt = -1.2 \times 10^{-11}yr^{-1}$ (note how both signs are possible!).

We emphasize that, at all times during the evolution, the only AM loss terms in the system are due to stellar winds. All other torque terms basically transfer AM between core and envelope zones, and between spin and orbital AM. The equation governing the change of the orbital eccentricity plays thereby a secondary role, although it allows for the simultaneous study of circularization effects in a (virtually) self-consistent manner. Of crucial importance is the use of Eq. (1) to calculate the semi-major axis (hence the orbital rotation rate) throughout the evolution, which is fed into the torque terms S_{ij}. This equation is valid as long as the orbital AM is considerably larger than the spin AM. At all times during the evolution, the ratio between the (total) spin AM of each component to the orbital AM is of the order $O(10^{-3})$, so that the validity is assured. For completeness, we show in Fig. 6 the AM balance: we plot the total AM, the evolution of the orbital AM J_{orb} (scaled to clarify the figure), and of the total AM contained in one component star J_* (also scaled to fit on the figure; all AM in 10^{55} cgs units). The latter is simply the sum of the AM contained in its rotating and evolving envelope (dotted line), and its rotating and evolving core (dash-dotted line). Throughout the PMS, as long as the stellar winds can be ignored, the total AM is conserved, so that what is initially gained by the component stars (due to the tidal effects and the spin-up), and then lost (due to the overall contraction), is exactly balanced by an initial slight decrease (in the first 1 Million years) followed by a slight increase in the orbital AM (during the rest of the PMS). During the MS however, the stellar winds can no longer be ignored, so that a continuous decrease in total AM occurs, most notably in J_{orb}.

As far as the circularization time is concerned, the evolution of e and τ_{circ} evidences that circularization takes place within a billion years on the MS, so that the initially mildly eccentric system is most likely seen as a perfectly circularized binary system on the MS (see Fig. 4 and 5). Note also that the efficiency of the tidal torques at the start of our calculation (when the asynchronism is about a factor of four) causes an initial, relatively fast drop in the eccentricity. The circularization time is about 10 Million years at t = 2 Myr, and this forces the eccentricity to low values within a comparable time. As we assumed a small initial value for e, the change from e = 0.05 to 0.04 may seem unimportant, but one may expect that larger initial values for the eccentricity will e-fold likewise, so that the short period (5 day) system will not allow for large eccentricities on the PMS. However, as pointed out above, exact circularization is only achieved on the MS within a billion years: the lengthening of the timescale τ_{circ} reflects the diminishing efficiency of the tidal torques, so that a non-zero PMS value for $e \approx 0.04$ within the 5 day period system remains.

Finally, a curious effect occurs when both stars start to ascend the giant branch. Contrary to what is expected from the expanding envelope, the envelope (and thereby, the orbital) rotation rate increases. This is an artifact of our choice of a constant (short) coupling timescale: indeed, in the final evolutionary stages discussed here, the core spins up due to a decrease in its moment of inertia (core collapse, see Fig. 3.1), but as it is coupled to the envelope in a timescale τ_c, the envelope region responds by a spin-up, although its moment of inertia increases in time. As the synchronization is perfect at this stage, this in

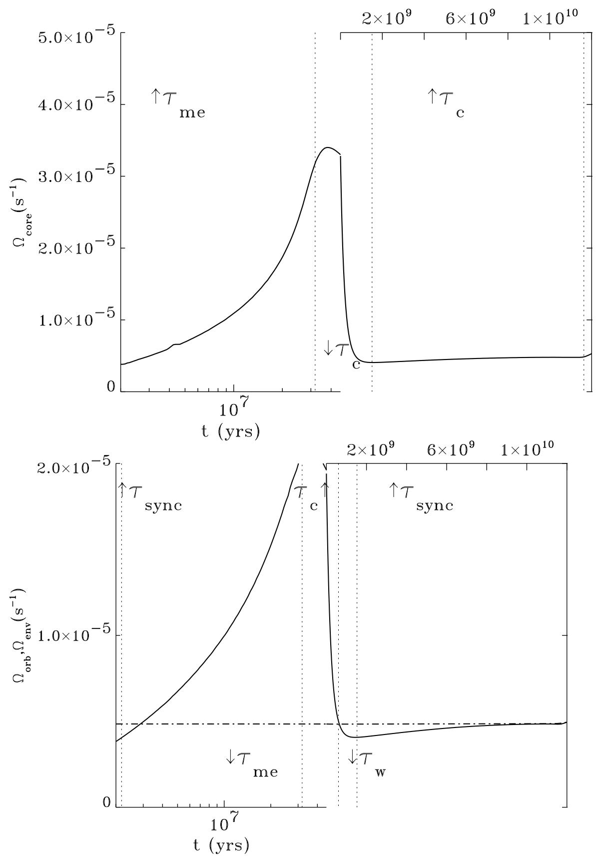

Fig. 7. As in Fig. 3, but with a longer initial orbital rotation period of 15 days. Note how asynchronism can be maintained for a considerable time on the MS.

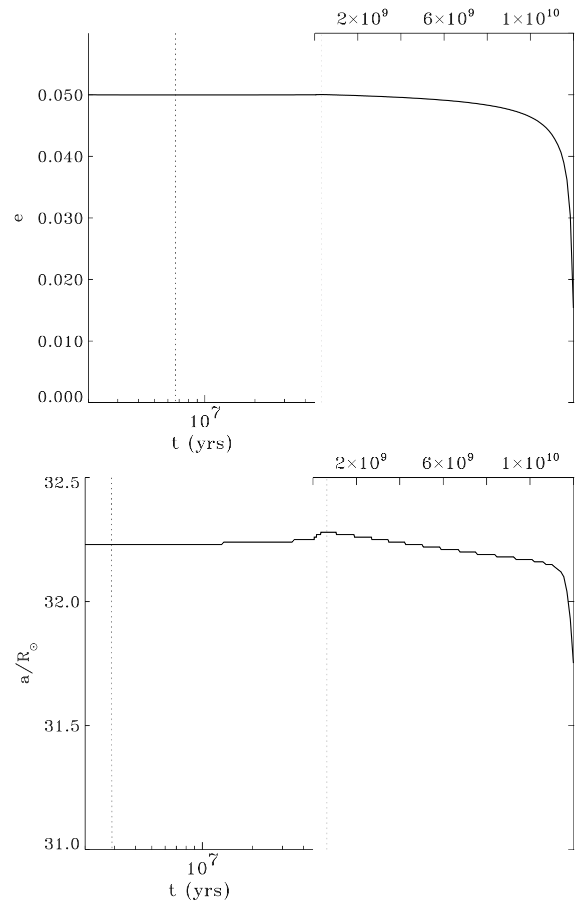

Fig. 8. For the system from Fig. 7, the evolution of the orbital eccentricity (top) and semi-major axis a (in units of $R_\odot$), as in Fig. 4. Note how the eccentricity only changes when structural changes indicate the ascent onto the giant branch.

turn causes the orbital rotation rate to increase in unison, with an accompanied dramatic shortening of the semi-major axis.

3.2. Case II: longer (15 day) period

If we evolve the same binary system of identical solar-mass stars, starting from unchanged initial conditions, with the important exception of a longer (15 days instead of 5 days) initial orbital rotation period, the AM evolution is significantly altered. Since the corresponding separation of the components amounts to more than $32R_\odot$ (ensuring the validity of our treatment of evolutionary and stellar wind effects), the tidal interactions are much less efficient. This leads to synchronization and circularization timescales that are roughly two orders of magnitude longer, as they scale with a large power of the ratio a/R_*. Therefore, the evolutionary effects pretty much dominate the changes in the core and envelope rotation rates during the PMS, and both are seen to increase, just as for a single star. Fig. 7 demonstrates that during most of the PMS, the mass reappointment from envelope to core regions represents the largest AM source term for the core, and AM sink term for the envelope, also identical to single star AM evolution. However, in the binary system, the envelope also tries to achieve synchronization within the timescale τ_{sync}, so that the overall contraction of the star does not lead to as large a ZAMS rotation rate as would be the case for an isolated star: in Fig. 7, the envelope rotation rate never exceeds $7.12\Omega_\odot$, while for a similar star (same mass, τ_c and wind braking timescale) in isolation[2], it would reach up to $9.21\Omega_\odot$. Note also that the core rotation rate for this Case II stays below $11.5\Omega_\odot$, in contrast to the maximal $13.5\Omega_\odot$ found in Case I (the 5 day period case). This is a consequence of the fact that the mass reappointment term increases linearly with the envelope rotation rate of the star, and the (almost) synchronized envelope zones from Case I represent larger AM source terms for the stellar cores during the initial ~ 20 Million years of the calculation (compare Fig. 3 with Fig. 7).

Beyond the ZAMS, the evolution of the rotation rates in the initial 15 day period case is qualitatively similar to the 5 day period one, as evidenced by the dominating torque terms indicated on both Fig. 3 and Fig. 7. But the weaker tidal interactions allow for a larger degree of asynchronism between orbital and spin rotation rates, which is ultimately caused by the braking action of the stellar wind. Indeed, it becomes possible to end up with

[2] The results for a star in isolation are embedded in our formalism, in the formal limit of $a \to \infty$.

an asynchronous system, when a balance is reached between the AM carried away by the wind, and the systems tendency to evolve towards synchronization according to expression (5). In Fig. 7, exact synchronism is eventually reached, but the system does not become circularized (Fig. 8) before considerable structural changes indicate the initial ascent onto the giant branch. Note further how a PMS time-interval during which the eccentricity could increase exists, as a result of the assumed linear dependence of the angles α on rotation rates, but due to the long timescale associated with circularization, the value of the eccentricity hardly changes during this period. The evolution of the semi-major axis in Fig. 8 again indicates how spin and orbital AM are exchanged alternately, and that the action of the stellar winds on the MS essentially forces the stars to close in onto each other. Similar to Case I, the effect of a collapsing core and a short coupling timescale in a synchronized system causes the final abrupt change in the semi-major axis.

4. Discussion

Although far from complete, the calculations presented in Sect. 3 already indicate that the study of the AM balance within binary star systems is a complicated matter. It is governed by the intricate interplay of evolutionary effects, mutual tidal interactions, internal visco-magnetic coupling mechanisms, and stellar winds, to name only those effects incorporated in our model.

As emphasized throughout this paper, none of these effects is thoroughly understood. At most, the timescale within which *one* of these mechanisms is believed to operate is known in order of magnitude, and possibly, its basic dependence on stellar properties firmly established from theory and comparisons with observational facts. In that respect, our model can be of considerable interest, since we are able to study how these timescales and their own evolution influences the entire AM balance within the system. Indeed, as evidenced by the illustrative calculations, we can trace every change in the observable parameters from a combined interpretation of evolutionary effects and the exerted torques.

The inclusion of evolutionary effects, especially during PMS and late MS stages, is an essential ingredient to the AM recipe (see also Zahn & Bouchet 1989; Habets & Zwaan 1989). We demonstrated this for a symmetric system of two solar-mass stars: the longer the orbital rotation period (and hence, separation), the more dominant is their effect upon the evolution of rotation rates during the PMS. For widely separated systems, the stars evolve as if in separation, and a spin-up follows from the overall contraction. For very short period binaries, tidal effects will dominate. With our model, we can explore the regime where both effects play an important role. Our results pertinent to an initial 5 day period system represent this regime, and it is clear that this bears implications for the many observational results pertaining to synchronization and circularization cut-off periods. The changes induced on the rotation rates due to evolutionary effects occur irrespective of the presence of a component star, so that exact synchronism may fail to be achieved. The point to note is that the increase in rotation rate due to overall contraction on the PMS can be balanced by a tidal spin-down torque by maintaining the right amount of asynchronism.

We witnessed a similar effect on the transition from PMS to MS, where the wind braking torque becomes extremely significant. In wider separated systems (as the initial 15 day period case), the stars rotation rate may lag the orbital rotation rate when a similar balance of braking and the tendency towards synchronism is achieved. This may put the whole question of pseudosynchronization and asynchronization observed in binary systems with CA components into a new perspective. As the degree of asynchronism that could be maintained on the MS in longer period binary systems ultimately relates to the combined action of stellar winds (well-understood in those separated systems) and the magnitude of the tidal torques (clearly depending on the viscous mechanism as measured by our parameter τ_V, which is very different depending on which tidal theory one favours), it may prove worthwhile to devote attention to the asynchronous binary systems in order to resolve the current disagreement between standard synchronization and circularization theories, no matter how ironic this may sound. Another aspect of the wind driven AM loss that can be adressed within the realms of our model, concerns the many observations pertaining to CA binary star systems (as catalogued by Strassmeier et al. 1993). When these systems are synchronized and circularized, the evolution of their orbital rotation period provides quantitative measures of the efficiency of their winds. This bears implications for wind models (for single star and binary star systems), for dynamo activity and saturation, for the magnetospheric structure (closed and open structures representing dead zones and wind zones, respectively) in relatively close binary systems, etc.

A final comment concerns the choice of appropriate initial conditions for evolving the system in time. The power of this model resides in its predictive nature, since we are able to tell what evolutionary scenario is plausible, starting from a particular initial state. When the calculation is started from time t_0, we must be able to tell the structure of both component stars at time t_0, the orbital eccentricity and semi-major axis (hence $\Omega_{orb}(t_0)$), and the individual rotation rates of the components (in so far as they differ from Ω_{orb}). Our choice for t_0 corresponded to an early-PMS phase, and observations of PMS binary systems should provide insights into (statistically likely) initial conditions. For example, the calculations presented started from slowly (with $\Omega_{rot} < \Omega_{orb}$ in particular) rotating component stars at t_0 = 2 Myr. For comparison, Zahn & Bouchet (1989) presented results with $t_0 \approx 100$ years, and initial spin rotation rates about 2.5 to 3 times faster than what an initial 5 day period yields as an orbital rotation rate at that age. They took $e(t_0 \approx 100\text{yr}) \approx 0.3$. Hence, they also considered the Hayashi contraction phase. Then, the component stars are fully convective and of even larger radius than the $\approx 2R_\odot$ with which our calculations begin. Through the a/R dependence of the relevant tidal timescales, this would ensure efficient tidal coupling during phases earlier than considered here. This is indeed confirmed by Zahn & Bouchet (1989). The evolutionary tracks of D. Vandenberg for a solar mass star do provide us a Hayashi phase from an initial t_0 = 3000 yr. As long as the star is fully convective however, we need to integrate fewer than the six equations used in this paper, since no core region exists before about 2 Myr. We discuss below how this represents a limiting case of the equations, through the parameter τ_c, so that the inclusion of this phase presents no serious difficulty.

In any case, it should already be evident from our calculations (and the results of Zahn & Bouchet 1989), that the shorter period PMS binaries (say $P_{orb} < 5$ days) with late-type components, will not allow for large asynchronism or considerable orbital eccentricities, due to the effective tidal interactions. This is evidenced by the rapid trend toward synchronization and fast decrease in orbital eccentricity as seen in Fig. 3 and Fig. 4 during the first few million years of our calculation. Clearly, as the Tassoul mechanism for tidal interaction is generally more effective (as it is a shorter-range mechanism), this result is firmly established. The exact value of the orbital period P_{orb} separating asynchronous from synchronous, and eccentric from circular binary systems should not (yet) be inferred from the illustrative calculations, however. First, as mentioned above, we started at an age before which tidal effects may have been effective; and second, our choice of the parameter τ_V did not identically mimic either Zahn's or Tassoul's tidal theory. Rather, it gave values for τ_{sync} and τ_{circ} that were somewhere in between both predictions.

Furthermore, we assumed identical components of solar-mass. Our model has the ability to consider components of different mass, and see how the unequal structural evolutions may influence the binary AM evolution.

The variety of other plausible initial conditions can alter the evolutionary scenarios in ways that may provide a better understanding of the observational results. In this respect, we mention the fact that the same binary system may show both increases and decreases in its orbital rotation period, depending on which torques and structural evolutionary effects are operative at the time, and which thereoff dominates the change in rotation rates.

The model presented uses a two-component representation of each star in the binary system. This representation forces us to introduce the extra parameter τ_c, modelling a typical timescale within which a radiative core and a convective envelope (for a solar-type star) try to achieve isorotation. It is possible to mimic uniformly rotating stars in binary systems, in the formal limit of $\tau_c \rightarrow 0$. The calculations presented assumed a constant but finite τ_c, which is inspired from AM studies of single stars (Keppens et al. 1995). It should be obvious in which way smaller or greater values for τ_c ensure, respectively, a more coupled or a more detached rotational evolutionary scenario for core and envelope regions. The slow structural evolution during the MS for solar-type stars will ultimately lead to isorotating core and envelope zones. Curiously, a sufficiently short coupling timescale may induce effects opposite to physical intuition: proof thereoff, the spin-up in rotation and orbital period while the stellar envelope expands when a component of the binary system ascends the red giant branch as seen in Figs. 3 and 7. In those figures, after about 11 billion years of 'quiet' evolution, the collapsing core is the ultimate regulator of the coupled, synchronized system, transfering its resulting increase in rotation rate within a coupling timescale to the envelope and thus indirectly to the binary system. Of course, it is questionable whether the same coupling mechanisms (thus, a constant τ_c) are effective at all evolutionary stages, but the mere possibility is quite intruiging. The associated dramatic decrease of the semi-major axis may well be suggestive of the formation of semi-detached and contact systems, where one component is an evolved star (recent results on the formation of contact systems are by Stepien 1995). It may also be of interest to the observations pertaining to synchronized CA binary systems, evolving from the MS up to the giant branch, where indications are found for both significant increases and decreases in orbital rotation period. Since the core collapse will eventually lead to a negligible moment of inertia for the core region, we expect that the expansion of the stellar envelope will halt and reverse the period change induced by this effect. We will investigate this possibility in a forthcoming paper (Keppens et al. 1996). In any case, it serves to point out that the assumption of rigid body rotation may not always be blindly applicable. The results from Habets & Zwaan (1989), as well as the late PMS phases discussed by Zahn & Bouchet (1989) did not include the possibility for differentially rotating core and envelope zones.

In this paper, we have focused on presenting the model, and did not intend to make definite claims relating our results to the observations. This will be the subject of a next paper (Keppens et al. 1996). However, we did demonstrate clearly, from illustrative calculations, that such can be done if physically meaningful prescriptions for the different timescales involved are available. In that respect, the prescriptions assumed in the cases discussed should serve as a useful example. It is further worth noting that this model treats both component stars in the binary system on an equal footing, so that other mechanisms of interest to the overall AM balance can in principle be incorporated. More specifically, one may include the possibility of mass exchange between stars in the binary system (through Roche lobe overflow of one component star). However, it is clear that this would require treatments for the structural evolution and the wind-driven AM losses appropriate for close (semi-detached and contact) binary systems.

Acknowledgements. Most of this work was done while R. Keppens was a Research Assistant of the Belgian National Fund of Scientific Research, on leave of the Center for Plasma-Astrophysics at the Catholic University of Leuven (K.U.L.). I thank Dr. S.K. Solanki and Dr. M. Schüssler for their encouragement and useful suggestions and discussions.

References

Boss, A.P., 1995, Sci. Am. 273, 4, 38

Bouvier, J., 1991, In: Catalano S., Stauffer J.R. (eds.) Angular Momentum Evolution of Young Stars. Kluwer Academic Publishers, Dordrecht, p. 41

Charbonneau, P., MacGregor, K.B., 1993, ApJ 417, 762

Collins, G.W., 1989, Astron. and Astroph. series 16: The Foundations of Celestial Mechanics. Pachart Publishing House, Tucson

Goode, P.R., Dziembowski, W.A., Korzennik, S.G., Rhodes, E.J. Jr., 1991, ApJ 367, 649

Habets, G.M.H.J., Zwaan, C., 1989, A&A 211, 56
Keppens, R., MacGregor, K.B., Charbonneau, P., 1995, A&A 294, 469
Keppens, R., Solanki, S.K., Schüssler, M., 1996, in preparation
Kopal, Z., 1969, Ap&SS 4, 330
MacGregor, K.B., Brenner, M., 1991, ApJ 376, 204
Mathieu, R.D., Walter, F.M., Myers, P.C., 1989, AJ 98, 987
Motz, L., 1952, ApJ 115, 562
Rieutord, M., 1992, A&A 259, 581
Schou, J., Tomczyk, S., Thompson, M.J., 1995, In: Hoeksema J.T., Domingo V., Fleck B., Battrick B. (eds.) Fourth Soho Workshop: Helioseismology. esa SP-376 Volume 2, p. 275
Shore, S.N., 1994, In: Nussbaumer H., Orr A. (eds.) Saas-Fee Advanced Course 22: Interacting Binaries. Springer-Verlag, Berlin Heidelberg, p. 1
Stawikowski, A., Glebocki, R., 1994, Acta Astron. 44, 33
Stawikowski, A., Glebocki, R., 1994, Acta Astron. 44, 393
Stepien, K., 1995, MNRAS 274, 1019
Strassmeier, K.G., Hall, D.S., Fekel, F.C., and Scheck, M., 1993, A&AS 100, 173
Stauffer, J.R., Caillault, J.-P., Gagné, M., Prosser, C.F., Hartmann, L.W., 1994, ApJS 91, 625
Tassoul, J.L., 1987, ApJ 322, 856
Tassoul, J.L., 1988, ApJ 324, L71
Tassoul, J.L., Tassoul, M., 1989, A&A 213, 397
Tassoul, J.L., Tassoul, M., 1992, ApJ 395, 259
van 't Veer, F., Maceroni, C., 1992, In: Duquennoy A., Mayor M. (eds.) Binaries as Tracers of Stellar Formation. Cambridge University Press, Cambridge, p. 237
Weber, E.J., Davis, L., 1967, ApJ 148, 217
Zahn, J.-P., 1977, A&A 57, 383
Zahn, J.-P., 1989, A&A 220, 112
Zahn, J.-P., Bouchet, L., 1989, A&A 223, 112

This article was processed by the author using Springer-Verlag LaTeX A&A style file *L-AA* version 3.