Astron. Astrophys. 318, 443–460 (1997)

ASTRONOMY AND ASTROPHYSICS

Critical study of the frequency of Be stars taking into account their outstanding characteristics

J. Zorec[1] **and D. Briot**[2]

[1] Institut d'Astrophysique de Paris, C.N.R.S. - 98bis, bd. Arago, F-75014 Paris, France (zorec@iap.fr)
[2] Observatoire de Paris - 61, av. de l'Observatoire, F-75014 Paris, France

Received 31 October 1995 / Accepted 13 June 1996

Abstract. A new determination of the frequency of Be stars among B stars is presented for each spectral type and luminosity class. The most outstanding characteristics of Be stars are taken into account: errors in spectral type determinations; difference of absolute magnitudes of B and Be stars due to the presence of a circumstellar envelope; volume differences of magnitude-limited samples of various spectral types; changes in magnitude, spectral type and luminosity class due to the fast rotation of Be stars; change in spectral type during constant mass evolution from the main sequence to luminosity class III. The mean frequency over the whole sample is at least 17 % and a frequency as high as 34 % is obtained for Be stars of spectral type B1e. The frequency is the highest for B1e whereas previous results found a maximum for B2e stars. The same frequency is obtained for each luminosity class. This result, as well as the high value of this frequency, imply that Be stars can hardly represent a given stage in the evolutionary track of every B star. This suggest that the Be character could be interpreted as existing since the formation of the star.

Key words: stars: emission-line; Be – stars: statistics; circumstellar matter – stars: evolution

1. Introduction

1.1. Aim of this study

Though Be stars have been known since 1866, and thousands of them have been detected up to now, the origin of the Be phenomenon still remains unexplained. Several ways by which an object could present a Be phenomenon are currently put forward in the literature: 1) due to an intrinsic property since stellar formation; 2) due to accelerated spinning during the secondary contraction phase (Crampin & Hoyle 1960; Schild & Romanishin 1976); 3) as a consequence of mass transfer in binary systems (Křiž & Harmanec 1975, Pols et al. 1991).

Send offprint requests to: J. Zorec

Hypothesis 1) considers the Be phenomenon as due to innate stellar characteristics, which favor the appearance of a circumstellar envelope at any evolutionary phase. Among these characteristics is strong rotation, either rigid (Endal 1982), or net differential rotation with a high content of angular momentum (Zorec et al. 1990). Although the mechanisms which determine the whole Be phenomenon are still unknown, a wide spectrum of known hydrodynamic instabilities could potentially be related to the Be phenomenon. As they can be active at any evolutionary stage, depending on the initial conditions, it is expected that the frequency of Be stars would then be about the same in all luminosity classes. Comparing the number of stars in each luminosity class of a sample of 179 Be stars, Slettebak (1982) concluded that there are roughly the same number of Be stars in all luminosity classes. The hypothesis of high rigid rotation as an innate characteristic of Be stars was studied by Endal (1982) for stars with masses $M = 5M_{\odot}$. He predicted a frequency not higher than 10% of Be stars with spectral types B4-B5. This proportion is rather low compared to that observed, which means that high rigid rotation might not be the unique innate characteristic of Be stars.

Hypothesis 2) and 3) consider the Be phenomenon as an evolutionary effect, either in a given stage of the star itself or due to that of the environment in a binary system. Concerning case 2), Hardorp & Strittmatter (1970) concluded that secondary contraction cannot account for the observed frequency of Be stars. In case 3) the Be phenomenon appears to be due either to an accreted disc during a mass-transfer phenomenon (Křiž & Harmanec 1975), or to transfer of angular momentum, the mass-gainer star becoming a rapid rotator (Packet 1981). This last mechanism was explored by Pols et al. (1991) who accounted for the shape of the distribution of Be stars of all luminosity classes together but only for half of their population.

Slettebak's (1982) observations, which support hypothesis 1), and the theoretically predicted proportions of Be stars clearly show that to obtain new insights on the actual status of the Be phenomenon in the evolutionary path of B stars, the frequency distribution of Be stars according to spectral type and separately for each luminosity class needs to be precisely determined. Although numerous determinations of the frequency of Be stars already exist, all luminosity classes are considered together. Moreover, this parameter was always obtained from direct counts in current catalogues, without taking into account the specific characteristics of Be stars liable to deeply modify its determination. For the first time, this paper presents a study of the frequency of Be stars made separately for each luminosity class. This study is also made in order to detect, estimate and take into account those phenomena which can modify the value of the fraction of stars presenting the Be characteristics among all B type stars. Among such phenomena, those analysed here are: 1) errors in spectral type determinations (Sect. 4); 2) difference of absolute magnitudes between B and Be stars due to the presence of circumstellar envelopes (hereafter CE) (Sect. 5.1); 3) rotational effects on magnitudes, spectral types and luminosity classes (Sect. 5.2); 4) changes in spectral types during stellar evolution from the main sequence towards luminosity class III (Sect. 6).

1.2. Previous studies and results obtained

Numerous determinations of the frequency of Be stars have already been made for more than sixty years. The set of results already published with their references is shown in Fig. 1. We report here only those studies which have used as much as possible unbiased samples. A biased sample can result when the study also includes those B stars which are considered as having a high probability of being a Be star (ex. B stars with high $V \sin i$, or showing some photometric variations), but have never been seen presenting emission lines. Even excluding such results, those presented in the literature frequently differ greatly according to the authors. These differences can be explained in several ways. The first obvious reason is the fraction of still undetected Be stars. Considering as a Be star every B star (excluding supergiants) which has shown some emission at least once, the number of Be stars can only increase since the first studies (see Fig. 1). The number of undetected Be stars can become particularly high when the sample extends to too faint stars. Such samples are necessarily made from poorly observed stars. As we shall see in Sect. 3, Coté & van Kerkwijk (1993) made an estimation of still undetected Be stars. It is known that the spectral type determination of a Be star is a difficult task. Different patterns of the frequency of Be stars as a function of spectral

type may result when spectral types are determined using different classification criteria. In some classification systems (see Sect. 4) certain subspectral types are more or less systematically discarded. In such systems, as well as in photometric classification systems, there may also be a systematic tendency to attribute too late spectral types, as for Be stars with strong "shell" characteristics (Divan & Zorec 1982). Results may also change greatly depending on the way the subspectral types are grouped together (Jaschek & Jaschek 1983, Abt 1987). Although Kogure & Suzuki (1986) use the same stellar sample as Jaschek & Jaschek (1983), these authors explain that their results are different because they included stars without any indication as to luminosity class and because they omitted Be/Ae stars considered as secondary components of binary systems. In spite of these rather large differences among studies, the general trend, from early to late B types, is as follows: there is an increase in the frequency of Be stars from B0 to B2, where it reaches the maximum value at about 20%, then it decreases toward late B types, for which there is only a small percentage. The mean frequency of Be stars considering all spectral type-luminosity classes, obtained from simple counts, is about 12% (Jaschek & Jaschek 1983; Kogure & Suzuki 1986) and 15%, as results from the most recently published work (Coté & van Kerkwijk 1993).

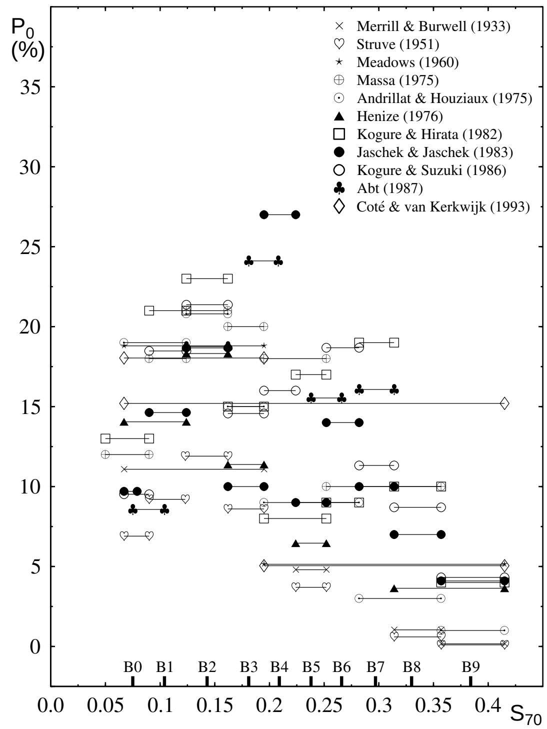

Fig. 1. Percentage of the number of Be among all B stars as a function of spectral type according to different authors. Spectral types are represented by the continuous parameter S_{70} (in dex; see Sect. 2.2)

2. Stellar samples

2.1. The sample used

The sample used in this work was taken from the Bright Star Catalogue (Hoffleit & Jaschek 1982, hereafter BSC) and from the Supplement to the Bright Star Catalogue (Hoffleit et al. 1983, hereafter SBSC) which concerns mainly field stars brighter than $V = 7.10$ mag. This catalogue is a good tool for this kind of study, because stars are, on the one hand, numerous enough to make statistically significant samples and, on the other hand, they are bright enough to have been rather well studied. The samples are made up of B and Be stars of luminosity classes in the range V-III, thus excluding supergiants. The sample referred to in this paper as "B stars" is made up not only of B stars currently considered as normal B stars, but also of particular ones: metallic, helium, magnetic Bp stars and special types such as Beta Cephei stars.

We considered as a Be star every object, single star or binary component, classified as such in the BSC and SBSC, including all those noted in the "Remarks" as having "Hα in emission" or "shell spectrum". We added 14 stars of the BSC whose emission characteristics were discovered after the BSC and SBSC publications (Barker 1982; Bidelman 1982; Singh 1982; Hirata et al. 1986; Ghosh et al. 1987; Herzog 1989; Coté & van Kerkwijk 1993). We included in our sample Be stars in binaries, because most of those in the BSC and SBSC have periods longer than 0.1 yr^{-1}, so that tidal forces could not prevent the formation of rapidly rotating stars (Abt & Cardona 1984). The few Be stars with forbidden lines reported in the BSC and SBSC were also considered in this study. Besides A stars noted as presenting emission/shell features in the BSC and SBSC, we also considered those listed by Jaschek et al. (1986, 1988). Counts were also extended to late O and early A type stars with emission and/or shell characteristics like those of Be stars in order to study the frequency of the Be phenomenon in both extremes of B subspectral types and to include the corrections made in Sects. 4, 5 and 6.

About 60 % of early B stars (B0-B5) and 30 % of late type (B6-B9) without emission lines in all luminosity classes, are considered in the BSC and SBSC as members of clusters, associations, stellar groups or streams, associated to nebulosities, etc. Nearly 40 % of Be stars of all spectral types and luminosity classes also share the same membership designation. Differences in the above given percentages of B and Be star memberships can be explained in terms of many uncertainties (in distance moduli, radial velocities, proper motions, etc.) due to the Be phenomenon, which makes impossible to determine unambiguously the association of these stars with a given stellar group. Hence, these fractions are not indicative of possible differences concerning distributions of B and Be stars among stellar groups. Moreover, Abt (1987) has also noted that: (a) locations of early B and Be Gould Belt stars are similar, which imply similar ages; (b) Be stars are roughly in constant frequency in clusters of all ages; (c) somewhat different frequencies of Be stars from one cluster to another may probably be due to detection bias. The above given percentages also show that it is almost impossible to isolate a statistically significant stellar sample for which the designation *field-star sample* can have some meaning. It is worth noting that the number of known and/or studied Be stars in any of the above mentioned groups, which can be considered as formed in the same environment, is too small up to now to have any statistical significance. Each stellar group (environment) may, in principle, be characterized by distinctive initial conditions of stellar formation, such as turbulence in the prostellar nebulae, amount and distribution of angular momentum, magnetic fields, etc. (Boss 1986; Capuzzo-Dolcetta et al. 1990; Catalano & Stauffer 1991; Lada & Kylafis 1991). They could determine: (a) that Be stars present their characteristics since the formation phases; (b) that instabilities (or whatever phenomenology) evolve, so that 'abilities' of a given object to develop the Be phenomenon become manifest at later evolutionary stages. Thus, having at the moment no convincing indications that B and Be stars distribute in a different way among all known stellar environments and/or that they are characterized by distinctive initial conditions, we can use the whole BSC+SBSC sample of Be stars to ask which of the two possibilities is the most probable.

2.2. The spectral type-luminosity class classification

In the present work the frequency of Be stars is determined for each spectral type, because the physical parameters (temperatures and mean $V \sin i$) differ greatly from B0 to B9 stars. In particular, emission characteristics vary strongly according to the stellar effective temperature, showing a monotonic decrease of mean intensity from B0 to B9 (Briot & Zorec 1981).

It is well known that spectral classification of Be stars is difficult to determine; first, because the emission/absorption produced by the circumstellar envelope perturbs the photospheric spectrum; second, because of the fast rotation (Slettebak & Kuzma 1979; Slettebak et al. 1980). As often as possible we used the BCD classification (Chalonge & Divan 1952), mainly for Be stars (94 objects). In this system spectral type-luminosity class determinations are not affected by the perturbing effects of the circumstellar envelope (Divan 1979; Zorec & Briot 1991, hereafter ZB and L. Divan, priv. comm.). In cases where the spectral classification is incomplete in the BSC or SBSC, we used data from Buscombe's (1977, 1980, 1981, 1988) compilations. The spectral types are represented in this work with a continuous parameter S_{70} defined in the BCD spectrophotometric system. It corresponds to the value $\mathscr{D}$ given in *dex* of the Balmer discontinuity (BD) at the continuous luminosity-class parameter $\lambda_1 - 3700 = 70$ Å, where λ_1 is the mean spectral position of the BD (Zorec et al. 1983, see also Fig. 10 of the present paper). The parameter S_{70} transforms the vertical curvilinear limits of spectral types in the $(\lambda_1, \mathscr{D})$ diagram into rectilinear ones. We could do the same operation for the limits of luminosity classes, but it is irrelevant for the present paper. The use of a continous parameter to describe spectral types is needed to perform the mathematical corrections we introduce to the "observed" counts.

To obtain statistically significant samples as a function of the luminosity class and to minimize as much as possible the errors in the luminosity class determinations, we classified our stellar samples into three wide groups: *dwarfs*, consisting of stars of luminosity class V only; *subgiants*, containing stars of luminosity classes IV-V and IV; *giants*, concerning stars of luminosity classes II-III, III and III-IV.

3. Preliminary stellar counts and frequency of Be stars including an estimation of undetected Be stars

The number of B and Be stars obtained by counts in the BSC and SBSC according to the various spectral type-luminosity classes is presented in Table 1. The general frequency of Be stars is 15 %.

Table 1. Number of B and Be stars in the *Bright Star Catalogue* and *Supplement*

		Be stars			All stars		
Spectral type	S_{70} (*dex*)	*dwarfs*	*subgiants*	*giants*	*dwarfs*	*subgiants*	*giants*
O8-O8.5	0.051	4		2	6		3
O9-O9.5	0.058	3	1	3	13	2	7
B0	0.075	3	4	1	11	6	11
B0.5	0.084	1	3	2	12	9	16
B1	0.104	10	4		44	8	13
B1.5	0.119	2	1	2	25	11	9
B2	0.143	28	21	10	99	110	26
B2.5	0.166	15	3	2	51	27	4
B3	0.181	20	15	8	137	54	40
B4	0.209	15	5	2	45	18	9
B5	0.238	15	1	4	134	33	38
B6	0.266	12	6	7	61	28	22
B7	0.297	10	6	4	59	24	43
B8	0.330	19	2	6	166	26	95
B8.5	0.347	2		1	13	1	1
B9	0.384	10	3	7	192	56	85
B9.5	0.423	9			104	10	22
A0	0.445	6	1	2	301	33	35
A1-A2	0.506	1		1	498	57	30
Total		185	76	64	1971	513	509

Note : All stars = *normal*, *peculiar* and *emission* stars. dwarfs = luminosity class V; *subgiants* = luminosity classes IV-V and IV ;*giants* = luminosity classes II-III, III and III-IV

We have to take into account an important parameter which can noticeably increase the estimated frequency of Be stars. This parameter is the fraction of those Be stars which have not yet been detected. Lack of detection can be due to several reasons: either because observations were inadequate, e.g. too low dispersion or the explored spectral range did not include the Hα line, or because the variable character had momentarily disappeared during the observation period. This parameter was studied by Coté & van Kerkwijk (1993) using stars of the BSC. It might well be considered that any star of this catalogue has been sufficiently observed for every possible Be star to have already been detected. However, Coté & van Kerkwijk (1993) observed again a sample of 168 "normal" B stars and detected among them 7 new Be stars. Hence, they inferred that there should be a mean value of 55^{+45}_{-24} (90 % confidence level) of still undetected Be stars among the 1318 "normal" B stars of the BSC. Nevertheless, this study treats Be stars as a whole and does not differentiate them into various subspectral type-luminosity classes. Unfortunately, the spectral classification of the 168 star sample studied is not given, and the sample of new detected Be stars is too small to deduce a correlation between the rate of undetected Be stars and the spectral type-luminosity class. Thus, we assume that the probability of undetected Be stars is the same for all subspectral type-luminosity classes. The factor deduced from the work of Coté & van Kerkwijk (1993), which corresponds to the mean probability of finding the still missing Be stars, by which our counts have to be multiplied to obtain a completed set of Be stars, is $m = 1.26$. Knowing that the total number of stars $N(\mathrm{Be}) + N(\mathrm{B})$ in each spectral type-luminosity class remains unchanged, we obtained the number ratios corrected for undetected stars $P_{\mathrm{m}} = m \times N(\mathrm{Be})/[N(\mathrm{Be}) + N(\mathrm{B})]$ presented in Table 2 [$N(\mathrm{B})$ and $N(\mathrm{Be})$ are the original counts given in Table 1; the suffix m indicates that correction has been done for "missing" or still undetected Be stars]. The number ratios P_{m} and P_o, the latter obtained directly from data of Table 1, as a function of spectral type and luminosity class are shown in Fig. 2. The left-hand ordinates are for P_{m} and the right-hand ones for P_o. In Fig. 2 we also plot the mean distribution obtained using stars of all luminosity classes together. We call them luminosity-mean ratios. The isolated point corresponds to the global mean ratio $P = 19^{+3}_{-2}$ when all spectral types and luminosity classes are considered together. In Fig. 2 we see that there is a global increase of the frequency of Be stars from B0 to B4, then a decrease to the latest B spectral types with an amazing low value for B5, which appears in all luminosity-class groups. The maximum in B4 was already found by Kogure & Suzuki (1986) and also very close to this spectral type by Abt (1987). The uneven distribution of the frequency of Be stars in the spectral type interval B4-B6 will be discussed in Sect. 4. We also see that surprisingly the frequency of Be stars does not actually dif-

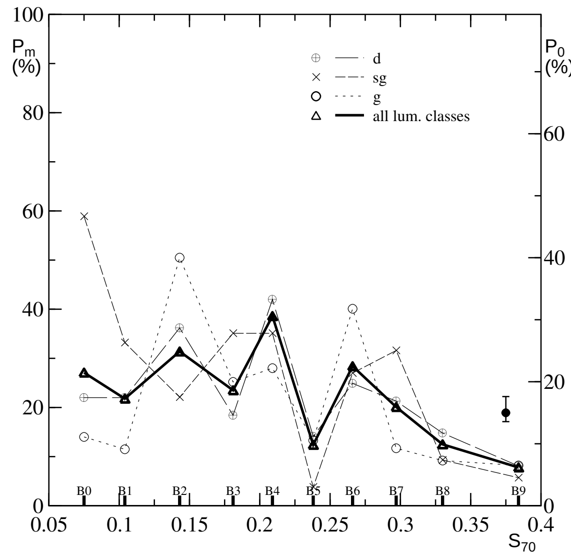

Fig. 2. Ratio P (as a percentage) of Be stars as a function of spectral type (S_{70}) for each luminosity class group (d: dwarfs; sg: subgiants; g: giants). Subindices of P are: m (stellar samples corrected for undetected Be stars); 0 (no correction was introduced to stellar counts). The isolated point corresponds to the global mean frequency of Be stars. Error bars are for uncertainties on the estimated missing stars at 90% confidence level.

fer in the various luminosity classes. Discrepancies among the calculated frequencies ensure, from statistical (mathematical) grounds, a similarity of frequency distributions in all studied luminosity class groups to more than 80 % confidence level in all sub-spectral types, except in the B4-B6 interval (only 50 % confidence level), which is probably affected by high uncertainties concerning spectral type assignations.

All remaining changes of frequencies of Be stars studied in this paper will be done considering counts corrected for undetected Be stars. The final number ratios obtained have then be considered as possible being upper limits. Dividing the obtained figures by m we can derive a good estimate of frequencies reflecting only the up to day known Be stars.

3.1. Correction for spectral type uncertainties

The number ratios presented in Fig. 2, and the respective values given in Table 2 for all B subspectral types are from unsmoothed stellar counts, which reflect the actual stellar population in the catalogues used in this work. It is known, however, that the MK classification system has no B4 comparison star, and there is only one of type B6. This incites a number of classifiers not to use them (Abt 1987, C. Jaschek, priv. comm.). The systematic lack of B4 and B6 type stars which thus results, (curiously stronger among B than Be stars), produces spurious maxima in the distribution of the frequency P around B4 and B6 and a deep minimum for the B5 spectral type. Although this error is rather of systematic nature, instead of grouping the B4 and B6 stars into larger neighbouring spectral-type intervals (Jaschek & Jaschek 1983; Abt 1987), which produces too wide sampling steps in the physical S_{70} scale, we introduced a statistical correction to the original stellar counts. Such a correction would nevertheless be necessary, because of differences existing among classifiers in determining a spectral type. If $N(S)_{\rm obs}$ is the observed count of stars of spectral type S, the corrected count from the errors of spectral classification $N(S)$ is obtained by solving the following integral equation (Smart 1958):

$$N(S)_{\rm obs} = \int_{-\infty}^{\infty} N(S - \delta S)\phi(\delta S){\rm d}\delta S \qquad (1)$$

where δS is the error of spectral classification in the S_{70} scale and $\phi(\delta S)$ is the error distribution function. We supposed $\phi(\delta S)$ to be Gaussian characterised by a dispersion $\sigma_{\delta S}$ = 0.6 (of sub-spectral type) for B stars without emission line stars (Jaschek & Jaschek 1987) and by a dispersion twice as large for Be stars. The solution of (1) produces smoothed counts, so that the deep minima around B4 and B6 spectral types are strongly shallowed as shown in Fig. 3. The corrected frequencies of Be stars from spectral type uncertainties $P_{\rm m,S}$ are given in Table 2. To each $P_{\rm m,S}$ value we give the expected statistical uncertainty related to the correction $\sigma_{\delta S}$. It can then be shown that the remaining 'bumps' near B4 and B6 might be considered as real facts only to a rough 80 % confidence level. For comparison, we also plotted in Fig. 3 the luminosity-mean frequency distribution $P_{\rm m}$. The isolated point represents the global mean frequency of Be stars with the corresponding error bars due to the lack of knowledge of still missing Be stars. More or less systematic errors of spectral classifications due to disturbances that cool circumstellar envelopes produce mainly in strong "shell" Be stars are very difficult to quantify. Although correction for this effect would probably increase somewhat the number of Be stars among the hotter sub-spectral types, we could not take them into account.

4. Effect due to the overluminosity of Be stars

Emission produced in the CE of Be stars and a brightening increase of Be stars in the visible range due to fast rotation have the consequence that apparent V magnitude-limited B and Be stars with the same mass and lifetime are not counted in the same space volume around the Sun. The resulting frequency of Be stars can then be systematically overestimated. We shall see that rotational brightening is also accompanied by a rotational spectral type-luminosity class deviation. The two effects have, as a mean, opposite consequences on counts. As the above mentioned stellar brightenings are different in nature, we shall consider their effects on counts in successive approximation steps.

4.1. Brightening due to the circumstellar envelope

4.1.1. Visible flux excess in Be stars

Be stars have an observed continuum energy distribution which is characterized by variable colours and flux excesses (in this section we neglect small-amplitude photometric variations

Table 2. Frequency P (in %) of Be stars after correction for missing stars (subindex "m"), specral type classification uncertainties (subindex "S"), reduction to common volume (subindex "ϵ"), circumstellar enveloppe emission (subindex "e"), rotational changes (subindex "ω") and evolution (subindex "evol.")

	$P_{\rm m}$				$P_{\rm m,S} \pm \delta P$				$P_{\rm m,S,\epsilon}$	$P_{\rm m,S,\epsilon,e}$
	d	sg	g	all	d	sg	g	all	all	
B0	22	59	14	27	23±14	52±21	16± 8	26± 4	26	20
B1	22	33	12	22	28± 5	27± 5	22± 7	27± 2	28	23
B2	36	22	51	31	31± 3	24± 3	39± 5	29± 1	29	26
B3	18	35	25	24	24± 4	30± 4	29± 5	27± 1	26	25
B4	42	35	28	39	26± 5	27± 8	22± 7	25± 2	24	24
B5	14	4	13	12	17± 4	12± 9	18± 6	17± 2	18	18
B6	25	27	40	28	21± 6	24±10	24± 5	22± 2	21	21
B7	21	32	12	20	19± 4	25±10	12± 3	18± 2	18	18
B8	15	9	9	13	13± 2	11± 7	9± 2	12± 1	12	12
B9	8	6	8	8	8± 2	6± 5	8± 2	8± 1	7	7
$\overline{P}$	19	22	17	19	20	23	18	20		
$\overline{P}(\epsilon)$					15	17	13		15.4	14.8

	$P_{\rm m,S,\epsilon,\omega}$ P(CTC)[P(CS)]				$P_{\rm m,S,\epsilon,\omega,e}$ P(CTC)[P(CS)]				$P_{\rm m,S,\epsilon,\omega,e,evol.}$ P(CTC)±δ[P(CS)±δ]
	d	sg	g	all	d	sg	g	all	all
B0	25[35]	54[59]	20[20]	29[37]	19[27]	44[49]	14[14]	22[28]	27±1[35 ± 2]
B1	36[48]	37[50]	31[43]	36[48]	29[40]	30[42]	24[35]	28[40]	34±1[45 ± 1]
B2	33[32]	27[27]	44[44]	32[31]	29[27]	23[23]	38[38]	27[26]	31±1[29 ± 1]
B3	24[23]	27[21]	25[17]	25[22]	21[20]	24[19]	23[15]	22[20]	24±1[23 ± 1]
B4	28[29]	27[28]	17[22]	27[29]	26[27]	25[26]	16[20]	25[27]	27±1[29 ± 1]
B5	20[20]	22[28]	15[22]	20[21]	19[19]	21[26]	14[20]	19[20]	20±1[21 ± 1]
B6	24[35]	29[32]	21[20]	25[33]	24[34]	29[31]	20[19]	24[32]	21±3[29 ± 4]
B7	26[35]	30[35]	10[11]	25[33]	26[35]	30[34]	9[11]	25[33]	22±3[29 ± 5]
B8	16[16]	21[21]	5[5]	16[15]	16[16]	20[20]	5[5]	15[15]	15±1[15 ± 2]
B9	8[5]	10[6]	4[2]	8[5]	8[5]	10[6]	4[2]	8[5]	8±1[5 ± 1]
$\overline{P}$	18[19]	23[24]	10[11]	18[19]	17[18]	22[23]	9[10]	17[18]	17[19]

Note: d = dwarfs; sg = subgiants; g = giants; all = all luminosity classes; δP (or δ) = uncertainty on P; $\overline{P} = \int(\partial N({\rm Be})/\partial S)dS/\int(\partial N({\rm Be+B})/\partial S)dS$; CTC = Collins et al. (1991); CS = Collins & Sonneborn (1977); $\overline{P}(\epsilon)$ = mean with common volume reduction

sometimes related to non-radial pulsations). The changes in the V magnitude in a "Be" phase imply brightenings which commonly are $\Delta V \sim -0.01$ to -0.2 mag (Feinstein 1975; Alvarez & Schuster 1987; Kozok 1985) but they can be as high as $\Delta V \sim -0.5$ to -1.5 mag (Huffer 1939; Alvarez & Schuster 1987; Divan et al. 1982, 1983; Apparao 1991; Mennickent & Vogt 1991). On the contrary, "Be-shell" phases are likely to be characterized by $0 \lesssim \Delta V \lesssim +0.2$ mag (Huffer 1939; Harmanec et al. 1980; Sharov & Lyutyi 1992; Zorec 1994). These photometric variations are not indicative by themselves of the total visible flux excess (positive or negative) produced by the CE. The $\Delta M_{\rm V}^{\rm e}$ absolute magnitude difference corresponding to the total visible luminosity excess produced by the CE was studied in ZB and in Ballereau et al. (1995), where it is shown that, on average, it is higher as stars are hotter. In Table 3 we reproduce the mean values of $\overline{\Delta M_{\rm V}^{\rm e}}$ as a function of the spectral type obtained in ZB which have been adopted for this work.

4.1.2. Correction of counts

Let us focus in this subsection on the brightening effect produced by the CE alone. Our aim is to determine the correction factor $\eta_{\rm e}$ able to transform the 'observed' counts of Be stars, so that they turn out representing the same objects as cleaned-up from the CE-effects. The effect of $\Delta M_{\rm V}^{\rm e}$ on counts of Be stars can be determined using the equation of stellar statistics (Mihalas & Binney 1981) which, for the purposes of this paper, we write for all directions around the Sun:

$$N(V|\mathscr{C}) = \int_{-\infty}^{V} {\rm d}m_V \int_{vol.} \Phi(M_V|\mathscr{C})\rho(\varpi, z, \ell|\mathscr{C})\varpi {\rm d}\varpi {\rm d}\ell {\rm d}z \quad (2)$$

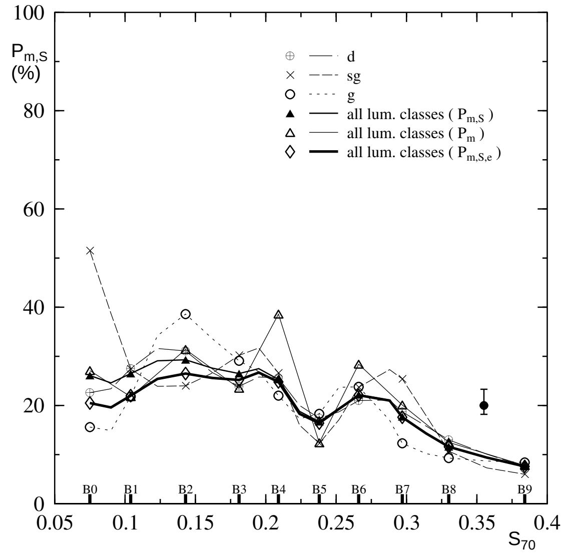

Fig. 3. Ratio P (as a percentage) of Be stars according to spectral type for each luminosity class group (d: dwarfs; sg: subgiants; g: giants). Subindices of P indicate successive corrections: m (stellar samples corrected for undetected Be stars); S (corrected for spectral type uncertainties); e (corrected for the overluminosity of Be stars due to CE). The isolated point corresponds to the global mean frequency of Be stars obtained using stellar samples with corrections "m" and "e". Error bars are for uncertainties in correction "m".

Table 3. Mean $\Delta M_{\rm V}^{\rm e}$ as a function of spectral type

Sp. type	$\overline{\Delta M_{\rm V}^{\rm e}}$ mag
B0	−0.54
B1	−0.46
B2	−0.32
B3	−0.22
B4	−0.15
B5	−0.10
B6	−0.18
B7	−0.03
B8	−0.02
B9	−0.00

where $N(V|\mathscr{C})$ is the number of stars of class $\mathscr{C}$ ($\mathscr{C}$ indicates B or Be stars of a given MK spectral type-luminosity class) brighter than the limiting apparent magnitude V; Φ is the luminosity function of stars of class $\mathscr{C}$; $M_{\rm V}$ is the visual absolute magnitude; ρ is the relative density function; (ϖ, z) are the positional components in the galactic heliocentric system (ϖ is measured on the galactic plane and z is the distance perpendicular to the galactic plane); ℓ is the galactic longitude (galactic center is for $\ell = 0^o$). We adopted for $\rho(\varpi, z, \ell)$ the density function derived by Bienaymé et al. (1987) for the youngest galactic disc. It thus corresponds to the entire population of B stars without any distinction concerning their particularities. Neglecting any perturbing effect produced by the CE on the underlying stellar photosphere (Höflich 1988), the following representation of the absolute magnitude of a Be star (central star plus envelope) then holds:

$$M_{\rm V}({\rm Be}) = M_{\rm V}({\rm B}) + \Delta M_{\rm V}^{\rm e} \qquad (3)$$

where $M_{\rm V}$(B) is the B normal-like absolute magnitude of Be stars during an emissionless phase. The $\Delta M_{\rm V}^{\rm e}$ magnitude difference is a strongly, mostly irregularly variable parameter. To take into account the possibility that Be stars of our sample are in any possible photometric variation phase, we suppose that $\Delta M_{\rm V}^{\rm e}$ follows a Gaussian distribution around a characteristic mean value $\overline{\Delta M_{\rm V}^{\rm e}}$ with dispersion σ_Δ. As no clear dependence of σ_Δ either on luminosity class or on the spectral type is known, we considered a unique value $\sigma_\Delta = 0.14$ mag (ZB). Adopting a Gaussian distribution for the $M_{\rm V}$ magnitudes, as for B stars of a given MK spectral type-luminosity class (see Appendix A), it can easily be shown that the luminosity function of Be stars is then given by:

$$\Phi(M_{\rm V}|{\rm Be}) = \frac{1}{\sqrt{2\pi}\sigma_{\rm Be}} {\rm e}^{-[M_{\rm V}-(\overline{M_{\rm V}({\rm B})}+\overline{\Delta M_{\rm V}^{\rm e}})]^2/2\sigma_{\rm Be}^2} \qquad (4)$$

where $\sigma_{\rm Be} = (\sigma_{\rm B}^2 + \sigma_\Delta^2)^{1/2}$. Integrals of type (2) are performed using Pogson's known formula: $V = M_{\rm V} - 5 + 2.5\log_{10}(\varpi^2 + z^2) + A_{\rm V}(\varpi, z, \ell)$, where $A_{\rm V}$ is the interstellar medium (ISM) extinction. Higher distances associated with the $\Delta M_{\rm V}^{\rm e}$ magnitude excess also imply larger interstellar medium (ISM) extinctions, which attenuates the luminosity excess effect. Thus, we paid some attention to the ISM absorption derivation. Its characteristics are given in Appendix B. Using the relations (2) to (4), the correction factor $\eta_{\rm e}$ is then obtained from:

$$\eta_{\rm e} = N(V|{\rm B})/N(V|{\rm Be}) \qquad (5)$$

where $N(V|{\rm B})$ represents counts of Be stars as they had lost their emissions. Because in the present approach, the count correction factor as derived from (5) only compares observed Be stars with the same stars cleaned-up from CE emission, it is sufficient to adopt for $M_{\rm V}$(B) the absolute magnitude of B stars without emission lines, so that rotational changes on $M_{\rm V}$ are neglected. This approximation introduces absolute errors on $\eta_{\rm e}$ not higher than $\delta(\eta_{\rm e}) \lesssim 0.007$. Considering that the limiting apparent magnitude of BSC and SBSC is $V = 7.1$ mag, the resulting function $\eta_{\rm e}$ against the spectral type is shown in Fig. 4 (pointed line). As the individual $\eta_{\rm e}(S_{70})$ curves for each luminosity class are very close to each other, within a dispersion $\sigma_\eta \lesssim 0.01$, in the diagram of Fig. 4 only the mean value of η was plotted. Thus, due to the CE brightening effect only, from Fig. 4 and counts of Table 1, it is expected that about 12% of 'observed' Be stars, mostly among the hottest ones, do not contribute to their global mean frequency determination. The luminosity mean frequency of Be stars corrected for CE brightening effect $P_{\rm m,S,e}$ (which also depicts, as an example, the effect for individual luminosity class groups) is shown in Fig. 3. The reduction of frequencies is the most pronounced in the B0-B4 spectral interval and it amounts to about 6 % for B0 type stars.

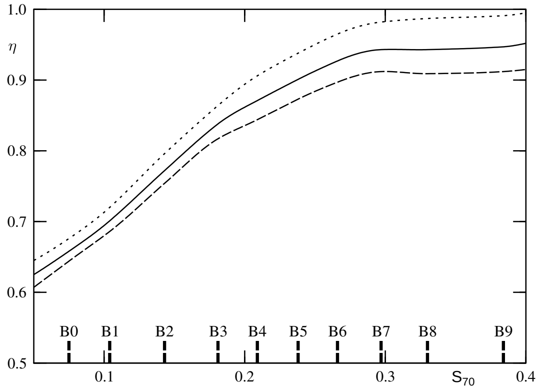

Fig. 4. Luminosity class-mean value of the correcting factor η due to the overluminosity of Be stars as a function of S_{70}. Pointed line: obtained using $\overline{\Delta M^{\rm e}_{\rm V}}$; full line: obtained using $\overline{\Delta M^{\rm e}_{\rm V}}$ and $\Delta M_{\rm V}(\omega)$ from CTC's models; dashed line: obtained using $\overline{\Delta M^{\rm e}_{\rm V}}$ and $\Delta M_{\rm V}(\omega)$ from CS' models.

4.1.3. Comments on the approximations used

The correction given by $\eta_{\rm e}$ aims at producing counts of Be stars as they were all observed without emission. We have, however, neglected in $\overline{M_{\rm V}({\rm B})}$ its dependence on the aspect angle i and on the rotational rate $\omega = \Omega/\Omega_{\rm c}$ (Ω and $\Omega_{\rm c}$ are respectively the angular velocity and the critical angular velocity), as it would be correct for rapid rotators. This approximation avoids to take into account in (4) the dependence of $M_{\rm V}({\rm B})$ on i and ω. It can easily be shown however, that in the present approach the effect of $\overline{M_{\rm V}({\rm B})}$ on the calculation of $\eta_{\rm e}$ is only of second order. Also, as in (5) the same density function weights both luminosity functions over the common part of the integration volume, possible uncertainties concerning the density function ρ will not introduce strong errors in the value of $\eta_{\rm e}$. Knowing that the z-height scale of the Galaxy is $h_{\rm z} \sim 70$ pc (Bienaymé et al. 1987), and, in the limiting case of small values of the magnitude V, where we can consider that $A_{\rm V} \simeq 0$, we can write $\rho(\varpi, z, \ell) \simeq \rho_o \delta(z)$ (δ is the 'delta' function). The following scaling relation independent of $\overline{M_{\rm V}({\rm B})}$ is then obtained from (5):

$$\eta_e \simeq \exp\Big(\frac{2}{5\mu}\overline{\Delta M^{\rm e}_{\rm V}}\Big); \quad \mu = 0.434 \qquad (6)$$

which can be used for a first order estimation of $\eta_{\rm e}$. Relation (6) can also be used to estimate the errors produced on counts due to uncertainties in the adopted values of $\Delta M^{\rm e}_{\rm V}$. We note again, that any comparison of counts of Be stars (rapid rotators) with those of B stars (low rotators), can however be made only after we had also taken into account the brightening effect and the change in spectral type-luminosity class induced in the central objects of Be stars by stellar rotation. In this case, differences in luminosity functions of B and Be stars are more important. We study these effects in the next section.

4.2. Effects due to stellar rotation

4.2.1. General remarks

Be stars rotate, on average, 1.5 to 2 times faster than B stars without emission lines, so that modal values of rotational rates are $\omega \sim 0.7$ to 0.8 [see also comments following relation (9)]. Rotation produces a geometrical deformation of the star, which induces non-uniformity of gravity and temperature on the stellar surface (Tassoul 1978; Moss & Smith 1981; Kippenhahn & Weigert 1990). Although the possibility of laws other than solid-body rotation should not be excluded (Collins & Smith 1985; Stoeckley and Buscombe 1987; Cranmer & Collins 1993; Zorec et al. 1987, 1990), we probably ought to prescribe statistical distributions of their characteristics, to take them properly into account for studying the induced effects on stellar counts. Because of the extent of uncertainties and non-uniqueness (Collins & Smith 1985) with which such study would be confronted, we shall limit the study of rotational effects to those related to rigid rotation. From Endal & Sofia's (1979) calculations, which assume that stars on the ZAMS are rigid rotators, and using relations (15) to estimate stellar radii in different evolutionary phases, we can see that the ratio of linear equatorial velocities $V_J/V_{\rm rigid}$ ($V_{\rm J}$ is the equatorial velocity considering angular momentum redistribution in the star by hydrodynamical instabilities; $V_{\rm rigid}$ equatorial velocity for complete angular momentum redistribution) evolve so that $V_J/V_{\rm rigid} \simeq 0.94$ for subgiants and $V_J/V_{\rm rigid} \simeq 0.90$ for giants. Rigid rotation seems then a good approximation. However, we still do not know what are the actual initial stellar rotation law and the total angular momentum stored in the star for this comparison to make sense. We should consider henceforth that our study probably gives only an indicative tendency of rotational effects on counts. Although the bolometric luminosity produced in the stellar core of rigid rotators is nearly the same (somewhat lower) as in non-rotating objects, the geometrical deformation and non-uniformity of gravity and temperature on the stellar surface produce a non-isotropic emerging radiation. The emitted stellar spectrum is then not only a function of the aspect angle, but in most cases the star looks globally brighter and more evolved-like than a non-rotating star. In the frame of rigid rotation, these effects have been extensively calculated by a number of authors [some of the most relevant references: Maeder & Peytremann 1970, 1972; Collins & Sonneborn 1977 (CS) and Collins et al. 1991 (CTC)]. Calculations for late B differential rotators were done by Collins & Smith (1985). In the present work we use the CTC's calculations as reference models. However, for comparison purposes we shall also estimate the rotational effects using the CS' models. The two calculations differ mainly in the attention given to line blanketing and rotational effects on stellar atmospheres, whose results produced significant quantitative differences in the predictions of magnitude and spectral type displacements.

4.2.2. Rotational effects on stellar counts

To calculate the correcting factor $\eta_{\rm e,\omega}$, which reduces counts of Be stars so that B and Be stars of a *given* mass can be considered as counted both in the same space volume, we shall take into account in this section the rotational brightening effect together with the CE emission. The reduction of counts of *all* masses to the same volume will be done in Sect. 5.2.3. As a function of ω and i, rotational displacements of magnitude and spectral type describe rather complicated curves in a $(M_{\rm V}, S_{70})$ diagram [see explanatory schema in Fig. 1 of Collins & Smith (1985)]. Consistent corrections of counts would then need to be done by considering both effects simultaneously. However, in all studied sub-spectral types, both pairs of characteristic values $(\omega_{\rm M}, i_{\rm M})$ and $(\omega_{\rm S}, i_{\rm S})$, which define the mean rotational deviations (weighted with distribution functions of ω and sin i) in $M_{\rm V}$ and S_{70}, are very similar. This suggests that, within a good approximation, corrections of counts for rotational effects on $M_{\rm V}$ and S_{70} can also be introduced in successive steps.

α) Rotational brightening effect and CE emission

Assuming that we can neglect in the luminosity class interval V to III the effects of rotation on stellar evolution and the CE-backwarming of the photosphere, we adopt the following representation for the absolute magnitude of a Be star (central star plus envelope):

$$M_{\rm V}(S'_{70}, \lambda'_1|{\rm Be}) \; = \; M_{\rm V}(S_{70}, \lambda_1|{\rm B}) + \Delta M_{\rm V}(\omega, i|S_{70}, \lambda_1) + \Delta M^{\rm e}_{\rm V}(S_{70}) \qquad (7)$$

In (7), the pair of parameters (S'_{70}, λ'_1) represents the spectral type-luminosity class of the rotating Be star [$S'_{70} = S_{70} + \Delta S_{70}(\omega, i|S_{70}, \lambda_1)$; $\lambda'_1 = \lambda_1 + \Delta\lambda_1(\omega, i|S_{70}, \lambda_1)$; ΔS_{70} and $\Delta\lambda_1$ are respectively the rotationally induced displacements of spectral type and luminosity class]. $M_{\rm V}(S_{70}, \lambda_1|{\rm B})$ is the absolute magnitude of a non-rotating B star whose spectral type-luminosity class is (S_{70}, λ_1) and which has the same mass and age as the studied Be star. $\Delta M_{\rm V}(\omega, i|S_{70}, \lambda_1)$ is the rotational change of the visible absolute magnitude. As in Sect. 5.1.1, $\Delta M^{\rm e}_{\rm V}(S_{70})$ represents the magnitude excess due to the CE. All displacements $\Delta M_{\rm V}(\omega, i)$, $\Delta S_{70}(\omega, i)$ and $\Delta\lambda_1(\omega, i)$ are estimated using CS' and CTC's models.

Let $Q(\omega|S_{70}, \lambda_1)$ be the distribution of angular velocity rates ω of stars with spectral type-luminosity class (S_{70}, λ_1) (see Appendix C for its derivation). Assuming that the aspect angles are randomly distributed, the luminosity function for Be stars can then be written as:

$$\Phi(M_{\rm V}|{\rm Be}) = \int_0^1 {\rm d}\omega \int_0^{\pi/2} \phi(M_{\rm V}, \omega, i|{\rm Be}) Q(\omega) \sin\, i\, {\rm d}i \qquad (8)$$

where we assumed a Gaussian distribution $\phi(M_{\rm V)}$ for the visual absolute magnitude around $M_{\rm V}$(Be) given by (7) with dispersion $\sigma_{\rm Be} = (\sigma^2_{\rm B} + \sigma^2_{\Delta})^{1/2}$. Supposing that populations of B and Be stars mirror each other, the factor $\eta_{{\rm e},\omega}$ which reduces the counts of rotating Be stars to a common volume shared with the emissionless, non-rotating B stars is obtained as indicated in (5) using (8) and $(A1.1)$ respectively for Be and B stars. In Fig. 4 we show $\eta_{{\rm e},\omega}$ (averaged over luminosity class) as a function of spectral type, obtained using the magnitude displacements $\Delta M_{\rm V}(\omega, i)$ calculated by CS (dashed line) and by CTC (full line).

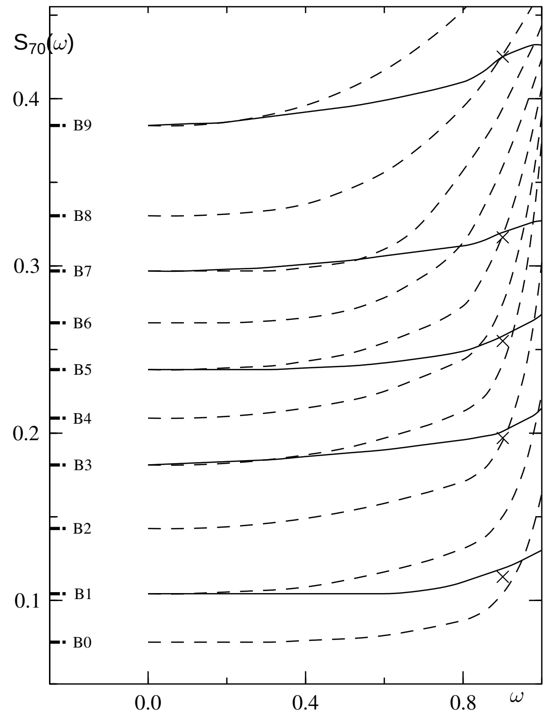

Fig. 5. The continuous spectral type parameter S_{70} (dex), averaged on aspect angle, as a function of the rotation rate ω. Full lines: using CTC's models; dashed lines: using CS' models; crosses: from CTC's models for luminosity class III.

β) Effect on counts due to spectral type displacement

As a consequence of rotational displacements of spectral types, stars with different masses and values of ω and i can be gathered together in a set characterized by the same *apparent* spectral type. So, the correction for spectral type displacement aims at transforming stellar counts, originally made as a function of *apparent* spectral types, into a new distribution given in terms of *rest* spectral types.

Using CS' and CTC's models, we determined $\mathscr{D}$, the value of the BD following the BCD method for all tabulated masses, aspect angles i and rotation rates ω. The BD were then transformed into the S_{70} index. To avoid cumbersome diagrams, we show in Fig. 5 only angle-averaged S_{70} curves [we used $P(i) = \sin\, i$ as weighting function] against ω; full lines are for CTC's models and dashed lines for CS's models. Even though we see in Fig. 5 a mean 'cooling' effect on the angle-averaged spectral types as ω increases, the published energy distributions in CS and CTC for $i \lesssim 30^{\circ}$ and $\omega \lesssim 0.9$, correspond to spectral types slightly hotter than those of non-rotating objects with same mass (CTC). In Fig. 5 we also plotted points (crosses) corresponding to the luminosity class III for $\omega = 0.9$ derived from CTC's models. As the effects for luminosity class III seem to be little different from those obtained for main sequence stars, we adopted the latter for all studied luminosity classes.

Let $\Delta(\omega, i|S)$ be the function which describes the change of a spectral type S at the rotational rate ω and aspect angle i. The relation between an observed distribution $N_{\rm obs}(S)$ of apparent spectral types, where stars of *different masses* contribute to the *same* spectrum S, and the distribution $N(S)$ of stars which all have the *same rest* spectrum S, can then be written as:

$$N_{\rm obs}(S) = \int_0^1 {\rm d}\omega \int_0^{\pi/2} N[S - \Delta(\omega, i|S)] Q(\omega|S) \sin\, i \, {\rm d}i \qquad (9)$$

where $Q(\omega|S)$ is the distribution of rates ω of stars with rest spectra S. Transformation (9) was used only for Be stars. We considered as negligible the fraction of high rotators among B stars without emission lines. A first approximation to $N(S)$ can be obtained supposing that in the interval $[S, S - \Delta(1, \pi/2|S)]$ the curve $N(S)$ is described with a second-order polynomial. Adopting the simplified representation of spectral type displacements: $\Delta(\omega, i) \simeq \Delta_o(\omega) \sin\, i$, the following mapping relation can readily be obtained:

$$N(S) \simeq N_{\rm obs}[S - \Delta(\omega_*, i_*|S)] \qquad (10)$$

where $i_*(S) \simeq 2\pi/7$ and $\omega_*(S) \lesssim 0.8$ for all cases studied in this paper. The parameter ω_* can be considered as characterizing a mean probable stellar deformation of Be stars by rigid rotation. A first approximation to $\eta_{\rm e,\omega}$ can be obtained using (6) by substituting $\overline{\Delta M^{\rm e}_{\rm V}} \rightarrow \overline{\Delta M^{\rm e}_{\rm V}} + \Delta M_{\rm V}(\omega_*, i_*)$. We also note that Slettebak et al. (1992) adopted as characteristic rate $\omega_* = 0.9$ to determine the photospheric like components of Balmer lines in Be stars. The effect on counts of rotational spectral type displacement is shown in Fig. 6. The 'observed' distribution (dashed line) concerns Be stars of all luminosity classes and counts in all spectral types were reduced to a common volume (see Sect. 5.2.3.). As expected, the main effect is to shift the observed distribution towards hotter spectral types. In the earlier spectral types (B0-B5) the CS' models (pointed line) produce a shift twice as large as CTC's models (full line), while in the latter spectral types (B6-B9) differences are in the same direction but they are of about a factor 3.

γ) Luminosity class displacement

The rotational displacement in luminosity class $\Delta\lambda_1(\omega, i)$ for each mass and luminosity class can be estimated by determining the effective surface gravity of the observed stellar hemisphere. Knowing that our stellar samples have a low luminosity class 'resolution', because they have been obtained by mixing rather wide luminosity class intervals, it would be sufficient to determine only a systematic mean displacement $\Delta\lambda_1(\omega_*, i_*|S_{70}, \lambda_1)$. This can easily be done using Slettebak's et al. (1980) results. Hence we obtained:

$$\log\ g(\omega_*, i_*) = \log\ g_o - (0.15 \pm 0.01) \qquad (11)$$

for all B sub-spectral types ($\log\ g_{\rm o}$ represents the surface gravity of non-rotating stars; see Appendix A). Considering that the observed luminosity class of Be stars corresponds to $\log\ g(\omega_*, i_*)$ and the corrected one to $\log\ g_{\rm o}$, the correction of counts for

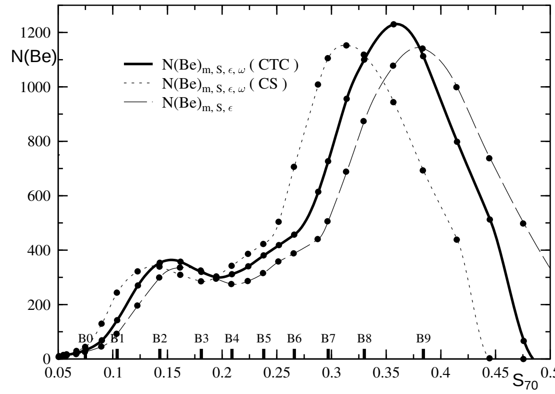

Fig. 6. Number of Be stars in all luminosity classes as a function of spectral type. Subindices indicate that stellar samples were corrected for: m (undetected Be stars); S (spectral type uncertainties), ϵ (stellar samples are volume-limited), ω (samples are corrected for effects on spectral type and luminosity class due to stellar rotation, but not on magnitudes).

luminosity class displacement is then simply determined by interpolation. As expected, although conserving the global mean frequency of Be stars over all luminosity classes, this effect slightly reduces the percentage of Be stars in the most evolved luminosity classes. These changes can be resumed in terms of luminosity mean frequencies calculated before and after the corrections are introduced, as follows: $P^{\rm after}_{\rm dwarfs} \gtrsim P^{\rm before}_{\rm dwarfs} \simeq 20\%$; $P^{\rm before}_{\rm subgiants} = 23\% \rightarrow P^{\rm after}_{\rm subgiants} = 24\%$; $P^{\rm before}_{\rm giants} = 18\% \rightarrow P^{\rm after}_{\rm giants} = 15\%$.

4.2.3. Volume-limited stellar samples

Before obtaining $N(S)$ from (9), it is convenient to reduce the observed counts $N_{\rm obs}(S)$, which are magnitude-limited, to a volume-limited distribution. In this way, stars with different apparent spectral types, which after (9) will form a set having the same rest spectral type, can be considered as having been all counted in the same space volume. In this work we considered a common limiting volume defined by the B0 giant stars. The sample-filling factor ϵ for each spectral type-luminosity class was obtained using (2) as:

$$\epsilon = N(V_L|{\rm B})/N(7.1|{\rm B}) \qquad (12)$$

where $V_{\rm L} = 7.1 + M_{\rm V} - M_{\rm V}({\rm B0})$. The values of ϵ obtained by (12) are given in Table 4.

Let us note that mean frequencies calculated with magnitude-limited stellar samples composed of both early and late type stars are affected by systematic error, because the hottest stars contribute an artificially enhanced weight, as compared to that they actually have regarding their low relative number.

4.2.4. Results

Corrections of counts for rotational effects were done in the following order: 1) all 'observed' counts corrected in Sect. 4

Table 4. Filling factors ϵ

Spectral type	d	sg	g
B0	2.1	1.4	1.0
B1	3.0	2.1	1.0
B2	4.5	3.1	1.4
B3	6.7	4.5	2.1
B4	9.9	6.6	3.0
B5	14.8	9.7	4.4
B6	22.0	10.0	4.5
B7	22.8	14.7	6.5
B8	33.9	15.0	6.6
B9	35.1	22.5	9.7

for spectral type uncertainties were first reduced to a common volume with the filling-factor ϵ given in Table 4 (Sect. 5.2.3); 2) counts of Be stars were then corrected for luminosity class rotational displacements (Sect. 5.2.2,γ); 3) corrections for spectral type displacement were then introduced for each luminosity class group (Sect. 5.2.2,β); 4) finally, using the factor $\eta_{\rm e,\omega}$ (Sect. 5.2.2,α) we added the correction for overluminosity of Be stars, considering the rotational effect and the CE emission together (Sect. 5.2.2,α).

Values of luminosity mean frequencies $P_{\rm m,S,\epsilon}$ and global mean frequencies obtained after reduction of counts to a common volume are given in Table 2. To have a quick insight on the lowering effect on frequencies produced by correction for CE emission, beside $P_{\rm m,S,\epsilon}$ we reproduced in Table 2 the luminosity mean frequencies $P_{\rm m,S,\epsilon,e}$ corrected for $\Delta M_{\rm V}^{\rm e}$ magnitude excess.

All rotational effects studied: brightening, spectral type and luminosity class displacement, can be appreciated together in frequencies $P_{\rm m,S,\epsilon,\omega}$ shown in Fig. 7 (in these frequencies the effect due to the CE is omitted). These frequencies are given for each luminosity class group and they were calculated using CTC's models. As counts seem to be rather sensitive to rotational effects, we also show in Fig. 7 the luminosity mean frequencies $P_{\rm m,S,\epsilon,\omega}$ determined with CS' models. To see clearly the effects due to rotation on P, we also reproduced in this figure the luminosity mean distribution $P_{\rm m,S,\epsilon}$. Hence, we may notice that rotational changes are responsible for a strong increase in the frequency of Be stars in masses corresponding to the B1-2 rest spectral type interval and to spectral types B6-7. We also see that rotational effects produce a well defined maximum of Be stars for the spectral type B1 (except perhaps for giants). Individual values of $P_{\rm m,S,\epsilon,\omega}$ corresponding to each spectral type-luminosity class, luminosity mean frequencies and global means for both models CTC and CS (in brackets) are given in Table 2. The additional brightening effect on counts due to the CE emission can be seen through the luminosity mean frequencies $P_{\rm m,S,\epsilon,\omega,e}$ shown in Fig. 8. Although only luminosity-mean frequencies are shown, it is worth noting that the effect of the CE acts in the same way in each luminosity class group. The corresponding values of frequencies, individual and means, derived

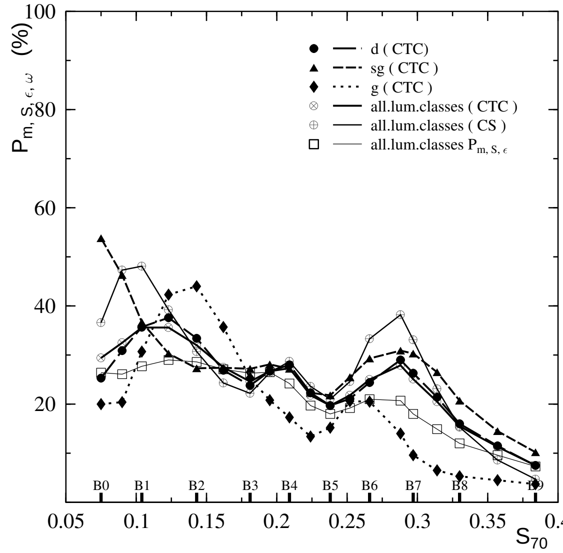

Fig. 7. Ratio P (as a percentage) of Be stars according to spectral type for each luminosity class group (d: dwarfs; sg: subgiants; g: giants). Subindices of P indicate successive corrections: m (stellar samples corrected for undetected Be stars); S (corrected for spectral type uncertainties); e (corrected for the overluminosity of Be stars due to CE); ϵ (stellar samples are volume-limited) and ω (effects on magnitude, spectral type and luminosity class due to stellar rotation using CTC's models and CS's models without CE emission effect).

using CTC's and CS' models are given in Table 2. In Fig. 8 we also plot the luminosity mean frequencies obtained after having introduced successively all effects quoted up to this stage of our work and using both CS' and CTC's models. We can see that in spite of the reduction of frequencies among the hotter spectral types produced by the correction for CE emission, the maximum of Be stars still remains for stars more massive than B2 rest spectral type, whatever the model of rotational effects used: CS or CTC. The wind-compressed disk (WCD) model (Bjorkman & Cassinelli 1993, Bjorkman 1994) predicts a disk formation rotation threshold so that Be stars are the most common at spectral type B2. Hence, our results imply that for the WCD model to be accepted as responsible for the Be phenomenon, the rotation threshold has to appear at 30 to 50% more massive stars than B2.

Within the statistical uncertainties quoted for $P_{\rm m,S}$, which are probably still higher for $P_{\rm m,S,\epsilon,\omega,e}$, because of all model-dependent corrections introduced, it is seen in Fig. 7 that distributions of frequencies of Be stars remain about the same in all luminosity class groups. After having introduced the correction for CE brightening, we see in Fig. 8 that many effects cancel each other out, so that in the spectral type interval B0-B5 we recover $P_{\rm m,S,\epsilon,\omega,e} \simeq P_{\rm m,S}$ if CTC's models are used. There is, however, an increase in the frequency of Be stars at latter spectral types than B5, due only to rotational effects. If there were to be no further modifications of proportions P, this result implies that in the whole spectral type interval B0-B7 is $\overline{P_{\rm m,S,\epsilon,\omega,e}}$ = 24 ± 3%, as the Be phenomenon had the same probability of appearing in

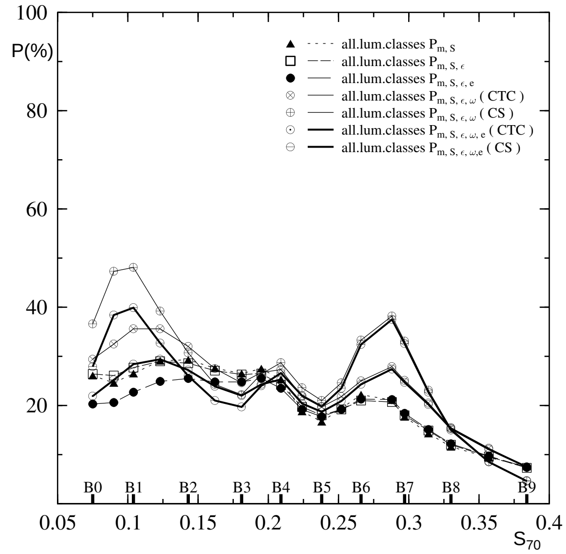

Fig. 8. Luminosity mean ratios P corrected successively for: m (undetected Be stars); S (spectral type uncertainties), ϵ (stellar samples are volume-limited), e (CE emission); ω (effects on magnitude, spectral types and luminosity class due to stellar rotation using CTC's models and CS's models).

any stellar mass and evolutionary stage of this spectral interval. However, this cannot perhaps be definitive because the uniformity is lost if rotational effects are discovered to be stronger than predicted by CTC's models, as suggested by results obtained with CS' models. The use of these models produces two well defined maxima: at B1 and B7 spectral types.

To a certain degree, all rotational effects studied in this paper can be scaled using the mean (i-averaged, ω-averaged or both) magnitude rotational displacement $< \Delta M_{\rm V} >$ as the reference parameter. Unfortunately, the existing attempts at determining $< \Delta M_{\rm V} >$ are affected by too strong uncertainties (empirical: Golay 1968, Maeder 1968, Smith 1971, in interpretation: Cotton and Smith 1983, Collins and Smith 1985) to be used for deciding which of the two, CTC's or CS's models, better represents the observations, or, whether rigid rotation is actually the most plausible rotational law. New determinations of $< \Delta M_{\rm V} >$ with HIPPARCOS parallaxes would perhaps give us better insights for such a choice, so that more reliable rotation-dependent corrections of frequencies of Be stars could be determined.

5. Spectral type change during constant mass evolution

For completeness, we also consider the effect on count distributions of spectral changes during stellar constant mass evolution, although this correction may be among the most uncertain ones. B and Be stars evolving from the main sequence towards luminosity class III have spectra that progressively correspond to cooler effective temperatures and lower surface gravities. In this section we aim at gathering together stars of different luminosity classes which have the same mass. To establish the relation among these objects, we should have to draw evolutionary tracks in an HR diagram for B and Be stars separately: (a) because of multiple effects that high rotation can produce on stellar interiors (Bodenheimer 1971, Endal & Sofia 1976, 1978, 1979, Sofia et al. 1994); (b) because emitted spectra of rotating stars (which are also functions of i and ω) may somehow reflect the evolution-rotation interaction, so that their evolutionary variation pattern is not necessarily the same as for non-rotating objects. The importance of these effects is also related to the value of stored rotational energy, currently measured with τ = [*rotational energy*]/[*gravitational potential energy*]. The evolutionary models of rotating stars calculated so far suppose that stars are rigid rotators in the ZAMS (Sofia et al. 1994), so that they are concerned only by small energy ratios $\tau \lesssim \tau_{\rm c} = 0.008$ ($\tau_{\rm c}$: rotational to gravitational energy ratio for critical rigid rotation, Zorec et al. 1990). Such low values of τ seem, from the existing calculations of intermediate massive stars (Kippenhahn et al. 1970a, Endal & Sofia 1979), to imply rather small changes in the evolutionary tracks (drawn using bolometric luminosity and effective temperature averaged over all possible orientations) as compared to non-rotating stars in the interval spanning the luminosity classes studied in this paper. These evolutionary tracks are nearly parallel to those of non-rotating B stars, and, within some percentages, lifetime scales are about the same. However, for objects with $\tau > \tau_{\rm c}$, or for those approaching the secular stability limit $\tau \lesssim 0.10$ (Clement 1979), such changes are still unknown in detail. For consistency with corrections for rotation made in Sect. 5, and considering the remarks in Sect. 5.2.1, we assume that stars began evolving as rigid rotators, so that $0 \lesssim \tau \lesssim \tau_{\rm c}$. However, as noted above, in an observed HR diagram spectra of rotating stars also depend on i and ω. Following the comments in Sect 5.2.2,β, we may consider that an evolutionary track described by the apparent spectrum $[\lambda_1(\omega, i), S_{70}(\omega, i)]$ is about parallel to that of a non-rotating star. As a consequence of this, B and Be stars with same apparent spectra (although somewhat different masses), should have the same apparent spectral progenetic relation.

Hence, in this work we separate evolution from rotation and use spectral progenetic relations which are obtained from evolutionary tracks of non-rotating stars and without mass-loss. They are shown in Fig. 10 and details on its determination are given in Appendix A. Thus, we obtained the number ratios $P_{\rm m,S,\epsilon,\omega,e,evol.}$ given in Table 2. As in previous cases, values were obtained using CTC's and CS' models, the last are given in brackets. Frequencies $P_{\rm m,S,\epsilon,\omega,e,evol.}$ are shown in Fig. 9. In this figure, we also plot the luminosity-mean ratio $P_{\rm m,S,\epsilon,\omega,e}$ for comparison. Within the statistical errors δP given in Table 2, we can conclude that the introduction of the evolutionary effect results in a monotonic decrease of P from hot spectral types to cooler ones, when the CTC's models are used for rotational effects. When CS' models are adopted, the evolutionary effects conserve both maxima at B1 and B7 type stars. In both cases the highest frequency of Be stars is well contrasted near B1 rest spectral type stars and it amounts to 34% (CTC) or 45% (CS).

As the evolutionary effect represents the last correction considered in this paper, the mixing in a given rest spectral type of stars of different apparent spectral types, due both to rotation

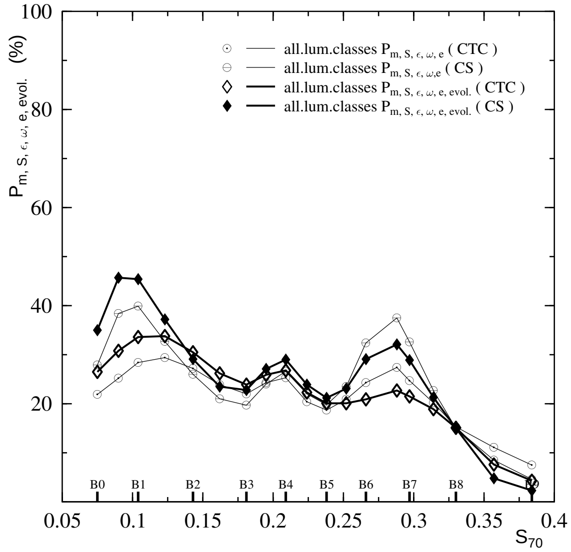

Fig. 9. Ratio P (as a percentage) of Be stars as a function of rest spectral types. Stars in all luminosity classes are grouped according to their mass. Subindices in P have the same meaning as in other figures; here we added "evol." which stands for effects on spectral types due to stellar evolution. CTC indicates that rotational effects on magnitude, spectral type and luminosity class are derived using CTC's models; CS idem from CS's models.

and evolution, is the highest. Such mixed sets are in principle sensitive to the volume-filling factor ϵ, which are particularly high for late type stars. Assuming, as a guess, that our values of ϵ are uncertain to $\pm 50\%$, we estimated the propagation of this error on the final derived frequencies $P_{\rm m,S,\epsilon,\omega,e,evol.}$. The resulting uncertainties are given in Table 2; they seem to be irrelevant.

The effect on ratios P due to stellar evolution seems globally small, although not negligible. Nevertheless, as corrections for B and Be stars were supposed to be the same and because the proportion of Be stars is the same in all luminosity class groups, also due to all possible uncertainties, including the simplified way in which the evolution of rotating stars was considered, results obtained in this section cannot perhaps show more than a global tendency. However, as in our sample dwarfs are on average 4 and 17 times more numerous than subgiants and giants respectively, it can be expected that a more detailed study of spectral changes of rotating stars during their evolution would hardly produce appreciably different effects on stellar counts than those obtained here.

6. Summary of results obtained and discussion

6.1. Summary of results obtained

From the new determinations of the frequency of Be stars, where some physical characteristics specific to these objects were taken into account, we obtained the following main results:

– A monotonic, rather steep decrease in the frequency of Be stars from the earlier to the later B subspectral types. The luminosity-mean ratio of Be stars is 34% for B1 and 8% for B9 (results depending on CTC's models of rotational effects). The global frequency of Be stars, considering all spectral types and luminosity classes, obtained after having introduced all corrections, amounts at least to 17%. From B0 to B4 [or $M \simeq 15$ to $5M_{\odot}$ (see Fig. 10)] there is 29% (CTC) or 32% (CS) of Be stars, which means that in this wide mass range, 1/3 of the whole B star population (excluding supergiants) show the Be phenomenon. These frequencies are probably upper limits. A rough estimate of their lower values can be calculated by dividing the obtained ratios P by m = 1.26.

– The absolute value of frequencies of Be stars obtained are model-dependent regarding rotational effects, which are among the most important determining their final distribution. However, considering separately three luminosity-class groups, a very similar frequency distribution of Be stars is obtained whatever the luminosity class group and whatever the model used.

We see that the maximal frequency of Be stars is found for the hottest B stars, and no longer in the spectral interval B2-B4, as obtained from simple counts. This result means that in the hotter Be stars, not only the emission characteristics are more pronounced (see e.g. Briot 1971, Briot & Zorec 1981, Zorec & Briot 1991), but also the proportion of these stars is the greatest.

6.2. What can be concluded about the status of Be stars

The monotonic decrease of the frequency of Be stars from the hot to late type stars was foreseen in some earlier studies (Massa 1975, Henize 1976) and the maximum frequency in B2 stars was hardly explained. Henize (1976) considers the possibility of misclassification of B0e and B1e stars due to the breadth of rotationally broadened lines. Comparing the number of Be stars in different luminosity classes, Slettebak (1982) has concluded that they are about equally numerous in all luminosity classes. Our results may be still more precise, because they show that the *frequency* distribution of Be stars as a function of the spectral type is also the same in all luminosity classes.

It was frequently considered in the past that Be stars may correspond to a given stage in the evolutionary track of a B star, this stage being generally the contraction phase following hydrogen exhaustion in the core of the star (Crampin & Hoyle 1960, Schild & Romanishin 1976). In this case, Be stars would be located only in the final phases of the main sequence or after the main sequence. Thus, our results, namely a similar frequency of Be stars in whatever the luminosity class, are at odds with this model. Hardorp & Strittmatter (1970) had already remarked that the ratio of observed Be stars is too large to correspond with this very short phase during the life of a star. This now turns out to be even more true, because of the even larger values of the frequency of Be stars obtained in the present paper. By studying cluster HR diagrams, it was noticed several times that Be stars seem to be more advanced than B stars (Schild and Romanishin 1976, Mermilliod 1982, Bessel & Wood 1992). Actually, location of Be stars in the HR diagram can be simply explained by the overluminosity of Be stars in the visible range due to the presence of the circumstellar envelope.

We have to note that two different processes exist which can increase the ratio of Be stars at various stages of stellar evolution. As the hypothesis of secondary contraction at the end of hydrogen burning in the stellar core would increase the number of post-main sequence Be stars, the hypothesis of Be stars created with fast rotation could imply longer lifetimes in the main sequence (Bodenheimer 1971, Sofia et al. 1994) and then a greater frequency of Be stars of luminosity class V, which is not found. Moreover, it is highly unlikely that these two effects, the one setting out the increase of the number of post-main sequence and the other the number of main sequence stars, are equally important and furthermore for every spectral type. It would then be concluded that fast rotation of Be stars does not involve a much longer lifetime in the main sequence. This seems to be confirmed by model calculations of the evolution of rigid rotators, which predict main sequence lifetimes not longer than 3-5% as compared to non-rotating stars (Kippenhahn et al. 1970a,b, Endal & Sofia 1976, 1978). This would also imply that the total angular momentum of Be stars is so that $\tau \lesssim \tau_c$ (cf. Sect 6.). However, this might be not so, if it can be demonstrated: (a) that stars with $\tau \gtrsim \tau_c$ may conserve their high rotational energy over lengthy evolutionary time scales; (b) that such objects are actually 'young', whose internal rotational law produces a stellar geometrical deformation so that their apparent spectral type-luminosity class resembles in all wavelength ranges that of stars with $\tau \lesssim \tau_c$ for all luminosity classes from V to III. As high rotation can also be the consequence of high stored angular momentum, it should not be excluded that these phenomena are present in Be stars. Knowing that the frequency of these objects is very high in masses $M \simeq 5$ to $15 M_{\odot}$ and that their frequency in late type spectra might be enhanced because of rotational effects, as shown in this work, studies of star formation with $\tau \gtrsim \tau_c$, their further internal angular momentum redistribution, stability, loss of kinetic energy, their evolution and the emitted energy distribution are still needed.

Let us finally comment on the possible contribution to the observed frequency distributions of the binary-modeled Be stars. Because no information exists on their contribution to each luminosity class, two extreme cases can be imagined: 1) they privilege a given luminosity class; 2) they are distributed in equal proportions in all of them. As the binary model accounts for half the whole Be star population (Pols et al. 1991), the consequence of favoring a given luminosity class would produce a very different distribution, which is not seen. If this model uniformly contributes to all luminosity classes, then the remaining half population of single Be stars should behave as objects which have distinctive characteristics since their formation phases.

7. Conclusion

We made a new determination of the proportion of the number of Be stars among B stars. Although numerous studies have long been made on this subject, for the first time each luminosity class is considered separately and the most striking physical characteristics of Be stars liable to deeply modify this parameter are taken into account. These characteristics are the overluminosity of Be stars as compared with B stars due to the CE emission and to the rapid rotation, errors in spectral type determination, differences of magnitude-limited volumes between early and late B and Be stars, effects on spectral type and luminosity class due to the fast rotation of Be stars, and changes in spectral types during constant mass evolution. We obtained a rather steep decrease of the frequency of Be stars from earliest to latest stars, and a similar frequency whatever the luminosity class. We note that the steepness of the frequency distribution is model dependent, mainly in what concerns rotational changes of magnitude and spectral type-luminosity class of stars, and different conclusions can be reached depending on the choice of models. For single Be stars, the results obtained suggest that the Be "phenomenon" does not correspond to a stage in the evolutionary track of every B star, but originates in the conditions of formation of the star, namely probably the fast rotation. However, a number of questions still remain open with this subject and further studies of rotational effects on stellar formation, spectra and evolution are needed.

Acknowledgements. Many thanks are due to Prof. C. Jaschek and Dr. L. Divan for useful information on spectral classifications and to Dr. O. Bienaymé, Dr. R. Cayrel, Dr. M. Crézé, Dr. A.M. Hubert-Delpace and Dr. O. Pols for helpful discussions. Criticisms and clarifying comments of the referee, Dr. G.W. Collins II, which helped to improve the presentation of this paper are greatly acknowledged. We are grateful to A. Garcia (IAP) for preparing the figures. This research has made use of the CDS (Strasbourg, France) database.

Appendix A: fundamental parameters

A.1. Luminosity function of B stars without emission lines

B stars of a given MK spectral type-luminosity class represent a set of objects with fundamental parameters that spread over more or less large intervals around the corresponding mean values [see the spectral $\lambda_1\mathscr{D}$ curvilinear classification boxes of the BCD spectrophotometric system (Chalonge & Divan 1973) recalling that $\mathscr{D}$ is a sensitive indicator of $T_{\rm eff}$ while λ_1 is a strong function of log g]. To a very good approximation, we can suppose that the absolute magnitudes $M_{\rm V}$ of B stars of a given MK spectral type-luminosity class scatter around a mean value $\overline{M_{\rm V}}$(B) following a Gaussian distribution with dispersion $\sigma_{\rm B}$ (Mihalas & Binney 1981):

$$\Phi(M_{\rm V}|{\rm B}) = \frac{1}{\sqrt{2\pi}\sigma_{\rm B}} {\rm e}^{-(M_{\rm V}-\overline{M_{\rm V}({\rm B})})^2/2\sigma_{\rm B}^2} \quad (13)$$

The values of $\overline{M_{\rm V}({\rm B})}$ and $\sigma_{\rm B}$ used in this work were obtained from the HR diagram defined in the BCD spectrophotometric system and its calibration in absolute magnitudes (see Fig. 2 in ZB). They were calculated supposing that each spectral type-luminosity class BCD curvilinear quadrilateral was uniformly populated by stars. We divided them into bands of constant $\delta M_{\rm V}$ absolute magnitude differences parallel to curves of $M_{\rm V}$ = constant. Then we calculated weighted mean values of the absolute magnitude $\overline{M_{\rm V}({\rm B})}$ and the corresponding dispersions $\sigma_{\rm B}$ which are given in Table 5.

Table 5. Mean absolute magnitude M_V and dispersion σ_M as a function of spectral type and luminosity class for the luminosity function $\Phi(M_V|B)$

	dwarfs		*subgiants*		*giants*	
Sp. Type	M_V mag	σ_M mag	M_V mag	σ_M mag	M_V mag	σ_M mag
B0	-3.46	0.24	-3.90	0.21	-4.48	0.31
B1	-2.86	0.26	-3.35	0.18	-3.98	0.35
B2	-2.15	0.32	-2.71	0.27	-3.50	0.47
B3	-1.57	0.23	-2.15	0.18	-3.08	0.60
B4	-1.20	0.20	-1.77	0.17	-2.79	0.68
B5	-0.72	0.23	-1.44	0.21	-2.48	0.64
B6	-0.40	0.21	-1.07	0.24	-2.18	0.49
B7	-0.10	0.20	-0.79	0.25	-1.92	0.61
B8	0.15	0.19	-0.55	0.23	-1.66	0.57
B9	0.42	0.22	-0.18	0.26	-1.30	0.54

A.2. Relations used between fundamental stellar parameters

A.2.1. Spectral type progenetic relations

To establish the progenetic relation of spectral types among different luminosity classes, we have drawn evolutionary tracks on the BCD HR diagram calibrated in MK spectral types (Chalonge & Divan 1952, 1973) using the evolutionary models of Maeder and Meynet (1988). They are for stars of Population I (chemical abundances Y = 0.28, Z = 0.02) and without mass loss. It is supposed that for stellar masses M $\lesssim$ 15M$_\odot$ the mass-loss phenomenon is not too significant either for rest stars or for rotating ones (Sreenivasan & Wilson 1978, Brunish & Truran 1982). Using the calibration of the BCD spectrophotometric parameters $\lambda_1\mathscr{D}$ in $T_{\rm eff}$ and log g (Divan & Zorec 1982b, Zorec 1986), we obtained the evolutionary tracks for stellar masses of 3 to 15$M_\odot$ in the $\lambda_1\mathscr{D}$-plane shown in Fig. 10. The spectral progenetic relations that we obtained by interpolation in this diagram, written in terms of the parameter S_{70} as a function of the stellar mass, are the following:

$$\left.\begin{array}{rcl} S_{70}^{\rm d} & = & 0.957\,(M/M_\odot)^{-0.893} \\ S_{70}^{\rm sg} & = & 0.987\,(M/M_\odot)^{-0.892} \\ S_{70}^{\rm g} & = & 1.295\,(M/M_\odot)^{-0.969} \end{array}\right\} \qquad (14)$$

where "d" stands for dwarfs, "sg" for subgiants and "g" for giants. As an example, it follows from Fig. 10 [or relations (14)], that a B4-5V ($M/M_\odot$ = 5.0) star progressively becomes B5 IV and then B7 III.

A.2.2. Surface gravities

The surface gravities used in Sect. 5.2.1 and in 5.2.2,γ to compare stellar radii in different luminosity classes were obtained with the same models of stellar evolution and calibrations of BCD spectrophotometric parameters as for establishing relations (14). The log g parameters for each luminosity class group thus obtained are:

$$\left.\begin{array}{rcl} \log\ g_{\rm d} & = & 4.09 + 0.14\ \log\ S_{70} \\ \log\ g_{\rm sg} & = & 3.88 + 0.07\ \log\ S_{70} \\ \log\ g_{\rm g} & = & 3.20 - 0.42\ \log\ S_{70} \end{array}\right\} \qquad (15)$$

From (15) it follows that for B0 to B9 stars $\overline{\log\ g_{\rm d}}$ = 4.00$\pm$0.03; $\overline{\log\ g_{\rm sg}}$ = 3.83 $\pm$ 0.02 and $\overline{\log\ g_{\rm g}}$ = 3.50 $\pm$ 0.10.

Appendix B: interstellar extinction

To take into account the effect of ISM extinction, we made a new estimation of the absorption $A_V = 3.1E(B-V)$ in the V magnitude as a function of the distance around the Sun up to d $\simeq$ 3500 pc. As B and Be stars are rather strongly concentrated towards the galactic plane, we determined A_V averaged over the galactic latitude as a function of the galactic longitude ℓ and the distance d to the Sun. This simplification is justified because the height scale of the density function D in z is small ($h_z \sim$ 70 pc). The extinction distribution $A_V(\ell,{\rm d})$ was determined for eight consecutive longitude zones each having an amplitude of $\Delta\ell = 45^o$. We used for that a sample of about 20000 O, B and A stars with distances $d \lesssim$ 3500 pc. All known stars with particularities like Be, Bp or Bep were excluded from calculation of the colour excess $E(B-V)$. Photometric data are from the catalogues of Nicolet (1975), Nicollier & Hauck (1978), Lanz (1986) and Deutschmann et al. (1976). Spectral classifications come from Buscombe's (1977, 1980, 1981, 1988), Houk & Cowley's (1975), Houk's (1978, 1982), Houk & Smith-Moore's (1988) compilation catalogues. Page's (1982) catalogue was used to identify Be, Bp and Bep stars. The calibration of absolute magnitudes used for O and B stars is from ZB and for A stars is from Grenier et al. (1985). Intrinsic UBV colours were taken from Schmidt-Kaler's (1982) compilation. From the $E(B-V) = f$(distance) diagrams corresponding to different longitude zones we determined mean absorption laws using interpolation functions of the type:

$$A_V(\ell,{\rm d}) = a(\ell)d^{b(\ell)} \qquad (16)$$

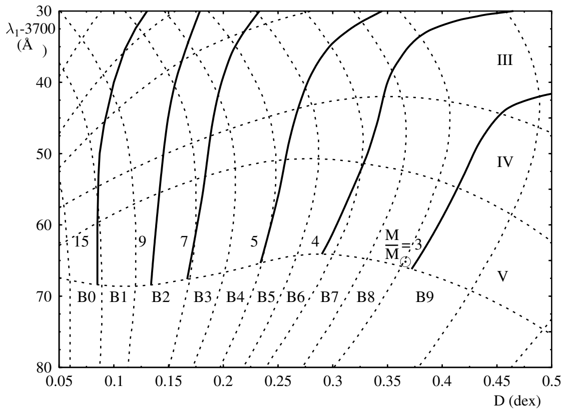

Fig. 10. HR diagram represented with the continuous parameters λ_1D of the BCD spectrophotometric system. D is the value of the Balmer discontinuity (BD) in *dex* and λ_1 the mean spectral position of BD in Å. The λ_1D plane is calibrated in terms of MK spectral type and stellar mass. Vertical pointed lines indicate the limits of MK spectral types and the horizontal pointed lines indicate the limits of MK luminosity classes. Heavy lines are curves of constant mass labeled with $M/M_{\odot}$.

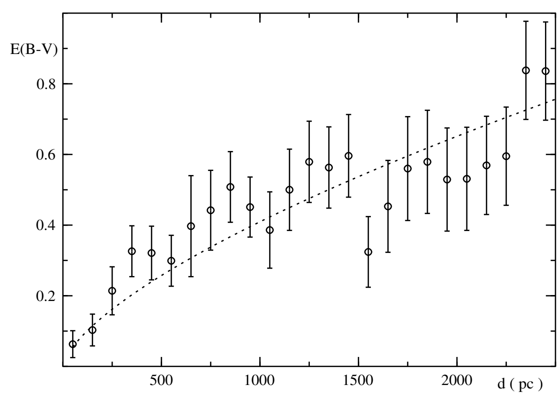

Fig. 11. Galactic latitude mean colour excess E(B-V) distribution as a function of distance d (in pc) in the galactic longitude zone $\ell = 157^o.5 \pm 22^o.5$. Error bars correspond to 1σ deviations in E(B-V).

The constants $a(\ell)$ and $b(\ell)$ are reported in Table 6. As an example, Fig. 11 shows the zone $\ell = 157^{\circ}.5 \pm 22.^{\circ}5$. In this figure, as for other longitude zones, each point is an average over 20 to 40 stars per interval of 100 pc. The error bars correspond to $1\sigma_{E(B-V)}$ deviations.

Table 6. Parameters of the absorption law $A_{\rm V}(\ell)$

ℓ	$a(\ell)\times 10^2$ mag	$b(\ell)$
$22^o.5$	1.80	0.63
$67^o.5$	0.81	0.73
$112^o.5$	1.54	0.66
$157^o.5$	1.24	0.67
$202^o.5$	0.33	0.80
$247^o.5$	0.54	0.67
$292^o.5$	0.83	0.65
$337^o.5$	2.60	0.52

Table 7. Parameters defining the distribution function $F(V|{\rm J}, {\rm X}_1)$

Sp. Types	X_1	$\overline{V}$ km s^{-1}	J^{-1} km s^{-1}
		dwarfs	
B0-B1.5	270	240	90
B2-B3	230	250	110
B4-B7	350	250	70
B8-B9.5	350	280	80
		subgiants	
B0-B1.5	380	220	60
B2-B3	230	190	80
B4-B7	450	280	60
B8-B9.5	580	290	50
		giants	
B0-B1.5	210	230	110
B2-B3	240	190	80
B4-B7	230	220	90
B8-B9.5	160	130	90

Appendix C: the distribution $Q(\omega|S)$ of Be stars

The stellar sample used is the same as for counts, excepting those objects for which no $V\ \sin\ i$ was found. As much as possible we used the $V\ \sin\ i$ determined by Slettebak (1982). In other cases, the $V\ \sin\ i$ parameters in Uesugi & Fukuda's (1982) compilation catalogue have been transformed into the new system of Slettebak et al. (1975), which gives smaller values of $V\ \sin\ i$ to about 15%. To gain some degree of statistical significance, we studied the $V\ \sin\ i$ distributions per spectral groups B0-B1.5, B2-B3, B4-B7, B8-B9.5 and luminosity class as already defined in Sect. 2.2. The obtained distributions of $V\ \sin\ i$ were then transformed into distributions of the true velocity V using the known Abel's type transformation (Chandrasekhar & Münch 1950) and van Dien's (1948) empirical function:

$$F(V|J,X_1) = \frac{J}{\sqrt{\pi}}[{\rm e}^{-J^2(V-V_1)^2} + {\rm e}^{-J^2(V+V_1)^2}] \quad (17)$$

The parameters J and $X_1 = JV_1$ are given in Table 7. In this table we also give, as a reference, the mean equatorial velocity $\overline{V} = (4/\pi)V\ \sin\ i$ characterizing each group. Adopting $R(\omega)/R_{\rm c}$ = 0.688 + 0.002 exp(4.87ω) (Zorec et al. 1988) [where $R(\omega)$ is the equatorial radius of a deformed object at ω; $R_{\rm c}$ is the stellar radius for ω = 1] and knowing that $V(\omega) = \omega R(\omega)(V_{\rm c}/R_{\rm c})$ ($V_{\rm c}$ is the linear equatorial velocity for ω = 1), we transformed (17) of each stellar group into a distribution function of the rotational rate ω :

$$Q(\omega|S) = F(V|J,X_1)\partial V/\partial\omega \quad (18)$$

so that $\int_0^1 Q(\omega|S)d\omega = 1$. The functions (18) were conserved in numerical form. They are similar in shape to the Maxwellian distributions studied by Deutsch (1970) and used by Collins & Smith (1985).

We note that recently Collins & Truax (1995) claimed that the V sin i of Be stars can be underestimated because of frequent use of wavelength independent limb darkening coefficients. On the other hand, Ballereau et al. (1995) and Zorec et al. (1996) showed that He I 4471 lines of Be stars can have emission components which lead to overestimations of the V sin i parameter as high as 20%. Having no means of checking these effects on the published values of V sin i, we could not take them into account.

References

Abt H.A. 1987, Galactic Distribution, Kinematics, Locations in Clusters and HR Diagrams, and Duplicity of Be Stars. In: Slettebak A., Snow Th. P. (eds.), IAU Coll. No 92, Physics of Be Stars, Cambridge Univ. Press, p. 470

Abt H.A., Cardona O. 1984, ApJ 285, 190

Alvarez M., Schuster W.J. 1987, Rev. Mex. Astron. Astrofis. 6, 163

Andrillat Y., Houziaux L. 1975, reported by Swings (1976). In: Slettebak A. (ed.) IAU Symp. No 70, Be and Shell Stars, Reidel Publ. Co., p. 219

Apparao K.M.V. 1991, ApJ 376, 256

Ballereau D., Chauville J., Zorec J. 1995, A&AS 111, 423

Barker P.K. 1982, IAU Circular N^o 3710

Bessel M.S., Wood P.R. 1992, Be Stars in Magellanic Clouds. In: Baschek B., Klare G., Lequeux J. (eds.), New Aspects of Magellanic Cloud Research, Springer-Verlag, Heidelberg, p. 271

Bidelman W.P. 1982, IAU Circular N^o 3756

Bienaymé O., Robin A.C., Crézé M. 1987, A& A 180, 94

Bjorkman J.E. 1994, The Wind-Compressed Disk Model. In: Balona L.A., Henrichs H.F., Le Contel J.M. (eds.) Proc. IAU Symp. No 162, Pulsation, Rotation and Mass Loss in Early-Type Stars, Kluwer Acad. Press, p. 455

Bjorkman J.E., Cassinelli J.P., 1993, ApJ 409, 429

Bodenheimer P. 1971, ApJ 167, 153

Boss A.P. 1986, ApJS 62, 519

Briot D. 1971, A&A 11, 57

Briot D., Zorec J. 1981, Spectral Type Dependence of the Characteristics of Be Stars. In: G.E.V.O.N. and Sterken C. (eds.), Workshop on Pulsating B Stars, Nice Observatory, p.109

Brunish W.M., Truran J.W. 1982, ApJS 49, 447

Buscombe W. 1977, MK Spectral Classification, Third General Catalogue, Evanston

Buscombe W. 1980, MK Spectral Classification, Fourth General Catalogue, Evanston

Buscombe W. 1981, MK Spectral Classification, Fifth General Catalogue, Evanston

Buscombe W. 1988, MK Spectral Classification, Seventh General Catalogue, Evanston

Capuzzo-Dolcetta R., Chiosi C., Di Fazio A. 1990, Physical Processes in Fragmentation and Star Formation. In: Capuzzo-Dolcetta R., Chiosi C., Di Fazio A. (eds.), Astrophysics and Space Science, Vol. 162

Catalano S., Stauffer J.R. 1991, Angular Momentum Evolution of Young Stars. In: Catalano S., Stauffer J.R. (eds.) Proc. of the NATO ASI Series C, Vol. 340, Kluwer Academic Publishers

Chalonge D., Divan L. 1952, Annales Astrophys. 15, 201

Chalonge D., Divan L. 1973, A&A 23, 69

Chandrasekhar S., Münch G. 1950, ApJ 111, 142

Clement M.J. 1979, ApJ 230, 230

Collins II G.W., Smith R.C. 1985, MNRAS 213, 519

Collins II G.W., Sonneborn G.H. 1977, ApJS 34, 41 (CS)

Collins II G.W, Truax R.J. 1995, ApJ 439, 860

Collins II G.W, Truax R.J., Cranmer S.R. 1991, ApJS 77, 541 (CTC)

Coté J., van Kerkwijk M.H. 1993, A&A 274, 870

Cotton A., Smith R.C. 1983, The Observatory 103, 8

Crampin J., Hoyle F. 1960, MNRAS 120, 33

Cranmer S.R., Collins II G.W. 1993, ApJ 412, 720

Deutsch A.J. 1970, Maxwellian Distributions for Stellar Rotations. In: A. Slettebak (ed.), Stellar Rotation, D. Reidel Publ. Co, Dordrecht-Holland, p. 207

Deutschmann W.A., Davis R.J., Schild R.E. 1976, ApJS 30, 97

Divan L. 1979, Quantitative Spectral Classification of Be Stars on Low Dispersion Spectra. In: McCarthy M.F., Philip A.G.D., Coyne G.V. (eds.), IAU Coll. No 47, Spectral Classification of the Future, Vatican Observatory, p. 247

Divan L., Zorec J. 1982a, BCD Spectrophotometry of the Be-Shell Star 88 Her. In: Jaschek M., Groth H.G. (eds.) Proc. IAU Symp. No 98, Be Stars, Reidel Publ. Co., Dordrecht-Holland, p. 61

Divan L., Zorec J. 1982b, Absolute Magnitudes and Other Basic Parameters of O and B Stars. In: Perrymann M.A.C., Guyenne T.D. (eds.), The Scientific Aspects of the Hipparcos Space Astrometry Mission, ESA-SP 177, 101

Divan L., Zorec J., Briot D. 1982, Correlation between BCD parameters of the continuous spectrum and the Balmer decrement of Be stars. In: Jaschek M., Groth H.G. (eds.) Proc. IAU Symp. No 98, Be Stars, Reidel Publ. Co., Dordrecht-Holland, p. 53

Divan L., Zorec J., Andrillat Y. 1983, A&A 126, L8

Endal A.S. 1982, The Evolution of Rapidly Rotating B/Be Stars. In: Jaschek M., Groth H.G. (eds.) Proc. IAU Symp. No 98, Be Stars, Reidel Publ. Co., Dordrecht-Holland, p. 299

Endal A.S., Sofia S. 1976, ApJ 210, 184

Endal A.S., Sofia S. 1978, ApJ 220, 279

Endal A.S., Sofia S. 1979, ApJ 232, 531

Feinstein A. 1975, PASP 87, 603

Golay M. 1968, Archives des Sciences (Genève) 21, 105

Ghosh K.K., Kuppuswamy K., Jaykumar K. et al. 1987, Inf. Bull. Var. Stars N^o 3056

Grenier S., Gómez A.E., Jaschek C. et al. 1985, A&A 145, 331

Hardorp J., Strittmatter P.A. 1970, Rotation and Evolution of Be Stars, In: Slettebak A. (ed.), Stellar Rotation, Reidel Publ. Co., Dordrecht-Holland, p. 48

Harmanec P., Horn J., Koubsky P. et al. 1980, Bull. Astr. Inst. Czech. 31, 144

Henize K.G. 1976, APJS 30, 491

Herzog K.P. 1989, Southern Stars 33, 144

Hirata R., Sadakane K., Nishimura S. et al. 1986, PASJ 38, 627

Hoffleit D., Jaschek C. 1982, The Bright Star Catalogue, Yale Univ. Observatory (BSC)

Hoffleit D., Saladyga M., Wlasuk P. 1983, A Supplement to the Bright Star Catalogue, Yale Univ. Observatory (SBSC)

Höflich P. 1988, A&A 191, 348

Houk N. 1978, Michigan Catalogue of the Two-Dimensional Spectral Types for the HD Stars, Vol. 2, Univ. Michigan

Houk N. 1982, Michigan Catalogue of the Two-Dimensional Spectral Types for the HD Stars, Vol. 3, Univ. Michigan

Houk N., Cowley A.P. 1975, Michigan Catalogue of the Two-Dimensional Spectral Types for the HD Stars, Vol. 1, Univ. Michigan

Houk N., Smith-Moore M. 1988, Michigan Catalogue of the Two-Dimensional Spectral Types for the HD Stars, Vol. 4, Univ. Michigan
Huffer C.M. 1939, ApJ 89, 139
Jaschek C., Jaschek M. 1983, A&A 117, 357
Jaschek M., Jaschek C. 1987, The Classification of Stars, Cambridge Univ. Press
Jaschek M., Jaschek C., Egret D. 1986, A&A 158, 325
Jaschek M., Jaschek C., Andrillat Y. 1988, A&AS 72, 505
Kippenhahn R., Meyer-Hofmeister E., Thomas H.C. 1970a, A&A 5, 155
Kippenhahn R., Meyer-Hofmeister E., Thomas H.C. 1970b, Evolution of Rotating Stars of $9M_{\odot}$, In: Slettebak A. (ed.), Stellar Rotation, Reidel Publ. Co., Dordrecht-Holland, p. 60
Kippenhahn R., Weigert A. 1990, Stellar Structure and Evolution, Astron. and Astrophys. Library, Springer-Verlag
Kogure T., Hirata R. 1982, Bull. Astron. Soc. India 10, 281
Kogure T., Suzuki M. 1986, Astron. Astrophys. Sp. Sci. 120, 253
Kozok J.R. 1985, A&AS 61, 387
Křiž S., Harmanec P. 1975, Bull. Astron. Inst. Czech. 26, 65
Lada C.J., Kylafis N.D. 1991, The Physics of Star Formation and Early Stellar Evolution. In: Lada C.J., Kylafis N.D. (eds.) Proc. of the NATO ASI Series C, Vol. 342, Kluwer Academic Publishers
Lanz T. 1986, A&AS 65, 195
Maeder A. 1968, Archives des Sciences (Genève) 21, 125
Maeder A. 1974, A&A 34, 409
Maeder A., Meynet G. 1988, A&AS 76, 411
Maeder A., Peytremann E. 1970, A&A 7, 120
Maeder A., Peytremann E. 1972, A&A 21, 279
Massa D. 1975, PASP 87, 777
Mennickent R.E., Vogt N. 1991, Rev. Mex. Astron. Astrofis. 22, 310
Meadows A.J. 1960, AJ 65, 335
Mermilliod J.-C. 1982, A&A 109, 48
Merrill P.W., Burwell C.G. 1933, ApJ 78, 87
Mihalas D., Binney J. 1981, Galactic Astronomy, Freeman and Company
Moss D., Smith R.C. 1981, Rep. Prog. Phys. 44, 831
Nicolet N. 1975, A&AS 22, 239
Nicollier C., Hauck B. 1978, A&AS 31, 437
Packet W. 1981, A&A 102, 17
Page A.A. 1982, Catalogue, Spectrum and Magnitude Data Bank of Be, Bp and Bep Stars, Mount Taborine Observatory, CDS-version 1984
Pols O.R., Coté J., Waters L.B.F.M. et al. 1991, A&A 241, 419
Press W.H., Flannerey B.P., Teukolsky S.A. et al. 1987, Numerical Recipes, Cambridge Univ. Press
Schild R., Romanishin W. 1976, ApJ 204, 493
Schmidt-Kaler T. 1982, In: Landolt-Börnstein Vol 2, 14
Sharov A.S., Lyutyi V.M. 1992, SvA 36
Singh M. 1982, Inf. Bull. Var. Stars N^o 2188
Slettebak A. 1982, ApJS 50, 55
Slettebak A., Collins II G.W., Boyce P.B. et al. 1975, ApJS 29, 137
Slettebak A., Collins II G.W., Truax R. 1992, ApJS 81, 335
Slettebak A., Kuzma T.J. 1979, Effects of Stellar Rotation on Spectral Classification. In: McCarthy M.F., Philip A.G.D., Coyne G.V. (eds.), IAU Coll. No 47, Spectral Classification of the Future, Vatican Observatory, p. 87
Slettebak A., Kuzma T.J., Collins II G.W. 1980, ApJ 242, 171
Smart W.M. 1958, Combination of Observations, Cambridge Univ. Press
Smith R.C. 1971, MNRAS 151, 463
Sofia S., Howard J.M., Demarque P. 1994, The Evolution of Rotating Stars. In: Balona L.A., Henrichs H.F., Le Contel J.M. (eds.) Proc. IAU Symp. No 162, Pulsation, Rotation and Mass Loss in Early-Type Stars, Kluwer Acad. Press, p. 131
Sreenivasan S.R., Wilson W.J.F.,1978, A&A 70, 755
Stoeckley T.R., Buscombe W. 1987, MNRAS 227, 801
Struve O. 1951, The Analysis of Peculiar Stellar Spectra. In: Hynek J.A. (ed.) A Topical Symposium, Astrophysics, McGraw-Hill Book Co., p.85
Tassoul J.-L. 1978, Theory of Rotating Stars, Princeton Univ. Press
Uesugi A., Fukuda I. 1982, Revised Catalogue of Stellar Rotational Velocities, Dep. Astron. Kyoto University
van Dien E. 1948, J.R. Astron. Soc. Can. 42, 249
Zorec J. 1986, Thèse d'Etat, Université Paris VII
Zorec J. 1994, The Photosphere of Pleione. In: Balona L.A., Henrichs H.F., Le Contel J.M. (eds.) Proc. IAU Symp. No 162, Pulsation, Rotation and Mass Loss in Early-Type Stars, Kluwer Acad. Press, p. 362
Zorec J., Briot D., Divan L. 1983, A&A 126, 192
Zorec J., Briot D. 1991, A&A 245, 150 (ZB)
Zorec J., Divan L., Mochkovitch R., et al. 1987, Differential Rotation in B and Be Stars. In: Slettebak A., Snow Th. (eds.) Proc. IAU Coll. No 92, Physics of Be Stars, Cambridge University Press, p. 68
Zorec J., Israelian G., Ballereau D., Chauville J. 1996, A&A 308, 852
Zorec J., Mochkovitch R., Divan L. 1988, C.R. Acad. Sci. Paris, 306, 1265
Zorec J., Mochkovitch R., Garcia A. 1990, The Angular Momentum-Loss and the Differential Rotation in B and Be Stars. In: Willson L.A., Stalio R. (eds.) NATO ASI Series C, Vol. 316, Angular Momentum and Mass Loss for Hot Stars, Kluwer Academic Publishers, p. 239

This article was processed by the author using Springer-Verlag LaTeX A&A style file *L-AA* version 3.