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On the confinement of one-armed oscillations in discs of Be stars

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Abstract. We discuss the effects due to the rotational deformation and the optically-thin line force on the confinement of one-armed oscillations in the inner part of Be-star discs. The period of these oscillations is identified with the observed V/R variations in Balmer emission lines. We take into account the effect of rotation by including the quadrupole contribution to the potential around the rotationally-distorted central star. For the radiative force due to an ensemble of optically thin lines, we adopt the parametric form proposed by Chen & Marlborough (1994). The disc is assumed to be isothermal. Based on these assumptions, we examine the linear, one-armed eigenmodes confined to the inner part of the disc. Our study strongly suggests that the mechanism that causes the confinement of one-armed oscillations in early-type Be stars is different from that in late-type Be stars. In late-type Be stars, the confinement occurs because of the deviation from the point-mass potential around the rotationally deformed star. On this point, we confirm the conclusions obtained by Papaloizou et al. (1992). In early-type Be stars, however, it is the weak-line force that mainly contributes to the confinement. The rotational effect plays a much smaller role for these stars. The period of the eigenmode confined to the disc depends sensitively on the effect by rotation or radiation. This sensitiveness, together with the range of the period of observed V/R variations, places rather narrow constraints on the parameters characterizing these effects. We compare our results with observed V/R properties, for which a list of 53 stars has been compiled.

Key words: global disc oscillation – hydrodynamics – radiative transfer – stars: circumstellar matter – stars: emission line, Be

1. Introduction

Be stars are non-supergiant B-type stars with Balmer emission lines. The emission lines arise from a cool envelope surrounding the star. Interferometric observations have confirmed that the cool envelope has a disc-like geometry with a rotational velocity distribution closer to a Keplerian distribution than to a distribution in the flow with a constant angular momentum (Mourard et al. 1989; Hirata 1994; Quirrenbach 1994; Vakili et al. 1994).

Many (but not all) Be stars exhibit long-term quasi-cyclic variations in the relative intensities of the violet and red components in double-peaked Balmer emission-line profiles (V/R variations). Typical features of V/R variations are summarized in e.g., Dachs (1987) or Hubert (1994) and references therein (see also Appendix). The periods of V/R variations range from years to a decade, which is much longer than the rotation periods of the central stars and cool disc-like envelopes. These periods do not depend on the spectral type of the central star. Moreover, these periods can vary from one cycle to the next for individual stars. In addition, the profile as a whole shifts blueward when the red component is stronger and shifts redward when the violet component is stronger. The present paper is mainly concerned with the physical mechanisms that cause the long-term V/R variations. A satisfactory model should account for (1) the range of V/R periods, (2) the dependence of the period on the properties of the Be star, and (3) the time evolution of the V/R cycle.

Okazaki (1991) constructed a model first suggested by Kato (1983) in which the long-term V/R variations are due to the phenomena caused by global one-armed (i.e., m = 1) oscillations in the equatorial discs of Be stars. Here, m is the azimuthal wave number. The model is based on the theoretical result that the low-frequency, one-armed oscillations are the only possible global oscillations in geometrically thin (i.e., near Keplerian) discs (Kato 1983, 1989; Okazaki & Kato 1985; Adams et al. 1989). Studying the linear m = 1 eigenmodes in isothermal equatorial discs, Okazaki (1991) found that the one-armed oscillation model well explains the observed periodicity of the long-term V/R variations.

Since then, a number of studies which support the one-armed oscillation model have appeared. Based on 3D radiative transfer calculations, Hummel & Hanuschik (1994, 1996) showed that calculated line profiles from discs with m = 1 perturbation patterns agree with observed line-profile variability. Using high-resolution Hα and Fe II spectroscopic data of five selected Be stars, Hanuschik et al. (1995) demonstrated that the one-armed

Send offprint requests to: A.T. Okazaki at Sapporo address oscillation model can account for the asymmetric profile shapes as well as the profile variations. Hummel & Vrancken (1995) analysed the line-profile shapes of Be stars in detail and derived several kinematical and geometrical constraints on the structure of individual discs. They found strong evidence that the asymmetric profiles originate from discs with an eccentric (i.e., m = 1) density distribution. Moreover, Telting et al. (1994) clearly demonstrated that the long-term variability of Balmer lines of the Be star β^1 Mon is caused by a prograde one-armed density structure in the disc. In addition, the following circumstantial evidence also seems to support the one-armed oscillation model: The activity of discrete absorption components in the stellar wind of γ Cas is associated with the cyclic V/R variability (Doazan et al. 1987; Telting & Kaper 1994). Telting & Kaper (1994) found that this correlation is consistent with the one-armed oscillation model.

In the one-armed oscillation model, the slow pattern speed of the m = 1 density wave results from the slight deviation of the rotational velocity distribution in the disc from the Keplerian distribution. Okazaki (1991) considered the pressure force in the disc as the only mechanism that gives rise to this deviation. In such a disc, the m = 1 waves can propagate throughout the disc. This means that the period of the finite-order m = 1 mode asymptotically goes to infinity as the disc size goes to infinity. In order to make oscillation periods finite, Okazaki (1991) needed to impose a disc outer radius and obtained the spectrum of the retrograde eigenmodes of which the periods depend on the size of the disc outer radius. When the disc outer radius is $5-20$ stellar radii, the periods of the eigenmode agreed with the observed period range of V/R variations. At present, however, imposing an outer radius to avoid the confinement problem seems inadequate, because there is no observational evidence that the edge of the Hα emitting region is also the edge of the equatorial disc, although the above range covers the observed sizes of Hα emitting regions quite well. On the contrary, from IR and radio observations, the discs seem to extend far beyond the area where the Hα emission arises (Waters et al. 1991). Moreover, the prograde one-armed density structure found in the disc of β^1 Mon (Telting et al. 1994) cannot be explained by Okazaki's model.

Papaloizou et al. (1992, henceforth PSH) pointed out this shortcoming of Okazaki's model and suggested that prograde one-armed oscillations are confined to the inner part of Be-star discs by including the quadrupole contribution to the potential of the star which is distorted by the rapid rotation. Savonije & Heemskerk (1993, henceforth SH) constructed a model in which the effect of the rotational deformation is much stronger than the pressure effect, and obtained the results that confirm the above conclusion of PSH.

In their studies, however, the effect of the rotational deformation of the star is overestimated by the pressure effect being underestimated. The range of the disc temperature they studied is too low to apply to Be-star discs. PSH adopted $H/r \sim 10^{-2}$, where r is the distance from the stellar center and H the half-thickness of the disc. This value of disc thickness corresponds to a disc temperature $T_{\rm d} \sim 3,300(r/r_{\rm s})^{-1}$K for a B0-type star and $T_{\rm d} \sim 2,300(r/r_{\rm s})^{-1}$K for a B5-type star, where $r_{\rm s}$ is the stellar radius. Even in the innermost region of the disc, the disc temperature PSH adopted is thus about an order of magnitude lower than typical temperatures of Be-star discs. SH studied m = 1 oscillations in discs with $10^{-2} \leq H/r \leq 2 \times 10^{-2}$ around a star of $M = 8M_\odot$ and $r_{\rm s} = 5R_\odot$. This parameter range covers $2,200(r/r_{\rm s})^{-1}{\rm K} \leq T_{\rm d} \leq 8,900(r/r_{\rm s})^{-1}$K, which is also too low to apply to Be-star discs. Consequently, it is worth examining the effect of the rotational deformation of the central star to investigate whether it is large enough to give rise to the confinement of m = 1 oscillations in the inner part, say $r \lesssim 10\,r_{\rm s}$, of the discs of Be stars.

Since the proposal of the rotation hypothesis for Be stars by Struve (1931), many studies have been done to test this hypothesis [see Slettebak (1976) for a historical review]. Statistical studies of the distribution of rotation velocities of Be stars have basically confirmed the rotation hypothesis. They have also revealed, however, that the rapid rotation cannot be the sole factor responsible for the Be phenomena (Massa 1975; Slettebak 1976; Fukuda 1982; Kogure & Hirata 1982). Be stars rotate significantly below the critical velocity. The ratio of the rotation velocity to the critical velocity (the rotation parameter) is, on average, smaller for earlier-type Be stars [see Fig. 4 of Kogure & Hirata (1982), and other references]. The distribution of the rotation velocities of Be stars earlier than B3 mostly overlaps that of normal B stars, while Be stars in the spectral range B3–B9 have rotation velocities distinctly higher than normal B stars. This property, together with the observed fact that the fraction of Be stars among B stars has a maximum at B2 (Kogure & Hirata 1982), strongly suggests that another mechanism also has to work to trigger the Be phenomena, at least in the spectral range B0–B2. It is the radiative force that we consider to be the promising candidate. If the radiative force plays an important role for the Be phenomena, it should also have a strong influence on the confinement of the one-armed oscillations in Be-star discs.

The purpose of this paper is to examine the effects of the radiative force and the rotational deformation of the star on the confinement of one-armed oscillations in Be-star discs. Since the radial flow in the inner part of the Be-star disc is considered to be subsonic (Hanuschik 1994), the radiative force in the disc would arise not from the optically-thick strong lines but from an ensemble of optically-thin weak lines (and from the optically-thin continuum). Hence, for the radiative force, we adopt the parametric form proposed by Chen & Marlborough (1994). Examining the eigenvalue problem for a wide range of parameters characterizing the effects of rotation and radiation, we find that, in late-type Be stars, the effect due to rotational deformation of the star is the cause for the confinement of m = 1 oscillations. Our results thus show that the mechanism proposed by PSH and confirmed by SH effectively works in discs around late-type Be stars. We also find, however, that the rotational effect is much less important in discs of early-type Be stars. For these stars, it is the weak line force that mainly contributes to the confinement of the m = 1 oscillations. Finally, we compare our results with observed V/R cycles of an extensive number of stars.

2. The unperturbed disc model and equations for one-armed isothermal perturbations

2.1. Unperturbed disc model

We take a geometrically thin, axisymmetric disc as an unperturbed equilibrium disc. For simplicity, the disc is assumed to be isothermal with a temperature range of $1/2 \leq T_\mathrm{d}/T_\mathrm{eff} \leq 1$. Here, T_eff is the effective temperature of the star. We also assume that the unperturbed disc is in hydrostatic equilibrium in the vertical direction and extends from the stellar surface ($r = r_\mathrm{s}$) to infinity in the radial direction. We neglect radial advective motions and viscous effects. The adopted value of the mean molecular weight is 0.6. We use a cylindrical coordinate system (r, ϕ, z) to describe the disc; the origin is at the center of the star and the z-axis is perpendicular to the disc midplane.

When the distortion of the star is weak, the potential of the star of mass M with apsidal motion constant k_2 can be approximated as

$$\psi \simeq -\frac{GM}{(r^2+z^2)^{1/2}} \times \left[1 + \frac{k_2 f^2}{3}\frac{r_\mathrm{s}^2}{r^2+z^2}\left(1-\frac{3z^2}{r^2+z^2}\right)\right] \quad (1)$$

(PSH; SH). Here, f is the rotation parameter, defined by the ratio of the rotation velocity of the star to the Keplerian velocity at the stellar surface. According to Kogure & Hirata (1982), $0.2 \lesssim f \lesssim 0.6$ for B0-type Be stars and $0.5 \lesssim f \lesssim 1.0$ for B5-type Be stars. The values of k_2 for rapidly rotating stars are not well known. Theoretically, k_2 for non-rotating main-sequence stars has a maximum value of $\sim 10^{-2}$ at a mass between 7 and $10 M_\odot$ (Stothers 1974). In general, k_2 is a decreasing function of the degree of the central condensation of the star: The value of k_2 is smaller for a more evolved star or for a star with larger uniform rotation (Stothers 1974; Claret & Giménez 1993), while it can be large for a star of which the inner part rotates more rapidly than the outer part. PSH adopted $f = 0.5$ and $k_2 = 5-6\times10^{-3}$ and SH studied the rotation effect for $2\times10^{-3} \leq k_2 f^2 \leq 10^{-2}$.

For the radiative force we adopt the parametric form proposed by Chen & Marlborough (1994):

$$F_\mathrm{rad} = \frac{GM\Gamma}{r^2+z^2} + \frac{GM(1-\Gamma)}{r^2+z^2}\eta\left(\frac{r^2+z^2}{r_\mathrm{s}^2}\right)^{\epsilon/2}, \quad (2)$$

where η and ϵ are parameters which characterize the force due to an ensemble of optically thin lines. According to Chen & Marlborough (1994), ϵ should be a positive number which is much smaller than unity. In the above expression, $\Gamma (= \kappa_\mathrm{e} L/4\pi c GM)$ is the Eddington factor that accounts for the reduction in the effective gravity due to electron scattering. Here, κ_e is the opacity due to electron scattering per mass unit and L is the total luminosity of the star. In the rest of this paper, we neglect the Eddington factor Γ. This approximation is valid as long as Γ can be regarded as a constant throughout the disc and is much smaller than unity. Both conditions hold well in Be-star discs. Since the disc is considered to be optically thin for electron scattering, the radial dependence of Γ can be neglected. Moreover, using the typical values for Be stars, Γ is as small as ~ 0.03 for a B0V star and ~ 0.003 for a B5V star. The radiative force is then written as

$$F_\mathrm{rad} \simeq \frac{GM}{r^2+z^2}\eta\left(\frac{r^2+z^2}{r_\mathrm{s}^2}\right)^{\epsilon/2}. \quad (3)$$

In the present model, the equation describing the hydrostatic balance in the vertical direction is written as

$$\frac{1}{\rho_0}\frac{\partial p_0}{\partial z} + \frac{GMz}{r^3} \times \left[1 + 3k_2 f^2\left(\frac{r}{r_\mathrm{s}}\right)^{-2} - \eta\left(\frac{r}{r_\mathrm{s}}\right)^{\epsilon}\right] \simeq 0, \quad (4)$$

where ρ_0 and p_0 are the unperturbed density and pressure, respectively. Since the disc is isothermal, Eq.(4) is integrated to give the density distribution in the vertical direction:

$$\rho_0(r,z) = \rho_{00}(r)\exp\left(-\frac{z^2}{2H^2}\right), \quad (5)$$

where ρ_{00} is the unperturbed density in the equatorial plane and H the vertical scale-height of the disc given by

$$H(r) = \frac{c_\mathrm{s}}{\Omega_\mathrm{K}(r)}\left[1 + 3k_2 f^2\left(\frac{r}{r_\mathrm{s}}\right)^{-2} - \eta\left(\frac{r}{r_\mathrm{s}}\right)^{\epsilon}\right]^{-1/2}. \quad (6)$$

Here, c_s and Ω_K are the isothermal sound speed and the Keplerian angular velocity, respectively.

Since the density profiles of Be-star discs are not well determined, we adopt a simple power-law form for ρ_{00},

$$\rho_{00}(r) \propto \left(\frac{r}{r_\mathrm{s}}\right)^{-\alpha}, \quad (7)$$

where the index α is a constant. Observationally, the density-gradient indices for Be stars have been derived by a curve of growth method, in which the disc is assumed to have a constant opening angle and a density distribution proportional to $(r^2+z^2)^{-n/2}$ (e.g., Waters 1986). The obtained value of n is different from star to star, but is roughly in the range of $2 \lesssim n \lesssim 3.5$ (Waters et al. 1987). Considering the difference between a disc geometry assumed by the curve of growth method and the disc geometry adopted in this paper, we infer the range of α to be $2.5 \lesssim \alpha \lesssim 4.0$. Theoretically, $\alpha \sim 7/2$ in the subsonic region of the isothermal disc, if the angular momentum distribution of the disc is due to viscous stress [see Eq. (11) of Lee et al. (1991)].

The radial distribution of the rotational angular velocity $\Omega(r)$ is derived from the equation of motion in the radial direction

$$r\Omega^2 - \frac{1}{\rho_0}\frac{\partial p_0}{\partial r} - \frac{\partial\psi}{\partial r} + F_\mathrm{rad}\frac{r}{(r^2+z^2)^{1/2}} = 0, \quad (8)$$

and is written explicitly in the form

$$\Omega(r) \simeq \Omega_\mathrm{K}(r) \times \left[1 - \alpha\mathscr{M}_\mathrm{s}^{-2}\frac{r}{r_\mathrm{s}} + k_2 f^2\left(\frac{r}{r_\mathrm{s}}\right)^{-2} - \eta\left(\frac{r}{r_\mathrm{s}}\right)^{\epsilon}\right]^{1/2} \quad (9)$$

under the approximation $z^2/r^2 \ll 1$. Here, $\mathscr{M}_{\rm s}$ is the Mach number of disc rotation at $r = r_{\rm s}$. For typical Be stars, $\mathscr{M}_{\rm s}$ is given by

$$\mathscr{M}_{\rm s} = \begin{cases} 34.3 \left(\dfrac{T_{\rm d}}{T_{\rm eff}}\right)^{-1/2} & \text{for a B0V star} \\ 38.8 \left(\dfrac{T_{\rm d}}{T_{\rm eff}}\right)^{-1/2} & \text{for a B5V star,} \end{cases} \tag{10}$$

where we adopted $M = 17.8 M_\odot$, $r_{\rm s} = 7.41 R_\odot$, and $T_{\rm eff} = 2.80 \times 10^4$ K for a B0V star and $M = 6.46 M_\odot$, $r_{\rm s} = 3.80 R_\odot$, and $T_{\rm eff} = 1.55 \times 10^4$ K for a B5V star (Allen 1973). In the present model, the epicyclic frequency $\kappa(r)$ is given by

$$\kappa(r) \simeq \Omega_{\rm K}(r) \times \left[1 - 2\alpha \mathscr{M}_{\rm s}^{-2} \frac{r}{r_{\rm s}} - k_2 f^2 \left(\frac{r}{r_{\rm s}}\right)^{-2} - \eta(1+\epsilon)\left(\frac{r}{r_{\rm s}}\right)^{\epsilon}\right]^{1/2}. \tag{11}$$

Note that the second, third, and fourth terms between the square brackets in Eqs. (9) and (11) denote the contributions from the pressure gradient force in the disc, the deviation from the point-mass potential due to the rotational deformation of the star, and the radiative force due to an ensemble of optically thin lines, respectively. The parameters which characterize the problem are α, $\mathscr{M}_{\rm s}$, $k_2 f^2$, η, and ϵ.

2.2. *Equations for one-armed isothermal perturbations*

A linear $m = 1$ perturbation which varies as $\exp[{\rm i}(\omega - \phi)]$ is superposed on the unperturbed disc described above. The perturbation is assumed to be isothermal, because in a Be-star disc the thermal time-scale is much shorter than the dynamical time-scale. In the following equations describing the perturbation, we treat only the lowest-order terms with respect to the vertical coordinate z. These assumptions and procedures are the same as those adopted in Okazaki (1991). Then, the linearized equations describing the mass, momentum, and angular momentum conservation are

$${\rm i}(\omega - \Omega)\frac{\rho_1}{\rho_0} + \frac{1}{r\sigma_0}\frac{\rm d}{{\rm d}r}(r\sigma_0 u_r) - \frac{{\rm i}u_\phi}{r} = 0, \tag{12}$$

$${\rm i}(\omega - \Omega)u_r - 2\Omega u_\phi = -c_{\rm s}^2 \frac{\rm d}{{\rm d}r}\frac{\rho_1}{\rho_0}, \tag{13}$$

$${\rm i}(\omega - \Omega)u_\phi + \frac{\kappa^2}{2\Omega}u_r = {\rm i}\frac{c_{\rm s}^2}{r}\frac{\rho_1}{\rho_0}, \tag{14}$$

where ρ_1 is the Eulerian perturbation of the density, u_r and u_ϕ the horizontal velocity components associated with the perturbation, and σ_0 the surface density given by

$$\sigma_0 = \int_{-\infty}^{+\infty} \rho_0 \, dz = \sqrt{2\pi}\rho_{00}(r) H(r) \tag{15}$$

[for details of deriving Eqs. (12)–(14), see Okazaki (1991)].

Eliminating u_ϕ from Eqs. (12)–(14), we obtain the following set of equations:

$$\frac{\rm d}{{\rm d}r}\frac{\rho_1}{\rho_0} = \frac{2\Omega}{r(\omega - \Omega)}\frac{\rho_1}{\rho_0} - \frac{(\omega - \Omega)^2 - \kappa^2}{(\omega - \Omega)c_{\rm s}^2}{\rm i}u_r, \tag{16}$$

$$\frac{\rm d}{{\rm d}r}{\rm i}u_r = \left[\omega - \Omega - \frac{c_{\rm s}^2}{r^2(\omega - \Omega)}\right]\frac{\rho_1}{\rho_0} - \left[\frac{\kappa^2}{2r\Omega(\omega - \Omega)} + \frac{\rm d}{{\rm d}r}\ln(r\sigma_0)\right]{\rm i}u_r. \tag{17}$$

Equations (16) and (17) are the basic equations for the linear, one-armed isothermal oscillations in the disc. Note that Eqs. (16) and (17) are essentially the same as those used in Okazaki (1991) and SH, except that, through Ω and κ, our equations implicitly depend on the effect due to the radiative force as well as due to the rotational deformation of the star.

We now consider the boundary conditions to be imposed on Eqs. (16) and (17). The eigenmode of interest is the mode confined to the inner part of the disc. Since the mode must be evanescent in the outer part, we adopt $u_r = 0$ at some large radius $r = r_{\rm out}$ as the outer boundary condition. We impose the inner boundary condition at the star/disc interface $r = r_{\rm s}$. Since the pressure scale-height of the star near the interface is much smaller than that of the disc, the waves in the disc do not penetrate the stellar surface. Hence, we take $u_r = 0$ at $r = r_{\rm s}$ as the inner boundary condition.

3. Constraints upon the confinement of one-armed oscillations

It is instructive to examine the set of Eqs. (16) and (17) semi-analytically by an asymptotic method, before solving them numerically. For this purpose, we transform Eqs. (16) and (17) into the form

$$\frac{{\rm d}^2 Y}{{\rm d}r^2} + k^2(r)\,Y = 0, \tag{18}$$

where

$$Y = r^{11/4 - \alpha/2} \times \left[1 + 3k_2 f^2 \left(\frac{r}{r_{\rm s}}\right)^{-2} - \eta\left(\frac{r}{r_{\rm s}}\right)^{\epsilon}\right]^{-1/4} {\rm i}u_r, \tag{19}$$

$$k^2(r) = \frac{(\omega - \Omega)^2 - \kappa^2}{c_{\rm s}^2} - \frac{(\alpha - \frac{1}{2})(\alpha + \frac{3}{2})}{4r^2} \simeq -\frac{2\Omega(\omega - \Omega_{\rm p})}{c_{\rm s}^2} - \frac{(\alpha - \frac{1}{2})(\alpha + \frac{3}{2})}{4r^2} \tag{20}$$

under the approximations $|\omega| \ll \Omega$, $|\Omega - \kappa| \ll \Omega$, and $(c_{\rm s}/r\Omega)^2 \ll 1$. In Eq. (20), $\Omega_{\rm p}$ is the local precession frequency given by

$$\Omega_{\rm p} = \Omega - \kappa \simeq \frac{\Omega_{\rm K}(r)}{2} \times \left[\alpha \mathscr{M}_{\rm s}^{-2}\frac{r}{r_{\rm s}} + 2k_2 f^2 \left(\frac{r}{r_{\rm s}}\right)^{-2} + \eta\epsilon\left(\frac{r}{r_{\rm s}}\right)^{\epsilon}\right]. \tag{21}$$

Equation (20) expresses the dispersion relation by which we can converniently discuss some of the characteristics of the eigenmodes. Local oscillations can propagate in regions where $k^2 > 0$, while they are evanescent in regions where $k^2 < 0$. The value of k^2 increases monotonically with decreasing ω over the frequency range $|\omega| \ll \Omega$ in the current problem. When $\omega > \Omega_{\rm p}(r_{\rm s})$, the value of k^2 is negative throughout the disc. No eigenmodes are thus present within this frequency range. On the other hand, when $\omega < 0$, the propagation region extends over the entire disc if a moderate value of α is adopted. As pointed out by PSH and SH, the retrograde modes (i.e., modes with $\omega < 0$) are thus not confined to the inner part of the disc. Therefore, the eigenfrequency ω for the m = 1 oscillation confined to the inner part of the disc is found in the range $0 < \omega < \Omega_{\rm p}(r_{\rm s})$, for which the propagation region lies between $r = r_{\rm s}$ and the turning-point radius at which $k^2 = 0$.

The asymptotic form of the eigenfunction Y, which satisfies the boundary conditions described in the previous section, is written as

$$Y \sim \begin{cases} \dfrac{1}{\sqrt{k}} \cos\left(\displaystyle\int_r^{r_{\rm TP}} k dr - \dfrac{\pi}{4} \right) & \text{for } r \ll r_{\rm TP} \\ \dfrac{1}{2\sqrt{|k|}} \exp\left(- \displaystyle\int_{r_{\rm TP}}^{r} |k| dr \right) & \text{for } r \gg r_{\rm TP}, \end{cases} \tag{22}$$

where k, given by Eq. (20), must satisfy the following eigenvalue condition

$$\int_{r_{\rm s}}^{r_{\rm TP}} k dr = \left(n + \frac{3}{4} \right) \pi \tag{23}$$

with a non-negative integer n. In the above equations, $r_{\rm TP}$ is the radius of the turning point. Note that $r_{\rm TP}$ approximately coincides with the radius of the inner Lindblad resonance $r_{\rm ILR}$ at which $\omega = \Omega - \kappa$ [for details of the modal analysis by an asymptotic method, see Sect. 16 of Unno et al. (1989)].

Equations (20)–(23) show that, with increasing $\alpha \mathscr{M}_{\rm s}^{-2}$, k^2 decreases and the mode becomes less confined. This confirms the discussion by PSH and SH that the pressure effect acts against the confinement. This also means that, for a given value of $\alpha \mathscr{M}_{\rm s}^{-2}$, there is a critical curve in the ($k_2 f^2$, $\eta\epsilon$)-plane below which no confinement of oscillations occurs. Our interest is then how large a value of $k_2 f^2$ or $\eta\epsilon$ is needed to give rise to the confinement of the m = 1 oscillations. Fig. 1 shows such critical curves in the ($k_2 f^2$, $\eta\epsilon$)-plane below which we find no numerical solution for Eqs. (16) and (17) given the boundary conditions. Fig. 1a is for a B0V star and Fig. 1b for a B5V star. The values of the disc parameters are attached to each line. The thick line in each figure denotes the critical curve for $T_{\rm d} = \frac{2}{3} T_{\rm eff}$ and $\alpha = 7/2$, which we regard as typical values for Be-star discs. The raggedness of each line reflects that we surveyed the ($k_2 f^2$, $\eta\epsilon$)-space with a resolution of 0.02dex. For the sake of convenience, we also show the scales of k_2 for f = 0.4 [Fig. 1a] and for f = 0.75 [Fig. 1b], which are taken as typical values for actual Be stars. Though Fig. 1 is plotted for ϵ = 0.1, the results are insensitive to the value of ϵ, as long as η, $\epsilon \ll 1$.

From Fig. 1 we observe that the critical curve in the ($k_2 f^2$, $\eta\epsilon$)-plane above which the confinement occurs moves upward with increasing importance of the pressure effect. We also observe that, for parameters of a typical B0-type Be star [Fig. 1a], no m = 1 oscillations are confined without the contribution of the radiative force, unless $k_2 f^2 \gtrsim 8 \times 10^{-3}$ (i.e., $k_2 \gtrsim 5 \times 10^{-2}$ for $f \sim 0.4$) for $T_{\rm d} = \frac{2}{3} T_{\rm eff}$ and $\alpha = 7/2$ (thick line). Even for the case of the weakest pressure effect ($T_{\rm d} = \frac{1}{2} T_{\rm eff}$ and $\alpha = 5/2$), the confinement of oscillations without the radiative effect requires $k_2 \gtrsim 2 - 3 \times 10^{-2}$. It is unlikely that B0-type Be stars have such large values of k_2, because they appear not to be extreme rotators. Adopting $k_2 \lesssim 10^{-2}$, $T_{\rm d} = \frac{2}{3} T_{\rm eff}$, and $\alpha = 7/2$, we obtain that the confinement of the m = 1 oscillations requires a radiative effect as large as $\eta\epsilon \gtrsim 4 \times 10^{-3}$. Note that this value of $\eta\epsilon$ is not unreasonable, if $\epsilon \gtrsim 10^{-2}$. We conclude that it is the radiative effect that is important for the confinement of the m = 1 oscillations in discs around B0-type Be stars. The rotational effect is minor for these stars.

On the other hand, for B5-type Be stars [Fig. 1b], the rotational effect seems large enough to confine the m = 1 oscillations. In a typical B5-type Be star disc ($f \sim 0.75$, $T_{\rm d} = \frac{2}{3} T_{\rm eff}$, and $\alpha = 7/2$), the m = 1 oscillations are confined without any contribution of the radiative effect, if $k_2 \gtrsim 10^{-2}$. This value of k_2 is reasonable, because the rotational deformation of a B5-type Be star is expected to be, on average, much larger than that of a B0-type Be star. Moreover, since the luminosity of a B5-type star is about 10 times smaller than that of a B0-type star, the effect due to the radiative force would play a minor role in a B5-type star. In such discs, the effect of rotational deformation of the star is therefore much more important than the radiative effect.

4. Eigenmodes confined to the inner part of the disc

In order to examine the characteristics of the m = 1 eigenmodes confined to the inner part of the disc, we have numerically solved Eqs. (16) and (17) with boundary conditions for a wide range of parameter values: We surveyed eigenmodes for a deformation factor of $10^{-3} \leq k_2 f^2 \leq 10^{-1}$, radiative parameters of $10^{-3} \leq \eta \leq 0.5$ and $10^{-2} \leq \epsilon \leq 10^{-1}$, disc temperature of $\frac{1}{2} \leq T_{\rm d}/T_{\rm eff} \leq 1$, density gradient index of $5/2 \leq \alpha \leq 4$, and central stars with spectral types of B0V and B5V. Since we found that the obtained eigenmodes are roughly similar for different values of disc parameters and ϵ, we restrict our attention to cases with $T_{\rm d} = \frac{2}{3} T_{\rm eff}$, $\alpha = 7/2$, and $\epsilon = 10^{-1}$.

Fig. 2 shows the distribution of the two characteristic quantities, the period of the oscillation and the size of the propagation region, of the fundamental m = 1 eigenmodes confined to the inner part of the disc. Fig. 2a is for a B0V star and Fig. 2b for a B5V star. Other values of parameters are annotated in the figure. In each figure thick contours denote the oscillation period in units of years and thin contours denote $r_{\rm ILR}/r_{\rm s} - 1$, where $r_{\rm ILR}$ is the radius of the inner Lindblad resonance. Recall that $r_{\rm ILR}$ is a good measure of the width of the propagation region of the eigenmode. The interval of contours is 0.5dex. At the top of each panel we show the scale for k_2 for (a) f = 0.4 and

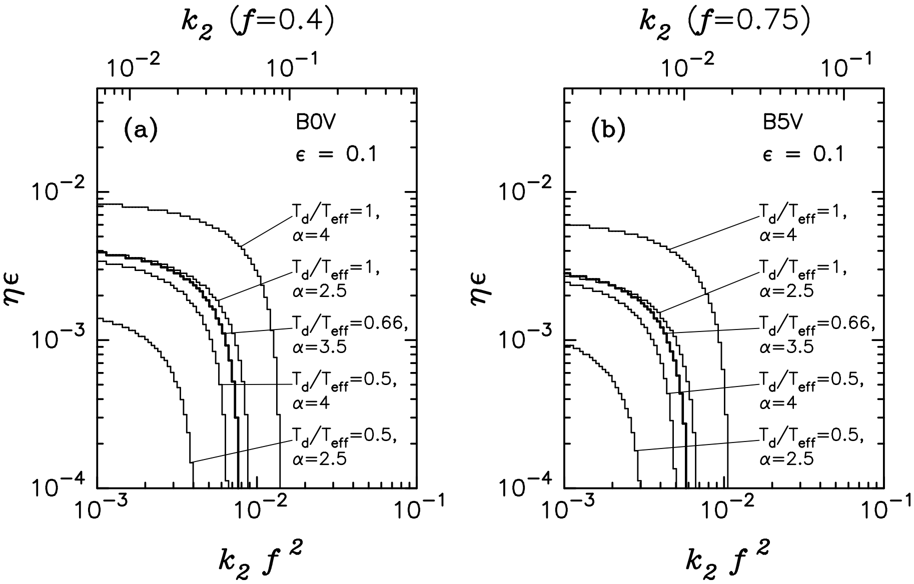

Fig. 1a and b. Critical lines in the $(k_2 f^2, \eta\epsilon)$-plane below which no confinement of m = 1 oscillations occurs in discs around **a** a typical B0V star and **b** a typical B5V star. The values of the disc parameters are annotated. The value of ϵ is fixed to be 0.1. Scales for k_2 are also shown for **a** f = 0.4 and **b** f = 0.75

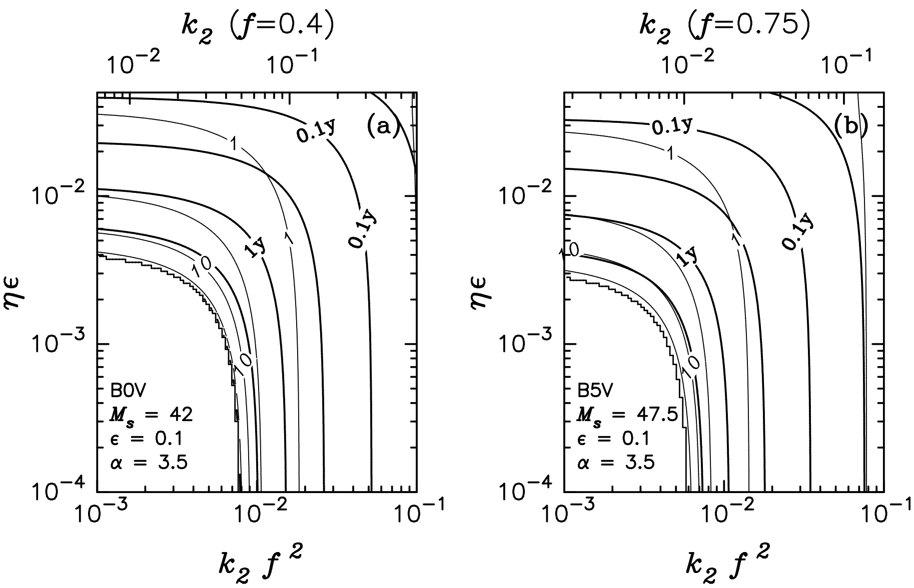

Fig. 2a and b. Distribution of the period and the propagation-region width of the fundamental mode confined to the inner part of the disc around **a** a B0V star and **b** a B5V star. Thick contours denote the oscillation period in units of years and thin contours denote $r_{\rm ILR}/r_{\rm s} - 1$. The interval of contours is 0.5dex. Scales for k_2 are also shown for **a** f = 0.4 and **b** f = 0.75. The ragged boundary denotes the critical curve below which no confined eigenmode is found

(b) f = 0.75. The ragged boundary in each figure denotes the critical curve discussed in the previous section; we found no confined eigenmode for the values of $k_2 f^2$ and $\eta\epsilon$ below this line. As mentioned earlier, the raggedness of the critical curve reflects that we surveyed the $(k_2 f^2, \eta\epsilon)$-space with a resolution of 0.02dex.

From Fig. 2, we note that the period of the eigenmode is sensitive to the parameters characterizing the rotational deformation and the weak line force. It rapidly decreases with increasing values of $k_2 f^2$ or $\eta\epsilon$. The dependence of the period on these parameters is similar for different values of α and $T_{\rm d}/T_{\rm eff}$ (or equivalently $\mathscr{M}_{\rm s}$), except that, as the pressure effect decreases, the critical curve in the $(k_2 f^2, \eta\epsilon)$-plane moves to the lower left and modes with longer periods appear. Needless to say, the period of the confined mode is independent of the disc outer radius as long as it is located far outside the propagation region of the mode. Consequently, the range of observed periods of the V/R variations places a rather narrow constraint on $k_2 f^2$ and $\eta\epsilon$. Suppose that the observed periods range from a year to a decade both for B0- and for B5-type Be stars. Suppose also that the radiative effect is dominant in discs of B0-type Be stars, while the rotational effect is dominant in discs of B5-type Be stars. Then, the range of $\eta\epsilon$ for B0-type Be stars and the range of $k_2 f^2$ for B5-type Be stars are, respectively, restricted to $4 \times 10^{-3} \lesssim \eta\epsilon \lesssim 10^{-2}$ and $6 \times 10^{-3} \lesssim k_2 f^2 \lesssim 10^{-2}$. If a star exhibits a V/R variation with a period longer than a decade, it indicates that the pressure effect in the disc would also be weaker than that adopted in Fig. 2, i.e., $\alpha < 7/2$ and/or $T_{\rm d} < \frac{2}{3} T_{\rm eff}$.

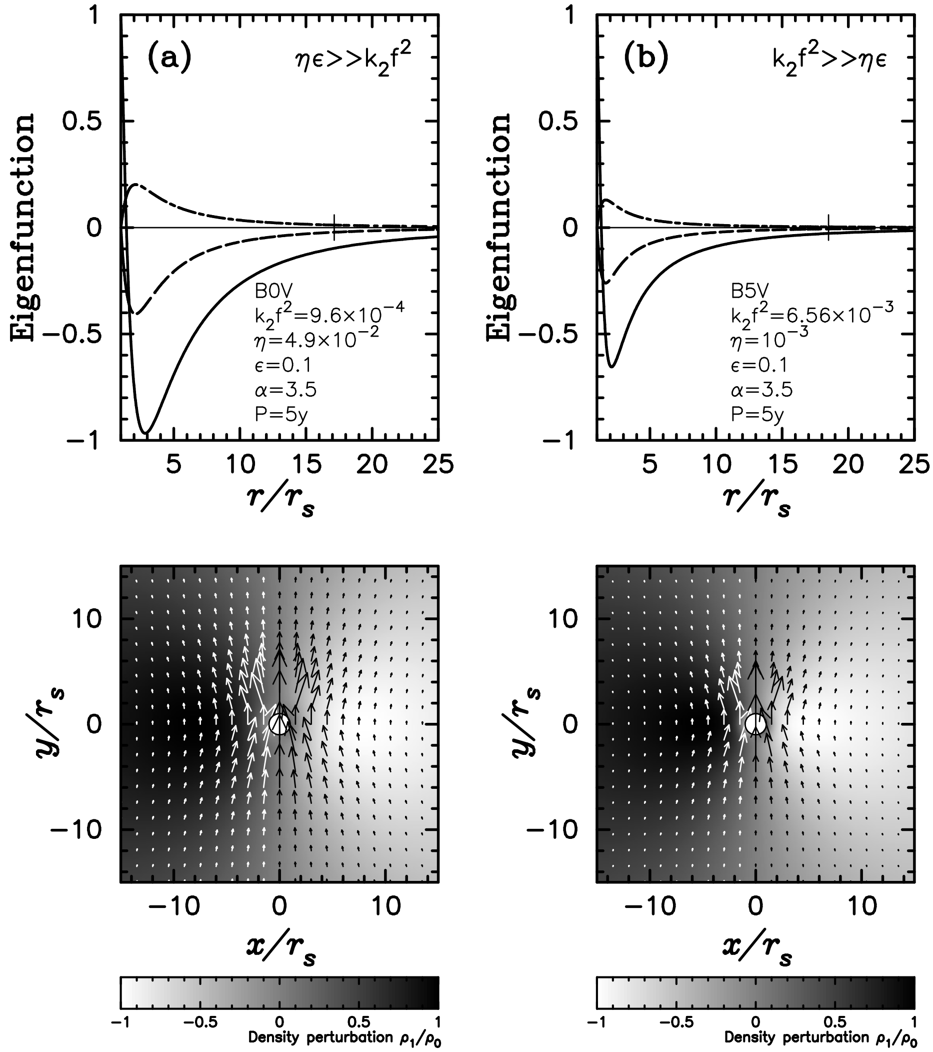

Fig. 3a and b. Fundamental mode for **a** $\eta\epsilon \gg k_2 f^2$ and **b** $k_2 f^2 \gg \eta\epsilon$. The disc temperature is $\frac{2}{3}T_{\rm eff}$. The values of the other parameters are indicated in the upper panel. In the upper panel, the solid, dashed, and dash-dotted lines denote ρ_1/ρ_0, $iu_r/r\Omega$, and $u_\phi/r\Omega$, respectively. The location of the inner Lindblad resonance is shown by the vertical bar. In the lower panel, the gray-scale plot denotes the density perturbation ρ_1/ρ_0 in the (r,ϕ)-plane, while arrows denote the perturbed-velocity vectors $(u_r/r\Omega,\, u_\phi/r\Omega)$

Now we discuss the characteristics of the eigenmodes confined to the inner part of Be-star discs. Fig. 3 presents two examples of the fundamental $m = 1$ modes. Fig. 3a is for the radiation-effect dominant case for a B0V star, while Fig. 3b is for the rotation-effect dominant case for a B5V star. In this example, both modes have the same period of 5 years. The adopted disc temperature is $\frac{2}{3}T_{\rm eff}$. The values of the other parameters are indicated in the upper panels. In each upper panel, the solid, dashed, and dash-dotted lines denote ρ_1/ρ_0, $iu_r/r\Omega$, and $u_\phi/r\Omega$, respectively. They are normalized such that the maximum value of ρ_1/ρ_0 is unity. The vertical bar denotes the location of the inner Lindblad resonance. Note that ρ_1 and u_ϕ vary as $\cos(\omega t - \phi)$, while u_r varies as $\sin(\omega t - \phi)$. The lower panels show the distribution of these quantities in the inner 15 $r_{\rm s}$ in the (r,ϕ)-plane. The circle at the center denotes the location of the star/disc interface. The perturbation pattern as well as the unperturbed disc rotates counterclockwise. The density perturbation ρ_1/ρ_0 is denoted by a gray-scale representation. Arrows superposed on the gray-scale plot are the perturbed-velocity vectors $(u_r/r\Omega,\, u_\phi/r\Omega)$. The length of each arrow is proportional to the strength of the perturbation.

From the upper panels of Fig. 3, we note that in both cases the fundamental $m = 1$ modes are well confined to the inner part of the disc. We also note that the global features of the eigenfunctions are very similar in both cases, except that the region in which the amplitude of the perturbation is large is slightly narrower in the rotation-effect dominant case than in the radiation-effect dominant case. The azimuthal component of the velocity perturbation u_ϕ anticorrelates with the density perturbation ρ_1, except in the innermost narrow part of the disc. Note that this property, which was also seen in the modes studied in Okazaki (1991), has been shown to cause the observed line-profile variability of the V/R variations (Hummel & Hanuschik 1994, 1996; Hanuschik et al. 1995; Okazaki 1996).

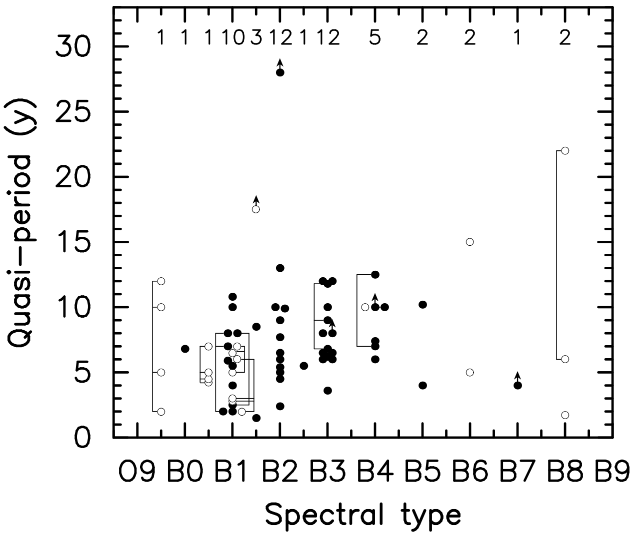

Fig. 4. Distribution of observed V/R periods. Filled circles denote periods for stars which show no evidence for binarity, while open circles denote periods for stars known as (or suspected to be) binaries. Circles with upward arrows indicate that possible periods are longer than the values shown. Different periods in different epochs for individual stars are shown by data points connected by lines

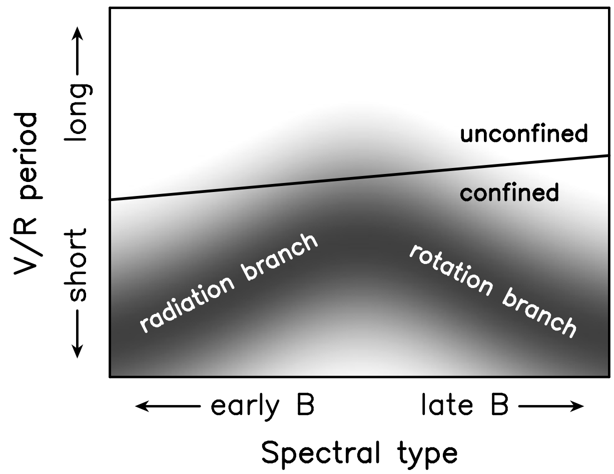

Fig. 5. Schematic distribution of the V/R period expected from the hybrid scenario of the confinement of one-armed oscillations

5. Observational tests of the hybrid scenario of the confinement of one-armed oscillations

For a total of 28 Be stars, Hirata & Hubert-Delplace (1981) showed that the period of the long-term V/R variation does not depend on the spectral type of the star. This conclusion was confirmed by Mennickent & Vogt (1991) for a total of 29 Be stars. Since the sample stars of Mennickent & Vogt (1991) only partially overlap those of Hirata & Hubert-Delplace (1981), we combined these two sets [we have missed four stars in the sample of Hirata & Hubert-Delplace (1981)], and also collected data for some other stars from the literature. Fig. 4 shows the resultant distribution of the observed V/R (quasi-) periods for 53 Be stars of spectral type O9.5–B8. Filled circles denote periods for stars which show no evidence for binarity, while open circles denote periods for stars known as (or suspected to be) binaries (in a binary system, the companion star possibly affects the confinement of the $m = 1$ oscilaltions through the tidal perturbation potential and the truncation of the disc). Circles with upward arrows indicate that possible periods are longer than the values shown in the figure. Some stars, which have exhibited different periods at different epochs, contribute two or more data points, which are connected by lines. The number of stars in each spectral bin is annotated at the top. For the sake of readers' convenience, we list detailed information for our sample stars in Table 1 in Appendix.

From Fig. 4, we observe the following features. First, as known before, the period of V/R variation shows no strong dependence on spectral type, although the sample of late-type Be stars is still scarce. Second, the data suggest a maximum around B3–B4. Third, the longer end of the period distribution is approximately independent of the spectral type, except for some stars with exceptionally long periods. Below we show that these features are consistent with the hybrid scenario of the confinement of the $m = 1$ oscillations discussed in the previous sections. We should keep in mind, however, that about 1/3 of the Be stars have not shown long-term V/R variations in Balmer line profiles (Copeland & Heard 1963; Mennickent & Vogt 1991), which obviously could not be included in Fig. 4. The difference between these stars and V/R variables should be clarified by future studies.

First, we discuss the feature that the observed V/R periods do not show strong spectral dependence. As mentioned earlier, the range of the rotation parameter f of B5-type Be stars is about twice as large as that of B0-type Be stars. Since the value of k_2 for the former stars is likely larger than that for the latter stars, the deformation factor $k_2 f^2$ of B5-type Be stars would be larger than that of B0-type Be stars by more than a factor of four. Consequently, if the confinement of the $m = 1$ oscillations is attributed solely to the rotational effect, the periods of V/R variation of B5-type Be stars would be shorter than those of B0-type Be stars by more than an order of magnitude [compare Fig. 2a with Fig. 2b]. It is obvious that this does not fit the observed data. Similarly, if the confinement is due only to the radiative effect, B0-type Be stars would exhibit V/R variations whose periods are much shorter than those of B5-type Be stars. Therefore, the observed absence of a significant difference between V/R periods for B0- and B5-type stars is in agreement with the hybrid scenario of the confinement. The rotational effect is dominant in discs of B5-type Be stars, while the radiative effect plays an important role in discs of B0-type Be stars. Based on this scenario, the insensitiveness of the period to spectral type is the result of the different mechanisms acting in early- and late-type Be stars.

Next, we show that the hybrid scenario of the confinement also agrees well with the second and the third features mentioned above. As discussed in Sect. 4, the period of the $m = 1$

oscillation confined to the inner part of the disc decreases with increasing values of radiative parameter $\eta\epsilon$ and deformation factor $k_2 f^2$. It is plausible to assume that the radiative parameter is, on average, a monotonically increasing function of the effective temperature of the central star. Then, we expect that, among early-type Be stars, the V/R period, on average, decreases towards earlier spectral type. On the other hand, as shown in Fig. 4 of Kogure & Hirata (1982), the average value of rotation parameter f monotonically increases towards later spectral type. The deformation factor is, thus, larger for a later spectral type, unless the apsidal motion constant k_2 decreases with f more rapidly than f^{-2}. Hence, it is likely that, among late-type Be stars, the V/R period decreases, on average, towards later spectral type. As a result, as schematically shown in Fig. 5, the distribution of the V/R period is expected to have a maximum at a transitional spectral type around which the rotational effect is comparable to the radiative effect.

Actually, the values of the radiative parameter, the deformation factor, and the parameter $\alpha \mathscr{M}_s^{-2}$ characterizing the pressure effect would be different for individual stars. Since the period of the m = 1 eigenmode is sensitive to these values, the distribution of the observed V/R periods is expected to have a wide spread even for the same spectral type, and to be limited below the critical period above which no oscillations are confined. The critical period would be roughly independent of the spectral type, because it depends mainly on the lower end of the distribution of $\alpha \mathscr{M}_s^{-2}$, which is considered to be not so much different between early- and late-type Be stars. It should be noted that these properties expected from the hybrid scenario of the confinement of oscillations agree well with the observed values shown in Fig. 4. It should also be noted that in this scenario the V/R period can vary from one cycle to the next for individual stars, which have been observed for some stars. For late-type Be stars, the V/R period may change because of a change in the magnitude of the pressure effect, while for early-type Be stars it may change because of a change in the combined radiative and pressure effects. This prediction can be tested, and will pose constraints on the model parameters.

Finally, we comment on the another observational aspect predicted by the hybrid scenario of the confinement. If the radiative effect is dominant, the V/R variations have periods independent of the value of the rotation parameter f. In contrast, if the rotational effect dominates, the period of the V/R variation should depend on the value of f. Thus, this hybrid scenario for the mechanisms which cause the confinement of the m = 1 oscillations predicts that the periods of V/R variations of early-type Be stars have little correlation with the rotation parameter, while late-type Be stars should exhibit either the V/R variations with periods anticorrelated with the rotation parameter, or no V/R variation if the stars rotate too slowly.

Unfortunately, the data available at present are too scarce to test this prediction over the whole spectral range. It seems possible, however, to perform a rough test for a restricted range of spectral types. Table 1 lists 8 B1-type Be stars of which the value of $v \sin i$ is known. Three stars with shell spectra, ζ Tau, V1294 Aql, and 59 Cyg, have equatorial rotation velocities $\sim 260\,\mathrm{km\,s^{-1}}$, since stars with shell spectra are considered to be seen nearly equator on. The V/R periods for these stars range from 2 to 7 years. On the other hand, the V/R periods for π Aqr, o Pup, 8 Lac, and 25 Ori, which have equatorial rotation velocities $> 300\,\mathrm{km\,s^{-1}}$ range from 2 to 10 years. Therefore, no dependence of the V/R period on the rotation parameter is seen for these B1-type stars. Moreover, 48 Lib, which is the fastest rotator among five B3-type stars with shell spectra listed in Table 1, does not show V/R periods shorter than other four less-rapidly-rotating stars. These results seem to support the hybrid scenario of the confinement, although we need more data for a definite conclusion.

6. Conclusions

We have examined how the weak-line force and the rotational deformation of the star affect the confinement of one-armed oscillations in near-Keplerian discs of Be stars. For simplicity, we assumed an isothermal disc with $1/2 \leq T_\mathrm{d}/T_\mathrm{eff} \leq 1$ and $5/2 \leq \alpha \leq 4$, where T_d and T_eff are the disc temperature and the effective temperature of the star, respectively, and α the density gradient index of the disc. The advective motion in the unperturbed disc was neglected. We took into account the effect of rotation by including the quadrupole contribution to the potential around the rotationally-distorted central star. For the radiative force due to an ensemble of optically thin lines, we adopted the parametric form proposed by Chen & Marlborough (1994).

Based on these assumptions, we have numerically studied the linear, isothermal m = 1 eigenmodes confined to the inner part of the disc. Examining the eigenvalue problem for a wide range of parameters characterizing the effects of rotation and radiation, we have derived the following conclusions:

1. The present study strongly suggests a scenario in which the mechanism that causes the confinement of one-armed oscillations in early-type Be stars is different from that in late-type Be stars. In late-type Be stars, the confinement of oscillations in the inner part of the disc occurs because of the deviation from the point-mass potential around the rotationally-deformed star. The radiative effect plays a minor role in these stars. On this point, we confirm the conclusions obtained by Papaloizou et al. (1992) and Savonije & Heemskerk (1993). In early-type Be stars, however, it is the weak-line force that mainly contributes to the confinement. For these stars, the rotational effect plays a minor role.
2. The period of a one-armed eigenmode confined to the inner part of the disc is sensitive to the value of the parameter characterizing the effect of rotation or radiation. This sensitiveness, together with the period range of observed V/R variations, enables us to place rather narrow constraints on the parameter values. In particular, the distribution of the V/R periods expected from the hybrid scenario agrees well with the distribution of the observed V/R periods. Furthermore, this scenario predicts that the periods of V/R variations of early-type Be stars have little correlation with the rotation parameter of the central star, whereas, in late-type Be stars,

a star with smaller rotation parameter exhibits either a V/R variation with a longer period or no V/R variation if the star rotates too slowly. Observational tests of this prediction are desired.

3. The characteristics of the eigenfunction are very similar in early- and late-type Be stars, in spite of the fact that the mechanism that causes the confinement of the mode is different. They agree well with the periodicity and line-profile variability of the observed V/R variations.

In the formulation of the current problem, we adopted the parametric form of the radiative force due to an ensemble of optically thin lines, and assumed that the radiative force decreases, with increasing radius, less rapidly than the gravity. Some of the above conclusions have been derived on the basis of this assumption. If the radiative force decreases with radius more rapidly than the gravity, it never contributes to the confinement of the m = 1 oscillations. We need, then, another unknown mechanism for the confinement of oscillations in discs of early-type Be stars. It is, therefore, highly desirable to derive a more detailed form of the radiative force due to an ensemble of optically thin lines.

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Appendix: list of stars showing long-term V/R variations

In their statistical study of long-term V/R variations, Hirata & Hubert-Delplace (1981) used a total of 28 sample stars collected from McLaughlin (1932, 1958), Copeland & Heard (1963), and Hubert-Delplace et al. (1982). Later, Mennickent & Vogt (1991) statistically analyzed short- and long-term V/R variations based on their plates for southern Be stars and data collected by a literature search. Among their sample stars, a total of 29 stars show long-term V/R variations. Since the sample stars of Hirata & Hubert-Delplace (1981) and Mennickent & Vogt (1991) overlap only partly, we combined the data for these two sets to make a larger data set [we have missed data for four stars in the sample of Hirata & Hubert-Delplace (1981)]. We added data for some additional stars from the literature. The results are shown in Table 1 (on the following pages). Table 1 also includes information on shell episodes and binarity for individual stars, which were taken mainly from Hanuschik (1996) and Pols et al. (1991), respectively. Many stars studied by Copeland & Heard (1963) are fainter than $7\overset{m}{.}0$, for which we could not find recent studies. We consider this table as not exhaustive but rather fairly representative, except for stars later than B4, for which there are not sufficient data available. We also note that about 1/3 of the Be stars have not shown long-term V/R variations (Copeland & Heard 1963; Mennickent & Vogt 1991).

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Table 1. Periods of long-term V/R variations for 53 stars

HD	Name or HR	MK type [a]	$v \sin i$ [a] $(\mathrm{km\,s^{-1}})$	shell episode [b]	orbital period [c] (days)	Epoch	Quasi-period (years)	References to quasi-periods
24534	X Per	O9.5 III	200	no	— [d]	1917–31	~ 5	38
						1932–36	2:	39
						1949–62	12	9
						1971–80	10	30
206773		B0 Vpnne [e]	390:: [f]			1946–61	6.8	7
5394	γ Cas	B0.5 IVe	230	yes	— [d]	1932–42	$4 - 5$	9
						1970–73	$3.5 - 5$	17
						1973–78	5	17
						1978–87	7	17, 52
212571	π Aqr	B1 III-IVe	300	no	— [g]	1912–24	$V = R$	42
						1925–36	2.8	42
						1936–40	$V = R$	42
						1940–44	3	42
						1944–56	$V = R$	42
						1976–85	2	51, 11, 24, 45
						1985–89	6	45, 46
37202	ζ Tau	B1 IVe-shell	260 [f]	yes	132.91	1960–67	7	12
						1967–81	5	30
						1989–93	$6 - 7$	26
63462	o Pup	B1 IVe	320	no		1980–89	2.5	51, 11, 45, 46
						1985-93	~ 8	26
192445		B1 IVe [34]				1946–60	5.9	7
214168	8 Lac	B1 IVe	320	yes [e]		1912–32	~ 10	38
184279	V1294 Aql	B1 IV-V [5]	256 [5]	yes [5]		1971–84	5.5	5
28497	1423	B1 Ve	230	no		1960–80	$7 - 9$	30
						1980–85	7	51, 11
						1985–87	2:	45
35439	25 Ori	B1 Ve	320	no		1915–35	$3 - 5$	38, 18
193009		B1 Vnne [34]				1946–61	10.8	7
200120	59 Cyg	B1 Ve	260	yes		1979–84	2	16
44458	2284	B1.5 IVe	200	no		1930–32	$1 - 2$	38
68980	3237	B1.5 IVe	115	no		1964–93	$8 - 9$	6, 50, 11, 45, 46, 25
148184	χ Oph	B1.5 Ve	140	no	138.8	1985–93	$> 15 - 20$	26
48917	10 CMa	B2 IIIe	200	no		1961–67	$V = R$	32, 31, 33
						1980–87	9	51, 4, 45, 46
113120	4930	B2 IIIe	300	no		1987–93	~ 10	26
20336	985	B2 (IV:)e	250	no		1912–37	4.5	38,
								41
						1938–59	$V \sim R$	41
45995A	2370	B2 IVe	250	no		1954–61	5.4	7
50013	κ CMa	B2 IVe	220	no		1965–93	> 28	32, 31, 33, 51, 11, 45, 26
88661	4009	B2 IVe	220	no		1978–87	6	10, 51, 11, 45, 46
105435	δ Cen	B2 IVe	220	no		1953–87	13	32, 10, 51, 11, 45
13661		B2 IV-Ve [49]	200 [f]			1946–62	7.7	7
164284	66 Oph	B2 IV-Ve	240	no		1989–94	5	25
177648		B2 Ve [34]				1951–56	2.4	7
208682	8375	B2 Ve	250			1946–61	6.5	7
225095		B2 Ve [36]	125 [f]			1946–61	9.9	7
157042	ι Ara	B2.5 IVe	320	no		1978–87	5.5	10, 51, 11, 45
205637	ϵ Cap	B3 IIIe	250	yes		1971–82	6	10, 19, 51, 1, 11, 24
105521	4625	B3 IVe	130			1980–87	8:	51, 45, 46
137387	κ^1 Aps	B3 IVe-2 sp.?	250–350	yes		1980–88	> 8 :	51, 11, 45, 46

Table 1. (continued)

HD	Name or HR	MK type [a]	$v \sin i$ [a] (km s^{-1})	shell episode [b]	orbital period [c] (days)	Epoch	Quasi-period (yr)	References to quasi-periods
142983	48 Lib	B3: IV:e-shell	400	yes		1953–72	11.8	53, 20, 29, 3
						1972–79	6.8	29, 3
						1979–94	~ 9	45, 46, 25
217050	EW Lac	B3: IV:e-shell	300	yes		1976–84	6	28, 37
12302		B3 IV-Ve [49]	200 [f]			1946–61	6.5	7
32343	11 Cam	B3 Ve	100	no		1916–32	~ 12	38
32991	105 Tau	B3 Ve	200	no		1930–65	10	43
						1982–93	~ 10	26
60606	2911	B3 Ve	230	no ?		1980–87	12:	51, 11, 45
180398		B(3) ne [7]				1946–61	6.2	7
195407		B3 e [7]				1946–61	6.5	7
217543	8758	B3 Vpe [e]	295 [f]	yes [e]		1946–59	3.6	7
15472		B4 ne [7]				1946–61	7.4	7
25940	48 Per	B4 Ve	200	no	16.59	1911–31	~ 10	38
45725	β^1 Mon	B4 Ve-shell	300	yes		1905–25	$V = R$	40, 8
						1928–67	12.5	40, 8
						1969–72	$V = R$	8
						1980–87	7	2, 51, 11, 23, 45
91465	4140	B4 Ve	250	no		1953–63	$> 10\,(V > R)$	32, 31, 27
						1976–84	10	44, 10, 48, 11
224559	LQ And	B4 Vne [e]	250 [47]	no		1960–80	$5 - 7$	30
112091	μ^2 Cru	B5 Ve	220	no		1985–89	4:	45, 46
	BD+5°4285	B5 ne [7]				1946–61	10.2	7
45542	ν Gem	B6 IVe	170	yes	40.198	1989–93	~ 5	26
162732	88 Her	B6 IVe [13]	300 [e]	yes	86.59	1955–81	15	14, 15
214748	ϵ PsA	B7 IVe	180	no		1974–83	$V \sim R$	10, 51, 11
						1985–89	$> 4\,(V \leq R)$	45, 46
23862	28 Tau	B8 (V:)e-shell	320	yes	218.0 [35]	1912–31	$V = R$	38
						1938–54	22:	22
						1954–72	~ 6	21
58715	β CMi	B8 Ve	245	no	218.498	1950–80	$V = R$	51
						1987–89	1.7	45, 46

[a] taken from Slettebak (1982), except noted otherwise
[b] taken from Hanuschik (1996), except noted otherwise
[c] taken from Pols et al. (1991), except noted otherwise
[d] Orbital elements are not known
[e] Bright Star Catalogue; Hoffleit & Jaschek (1982)
[f] 19% smaller than the value listed in Uesugi & Fukuda (1982) in order to convert to the new Slettebak system, following Kogure & Hirata (1982)
[g] suspected to be an interacting binary (Hanuschik et al. 1996)

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