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Corrections and new developments in rigid-Earth nutation theory

II. Influence of second-order geopotential and direct planetary effect

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Abstract. In a recent paper, Souchay & Kinoshita (1996) recalculated the coefficients of the nutation for a rigid-Earth model due to the main luni-solar potential (coming from J_2), with respect to the values given by Kinoshita & Souchay (1990), by taking into account an updated value of the general precession in longitude. In the following we complete this study by computing the coefficients coming from the lunar influence on the second-order geopotential (J_3, $C_{2,2}$, $S_{2,2}$, J_4) and those coming from the direct torque exerted by the planets. We notice some corrections to be done with respect to the tables of Kinoshita & Souchay(1990), and we show that the agreement with computations done by Hartmann et al. (1995) for the J_3 part and Williams (1995) for the direct planetary effect is remarkable, although the way of calculation is quite different.

Key words: reference systems – Earth – celestial mechanics

1. Introduction

The dramatic improvement of the accuracy of the VLBI observations in the last decade leads to some estimations of the coefficients of the nutation of the Earth at a few microarcseconds accuracy. Thus it became necessary to be confident in the values of the coefficients of the nutation of the Earth for a simplified rigid model, at less at the microarcsecond level. In the present paper we check the analytically determined coefficients of the nutation for a rigid Earth model as calculated by Kinoshita & Souchay (1990), due to small influences of the potential of the Earth (J_3, $C_{2,2}$, $S_{2,2}$, J_4), and to the direct torque exerted by the planets (also quoted as the direct planetary effect) up to their level of truncation, that is to say 5 microarcseconds. Moreover we calculate also the coefficients ranging between 0.1 microarcseconds and 5 microarcseconds, in order to perform a comparison with recent calculations done by Hartmann et al. (1995) and Williams (1995), respectively for the two kinds of effects above.

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2. Terms of nutation coming from the J3 geopotential

As explained by Kinoshita (1977) the coefficient J_3 of the geopotential is small, so that when calculating its contribution to the nutation due to the Moon, the motion of the Moon can be considered as Keplerian. Nevertheless, we recompute here these coefficients by including all the perturbations in the coordinates λ_M, β_M, and r_M of the Moon given by their analytical expressions in the form of Fourier series (Chapront-Touzé and Chapront 1988). Notice that Kinoshita (1977) did not calculate the terms coming from the J3 geopotential, because they could be considered as negligible at that time. Notice also that the J_3 geopotential does not bring any significant contribution to the precession (see Hartmann et al. 1996).

In fact, Kinoshita & Souchay (1990) showed that only one term in longitude, that is to say: $\delta\psi = -0.105_{\text{(mas)}} \cos(F - l_M + \Omega)$, of period 8.85 y, is above the level of truncation of the series of Kinoshita (1977), that is to say 0.1 milliarcsecond (mas), whereas they found several other coefficients up to the level of truncation of their own series, that is to say 5 microarcseconds. Recently Hartmann et al. (1995) computed the nutations $\Delta\psi$ and $\Delta\varepsilon$ for the figure axis, starting from the harmonic tidal development of the torque exerted by the Moon, as given by Hartmann & Wenzel (1995). Their results is in very good agreement with those of Kinoshita & Souchay (1990), with a 3.3 and 2.6 microarecseconds *rms* respectively in longitude and in obliquity. By contrast, their results do not fit with those given by Zhu & Groten (1989), with a respective 41.7 and 86.8 microarcseconds *rms*, although the method used is quite equivalent. This is due in some errors in Zhu & Groten calculations, as explained by Hartmann et al. (1995), who could also avoid the difficulty to compute long-periodic components of the nutation starting from tidal quasi-diurnal harmonics, as explained by Souchay (1993). The expression of the lunar potential related to the J_3 harmonic can be expressed as follows (Kinoshita & Souchay 1990):

$$U_{J3} = \Big(\frac{\kappa^2 M_{\rm M} M_{\rm E} a_{\rm E}^3}{r_{\rm M}^4}\Big) J_3 P_3^0(\sin\delta_{\rm M}) \quad (1)$$

κ^2 is the Gauss constant, $M_{\rm M}$ and $M_{\rm E}$ are respectively the masses of the Moon and of the Sun, and $a_{\rm E}$ is the mean equatorial radius of the Earth.

The Legendre polynomial $P_3^0(\sin\delta_{\rm M})$ can be expressed itself as a function of the associated Legendre polynomials $P_3^i(\sin\beta_{\rm M})$, where $\delta_{\rm M}$ is the declination of the Moon, $\beta_{\rm M}$ its latitude with respect to the equator, and $r_{\rm M}$ the distance between the center of mass of the Earth and the center of mass of the Moon. Thus we have:

$$P_3^0(\sin\delta_{\rm M}) = \frac{1}{2}\cos I(-3 + 5\cos^2 I)P_3^0(\sin\beta_{\rm M})$$

$$+\frac{1}{4}\sin I(1-5\cos^2 I)\sin(\lambda_{\rm M} - h)P_3^1(\sin\beta_{\rm M})$$

$$-\frac{1}{4}\cos I\sin^2 I\cos 2(\lambda_{\rm M} - h)P_3^2(\sin\beta_{\rm M})$$

$$+\frac{1}{24}\sin^2 I\sin 3(\lambda_{\rm M} - h)P_3^3(\sin\beta_{\rm M}) \quad (2)$$

The P_3^i polynomials have the classical following expressions:

$$P_3^0(\sin\beta_{\rm M}) = (\frac{5}{2}\sin^2\beta_{\rm M} - \frac{3}{2})\sin\beta_{\rm M} \quad (3.1)$$

$$P_3^1(\sin\beta_{\rm M}) = (\frac{15}{2}\sin^2\beta_{\rm M} - \frac{3}{2})\cos\beta_{\rm M} \quad (3.2)$$

$$P_3^2(\sin\beta_{\rm M}) = 15\sin\beta_{\rm M}\cos\beta_{\rm M} \quad (3.3)$$

$$P_3^3(\sin\beta_{\rm M}) = 15\cos^3\beta_{\rm M} \quad (3.4)$$

The variable I is related to the canonical variables H and G in the Hamitonian theory (Kinoshita 1977), by the equation: $H = G\cos I$, where G is the angular momentum of the rotation of the Earth, and H its projection along the axis of the ecliptic of the epoch. I corresponds to $-\varepsilon$, where ε is the mean value of the obliquity of the axis of angular momentum with respect to the axis of the ecliptic of the date.

In a similar way, the canonical variable h represents the opposite of the general precession in longitude of the plane perpendicular to the angular-momentum vector (Kinoshita 1977): $h = -p$, where p is the linear component of the general precession in longitude $p_{\rm A}$ (Lieske et al. 1977).

Following the equations above, we calculate the determining function W_{J3} by the way of integration:

$$W_{J3} = \int U_{J3} \times dt \quad (4)$$

Then the coefficients of the nutation coming from J_3 are calculated by the direct partial derivatives (Kinoshita & Souchay 1990):

$$\Delta\varepsilon_{J3} = -\Delta I = -\frac{1}{G\sin I} \times \frac{\partial W_{J3}}{\partial h} \quad (5.1)$$

$$\Delta\psi_{J3} = -\Delta h = \frac{1}{G\sin I} \times \frac{\partial W_{J3}}{\partial I} \quad (5.2)$$

For our calculations, the constant term k_{J3}, which serves as a scaling factor, is given by the following formula (Kinoshita & Souchay 1990):

$$k_{J3} = \big(\frac{J_3}{J_2}\big)\big(\frac{a_{\rm E}}{a_{\rm M}}\big) \times k_{\rm M} \quad (6)$$

$a_{\rm M}$ is the semi-major axis of the Earth. The direct correspondence between the value of the general precession in longitude $p_{\rm A}$ and the scaling factors $k_{\rm M}$ and $k_{\rm S}$ which enable calculating the coefficients of the nutation due respectively to the Moon and the Sun has been studied in detail by Kinoshita & Souchay (1990) by using the conventional value of $p_{\rm A}$ (Lieske et al. 1977). Williams (1994) and Souchay & Kinoshita (1996) repeated this study by choosing an updated value of the linear component $p_{\rm A}^1$, as confirmed by various authors (see Williams 1994), and corresponding to a correction of $\delta p_{\rm A}^1 = -0.3266''$/cy. Notice that the consequence is a respective change of about 1 mas for the amplitude of leading term of nutation $\delta\psi$ of argument Ω (Souchay & Kinoshita 1996) due to J_2.

$k_{\rm M}$ being the scaling factor related to the first-order nutation coefficients (Kinoshita 1977; Kinoshita & Souchay 1990) In Souchay & Kinoshita (1996) we have calculated a new value of $k_{\rm M}$ by taking into account a recent evaluation of the general precession in longitude (Williams 1994) different from the conventional one (Lieske et al. 1977). This new value is used in our calculations of k_{J3} starting from (6), that is to say: $k_{\rm M} = 7546.7173289''/J.cy.$

Moreover, by choosing an updated value of J_3 as taken from Lerch et al. (1994), as Hartmann et al. (1995) did, that is to say: $J_3 = -2.533 \times 10^{-6}$ instead of: $J_3 = -2.54 \times 10^{-6}$, we find the value: $k_{J3} = -0.2932''$ instead of: $k_{J3} = -0.2961''$ (Kinoshita & Souchay 1990).

In Table 1 we are listing our results $\Delta\psi_{\rm new}$ and $\Delta\varepsilon_{\rm new}$ and comparing them with those of Kinoshita & Souchay (1990) and Hartmann et al. (1996), by retaining all the coefficients bigger than 0.5 microarcseconds (μas) both for $\Delta\psi_{\rm new}\sin\varepsilon$ and $\Delta\varepsilon_{\rm new}$. We can observe the remarkable agreement between the authors, and especially between our results and those of Hartmann et al. (1995): the amplitude difference is bigger than 0.1 μas for only 3 terms among the 20 terms to be compared, and anyway does not exceed 0.6 μas. The comparison is all the more convincing since the computation method used by Hartmann et al. (1995) is quite different from ours, that is to say based on a recent expansion on the tesseral part of the tidal potential (Hartmann & Wenzel 1995).

Table 1. List of the coefficients of rigid Earth nutation coming from the J_3 geopotential. Comparison of new results (this paper) is made with results of Kinoshita & Souchay (1990) and Hartmann et al. (1995), both in longitude and in obliquity

$l_{\rm M}$	$l_{\rm S}$	F	D	Ω	Period(d) (day)	$\Delta\psi_{\rm KS}$ (cos) (μas)	$\Delta\psi_{\rm HWS}$ (cos) (μas)	$\Delta\psi_{\rm new}$ (cos) (μas)	$\Delta\varepsilon_{\rm KS}$ (sin) (μas)	$\Delta\varepsilon_{\rm HWS}$ (sin) (μas)	$\Delta\varepsilon_{\rm new}$ (sin) (μas)
0	−1	1	−1	1	7646846.820			−196.9			−167.9
−1	0	1	0	3	65502.153	−6.0	−6.3	−6.2		−2.7	−2.7
−1	0	1	0	2	6159.142	36.0	36.6	36.0	18.0	17.5	17.6
−1	0	1	0	1	3231.495	−105.0	−104.0	−104.5	−89.0	−89.0	−89.1
−1	0	1	0	0	2190.344	33.0	33.2	32.8			
1	0	−1	0	1	−1656.609			0.5			−0.4
1	0	1	−2	1	193.559		−1.2	−1.2		−1.0	−1.0
1	0	1	−2	2	199.232						0.1
−1	0	−1	2	0	−188.202			0.5			
0	0	−1	2	−1	32.128						−0.2
0	0	1	0	2	27.432		3.0	3.0		1.4	1.4
0	0	1	0	1	27.322	−16.0	−16.2	−16.2	−14.0	−14.0	−13.8
0	0	1	0	0	27.212	8.0	7.5	7.4			
0	0	1	0	−1	27.104						−0.1
−1	0	1	2	1	14.698			−0.3			−0.2
1	0	1	0	2	13.747						0.1
1	0	1	0	1	13.719		−1.3	−1.3		−1.1	−1.1
1	0	1	0	0	13.691			0.6			
0	0	1	2	1	9.585						−0.1
0	0	3	0	3	9.107		−2.6	−2.6		−1.1	−1.1
0	0	3	0	2	9.095		−1.0	−1.0		−0.5	−0.5
1	0	3	0	3	6.845			−0.5			

3. Terms of nutation coming from the sectorial parts $C_{2,2}$ and $S_{2,2}$ of the geopotential

A, B, and C being the principal moments of inertia of the Earth, the dimensionless coefficients $C_{2,2}$ and $S_{2,2}$ of the geopotential are characterzing the dynamical triaxiality of the Earth, by the way of the following equation:

$$\frac{B-A}{4Ma_{\rm E}^2} = \sqrt{\frac{5}{12}} \times \sqrt{\bar{C}_{2,2}^2 + \bar{S}_{2,2}^2} \quad (7)$$

Although the Earth is relatively close to an axisymetric body, with: $(B-A)/(C-A) \approx 0.0067$, the influence of the difference $(B-A)$ on the nutation of the Earth is not negligible. It was calculated firstly by Kinoshita (1977) for the 3 biggest terms and by Kinoshita & Souchay (1990), up to 0.005 milliarcsecond (mas). The expression of the luni-solar potential related to the triaxiality can be expressed as follows (Kinoshita 1977):

$$U_{\rm Tr.} = \left(\frac{\kappa^2 M_{\rm M,S}}{r_{\rm M,S}^3}\right)\left(\frac{A-B}{4}\right) P_2^2(\sin\delta_{\rm M,S})\cos 2\alpha_{\rm M,S} \quad (8)$$

where $r_{\rm M,S}$, $\delta_{\rm M,S}$ and $\alpha_{\rm M,S}$ are the geocentric distance, latitude and longitude of the perturbing body (Moon or Sun).

$P_2^2(\sin\delta_{\rm M,S})\cos 2\alpha_{\rm M,S}$ is given by the following formula (Kinoshita 1977):

$$\begin{aligned}P_2^2(\sin\delta_{\rm M,S})\cos 2\alpha_{\rm M,S} = \sum_{\rho=\pm1} \frac{1}{4}(1+\rho\cos J)^2 \\ \times\Big[-3\sin^2 I P_2^0(\sin\beta_{\rm M,S}) \times \cos(2l+2\rho g) \\ - \sum_{\varepsilon=\pm1} \varepsilon \sin I(1+\varepsilon\cos I) \times P_2^1(\sin\beta_{\rm M,S}) \\ \times \sin(\lambda_{\rm M,S} - h - 2\rho\varepsilon l - 2\varepsilon g) \\ + \sum_{\varepsilon=\pm1} \frac{1}{4}(1+\varepsilon\cos I)^2 \times P_2^2(\sin\beta_{\rm M,S}) \\ \times \cos 2(\lambda_{\rm M,S} - h - \rho\varepsilon l - \varepsilon g)\Big] + O(\sin J) \quad (9)\end{aligned}$$

h, l and g are the canonical variables in Kinoshita (1977) theory. Three planes are involved in order to define these variables. h is defined along the ecliptic of the date from the mean equinox of the date to the intersection with the plane perpendicular to the angular-momentum vector. g is the angle from that intersection along the angular momentum plane and its intersection with the equator. And l is from that last intersection to the prime meridian. Thus the sum $(l+g)$ is equivalent to the sidereal angle of rotation ϕ.

The P_2^i polynomials have the following expressions:

$$P_2^0(\sin\beta_{\rm M,S}) = \frac{1}{2}(1-\sin^2\beta_{\rm M,S}) \quad (10.1)$$
$$P_2^1(\sin\beta_{\rm M,S}) = 3\sin\beta_{\rm M,S}\cos\beta_{\rm M,S} \quad (10.2)$$
$$P_2^2(\sin\beta_{\rm M,S}) = 3\cos^2\beta_{\rm M,S} \quad (10.3)$$

Notice that for the Sun, we can adopt the approximations: $\sin\beta_{\rm S} \approx 0$ and: $\cos\beta_{\rm S} \approx 1$. The coefficients of the nutation due

Table 2. Coefficients of rigid Earth nutation coming from the Earth triaxiality. Comparison of new results (this paper) is made with results of Kinoshita & Souchay (1990), both in longitude and in obliquity

ϕ	l_M	l_S	F	D	Ω	Period(d) (day)	$\Delta\psi_{KS}$ (sin) (μas)	$\Delta\psi_{new}$ (sin) (μas)	$\Delta\varepsilon_{KS}$ (cos) (μas)	$\Delta\varepsilon_{new}$ (cos) (μas)	Contr.
2	−2	0	−2	0	−2	0.538		0.7		−0.3	Moon
2	0	0	−2	−2	−2	0.536		0.8		−0.3	Moon
2	0	0	−3	0	−3	0.529	5.0	5.0			Moon
2	−1	0	−2	0	−2	0.527	5.0	5.2		−2.1	Moon
2	−1	0	−2	0	−1	0.527		1.0		−0.4	Moon
2	1	0	−2	−2	−2	0.526		0.7		−0.4	Moon
2	0	0	−2	0	−2	0.518	26.0	27.1	−10.0	−11.0	Moon
2	0	0	−2	0	−1	0.518	5.0	5.0		−2.0	Moon
2	0	0	−2	0	0	0.518		−0.2			Moon
2	−2	0	0	0	0	0.517		−0.2			Moon
2	0	0	0	−2	0	0.516		−0.3		0.1	Moon
2	1	0	−2	0	−2	0.508		−0.7		0.3	Moon
2	−1	0	0	0	0	0.508		−2.0		0.8	Moon
2	−1	0	0	0	−1	0.508		−0.4		0.2	Moon
2	−1	0	0	0	1	0.508		−0.1			Moon
2	1	0	0	−2	0	0.507		−0.4		0.2	Moon
2	0	−1	−2	2	−2	0.503		0.7		−0.3	Sun
2	0	0	−2	2	−2	0.500	12.0	12.5	−5.0	−4.7	Sun
2	0	0	0	0	0	0.499	−36.0	−37.8	14.0	15.0	Moon/Sun
2	0	1	0	0	0	0.499		−0.3		0.1	Sun
2	0	0	0	0	1	0.499		−0.7		0.3	Moon
2	0	0	0	0	−1	0.499	5.0	−4.9		1.9	Moon
2	0	−1	0	0	0	0.498		−0.3		0.1	Sun
2	0	0	2	−2	2	0.497	−5.0	−0.5		0.2	Sun
2	−1	0	0	2	0	0.491		−0.4		0.2	Moon
2	1	0	0	0	1	0.490		−2.0		0.8	Moon
2	−2	0	−2	0	−2	0.490		−0.1			Moon
2	0	0	0	2	0	0.482		−0.3		0.1	Moon
2	2	0	0	0	0	0.481		−0.2			Moon
2	0	0	2	0	0	0.481		−0.1			Moon
2	0	0	2	0	1	0.481		0.7		−0.3	Moon
2	0	0	2	0	2	0.481		−1.1		0.4	Moon
2	1	0	2	0	2	0.473		−0.2		0.1	Moon
2	1	0	2	0	1	0.473		0.1		−0.1	Moon

to the triaxiality can be calculated starting from the determining function, by integrating the potential as given by (9):

$$W_{Tr.} = \int U_{Tr.} \times dt \tag{11}$$

Then, by the following partial derivatives:

$$\Delta\varepsilon_{Tr} = -\Delta I = -\frac{1}{G\sin I} \times \left[\frac{\partial W_{Tr}}{\partial h} + \cos I \frac{\partial W_{Tr}}{\partial g}\right] \tag{12.1}$$

$$\Delta\psi_{Tr} = -\Delta h = \frac{1}{G\sin I} \times \frac{\partial W_{Tr}}{\partial I} \tag{12.2}$$

The constant terms k^M_{Tr} and k^S_{Tr}, which serve as a scaling factor respectively for the action of the Moon and of the Sun, are given by the following relationships:

$$k^M_{Tr} = k_M \times \frac{(B-A)}{(2C-A-B)} \tag{13.1}$$

$$k^S_{Tr} = k_S \times \frac{(B-A)}{(2C-A-B)} \tag{13.2}$$

where the value of the constant term k_M has been given in Sect. 2, when calculating the terms of the nutation due to J_3, and where in a similar manner an accurate value of k_S has been recently calculated with an updated value of the general precession in longitude by Souchay & Kinoshita (1996), that is: $k_S = 3475.1883295''/J.cy$. We choose for the ratio $R = (B-A)/(2C-A-B)$ the same value as in (Hartmann et al. 1995) which is significantly different from the value in Kinoshita & Souchay (1990), that is to say: $R = 0.0033536$ instead of $R = 0.003272$.

The results of the computations are listed in Table 2. Notice that all the terms have a quasi semi-diurnal period, because of the presence of 2ϕ in their argument. We have 34 coefficients up to 0.1 μas instead of the 7 coefficients up to 5 μas found by

Table 3.1. Coefficients of rigid Earth nutation due to the direct action of Venus, longitude part. Comparison of new results (this paper) is made with results of Kinoshita and Souchay (1990) and Williams (1995)

$\lambda_{\rm Ve}$	$\lambda_{\rm Ea}$	$p_{\rm A}$	Period(d)	KS (1990) sin (μas)	KS (1990) cos (μas)	Williams (1995) sin (μas)	Williams (1995) cos (μas)	This paper sin (μas)	This paper cos (μas)
8	−13	−2	88914.033			6.1	21.7	4.9	20.6
8	−13	−1	88082.037			4.1	−2.2	4.0	−1.8
3	−5	0	−2959.207			7.2	−2.7	7.2	−2.7
3	−5	−1	−2958.281	−10.0	−45.0	−10.3	−43.1	−10.2	−43.0
3	−5	−2	−2957.350	220.0	0.0	215.0	0.1	215.0	0.1
5	−8	−2	2863.895	0.0	−26.0	1.1	−26.8	1.1	−26.8
5	−8	−1	2863.023			−5.7	1.1	−5.7	1.1
5	−8	0	2862.152			−0.3	−0.9	−0.3	−0.8
6	−10	−1	−1479.371			0.2	0.5	0.2	−0.5
6	−10	−2	−1479.142			−2.4	0.5	−2.4	0.5
2	−3	−2	1455.382			−0.5	−1.1	−0.5	−1.1
2	−3	−1	1455.160			−1.8	−0.3	−1.8	0.3
2	−3	0	1454.939	0.0	14.0	−0.2	13.7	0.2	13.8
2	−3	1	1454.708			1.3	−0.3	1.3	−0.3
1	−2	1	−975.480			−0.9	0.2	−1.0	0.2
1	−2	0	−975.376	0.0	−9.0	0.1	−8.7	0.1	8.7
1	−2	−1	−975.277			0.7	0.2	0.7	0.2
7	−11	−2	964.793			1.6	−0.3	1.6	−0.3
4	−7	−1	−733.527			1.5	−0.3	1.5	−0.3
4	−7	−2	−733.468	0.0	6.0	−0.3	6.6	−0.3	6.6
4	−6	−2	727.580	−52.0	0.0	−50.3	0.0	−50.4	0.0
4	−6	−1	727.525	0.0	9.0	2.2	9.3	2.2	9.3
4	−6	0	727.467			−1.6	0.6	−1.6	0.6
1	−1	−1	583.957	0.0	9.0	−2.2	9.5	−2.2	9.5
1	−1	0	583.921	87.0	0.0	84.6	0.0	84.6	0.0
1	−1	1	583.886	0.0	−6.0	−1.5	−6.4	−1.5	6.4
2	−4	0	−487.689			1.3	−0.5	1.3	−0.5
2	−4	−1	−487.663	0.0	−8.0	−1.9	−7.9	−1.9	−7.9
2	−4	−2	−487.638	36.0	0.0	34.9	0.0	34.9	0.0
6	−9	−2	485.029			0.2	−4.5	0.2	−4.5
6	−9	−1	485.003			−0.9	0.2	−0.9	0.2
5	−9	−2	−418.651			−0.7	0.2	−0.7	0.2
3	−4	−2	416.725			−0.1	−0.5	0.0	−0.5
3	−4	−1	416.707			−0.7	−0.1	−0.7	−0.1
3	−4	0	416.689			−0.1	3.9	−0.1	3.9
0	1	0	365.256			0.0	−2.7	0.0	−2.7
8	−12	−2	363.762			0.6	−0.1	0.6	−0.1
3	−6	−1	325.115			0.7	−0.1	0.7	−0.1
3	−6	−2	325.103			−0.1	2.6	−0.1	2.6
5	−7	−2	323.941	−21.0	0.0	−20.4	0.0	−20.5	0.0
5	−7	−1	323.930			0.9	3.5	0.8	3.5
5	−7	0	323.919			−0.6	0.2	−0.6	0.2
2	−2	−1	291.970	0.0	5.0	−1.2	5.1	−1.2	5.1
2	−2	0	291.961	36.0	0.0	35.0	0.0	35.0	0.0
2	−2	1	291.952			−0.6	−2.5	−0.6	−2.5
1	−3	0	−265.742			0.8	−0.3	0.8	−0.3
1	−3	−1	−265.734	0.0	−5.0	−1.1	−4.6	−1.1	−4.6
1	−3	−2	−265.727	17.0	0.0	17.0	0.0	16.9	0.0
7	−10	−2	264.950			0.1	−2.4	0.0	−2.4
4	−5	−1	243.171					−0.5	0.0
4	−5	0	243.165			0.0	2.1	0.0	2.1
1	0	0	224.700			0.0	−1.2	0.0	−1.2
2	−5	−2	−208.833			−0.1	1.4	−0.1	1.4
6	−8	−2	208.354	−12.0	0.0	−11.6	0.0	−11.7	0.0

Table 3.1. (continued)

				KS (1990)		Williams (1995)		This paper	
λ_{Ve}	λ_{Ea}	p_A	Period(d)	sin (μas)	cos (μas)	sin (μas)	cos (μas)	sin (μas)	cos (μas)
6	−8	−1	208.354			0.5	1.9	0.5	1.9
3	−3	−1	194.644			−0.8	3.4	−0.8	3.4
3	−3	0	194.641	19.0	0.0	18.7	0.0	18.7	0.0
3	−3	1	194.636			−0.3	−1.3	−0.3	−1.3
0	2	0	182.627			−0.5	−0.2	−0.5	−0.2
0	2	1	182.625			0.8	−3.2	0.7	−3.2
0	2	2	182.622	−9.0	0.0	−8.7	0.0	−8.6	0.0
8	−11	−2	182.253			0.1	−1.5	0.0	−1.5
5	−6	0	171.674			0.0	1.3	0.0	1.3
2	−1	0	162.261			0.0	−0.6	0.0	−0.6
1	−4	−2	153.821			0.0	0.8	0.0	0.8
7	−9	−2	153.561	−7.0	0.0	−7.3	0.0	−7.4	0.0
7	−9	−1	153.558			0.3	1.2	0.3	1.2
4	−4	−1	145.983			−0.6	2.4	−0.6	2.4
4	−4	0	145.981	11.0	0.0	11.0	0.0	11.0	0.0
4	−4	1	145.978			−0.2	−0.7	−0.2	−0.7
1	1	1	139.116			0.5	−2.3	0.5	−2.3
1	1	2	139.114			−3.3	0.0	−3.4	0.0
9	−12	−2	138.900			0.0	−1.0	0.0	−1.0
6	−7	0	132.669			0.0	0.9	0.0	0.9
8	−10	−2	121.586	−5.0	0.0	−5.0	0.0	−5.0	0.0
8	−10	−1	121.584			0.2	0.7	0.2	0.7
5	−5	−1	116.786			−0.4	1.7	−0.4	1.7
5	−5	0	116.785	7.0	0.0	6.8	0.0	6.8	0.0
2	0	0	112.350			−0.3	0.0	−0.3	0.0
2	0	1	112.349			0.4	−1.5	0.4	−1.5
2	0	2	112.347			−1.6	0.0	−1.6	0.0
10	−13	−2	112.210			0.0	−0.8	0.0	−0.8
7	−8	0	108.107			0.0	0.6	0.0	0.6
9	−11	−2	100.632			−3.4	0.0	−3.4	0.0
9	−11	−1	100.631			0.1	0.5	0.1	0.5
6	−6	−1	97.321			−0.3	1.3	−0.3	1.3
6	−6	0	97.320			4.4	0.0	4.4	0.0
3	−1	1	94.221			0.2	−1.0	0.0	−1.0
3	−1	2	94.220			−0.9	0.0	−0.9	0.0
11	−14	−2	94.122			0.0	−0.6	0.0	−0.6
10	−12	−2	85.838			−2.4	0.0	−2.4	0.0
7	−7	−1	83.418			−0.2	0.9	−0.2	0.9
7	−7	0	83.417			2.8	0.0	2.8	0.0
4	−2	1	81.130			0.2	−0.7	0.0	−0.7
4	−2	2	81.129					−0.5	0.0
11	−13	−2	74.837			−1.7	0.0	−1.7	0.0
8	−8	−1	72.991			−0.2	0.7	−0.2	0.7
8	−8	0	72.990			1.9	0.0	1.9	0.0
5	−3	1	71.233					0.0	−0.5
12	−14	−2	66.335			−1.2	0.0	−1.2	0.0
9	−9	−1	64.881			−0.1	0.5	0.0	0.5
9	−9	0	64.880			1.2	0.0	1.3	0.0
13	−15	−2	59.568			−0.9	0.0	−0.9	0.0
10	−10	0	58.392			0.8	0.0	0.8	0.0
14	−16	−2	54.054			−0.6	0.0	−0.6	0.0
11	−11	0	53.084			0.6	0.0	0.6	0.0
15	−17	−2	49.474					−0.5	0.0

Table 3.2. Coefficients of rigid Earth nutation due to the direct action of Venus, obliquity part. Comparison of new results (this paper) with results of Kinoshita & Souchay (1990) and Williams (1995)

				KS (1990)		Williams (1995)		This paper	
λ_{Ve}	λ_{Ea}	p_A	Period(d)	sin	cos	sin (μas)	cos (μas)	sin (μas)	cos (μas)
8	−13	−2	88914.033			−9.4	2.6	−9.0	2.1
8	−13	−1	88082.037			1.2	2.2	1.0	2.1
3	−5	1	−2960.139			−0.2	0.1	−0.2	0.1
3	−5	−1	−2958.281	23.0	−6.0	23.0	−5.5	23.0	-5.5
3	−5	−2	−2957.350	12.0	96.0	−0.1	93.2	0.0	93.2
5	−8	−2	2863.895	13.0	0.0	11.6	0.5	11.6	0.5
5	−8	−1	2863.023			−0.6	−3.0	−0.6	−3.0
6	−10	−1	−1479.371			−0.3	0.1	−0.3	0.1
6	−10	−2	−1479.142			−0.2	−1.1	−0.2	−1.1
2	−3	−2	1455.382			0.5	−0.2	0.5	−0.2
2	−3	−1	1455.160			0.2	−1.0	0.2	−1.0
2	−3	1	1454.708			−0.2	−0.7	−0.2	−0.7
1	−2	1	−975.480			0.1	0.5	0.1	0.5
1	−2	−1	−975.277			0.0	0.4	0.0	0.4
7	−11	−2	964.793			0.1	0.7	0.1	0.7
4	−7	−1	−733.527			0.2	0.8	0.2	0.8
4	−7	−2	−733.468			−2.9	−0.1	−2.9	−0.1
4	−6	−2	727.580	0.0	−23.0	0.0	−21.9	0.0	−21.9
4	−6	−1	727.525	−5.0	1.0	−4.9	1.2	−4.9	1.2
1	−1	−2	583.994	0.0	9.0			0.0	0.1
1	−1	−1	583.957	−5.0	−1.0	−5.1	−1.2	−5.1	−1.2
1	−1	1	583.886	0.0	−6.0	−3.4	0.8	−3.4	0.8
2	−4	−1	−487.663	0.0	−8.0	4.2	−1.0	4.2	−1.0
2	−4	−2	−487.638	0.0	15.0	0.0	15.1	0.0	15.1
6	−9	−2	485.029			2.0	0.1	2.0	0.1
6	−9	−1	485.003			−0.1	−0.5	0.0	−0.5
5	−9	−2	−418.651			−0.1	−0.3	0.0	−0.3
3	−4	−2	416.725			0.2	−0.1	0.2	0.0
3	−4	−1	416.707			0.1	−0.4	0.0	−0.4
8	−12	−2	363.762			0.0	0.2	0.0	0.2
3	−6	−1	−325.115			0.0	0.4	0.0	0.4
3	−6	−2	−325.103			−1.1	0.0	−1.1	0.0
5	−7	−2	323.941	0.0	−9.0	0.0	−8.9	0.0	−8.9
5	−7	−1	323.930			−1.9	0.5	−1.9	0.5
2	−2	−1	291.970			−2.7	−0.6	−2.7	−0.6
2	−2	1	291.952			−1.3	0.3	−1.3	0.3
1	−3	−1	−265.734			2.4	−0.6	2.4	−0.6
1	−3	−2	−265.727	0.0	7.0	0.0	7.3	0.0	7.3
7	−10	−2	264.950			1.0	0.0	1.0	0.0
4	−5	−1	243.171			0.0	−0.2	0.0	−0.2
2	−5	−1	−208.837					0.0	0.2
2	−5	−2	−208.833			−0.6	0.0	−0.6	0.0
6	−8	−2	208.354	0.0	−5.0	0.0	−5.1	0.0	−5.1
6	−8	−1	208.348			−1.0	0.2	−1.0	0.2
3	−3	−1	194.644			−1.8	−0.4	−1.8	−0.4
3	−3	1	194.636			−0.7	0.2	−0.7	0.2
0	2	1	182.625			−1.7	−0.4	−1.7	−0.4
0	2	2	182.622			0.0	3.7	0.0	3.7
8	−11	−2	182.253			0.7	0.0	0.7	0.0
5	−6	−1	171.678					0.0	−0.2
1	−4	−2	153.823					0.0	0.2
1	−4	−2	153.821			−0.3	0.0	−0.3	0.0
7	−9	−2	153.561			0.0	−3.2	0.0	−3.2

Table 3.2. (continued)

				KS (1990)		Williams (1995)		This paper	
λ_{Ve}	λ_{Ea}	p_{A}	Period(d)	sin	cos	sin (μas)	cos (μas)	sin (μas)	cos (μas)
7	−9	−1	153.558			−0.6	0.2	−0.6	0.2
4	−4	−1	145.983			−1.3	−0.3	−1.3	−0.3
4	−4	1	145.978			−0.4	0.0	−0.4	0.0
1	1	1	139.116			−1.2	−0.3	−1.2	−0.3
1	1	2	139.114			0.0	1.5	0.0	1.5
9	−12	−2	138.900			0.5	0.0	0.5	0.0
0	3	2	121.750					0.2	0.0
8	−10	−2	121.586			0.0	−2.2	0.0	−2.2
8	−10	−1	121.584			−0.4	0.0	−0.4	0.0
5	−5	−1	116.786			−0.9	−0.2	−0.9	−0.2
5	−5	1	116.783					−0.2	0.0
2	0	1	112.349			−0.8	0.2	−0.8	0.2
2	0	2	112.347			0.0	0.7	0.0	0.7
10	−13	−2	112.209			0.3	0.0	0.3	0.0
9	−11	−2	100.632			0.0	−1.5	0.0	−1.5
9	−11	−1	100.631			−0.3	0.0	−0.3	0.0
6	−6	−1	97.321			−0.7	−0.2	−0.7	−0.2
6	−6	1	97.319					−0.2	0.0
3	−1	1	94.221			−0.5	−0.1	−0.6	−0.1
3	−1	2	94.220			0.0	0.4	0.0	0.4
11	−14	−2	94.122			0.2	0.0	0.2	0.0
10	−12	−2	85.838			0.0	−1.0	0.0	−1.0
10	−12	−1	85.838					−0.2	0.0
7	−7	−1	83.418			−0.5	−0.1	−0.5	−0.1
4	−2	1	81.130			−0.4	0.0	−0.4	0.0
4	−2	2	81.129					0.0	0.2
12	−15	−2	81.056					0.2	0.0
11	−13	−2	74.837			0.0	−0.7	0.0	−0.7
11	−13	−1	74.837					−0.1	0.0
8	−8	−1	72.991			−0.4	0.0	−0.4	0.0
5	−3	1	71.233					−0.3	0.0
12	−14	−2	66.335			0.0	−0.5	0.0	−0.5
9	−9	−1	64.881			−0.3	−0.1	−0.3	0.0
6	−4	1	63.488					−0.2	0.0
13	−15	−2	59.568			0.0	−0.4	0.0	−0.4
10	−10	−1	58.392					−0.2	0.0
14	−16	−2	54.054			0.0	−0.3	0.0	−0.3
15	−17	−2	49.474			0.0	−0.2	0.0	−0.2
12	−12	−1	48.660			−0.1	0.0	−0.1	0.0
16	−18	−2	45.610			0.0	−0.1	0.0	−0.1
17	−19	−2	42.305			0.0	−0.1	0.0	−0.1

Kinoshita & Souchay (1990). A sign error has been detected in this last work, for the term of amplitude $-4.9\ \mu$as and argument $2\phi - \Omega$. Moreover, a big correction concerns the amplitude of the term of argument $2\phi + 2F - 2D + 2\Omega$ which is $-0.5\ \mu$as instead of $-5\ \mu$as. Except this term all the other ones have been confirmed.

4. Terms of nutation coming from the J_4 geopotential

As it can be expected the influence of the coefficient J_4 of the geopotential on the nutation is very small. Kinoshita (1977) made a brief study which showed that this influence was negligible in regards of the accuracy of the precession-nutation observations, and anyway far under his level of truncation for his series of nutation, that is to say 0.1 milliarcsecond. Kinoshita & Souchay (1990) calculated only the influence of J_4 on the precession, and found the value: $\dot{\psi}_{\mathrm{J4}} = -0.00257''/cy$ which is in agreement with Kinoshita's value: $\dot{\psi}_{\mathrm{J4}} = -0.0025''/cy$. This value is taken into account for when studying the connection between the scaling factors k_{M} and k_{S}, and the value of the general precession in longitude (Souchay & Kinoshita 1996).

Table 4.1. Coefficients of rigid Earth nutation due to the direct action of Mars, longitude part. Comparison of new results (this paper) is made with results of Kinoshita & Souchay (1990) and Williams (1995)

				KS (1990)		Williams (1995)		This paper	
$\lambda_{\rm Ea}$	$\lambda_{\rm Ma}$	$p_{\rm A}$	Period(d)	sin	cos	sin (μas)	cos (μas)	sin (μas)	cos (μas)
8	−15	−2	14812.425			1.2	0.9	1.1	0.8
1	−2	1	−5767.541			−0.8	−0.3	−0.8	−0.3
1	−2	0	−5764.007	−2.0	−3.0	−8.9	5.0	−8.8	5.0
1	−2	−1	−5760.476			−0.2	1.3	−0.2	1.3
7	−13	−2	4149.355			0.4	0.8	0.4	0.8
2	−4	0	−2882.003			−1.0	1.6	−1.0	1.6
2	−4	−1	−2881.120			0.6	0.9	0.6	0.9
2	−4	−2	−2880.238	5.0	0.0	5.2	0.1	5.1	0.1
6	−11	−2	2412.593			−0.1	1.1	−0.1	1.1
3	−6	−1	−1920.945			0.5	0.2	0.5	0.2
3	−6	−2	−1920.559			2.4	−1.3	2.4	−1.3
5	−9	−2	1700.732			−0.8	1.2	−0.8	1.2
4	−8	−2	−1440.564			0.6	−1.0	0.6	−1.0
4	−7	−2	1313.244			−1.6	0.9	−1.6	0.9
5	−10	−2	−1152.521					0.0	−0.5
3	−5	−2	1069.560			−1.7	0.0	−1.7	0.0
3	−5	0	1069.316			0.4	−0.6	0.4	−0.6
2	−3	0	901.987			1.4	−0.8	1.4	−0.8
1	−1	0	779.937			3.2	0.0	3.1	0.0
0	1	0	686.980			0.9	0.5	0.9	0.5
1	−3	−2	−613.742			1.1	0.0	1.1	0.0
2	−5	−2	−554.680			0.6	−0.4	0.6	−0.3
5	−8	−2	489.326			−0.5	0.3	−0.5	0.3
4	−6	−2	451.036			−0.6	0.0	−0.6	0.0
3	−4	0	418.266			0.6	−0.3	0.6	−0.3
2	−2	0	389.968			1.2	0.0	1.2	0.0
0	2	2	343.465					−0.5	0.0
3	−3	0	259.979			0.6	0.0	0.6	0.0

Hartmann et al. (1995) showed that the biggest term of nutation due to J_4 concerns the leading component at the first-order, that is to say the Ω component. Contrary to the general case, the amplitude in obliquity: $\Delta\varepsilon_{\rm J4} = 6.8(\mu{\rm as})\cos\Omega$ is relatively much larger than the amplitude in longitude: $\Delta\psi_{\rm J4} = -0.7(\mu{\rm as})\sin\Omega$.

In the following we propose to calculate the values of the nutation due to the J_4 geopotential, by the same way as precedently, that is to say starting from the theory derived from Hamiltonian formulation (Kinoshita 1977; Kinoshita & Souchay 1990). We choose the same value as Hartmann et al. (1995), that is to say: $J_4 = -1.61610^{-6}$.

The solar part of the potential related to J_4 being completely negligible, only the lunar part is considered here, which can be written classically as follows:

$$U_{\rm J4} = \left(\frac{\kappa^2 M_{\rm M} M_{\rm E} a_{\rm E}^4}{r_{\rm M}^5}\right) J_4 P_4^0(\sin\delta_{\rm M}) \tag{14}$$

Rigorously, $P_4^0(\sin\delta_{\rm M})$ can be expressed in function of the modified Jacobi polynomials (Kinoshita et al. 1974):

$$P_4^0(\sin\delta_{\rm M}) = \sum_{\varepsilon=\pm1}\sum_{\rho=\pm1}\sum_{m',m''=0}^{4} \bar{Q}_4^{(0,m')}(\rho,\cos J)\bar{Q}_4^{(m',m'')}(\varepsilon,\cos I) \times P_4^{m''}(\sin\beta_{\rm M}) \times \cos\left[m''\rho\varepsilon(\lambda_{\rm M}-h) - ml - m'(\rho g + \frac{\pi}{2}) + \frac{\pi}{2}\rho(m'-m'')\right] \tag{15}$$

The modified Jacobi polynomials $\bar{Q}_{\rm n}^{(p,q)}(\tau,\cos\alpha)$ are given by:

$$\bar{Q}_{\rm n}^{(p,q)}(\tau,\cos\alpha) = \tau^{n-p} Q_{\rm n}^{(p,q)}(\tau\cos\alpha)\,. \tag{16}$$

Thus, we have:

$$\bar{Q}_4^{(0,m')}(\rho,\cos J) = Q_4^{(0,m')}(\rho\cos J) \tag{17.1}$$

$$\bar{Q}_4^{(m',m'')}(\varepsilon,\cos I) = \varepsilon^{4-m'} Q_4^{(m',m'')}(\varepsilon\cos I) \tag{17.2}$$

For $m'\neq0$, we can neglect the polynomials $Q_4^{(0,m')}(\rho,\cos J)$ because they have $\sin J$ as a factor in their developement, and $\sin J$ is very small, its value being of the order of a few 10^{-6} (J represents the angle between the axis of figure and the axis of

Table 4.2. Coefficients of rigid Earth nutation due to the direct action of Mars, obliquity part. Comparison of new results (this paper) is made with results of Williams (1990)

				Williams (1995)		This paper	
λ_{Ea}	λ_{Ma}	p_{A}	Period(d)	sin (μas)	cos (μas)	sin (μas)	cos (μas)
8	−15	−2	14812.425	−0.4	0.5	−0.3	0.5
9	−17	−2	9435.783			0.0	−0.2
1	−2	1	−5767.541	−0.2	0.4	−0.2	0.4
1	−2	−1	−5760.476	−0.7	−0.1	−0.7	0.0
7	−13	−2	4149.355	−0.4	0.2	−0.3	0.2
2	−4	−1	−2881.120	−0.5	0.3	−0.5	0.3
2	−4	−2	−2880.238	0.0	2.2	0.0	2.2
6	−11	−2	2412.593	−0.5	0.0	−0.5	0.0
3	−6	−1	−1920.945	−0.1	0.3	−0.1	0.3
3	−6	−2	−1920.559	0.6	1.0	0.6	1.0
5	−9	−2	1700.732	−0.5	−0.3	−0.5	−0.3
4	−8	−2	−1440.564	0.4	0.3	0.4	0.3
4	−7	−2	1313.244	−0.4	−0.7	−0.4	−0.7
4	−7	−1	1313.064			0.0	−0.2
5	−10	−2	−1152.521			0.2	0.0
3	−5	−2	1069.560	0.0	−0.7	0.0	−0.7
3	−5	−1	1069.440			0.2	−0.1
1	−3	−2	−613.742	0.0	0.5	0.0	0.5
2	−5	−2	−554.680	0.2	0.3	0.2	0.3
6	−10	−2	534.720			−0.2	−0.1
5	−8	−2	489.326	−0.1	−0.2	−0.1	−0.2
4	−6	−2	451.036	0.0	−0.2	0.0	−0.2
0	2	2	343.465			0.0	0.2

angular momentum). Thus the Legendre polynomial $P_4^0(\sin\delta_{\mathrm{M}})$ can be restricted to the following expression:

$$P_4^0(\sin\delta_{\mathrm{M}}) \approx \sum_{\varepsilon=\pm1}\sum_{\rho=\pm1}\sum_{m=0}^{4} Q_4^{(0,0)}(\cos J) Q_4^{(0,m)}(\varepsilon\cos I) P_4^m(\sin\beta_{\mathrm{M}}) \times \cos\left[m\rho\varepsilon\cos(\lambda_{\mathrm{M}}-h) - m\rho\frac{\pi}{2}\right] \tag{18}$$

where the Legendre polynomials $P_4^m(\sin\beta_{\mathrm{M}})$ have the following expressions:

$$P_4^0(\sin\beta_{\mathrm{M}}) = \frac{1}{8}\times(3-30\sin^2\beta_{\mathrm{M}}+35\sin^4\beta_{\mathrm{M}}) \tag{19.1}$$

$$P_4^1(\sin\beta_{\mathrm{M}}) = \frac{5}{2}\times\sin\beta_{\mathrm{M}}\cos\beta_{\mathrm{M}}(-3+7\sin^2\beta_{\mathrm{M}}) \tag{19.2}$$

$$P_4^2(\sin\beta_{\mathrm{M}}) = \frac{15}{2}\times\cos^2\beta_{\mathrm{M}}(-1+7\sin^2\beta_{\mathrm{M}}) \tag{19.3}$$

$$P_4^3(\sin\beta_{\mathrm{M}}) = 105\times\cos^3\beta_{\mathrm{M}}\sin\beta_{\mathrm{M}} \tag{19.4}$$

$$P_4^4(\sin\beta_{\mathrm{M}}) = 105\times\cos^4\beta_{\mathrm{M}} \tag{19.5}$$

There we have neglected the terms for which: $l \neq 0$ in Eq. (15), because their amplitude after integration becomes very small, the canonical variable l having a quasi-diurnal period.

By the means of (18) and (19.1–5) we finally get:

$$\begin{aligned} P_4^0(\sin\delta_{\mathrm{M}}) = {} & \frac{1}{2}(3-30\cos^2 I+35\cos^4 I)(3-30\sin^2\beta_{\mathrm{M}}+35\sin^4\beta_{\mathrm{M}}) \\ & +\frac{5}{4}\sin I\cos I(-3+7\cos^2 I)\sin\beta_{\mathrm{M}}\cos\beta_{\mathrm{M}} \\ & \times(-3+7\sin^2\beta_{\mathrm{M}})\times\sin(\lambda_{\mathrm{M}}-h) \\ & +\frac{15}{24}\sin^2 I(-1+7\cos^2 I)\cos^2\beta_{\mathrm{M}} \\ & \times(-1+7\sin^2\beta_{\mathrm{M}})\times\cos 2(\lambda_{\mathrm{M}}-h) \\ & +\frac{105}{12}\sin^3 I\cos I\cos^3\beta_{\mathrm{M}}\sin\beta_{\mathrm{M}}\times\sin 3(\lambda_{\mathrm{M}}-h) \\ & +\frac{105}{96}\sin^4 I\cos^4\beta_{\mathrm{M}}\times\cos 4(\lambda_{\mathrm{M}}-h) \end{aligned} \tag{20}$$

Then the determining function W_{J4} related to J_4 can be written:

$$W_{\mathrm{J4}} = \int U_{\mathrm{J4}}\times dt \tag{21}$$

And the coefficients of the nutation are given by:

$$\Delta\varepsilon_{\mathrm{J4}} = -\Delta I = -\frac{1}{G\sin I}\times\frac{\partial W_{\mathrm{J4}}}{\partial h} \tag{22.1}$$

$$\Delta\psi_{\mathrm{J4}} = -\Delta h = \frac{1}{G\sin I}\times\frac{\partial W_{\mathrm{J4}}}{\partial I} \tag{22.2}$$

$$\begin{aligned} \Delta\psi_{\mathrm{J4}} = {} & k_{\mathrm{J4}}\left(\frac{15}{16}\cos I-\frac{35}{16}\cos^3 I\right) \\ & \times\int\left(\frac{a_{\mathrm{M}}}{r_{\mathrm{M}}}\right)^5(3-30\sin^2\beta_{\mathrm{M}}+35\sin^4\beta_{\mathrm{M}})dt \\ & +k_{\mathrm{J4}}\Big(\frac{\cos 2I}{\sin I}\times\left(\frac{15}{8}-\frac{35}{8}\cos^2 I\right)+\frac{35}{4}\sin I\cos^2 I\Big)\times \\ & \times\int\left(\frac{a_{\mathrm{M}}}{r_{\mathrm{M}}}\right)^5(-3+7\sin^2\beta_{\mathrm{M}})\cos\beta_{\mathrm{M}}\sin\beta_{\mathrm{M}}\sin(\lambda_{\mathrm{M}}-h)dt \\ & +k_{\mathrm{J4}}\frac{\cos I}{8}\big(5-35\cos 2I\big) \\ & \times\int\left(\frac{a_{\mathrm{M}}}{r_{\mathrm{M}}}\right)^5(-1+7\sin^2\beta_{\mathrm{M}})\cos^2\beta_{\mathrm{M}}\cos 2(\lambda_{\mathrm{M}}-h)dt \\ & +k_{\mathrm{J4}}\frac{105}{24}\big(-\sin^3 I+3\sin I\cos^2 I\big) \\ & \times\int\left(\frac{a_{\mathrm{M}}}{r_{\mathrm{M}}}\right)^5\cos^3\beta_{\mathrm{M}}\sin\beta_{\mathrm{M}}\sin 3(\lambda_{\mathrm{M}}-h)dt \\ & +k_{\mathrm{J4}}\frac{35}{16}\big(\sin^2 I\cos I\big) \\ & \times\int\left(\frac{a_{\mathrm{M}}}{r_{\mathrm{M}}}\right)^5\sin^4\beta_{\mathrm{M}}\cos 4(\lambda_{\mathrm{M}}-h)dt \end{aligned} \tag{23.1}$$

$$\Delta\varepsilon_{\mathrm{J4}} = k_{\mathrm{J4}}\frac{5}{4}\big(\cos I(-3+7\cos^2 I)\big)$$

$$\times \int (\frac{a_{\rm M}}{r_{\rm M}})^5(-3+7\sin^2\beta_{\rm M})\cos\beta_{\rm M}\sin\beta_{\rm M}\cos(\lambda_{\rm M}-h)dt$$
$$-k_{\rm J4}\frac{5}{4}\left(\sin I(-1+7\cos^2 I)\right)$$
$$\times \int (\frac{a_{\rm M}}{r_{\rm M}})^5(-1+7\sin^2\beta_{\rm M})\cos^2\beta_{\rm M}\sin 2(\lambda_{\rm M}-h)dt$$
$$-k_{\rm J4}\frac{105}{4}\left(\sin^2 I\cos I\right)$$
$$\times \int (\frac{a_{\rm M}}{r_{\rm M}})^5\cos^3\beta_{\rm M}\sin\beta_{\rm M}\cos 3(\lambda_{\rm M}-h)dt$$
$$+k_{\rm J4}\frac{35}{8}\sin^3 I\times\int (\frac{a_{\rm M}}{r_{\rm M}})^5\cos^4\beta_{\rm M}\sin 4(\lambda_{\rm M}-h)dt \quad (23.2)$$

By using the analytical developements in Fourier series of the coordinates $\lambda_{\rm M}$, $\beta_{\rm M}$ and $r_{\rm M}$, we find the following values for the nutation related to the J_4 geopotential, given in microarcseconds:

$$\Delta\psi_{\rm J4} = 0.73\sin\Omega - 0.61\sin 2\Omega + 0.13\sin(2F+2\Omega)$$
$$\Delta\varepsilon_{\rm J4} = 6.83\cos\Omega - 0.34\cos 2\Omega + 0.07\cos(2F+2\Omega)$$

They are quite in agreement with the values above calculated by Hartmann et al. (1995). Moreover, our determination of the precession rate related to J_4 is: $\dot\psi_{\rm J4} = -0.002515''/cy$, which is very close to the values of Kinoshita (1977) and Kinoshita & Souchay (1990), and exactly the same as the value found by (Hartmann et al. 1995), with a completely different way of computation.

5. Direct planetary effects on the nutation

Vondrak (1983) calculated the direct influence of the planets on the nutation, and showed that it could reach the 0.1 milliarcsecond level for individual coefficients. Kinoshita & Souchay (1990), by choosing an Hamiltonian formalism, as well as for the lunisolar case, and by using the analytical developments for the planetary parameters given in VSOP82 (Bretagnon 1982), found results very close to those of Vondrak after converting Vondrak's arguments to their own ones. Their count is 25, 2, 6 and 2 terms respectively for Venus, Mars, Jupiter and Saturn influence, down to 5 μas.

Recently, Williams (1995), calculated all the coefficients related to the direct influence of the planets, up to 0.5 μas, both for $\Delta\psi\cos\varepsilon$ and $\Delta\varepsilon$. At this level the influence of Mercury and Uranus has to be taken into account, and the count is 1, 103, 26, 22, 5 and 1 terms respectively for the Mercury, Venus, Mars, Jupiter, Saturn and Uranus contributions.

In the following we are calculating the direct planetary effects on the nutation starting from the same canonical equations as in Kinoshita & Souchay (1990), but by using recent Fourier developements of the rectangular coordinates of the planets, as they can be found in the ephemeris VSOP87 (Bretagnon & Francou, 1988). The geocentric coordinates X, Y, and Z with respect to the mean ecliptic and equinox of J2000.0 are then converted in the corresponding coordinates with respect to the mean ecliptic and equinox of the date, which are the conventional references to measure the nutation. Then the spherical coordinates λ, β and r of the perturbing planet can be expressed in a very straightforward and classical manner, with the trivial transformations:

$$r = (X^2+Y^2+Z^2)^{1/2} \quad (24.1)$$
$$\sin\beta = \frac{Z}{r} \quad (24.2)$$
$$\cos\beta\cos\lambda = \frac{X}{r} \quad (24.3)$$
$$\cos\beta\sin\lambda = \frac{Y}{r} \quad (24.4)$$

The perturbing function can be calculated easily from these last expressions. The big advantage of this procedure when compared with the procedure in Kinoshita & Souchay (1990) is that they started from the initial solution VSOP82, which means that they firstly had to calculate X, Y, Z starting from the Fourier series of a, λ, $k = e\cos\omega$, $h = e\sin\omega$, $q = \sin(i/2)\cos\Omega$ and $p = \sin(i/2)\sin\Omega$ (Bretagnon 1982).

The algorithm to calculate the coefficients of the nutation due to the direct action of the planets starting from Hamiltonian formalism is explained in detail in Kinoshita & Souchay (1990). We follow exactly the same procedure but the quality of our calculations with respect to these last work is improved by two facts: at first we keep all the terms in the intermediate Fourier series of our calculations with a relative 5×10^{-11} instead of 10^{-9}. At second, our level of truncation in the determination of the coefficients is 0.5 μas for $\Delta\psi\sin\varepsilon$ and $\Delta\varepsilon$ instead of 5 μas for $\Delta\psi$ and $\Delta\varepsilon$ as it was the case for Kinoshita & Souchay (1990).

Tables 3.1–3.2, 4.1–4.2, and 5.1–5.2 show our results concerning respectively the direct action of Venus, Mars and Jupiter, both on $\Delta\psi$ and $\Delta\varepsilon$, whereas tables 6.1–6.2 show the contribution due to the other planets (Mercury, Saturn and Uranus). Our coefficients are compared with Williams (1995) results. We can thus remark the quasi perfect agreement between them and the present calculations. The difference between the coefficients listed in the tables above never exceeds 0.1 μas, except for a few very long periodic terms, as we can observe for the two first coefficients of Table 3.1. We do not present here the comparison with the values found by Hartmann & Soffel (1994), but notice that they were also very closed to Williams'ones. Moreover we can remark that in the case of Jupiter (Tables 5.1 and 5.2) some coefficients were not present in Williams calculations, although their value is bigger than 0.5 μas.

6. The obliquity rate

The obliquity with respect to a fixed ecliptic (that we call here $\omega_{\rm A}$ with respect to the ecliptic of J2000.0) itself has a quasi linear term, which is a part of long-periodic terms. According to Williams (1995) the time derivative of the long periodic terms in $\omega_{\rm A}$ is called the obliquity rate. The quasi-linear term in the obliquity has two main origins: an indirect planetary origin and a direct planetary origin. Furthermore the indirect planetary origin

Table 5.1. Coefficients of rigid Earth nutation due to the direct action of Jupiter, longitude part. Comparison of new results (this paper) is made with results of Kinoshita & Souchay (1990) and Williams (1995)

					KS (1990)		Williams (1995)		This paper	
λ_{Ea}	λ_{Ju}	λ_{Sa}	p_A	Period(d)	sin (μas)	cos (μas)	sin (μas)	cos (μas)	sin (μas)	cos (μas)
0	2	−5	2	−346357.471					−1.2	0.0
0	2	−5	0	−322614.367			−2.4	−1.3	0.0	1.5
0	2	−5	−2	−301918.049					−1.1	−2.4
0	1	0	−1	4334.586			−0.3	−0.8	−0.3	−0.8
0	1	0	0	4332.589	34.0	−5.0	33.9	−5.2	33.4	−4.7
0	1	0	1	4330.594			0.0	0.6	0.0	0.5
0	1	0	2	4328.601			4.4	1.7	4.7	2.0
0	4	−5	2	2179.921			0.6	0.1	0.7	0.0
0	2	0	0	2166.299			1.2	−0.4	0.6	0.0
0	2	0	1	2165.788	0.0	5.0	0.9	4.9	0.9	4.9
0	2	0	2	2165.297	−106.0	0.0	−106.4	0.1	−106.2	0.0
0	0	5	2	2150.868			−0.6	−0.1	−0.6	0.0
0	3	0	1	1443.972			0.2	0.5	0.0	0.6
0	3	0	2	1443.754	−12.0	0.0	−12.0	2.2	−11.9	2.1
0	4	0	2	1082.899			−1.0	0.4	−0.9	0.3
1	−5	0	−2	631.495			−0.5	−0.2	−0.5	−0.2
1	−4	0	−2	551.161			−3.1	−0.6	−3.0	−0.6
1	−3	0	−2	488.958	−11.0	0.0	−11.3	0.0	−11.2	0.0
1	−3	0	−1	488.933			0.1	−0.5	0.0	−0.5
1	−2	0	−2	439.372			0.5	−0.1	0.5	0.0
1	−2	0	0	439.332			1.9	0.3	1.9	0.3
1	−1	0	0	398.884	12.0	0.0	11.7	0.0	11.6	0.0
1	0	0	0	365.256			0.6	−0.1	0.5	0.0
1	1	0	2	336.834			−1.6	0.0	−1.6	0.0
2	−5	0	−2	231.409			−0.6	−0.1	−0.5	0.0
2	−4	0	−2	219.676			−1.7	0.0	−1.7	0.0
2	−2	0	0	199.441			1.4	0.0	1.4	0.0

is divided into two parts. One is the motion of the mean ecliptic, which is caused by the mutual perturbations among planets and whose motion is expressed of combinations of long-periodic terms (the period is ranged from 50 thousand years to 2 million years). This slow ecliptic motion disturbs the motion of the Moon and then causes a very long periodic perturbation in obliquity. Other indirect origin is due to the fact that the Sun is not located in the mean ecliptic and moves up and down with respect to the mean ecliptic with short period. This motion also causes a small long periodic change in the obliquity. These two perturbations were neglected in Kinoshita & Souchay (1990). The direct planetary part originates from the fact that the planetary orbital planes are precessing due to the mutual planetary perturbations. This small effect was neglected in Kinoshita & Souchay (1990). Williams (1995) calls these perturbations tilt effects and also obtained the obliquity rate from another approach.

We recalculate the two effects above, starting from Hamiltonian equations and the recent ephemerides ELP2000 for the Moon (Chapront-Touzé & Chapront 1988) and VSOP87 (Bretagnon & Francou 1988). They are used to calculate the obliquity rate coming respectively from the first and second origins above.

Concerning the planetary tilt-effect, our results give: ω_A = 1041.7876 $\sin p_A$ + 121.6690 $\cos p_A$, and after replacing p_A by its polynomial developement in function of time (Lieske et al. 1977), this leads to the following rate: $\dot{\omega}_A = -0''.0254004/cy$, which is exactly in accordance with the value determined by Williams (1994), that is to say: $\dot{\omega}_A = -0''.0254/cy$.

The obliquity rate related to the direct planetary torque is characterized for each planet by expressions in the form $t \times \cos p_A$ and $t \times \cos 2p_A$, which can be assimilated to a polynomial expression in function of time, after a developement starting from J2000.0 (with a value of p_A set to $p_A = 0$ at this date).

In Table 7 we present our results of the obliquity rate due to the action of each planet. They are compared with the values of Williams (1994). The total amount is $\dot{\omega}_A = -0.0013478''/cy$ instead of $\dot{\omega}_A = -0.0014207''/cy$ for Williams (1994).

At last another contribution to the variation of ω_A has been identified in the present paper, following the output values of nutation (Souchay & Kinoshita 1996) when computing the effect of the solar potential. More precisely a term, with the following expression, in milliarsecond: $\Delta\omega_A = 11.8026 \sin p_A -$

Table 5.2. Coefficients of rigid Earth nutation due to the direct action of Jupiter, obliquity part. Comparison of new results (this paper) with results of Kinoshita & Souchay (1990) and Williams (1995)

					KS (1990)		Williams (1995)		This paper	
λ_{Ea}	λ_{Ju}	λ_{Sa}	p_A	Period(d)	sin (μas)	cos (μas)	sin (μas)	cos (μas)	sin (μas)	cos (μas)
0	2	−5	−2	−301918.049					1.1	−0.9
0	1	0	−1	4334.586					0.0	−0.2
0	0	2	2	5373.469			0.4	−0.1	0.4	−0.2
0	1	0	1	4330.594			0.3	0.0	0.3	0.0
0	1	0	2	4328.601			0.7	−1.9	0.9	−2.1
0	4	−5	2	2179.921			0.0	−0.3	0.0	−0.3
0	2	0	1	2165.788			2.6	−0.5	2.6	−0.5
0	2	0	2	2165.297	0.0	46.0	0.0	46.1	0.0	46.0
0	0	5	2	2150.868			0.0	0.3	0.0	0.3
0	3	0	1	1443.972			0.3	−0.1	0.3	−0.1
0	3	0	2	1443.754	0.0	5.0	1.0	5.2	0.9	5.2
0	4	0	2	1082.899			0.2	0.4	0.1	0.4
1	−5	0	−2	631.495			0.1	−0.2	0.0	−0.2
1	−4	0	−2	551.161			0.2	−1.3	0.2	−1.3
1	−3	0	−2	488.958			0.0	−4.9	0.0	−4.9
1	−3	0	−1	488.933			0.3	0.1	0.3	0.0
1	−2	0	−2	439.372			0.1	0.2	0.0	0.2
1	−1	0	−1	398.901					0.2	0.0
1	1	0	2	336.834			0.0	0.7	0.0	0.7
2	−5	0	−2	231.409			0.0	−0.2	0.0	−0.2
2	−4	0	−2	219.676			0.0	−0.7	0.0	−0.7

Table 6.1. Coefficients of rigid Earth nutation due to the planets Mercury, Saturn and Uranus, longitude part. Comparison of new results (this paper) is made with results of Kinoshita & Souchay (1990) and Williams (1995)

						KS (1990)		Williams (1995)		This paper	
λ_{Me}	λ_{Ea}	λ_{Sa}	λ_{Ur}	p_A	Period(d)	sin (μas)	cos (μas)	sin (μas)	cos (μas)	sin (μas)	cos (μas)
1	−4	0	0	−2	2402.790			0.4	1.0	0.4	1.0
1	−3	0	0	−2	317.059					−0.4	0.0
0	2	0	0	−2	182.622					−0.3	0.0
0	0	1	0	0	10759.232	−4.0	0.0	−0.2	−4.0	0.0	−3.9
0	0	1	0	2	10734.673			0.0	0.7	0.0	0.8
0	0	2	0	1	5376.541			0.4	0.9	0.4	0.9
0	0	2	0	2	5373.469	−12.0	0.0	−12.3	0.0	−12.3	0.0
0	0	3	0	2	3583.678			0.1	1.5	0.0	1.5
0	1	−1	0	0	378.092					0.3	0.0
0	0	0	2	2	15294.356			−0.7	0.0	−0.7	0.0

$1.0403\cos p_A$, can be converted in a secular variation of ω_A as above, that is to say: $\dot{\omega}_A = -0''.0002877/cy$.

The combination of the three components of $\dot{\omega}_A$ leads to the following total amount of: $\dot{\omega}_A = -0.0270359''/cy$. Williams (1994), by combinating the two first components, got: $\dot{\omega}_A = -0.0268''/cy$, and he added to this value the contribution of tidal torques amounting to: $0.0024''/cy$. Then his final estimation is: $\dot{\omega}_A = -0.0240''/cy$.

Notice that the long-periodic variation of the obliquity has been recently confirmed by VLBI analysis extending for 15 years of data (Souchay et al. 1995; Charlot et al. 1995). In the first work, the observational determination is $\dot{\omega}_A = -0.026''/cy$, whereas for the second one it ranges between $\dot{\omega}_A = -0.025''/cy$ and $\dot{\omega}_A = -0.021''/cy$ These values are quite in agreement with the analytical ones above.

7. Conclusion

In the present paper we have studied small contributions to the nutation for a rigid Earth model. One part concerns the influence of the coeffficients of the geopotential others than J_2. The other part is the direct action of the planets on the nutation. Our

Table 6.2. Coefficients of rigid Earth nutation due to the planets Mercury, Saturn and Uranus, obliquity part. Comparison of new results (this paper) is made with results of Kinoshita & Souchay (1990) and Williams (1995)

λ_{Me}	λ_{Ea}	λ_{Sa}	λ_{Ur}	p_A	Period(d)	KS (1990) sin (μas)	KS (1990) cos (μas)	Williams (1995) sin (μas)	Williams (1995) cos (μas)	This paper sin (μas)	This paper cos (μas)
1	−4	0	0	−2	2402.790			−0.4	0.2	−0.4	−0.2
1	−3	0	0	−2	317.059					0.0	−0.2
0	2	0	0	2	182.622					0.0	0.1
0	0	1	0	1	10746.938					0.0	−0.1
0	0	1	0	2	10734.673			0.3	0.0	0.4	0.0
0	0	2	0	1	5376.541			0.5	−0.2	0.5	−0.2
0	0	2	0	2	5373.469	0.0	5.0	0.0	5.3	0.0	5.3
0	0	3	0	2	3583.678			0.7	0.0	0.7	0.0
0	1	−3	0	−2	406.709					0.0	−0.1
0	0	0	2	2	15294.356			0.0	0.3	0.0	0.3

Table 7. Influence of the direct torque exerted by the planets on the precession in obliquity

Planet	Williams (1994) (″/cy)	This paper (″/cy)
Mercury	−0.000090	−0.000087
Venus	−0.017372	−0.016816
Mars	0.000255	0.000346
Jupiter	0.002782	0.002859
Saturn	0.000217	0.000218
Uranus	0.000001	0.000001
Neptune	0.000001	0.000001
Total	−0.014207	−0.013478

calculations are refined with respect to previous ones done by Kinoshita & Souchay (1990) mainly because of the change of the level of truncation of the series (0.5 μas instead of 5 μas), but also because we keep all the terms in the Fourier series of intermediate calculations with a relative 5×10^{-11} absolute value instead of 10^{-9} in (Kinoshita & Souchay 1990). A few errors found in this last paper are corrected.

Table 1 gives all the coefficients of nutation due to J_3, and Table 2 those due to the triaxiality of the Earth, whereas Tables 3 to 6 show the coefficients due to the direct influence of the planets. Moreover in end of Sect. 3, we have shown the three biggest components, both in $\Delta\psi$ and $\Delta\varepsilon$, coming from the J_4 influence.

Comparisons of our results with those for the coefficients of the nutation due to the J_3 and J_4 geopotential as calculated by Hartmann et al. (1995) and with those due to the direct action of the planets calculated by Williams (1995) show a quasi-perfect agreement, although the way of calculation is quite different.

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