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Contribution of spiral arms to the surface brightness distribution of disk galaxies

Arturo Serna

LAEC, Observatoire de Paris-Meudon, Université Denis Diderot, F-92195 Meudon Cedex, France

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Abstract. The usual bulge/disk decomposition of the surface brightness distribution in disk galaxies generally gives good results for S0 galaxies, but often fails for more advanced types of the Hubble sequence. We propose in this paper that, in the case of spiral galaxies, a third component must be considered: the contribution of spiral arms. The functional form of the luminosity law of spiral arms is here obtained by means of the density wave theory, in contrast with the essentially empirical nature of the bulge and disk laws. An advantage of this approach is that the fitting parameters appearing in our analytical expression are related to galactic dynamical properties as the mass-to-luminosity ratio, the angular velocity of the spiral pattern rotation, etc. Consequently, the fitting of surface brightness profiles provides an estimate of such quantities. Reciprocally, the requirement that galactic physical properties must be reasonable can be used as a condition limiting the freedom in the fitting of data. Our luminosity law has been applied to some spiral galaxies for which the simple bulge/disk decomposition did not give good results. We find that our expression provides excellent fits to the observed profiles of these galaxies, and implies physical properties which reasonably coincide with those obtained from other procedures.

Key words: galaxies: structure – galaxies: spiral – galaxies: photometry

1. Introduction

The morphological structure of galaxies contains very valuable information about the history of their formation and subsequent interaction with their environments. In particular, the surface brightness distribution has long been recognized as an essential clue to describe quantitatively the morphological features of galaxies, as well as to investigate possible relationships to other galactic properties.

The luminosity profile of disk galaxies is often considered as the sum of two overlapping components: the bulge and the disk. These components are parameterized by simple empirical fitting functions with free parameters which are determined by a non-linear fit to the observed profiles (see, e.g., the reviews by Simien 1989, and Capaccioli and Caon 1992).

The most frequently used function describing the radial surface brightness profile of the disks of spiral galaxies is an exponential function (de Vaucouleurs 1959, Freeman 1970):

$$\Sigma_{disk}(r) = \Sigma_D^0 e^{-r/h}, \tag{1}$$

where Σ_D^0 is the central surface luminosity and h is the scale length of the exponential disk.

In that concerning the bulge, the early papers used a de Vaucouleurs $r^{1/4}$ law to describe its brightness profile. However, since strong deviations from this law have been frequently observed in bulges of spirals, an exponential fitting function has been more recently proposed for this component (Frankston & Schild 1976, Kent el al. 1991, and Andreakis & Sanders 1994)

$$\Sigma_{bulge} = \Sigma_B^0 e^{-r/r_e}, \tag{2}$$

where Σ_B^0 and r_e are the bulge central surface luminosity and scale length, respectively.

Although the use of an exponential bulge improves the quality of fits, we note however that a single bulge/disk decomposition (whatever the bulge law is) still fails frequently for the most advanced types of the Hubble sequence, while it generally gives very good results for S0 galaxies (see, e.g., Kent 1985). Inspection of the observed profiles of Sa-Sc galaxies shows that a failure in the fitting of data is often produced by the existence of sinusoidal-like oscillations in the radial brightness distribution. This strongly suggests that such a failure originates from having neglected a third component whose importance could be crucial in many Sa-Sc galaxies: the spiral arms.

The aim of this paper is to deduce and to apply a luminosity law for the contribution of spiral arms. Unlike the purely empirical functions traditionally used for the bulge and unperturbed disk components, we will obtain the luminosity law of spiral arms by means of dynamical arguments based on the density wave theory. An advantage of this dynamical approach is that it provides the relationship between the fitting parameters and

Send offprint requests to: A. Serna

certain dynamical properties of the galaxy under consideration. This allows one to establish a connection between theory and observations much clearer than that obtained through purely empirical expressions.

The density wave theory is an extensive formalism describing the dynamics of differentially rotating disks (see, e.g., the reviews by Rohlfs 1977, Athanassoula 1984, and Binney and Tremaine 1987). This theory assumes that spiral arms are the excess matter associated with a wavelike oscillation that propagates through the galactic disk. This spiral pattern generates a non-axisymmetric component of the gravitational field which, in principle, can help to produce long-lived spiral arms. The main difficulty in giving a complete treatment of the problem lies in the long-range nature of gravitational forces: all parts of a galaxy are strongly coupled together and, in general, the wave features can only be determined numerically. Fortunately, there exists one important limit in which the analysis is much simpler: if the radial wavelength of waves is much smaller than the radius, the long-range coupling is negligible, the response is determined locally, and the relevant solutions are analytic. In addition to this limit, known as the WKB approximation, the simplest version of the density wave theory also assumes that waves are quasi-stationary (Lin and Shu 1966). Although the Lin-Shu theory yields a broad range of successful predictions, the validity of its hypotheses is still the subject of intense debate and it cannot be considered as a final solution to all spiral structure problems. In general, the WKB theory must be augmented with new physical concepts, as feedback loops and the swing amplifier (Toomre 1981), which require numerical programs to follow the non-linear evolution of wave packets.

The paper is arranged as follows. The basic equations of the density wave theory are briefly described in Sect. 2.1, as well as the approximations assumed in this paper. These expressions are then used in Sect. 2.2 to obtain an analytical function for the spiral arm contribution to the surface brightness profiles of galaxies. The fitting procedure using our bulge-disk-arms composite model and its application to some observed profiles is then presented in Sect. 3. Finally, Sect. 4 summarizes our main results and conclusions.

2. The luminosity law for spiral arms

2.1. Equations and basic assumptions

In order to determine the features of density waves in the Lin and Shu (1966) approximation, the surface density in a thin disk is represented as the sum of an unperturbed surface density $\sigma_0(r)$, and a small perturbation $\sigma_1(r,\theta,t)$, which represents the spiral pattern,

$$\sigma = \sigma_0(r) + \sigma_1(r,\theta,t). \tag{3}$$

When this expression is introduced into the motion equations, one finds that they admit solutions for σ_1 whose real part has the form:

$$\sigma_1(r,\theta,t) = \hat{\sigma}(r)\cos[\omega t - m\theta + \Phi(r)], \tag{4}$$

where $\hat{\sigma}(r)$ is the surface density amplitude, and $\Phi(r)$ is the shape function.

At any time t, Eq. (4) represents a density distribution with a geometric shape given by

$$\theta = \frac{1}{m}[\Phi(r) + \text{const}]. \tag{5}$$

It is then a spiral pattern, with m arms, which rotates with an angular velocity $\Omega_p = \omega/m$ and whose radial wavenumber $|k(r)|$ is given by

$$k(r) = d\Phi(r)/dr. \tag{6}$$

The condition that density waves are self-sustained (that is, the surface-density response is locally consistent with that required to support the spiral gravitational field) allows one to calculate two important relations: the dispersion relation and the equation for the density amplitude $\hat{\sigma}$. In the standard case of a thin disk with a Schwarzschild velocity distribution, these relations are, respectively (Lin and Shu 1966, Toomre 1969, Shu 1970):

$$\frac{2\pi G\sigma_0}{|k|\, c_r^2}\frac{\mathscr{F}_\nu(x)}{1-\nu^2}x = 0, \tag{7}$$

$$\frac{d}{dr}\left[\frac{r\hat{\sigma}^2}{k^2}\left(1 + 2\frac{\partial \ln \mathscr{F}_\nu}{\partial \ln x}\right)\right] = 0, \tag{8}$$

where

$$\nu = \frac{m}{\kappa}(\Omega_p - \Omega), \tag{9}$$

$$\kappa = 2\Omega\left(1 + \frac{r}{2\Omega}\frac{d\Omega}{dr}\right)^{1/2}, \tag{10}$$

$$x = \frac{k^2 c_r^2}{\kappa^2}, \tag{11}$$

$$\mathscr{F}_\nu(x) = \frac{1-\nu^2}{x}\left[1 - \frac{\nu\pi}{\sin(\nu\pi)}\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-x(1+\cos s)}\cos(\nu s)ds\right], \tag{12}$$

G being the gravitational constant, $\nu(r)$ the intrinsic frequency, $c_r^2(r)$ the radial velocity dispersion of stars, $\kappa(r)$ the epicyclic frequency, $\Omega(r)$ the angular velocity, $x(r)$ the stability parameter, and $\mathscr{F}_\nu(x)$ the reduction factor. This last function, $\mathscr{F}_\nu(x)$, given by Eq. (12) and tabulated by Lin, Yuan and Shu (1969), describes how much the response to a spiral perturbation is reduced with respect to that found for a cold disk ($c_r^2 = 0$).

In order to obtain a relatively simple analytical expression for the contribution of spiral arms to the brightness profile of galaxies, we have to adopt some approximations simplifying the theoretical problem. The set of approximations assumed here is the following:

1. In the regions dominated by the disk, the rotation curve can be approximated by a straight line. In other words, the angular velocity of the disk rotation is given by
$$\Omega(r) = V_0/r + \Omega_\infty, \tag{13}$$
where Ω_∞ is the slope of the straight rotation curve, and V_0 is the extrapolated velocity at $r = 0$.

It is well known that, except for small r values, Eq. (13) is roughly valid for many spiral galaxies. Since the most inner regions are dominated by the bulge, deviations from Eq. (13) in the limit of small r are not important in the total profile.

2. The shape of the spiral arms corresponds to a logarithmic spiral. That is, the wavenumber and the shape function are given by:

$$k = \frac{m}{r} \text{ cotan } \alpha_0, \quad (14)$$

$$\Phi(r) = m \text{ cotan } \alpha_0 \ln(r) + \text{const}, \quad (15)$$

where α_0 is the constant pitch angle.
Although the observed pitch angle often depends slightly on r and, hence, the shape of the spiral arms deviates something from a logarithmic spiral, this approximation is reasonably good for most spiral galaxies (Danver 1942, Kennicut 1981).

3. Since Eq. (1) gives very good fits for the disks of S0 galaxies, we will also admit here that the unperturbed disk has a surface density profile which verifies the exponential law (1). This hypothesis will allow us to obtain a contribution of spiral arms which is coherent with the profile commonly considered for the disk component.

Together with these approximations, we will also admit throughout this paper that the mass-to-luminosity ratio is a constant for each galaxy and that the hypotheses taken by the density wave theory are valid.

2.2. Contribution of the spiral arms

According to Eq. (13), the epicyclic and intrinsic frequencies (Eqs. 10 and 9) are given by

$$\kappa = \frac{\sqrt{2}V_0}{r} f^{1/2}, \quad (16)$$

$$\nu = \frac{m}{\sqrt{2}} \left(\frac{\Omega_p + \Omega_\infty}{V_0} r - 1 \right) f^{-1/2}, \quad (17)$$

where

$$f = 1 + 3(\Omega_\infty/V_0)r + 2(\Omega_\infty/V_0)^2 r^2. \quad (18)$$

On the other hand, considering the values tabulated by Lin, Yuan and Shu (1969) for the reduction factor $\mathscr{F}_\nu(x)$, and taking into account that we are here interested in obtaining relatively simple analytical expressions, we can approximate the $\mathscr{F}_\nu(x)$ function by

$$\mathscr{F}_\nu(x) \simeq \frac{1 - \nu^2}{1 - \nu^2 + x}. \quad (19)$$

Substituting Eq. (19) into the dispersion relation (Eq. 7), we find

$$\frac{|k|}{k_T} = 1 - \nu^2 + x, \quad (20)$$

where

$$k_T(r) \equiv \frac{\kappa^2(r)}{2\pi G \sigma_0(r)} \quad (21)$$

is the Toomre wavenumber scale.

Expression (20) is formally identical to the dispersion relation of a gaseous disk (Lin and Shu 1966). Hence, to the level of approximation that we consider here, the stellar and gaseous disks are dynamically similar. This explains why we have not tried of decomposing Eq. (7) into these two components.

Using Eqs. (11) and (21), and defining the Toomre stability parameter as

$$x_T(r) \equiv \frac{k_T^2(r) c_r^2(r)}{\kappa^2(r)}, \quad (22)$$

the dispersion relation can be written as

$$(1 - \nu^2) \left(\frac{k_T}{k} \right)^2 - \frac{k_T}{|k|} + x_T = 0, \quad (23)$$

which is an algebraic equation of second order in $k_T/|k|$ and, hence, its solution has two branches

$$\frac{k_T}{|k|} = \frac{1}{2(1 - \nu^2)} \left[1 \pm \sqrt{1 - 4x_T(1 - \nu^2)} \right]. \quad (24)$$

The solution (+) in Eq. (24) corresponds to the Long Wave Mode (LWM), while the solution (−) corresponds to the Short Wave Mode (SWM).

In order to find the functional form of the density amplitude $\hat{\sigma}(r)$ appearing in Eq. (4), we must solve Eq. (8). To that end, we consider again Eq. (19) and we use $x = x_T(k/k_T)^2$. The solution of Eq. (8) is then

$$\hat{\sigma}^2 = -\frac{Ck^2}{r} \frac{x_T + (k_T/k)^2(1 - \nu^2)}{x_T - (k_T/k)^2(1 - \nu^2)}. \quad (25)$$

Now, we define

$$\Delta \equiv 2\frac{k_T}{|k|}(1 - \nu^2) - 1 \quad (26)$$

which, according to Eq. (24), is positive in the LWM and negative in the SWM.

Using Eqs. (23) and (26) to eliminate x_T and k_T/k in Eq. (25), we obtain

$$\hat{\sigma}(r) = \frac{C\,|k|}{r^{1/2}} \frac{1}{|\Delta|^{1/2}}. \quad (27)$$

For patterns with m logarithmic spiral arms, the wavenumber k and the shape function $\Phi(r)$ are given by Eqs. (14) and (15). Substituting these equations into Eq. (4), we obtain the surface density profile (with fix t and θ) of the spiral perturbations:

$$\sigma_1 = \frac{a_1'}{r^{3/2}} \frac{1}{|\Delta|^{1/2}} \cos\{m \text{ cotan } \alpha_0 \ln(r/a_2)\} \quad (28)$$

and, assuming a constant M/L ratio for the galaxy disk, the surface brightness profile is

$$\Sigma_{arms} = \frac{a_1}{r^{3/2}} \frac{1}{|\Delta|^{1/2}} \cos\{a_3 \ln(r/a_2)\}, \quad (29)$$

where a_1 and a_2 are arbitrary constants, while a_3 is defined bellow.

The function Δ appearing in Eq. (29) contains the dependence of σ_1 on the rotation curve and on the unperturbed disk surface density (see Eqs. 21 and 26). For an exponential unperturbed disk (Eq. 1) with an outer differential rotation given by Eq. (13), this function reduces to

$$\Delta = a_4 f \frac{e^{r/h}}{r}(1-\nu^2) - 1, \tag{30}$$

where

$$\nu = \frac{m}{\sqrt{2}}[r/a_5 - 1]f^{-1/2}, \tag{31}$$

and constants a_i are defined by

$$\begin{aligned} a_3 &= m \text{ cotan } \alpha_0, \\ a_4 &= \frac{2V_0^2}{(M/L)_D \Sigma_D^0 \pi G m \text{ cotan } \alpha_0}, \\ a_5 &= \frac{V_0}{\Omega_p + \Omega_\infty}, \end{aligned} \tag{32}$$

$(M/L)_D$ and Σ_D^0 being the blue mass-to-luminosity ratio and the central surface brightness of the unperturbed disk.

The range of r values in which Eq. (29) must be taken into account is restricted by the condition $1 - \nu^2 > 0$ and by the requirement that the wavenumber k is real. From Eqs. (31) and (24), we see that these conditions imply

$$\begin{aligned} a_5(1 - \sqrt(2) f^{1/2}/m) < r \;&<\; a_5(1 + \sqrt(2) f^{1/2}/m), \\ 4x_T \;&<\; 1/(1-\nu^2), \end{aligned} \tag{33}$$

respectively. Consequently, we will consider that $\Sigma_{arms}(r)$ is zero for any value of r not satisfying some of conditions (33), while $\Sigma_{arms}(r)$ is given by Eq. (29) if both criteria are simultaneously satisfied.

3. Applications

3.1. Fitting procedure and determination of physical parameters

The expression (29) for the contribution of the spiral arms to the surface brightness profile of a galaxy introduces five new fitting parameters. In principle, this is not very satisfactory since a so high number of parameters can introduce a considerable degree of arbitrariness. Furthermore, as in the usual bulge/disk decomposition, very different sets of the free parameters can lead to very close values of the statistical measures (as the r.m.s.) for the fit quality. In order to choose the best approximating model, a criterion just based on such statistical measures is often inappropriate because other very different models could lead to fits with just a very slightly smaller goodness.

We remark however that Eq. (29) has been obtained through physical arguments and, consequently, it is not just a mathematical fitting function. As a matter of fact, the fitting parameters are related with measurable physical properties through (Eqs. 32):

$$\begin{aligned} \alpha_0 &= \text{atan}(m/a_3) \quad \text{[rad]}, \\ (M/L)_D &= 4.07 \times 10^{-12} \frac{V_0^2}{\Sigma_D^0 a_3 a_4} \quad [M_\odot/L_\odot], \\ \Omega_p &= \frac{V_0}{a_5} - \Omega_\infty \quad \text{[km/s/kpc]}, \end{aligned} \tag{34}$$

where V_0 is expressed in km/s and Σ_D^0 in $L_\odot$/arcsec^2.

Hence, the freedom in the choice of such parameters is limited by the requirement that they must imply reasonable physical properties for the galaxy under study. If this requirement cannot be satisfied for a given galaxy, we must consider that Eq. (29) does not introduce a reliable improvement with respect to the usual bulge/disk decomposition.

In the present study, we have used a fitting procedure in which reproducibility of reasonable physical properties, as well as the fit goodness, is also taken as a criterion to identify the best model. Furthermore, since the result of a non-linear fitting can depend on the value initially assigned to the free parameters, our fitting procedure consists of two stages where the best model is identified after exploring a grid of input parameters.

The first stage of this procedure is conceived to explore all the range of reasonable parameters and thus find the better choice of their input values. Toward this end, the bulge and disk parameters, as well as the arbitrary spiral arm constants a_1 and a_2, have input values generated through the grids:

$$a_i = \frac{2^n}{8} a_i^c \qquad (n = 1, ..., 5) \tag{35}$$

where a_i^c represent their input central values, chosen after performing some tests.

The input dynamical parameters (a_3, a_4, and a_5) are instead generated so that they cover a wide interval of values considered as physically reasonable. In other words, they are generated so that Eqs. (32) imply a grid of $(M/L)_D$, α_0 and Ω_p values covering all their corresponding allowed intervals. For example,

$$a_3 = m \text{ cotan } [\alpha_0^{min} + (\alpha_0^{max} - \alpha_0^{min}) 2^{(1-n)}], \tag{36}$$

and similarly for a_4 and a_5.

The allowed interval considered for the pitch angle is (Kennicut 1981) $\alpha_0 \in [3, 35]$, while the interval considered for $(M/L)_D$ covers from $(M/L)_\odot/10$ to $10(M/L)_\odot$ (see, e.g., Kent 1985, Forbes 1992). The Ω_p values are instead more poorly known. In the case of our Galaxy, Lin, Yuan and Shu (1969) obtained that $\Omega_p \simeq 11$ km/s/kpc fits well various observed data, while Marochnik, Mishurov and Shuchkov (1972) obtained $\Omega_p \simeq 23$ km/s/kpc. We have considered here a rather wide allowed interval $\Omega_p < 100$ km/s/kpc.

Starting from these input grids, we have then performed a standard non-linear fitting (based on the Levenberg-Marquard method). The selection of the best model a_i^{end} is carried out by taking, among all those implying reasonable physical properties, that which leads to the smallest r.m.s. For reasonable physical properties we mean, not only that the corresponding α_0, Ω_p, and $(M/L)_D$ values are within the above described intervals, but also that the resulting profile for the spiral arms does not violate one of the basic hypothesis of the density wave theory: $\sigma_1(r) \ll \sigma_0$. In fact, we see from Eq. (29) that density perturbations have a very large amplitude when $\Delta \to 0$. The possibility of obtaining large density amplitudes has been previously noted by Toomre (1969) and Shu (1970). According to Eq. (24), such values appear when the LWM and SWM solutions coincide ($\Delta = 0$). In that case, the wave number is marginally real and the disk is marginally stable. Since the density wave theory assumes that σ_1 is small as compared to the density σ_0 of the unperturbed disk, such models are not acceptable. In our numerical computation, we have rejected those models with $\max_r[|\sigma_1(r)| / \sigma_0(r)] > 1/2$.

The second stage of our procedure takes the values of a_i^{end} obtained in the previous stage as input central values to generate a new set of grids, but now using Eq. (35) for any parameter. From the same criteria and non-linear fitting algorithm as before, we select a new best model. The a_i^{end} values of this model are then used as input central values to generate new grids. This iterative procedure is followed until the output a_i^{end} values converge.

Obviously, this procedure consumes a considerable amount of computing time ($\approx$ 20 hr), but it reduces notably the possible arbitrariness of results. The obtained model is in fact rather insensitive to the initial central values adopted at the beginning of the first stage.

3.2. Results

In order to illustrate the application of Eq. (29) to some observed profiles, we have analyzed the surface brightness distribution of two galaxies in the Kent (1984, 1986) sample: UGC 2885 and IC 467.

UGC 2885 has been selected as being a typical Sc galaxy for which the usual bulge/disk decomposition provided rather bad results. Using the rotation curve measured by Rubin et al. (1985) (V_0 = 299.1 km/s and Ω_∞ = 0.09 km/s/kpc), and applying the above procedure, we find that the surface brightness profile of UGC 2885 is very well fitted by our triple decomposition (see Fig. 1). The physical parameters implied by this procedure are α_0 = 21.6°, Ω_p = 8.4 km/s/kpc and $(M/L)_D$ = 5.07 $(M/L)_\odot$. This $(M/L)_D$ value coincides within a very reasonable margin with those obtained from different techniques. For example, the maximum-disk and the constant-density halo solutions of the UGC 2885 rotation curve (Kent 1986) imply $(M/L)_D$ = $5.08 - 5.52$ $(M/L)_\odot$. In the same way, Roelfsema & Allen (1985) found the values $(M/L)_D$ = 4.7 $(M/L)_\odot$ and Ω_p = 6.4 km/s/kpc, which are just slightly smaller than ours. Comparison with Canzian's (1985) results is however much more difficult because he considered the existence of two superposed spiral patterns in UGC 2885, while our approach assumes a single two-armed spiral structure. Nevertheless, we note that the outer pattern in Canzian (1985) has Ω_p = 8 km/s/kpc, very close to our results (the inner pattern has instead a considerably higher speed, Ω_p = 17 km/s/kpc).

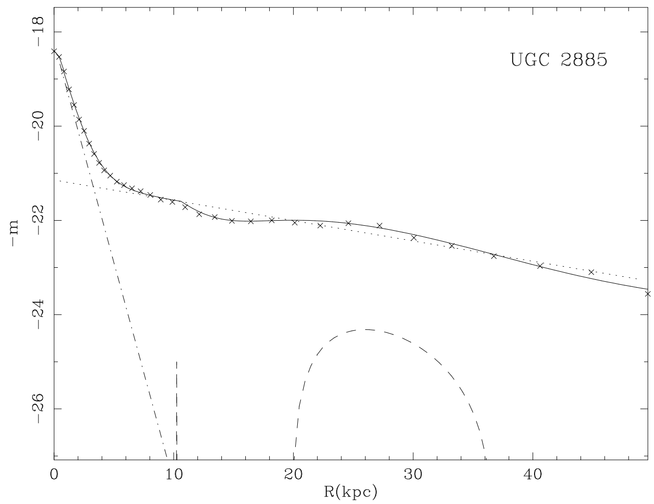

Fig. 1. Major-axis luminosity profile for UGC 2885. The theoretical profile (solid line) is decomposed into bulge (dot-dashed line), unperturbed disk (dotted line), and spiral perturbations (dashed line).

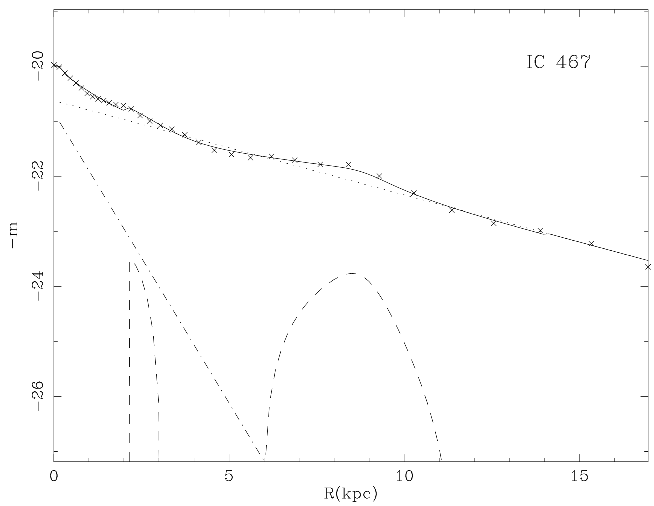

Fig. 2. Same as Fig. 1, but for IC 467

IC 467 has been selected as another Sc galaxy but, now, with a rising rotation curve (V_0 = 118.0 km/s, Ω_∞ = 1.37 km/s/kpc) also measured by Rubin et al. (1985). The fitting of its surface brightness distribution is again much better than that found from the usual double decomposition (see Fig. 2). The resulting dynamical parameters (α_0 = 22.7°, Ω_p = 14.4 km/s/kpc and $(M/L)_D$ = 2.9 $(M/L)_\odot$) are also reasonably close to those found from other procedures. The disk mass-to-luminosity ratio is in fact intermediate between that found by Forbes (1992), 2.5 $(M/L)_\odot$, and that obtained by Kent (1986), 3.4 $(M/L)_\odot$. It must be mentioned that, although IC 467 is considered as a galaxy with a normal spiral structure, it probably has a companion (NGC 2336) at a projected separation of about

135 kpc (van Moorsel 1987). The good modeling of its surface brightness by our WKB approximation suggests that possible tidal effects on IC 467 have a very small dynamical influence.

Another important consequence of the incorporation of spiral arms in this kind of analysis is that the relative importance attributed to the bulge with respect to the other components can be different from that obtained through the usual approach. In the examples shown above, our calculations imply that the luminosity of the bulge is greater by a factor of ~ 1.5 than that obtained by Andreakis and Sanders (1994), who did not consider the presence of spiral arms perturbing the disk component.

4. Conclusions

In this paper, we have used the density wave theory to deduce an approximate analytical expression which takes into account the contribution of spiral arms to the surface brightness distribution in disk galaxies (Eq. 29). Since the physics involved by the spiral structure of a galaxy is relatively complex, our expression introduces several additional parameters in the law of total luminosity. Fortunately, most of these constants are related to dynamical parameters which can be measured by using independent techniques. For example, the parameters related to the disk rotation can be directly extracted from the galactic rotation curve and, hence, they are not free constants. In the same way, other parameters can be directly measured (as the pitch angle) or used to limit the freedom in the value of constants appearing in our expression. Reciprocally, the fitting of surface brightness profiles can be used to obtain a rough estimate of several dynamical parameters.

The application of our expression to some observed profiles gives excellent fits even when the surface brightness distribution is considerably affected by the irregularities produced by spiral arms. Much more encouraging for us is the fact that the dynamical parameters implied by such fits coincide reasonably with those measured by using other techniques. Because of the approximations involved in our expression, and the uncertainties inherent to any non-linear fitting, we cannot however consider the outputs of this procedure as an accurate measure of such parameters, but just as a rough estimate.

As a consequence of considering the spiral arms as a third important component in the surface brightness distribution of disk galaxies, the relative importance attributed to the bulge can significantly differ from that obtained from a simple double decomposition. Such values have been used in several statistical studies analyzing problems as a possible universality in the central surface brightness of the bulge (Freeman 1970, Kormendy 1977), the analogies among bulges and elliptical galaxies (Jablonka, Martin & Arimoto 1996), etc. In a forthcoming paper, we will apply the expression obtained in this work to describe the surface brightness distribution of a large number of galaxies. We will be then able to analyze, from a statistical point of view, the full implications of our procedure.

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