

The intrinsic Ly α to H α ratio in M dwarf stars

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Abstract. Using Ly α line profiles generated from a grid of M dwarf model atmospheres we calculate the attenuation factor [i.e. Ly α (obs)/ Ly α (intrin)] due to the interstellar medium as a function of hydrogen column density. Then using selected model atmospheres, attenuation factors were calculated for those M dwarfs with available Ly α observations. The prime motivation in this work has been to look afresh at the intrinsic Ly α to H α flux ratio, an important constraint in the radiative transfer modelling of M dwarfs. For those active dMe stars where both lines were observed, the intrinsic ratio is $\sim 3 - 5$ (with $\sim 50\%$ variation). The major uncertainty in this work has been the interstellar hydrogen column density, emphasizing the need for further work in this area and in particular an accurate model of its variation in all directions.

Key words: lines: profiles – stars: activity: chromospheres; late-type – ISM: general

1. Introduction

The hydrogen Ly α line is one of the most important radiative cooling agents in the atmospheres of late-type stars. In the solar case the Ly α wings are formed throughout the chromosphere while the line center can be formed at temperatures as high as 40,000 K (Vernazza, et al. 1981, Fontenla et al. 1991). Furthermore, Ly α fluxes provide a key constraint for quantitative steady state models of low activity dwarfs that include heating by shock dissipation. This is because in low activity stars the hydrogen Balmer lines do not contribute significantly to the radiative output of the outer atmosphere and the Lyman lines become relatively more important (Mullan & Cheng 1993, Houdebine et al. 1995).

A vital constraint which can be used in the semi-empirical modelling of late-type dwarf atmospheres is the Ly α to H α ratio (Houdebine & Doyle 1994). These authors used a Ly α

to H α ratio of 1 in their modelling of the dMe star AU Mic. On the other hand Panagi (1990) constructed a semiempirical model for Gl 494, an object which has properties similar to AU Mic, and obtained a Ly α to H α ratio of 8. However, as noted by Houdebine et al. (1995) in their NLTE-radiative transfer calculations, the Ly α to H α ratio is model dependent, ranging in value from 1.4 to 21. In a more global study, Zirin (1978) has suggested that the Ly α to H α ratio is close to unity for astrophysical phenomena ranging from flares to quasars and the solar chromosphere.

Although it is relatively easy to determine the stellar Ly α flux in IUE high resolution spectra of M dwarfs, the analysis of low resolution spectra is somewhat more difficult due mainly to the contamination of the stellar flux by the geocoronal emission. Byrne & Doyle (1989) and Doyle et al. (1990) have described a technique for extracting the stellar component in low resolution IUE spectra and have presented Ly α fluxes for a large sample of late-type dwarfs. However, these were not corrected for attenuation by the interstellar medium and therefore can only be considered as lower limits.

In a related study, Landsman & Simon (1993) presented a catalogue of stellar Ly α fluxes corrected for the interstellar medium for a large sample of stars with spectral types ranging from late A to M. In this present paper we concentrate on M dwarfs and use a grid of intrinsic Ly α stellar profiles generated by chromospheric model calculations to compute attenuation factors [i.e. Ly α (obs)/ Ly α (intrin)] and correct for the interstellar absorption.

2. Attenuation factors at Ly α

In the calculation of the attenuation factors at Ly α , a number of different parameters were taken into account, including:

2.1. Intrinsic stellar profile/models

The Lyman alpha line was computed using the MULTI code (Carlsson 1986) to solve the combined radiative transfer, statistical equilibrium and hydrostatic equilibrium equations for a

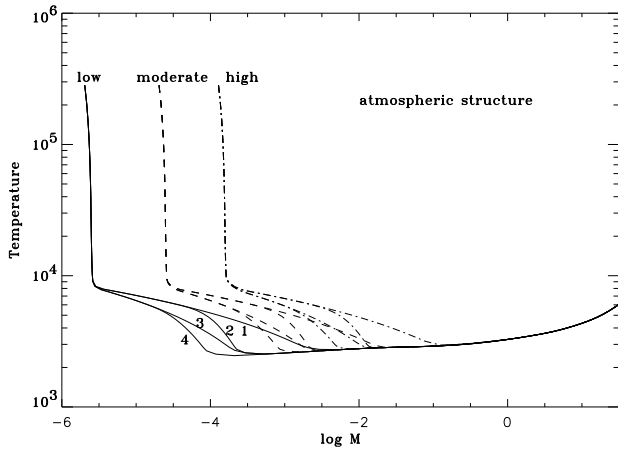


Fig. 1. Three sample model atmospheres with a log column mass at the base of the transition region corresponding to -5.5 , -4.5 and -3.8 (see text for details)

seven level model H I atom. The ground state of H II is included in the model. The radiative transfer problem is solved in detail for all 15 bound-bound transitions connecting the six H I states and for the photoionization continua of these states. The temperatures and densities just above T_{\min} in our model correspond to the partial ionization of H I which is the main e^- donor. Therefore, we have iterated the solution of the statistical equilibrium and hydrostatic equilibrium equations to convergence.

The calculation of the photoionizing radiation field at depths below T_{\min} includes line opacities computed with the PHOENIX model atmosphere code (Allard & Hauschildt 1995), corresponding to a star with effective temperature $T_{\text{eff}} = 3700$ K, $\log g = 4.7$ and solar metallic abundances. These were incorporated using a modified version of MULTI (Andretta et al. 1996).

In order to probe the effect of the lower atmosphere, four types of models were constructed. Models 1 and 3 (see Fig. 1) have a linear variation of temperature versus $\log m$ while models 2 and 4 are identical to the first series in the upper chromosphere but with the temperature gradient in the lower chromosphere steepened.

The most important parameter in determining the “activity” of a model chromosphere is its pressure. We therefore generated “active” states from the “quiet” chromosphere in Fig. 1 by translating the temperature structure toward higher column masses. Thus, the variation of pressure in the chromosphere and transition region is parameterized by the mass loading at the top of the chromosphere, m_{\circ} . The mass loading in Fig. 1 for the very quiet chromospheric model is $\log m_{\circ} = -5.5$. Two other series, corresponding to $\log m_{\circ} = -4.5$ and $\log m_{\circ} = -3.8$ are also shown, these are representative of a moderate and active chromosphere respectively. Another relevant parameter is the temperature gradient in the transition region, here we have chosen $\log \nabla_{\text{TR}} T = 6.5$ (see Andretta et al. 1996 for further details).

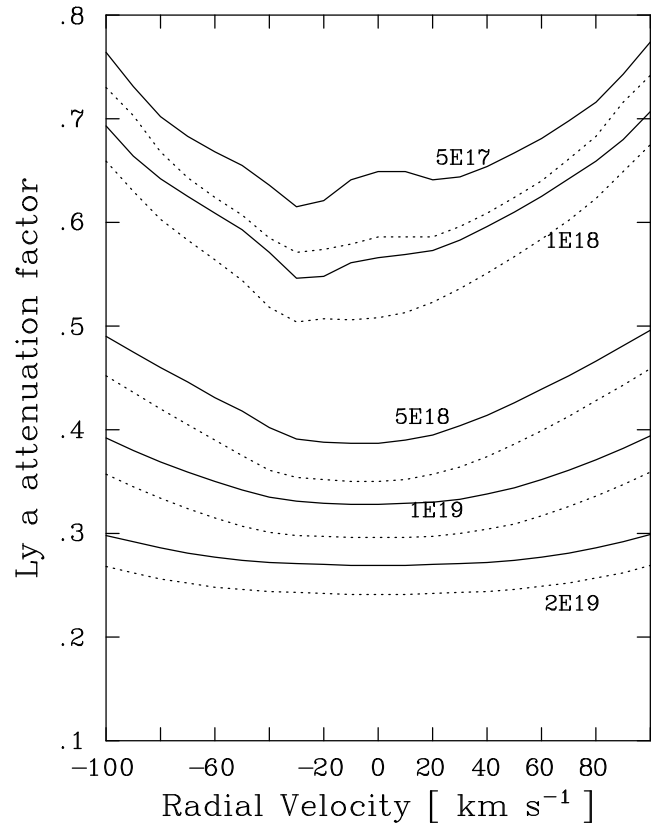


Fig. 2. Ly α attenuation factors as a function of radial velocity for interstellar hydrogen column densities of $\log N_H = 17.70, 18.00, 18.70, 19.00, 19.30$. The attenuation factors have been calculated for model $\log m_{\circ} = -4.8$ for model series 1 (solid line) and 2 (dashed line).

2.2. Interstellar Ly α profile

Using an accurate determination of the interstellar column density towards the white dwarf G191-B2B (which allows them to get a good fit to the Lyman edge), Hurwitz & Bowyer (1995) have shown that the interstellar profile has a broadening parameter $b_{HI} = 11 \pm 1 \text{ km s}^{-1}$. This value is in excellent agreement with the b_{HI} determined by Linsky et al. (1993) in the direction of Capella and is therefore used in the present study. The quoted uncertainty in b_{HI} causes a very small uncertainty in the attenuation factors, generally less than 2%. For the same interstellar hydrogen column density an increase in b_{HI} would mean a broader interstellar profile and therefore an increased attenuation. At higher interstellar hydrogen column densities (N_H), the attenuation factors are influenced much less by increases in b_{HI} . At sufficiently high column densities, the interstellar transmission profile is saturated to such a degree that a higher b_{HI} value causes an insignificant increase in the attenuation factor.

2.3. Hydrogen column density

For all stars the interstellar hydrogen column density to each object was computed using a model of the interstellar medium

Table 1. Ly α attenuation factors [i.e. Ly α (obs)/ Ly α (intrinsic)] as a function of the interstellar medium hydrogen column densities (in cm^{-2}) for various M dwarf intrinsic stellar profiles as a function of the column mass at the transition region temperature break 8500 K. Four different model atmospheres are considered as outlined in Fig. 1, at each value of mass loading at m_o . The attenuation factors have been determined assuming $b_{HI}=11 \text{ km s}^{-1}$ for the interstellar line profile, a zero radial velocity and a zero interstellar wind velocity

$\log m_o$ $g \text{ cm}^{-2}$	Model	$\log N_H$				
		17.70	18.00	18.70	19.00	19.30
-6.0	1	0.441	0.370	0.254	0.220	0.185
	2	0.363	0.304	0.209	0.177	0.146
	3	0.368	0.306	0.206	0.175	0.143
	4	0.355	0.296	0.201	0.170	0.139
-5.6	1	0.538	0.459	0.319	0.277	0.233
	2	0.478	0.405	0.280	0.239	0.197
	3	0.438	0.367	0.247	0.209	0.171
	4	0.447	0.374	0.252	0.212	0.173
-5.2	1	0.581	0.502	0.347	0.298	0.249
	2	0.537	0.463	0.323	0.276	0.228
	3	0.545	0.464	0.315	0.267	0.219
	4	0.501	0.425	0.287	0.242	0.197
-4.8	1	0.649	0.566	0.387	0.328	0.269
	2	0.586	0.508	0.349	0.295	0.241
	3	0.572	0.492	0.332	0.279	0.226
	4	0.561	0.481	0.325	0.272	0.219
-4.6	1	0.658	0.575	0.389	0.327	0.266
	2	0.622	0.540	0.367	0.307	0.249
	3	0.583	0.502	0.335	0.279	0.224
	4	0.574	0.494	0.330	0.274	0.219
-4.4	1	0.637	0.557	0.372	0.310	0.249
	2	0.598	0.518	0.346	0.287	0.230
	3	0.609	0.524	0.343	0.283	0.225
	4	0.574	0.493	0.324	0.266	0.211
-4.2	1	0.627	0.545	0.357	0.294	0.234
	2	0.603	0.518	0.337	0.277	0.220
	3	0.568	0.484	0.308	0.251	0.197
	4	0.580	0.493	0.313	0.255	0.200
-4.0	1	0.577	0.494	0.311	0.254	0.201
	2	0.531	0.450	0.284	0.232	0.184
	3	0.539	0.452	0.276	0.223	0.173
	4	0.528	0.442	0.271	0.219	0.170
-3.8	1	0.561	0.472	0.283	0.230	0.183
	2	0.522	0.436	0.266	0.217	0.172
	3	0.510	0.420	0.246	0.198	0.154
	4	0.519	0.427	0.251	0.202	0.158

(Jelinsky & Fruscione 1996) that employs a three-dimensional interpolation method on a large database of column densities (Fruscione et al. 1994) in order to estimate the amount of hydrogen for any given direction and distance. The accuracy of the method depends on the number of hydrogen column densities used for the fit in the direction of the source in question. For two of our sources the fit obtained was not satisfactory and for these we used a more recent catalogue of N_H presented by Welsh et al. (1994). It is rather difficult to give proper error estimates due to possible errors in the N_H determinations given in the Fruscione et al. (1994) and Welsh et al. (1994) catalogues as a

result of an inaccuracy in their distances estimates. However, since the above technique is based on an interpolation between N_H 's derived from several stars in the same general direction we estimate that the typical error in our determination of N_H is 0.3–0.4 dex. One of our stars, Gl 803 has perhaps the largest uncertainty due to poor sampling of column densities in this direction of the sky. For N_H less than 10^{18} cm^{-2} the above uncertainty could imply a 18% error in the attenuation factor with an even larger error for N_H greater than $5 \cdot 10^{18} \text{ cm}^{-2}$.

2.4. Accounting for Interstellar Deuterium

The interstellar line of deuterium (D) is located at -0.331 \AA from the center of the Ly α interstellar line and therefore contributes to the attenuation of the stellar Ly α flux. We have taken deuterium into account assuming a deuterium to hydrogen ratio of $1.65 \cdot 10^{-5}$. This value was obtained from the work of Linsky et al. (1993) and is representative for the line of sight to Capella. Given the lack of additional high quality D/H measurements we have assumed a constant D/H ratio. This is consistent with the work of McCullough (1992). The broadening parameter used for deuterium is $b_D = 7.8 \text{ km s}^{-1}$ (also from Linsky et al. 1993) and the corresponding atomic parameters are from Morton (1991). The additional attenuation introduced by D is rather small, being less than 2% at zero radial velocity. Its influence becomes less pronounced at higher N_H .

2.5. Stellar radial velocity

The radial velocity of the star will shift the stellar profile with respect to the interstellar profile and therefore affect the attenuation. The amount by which the attenuation will change depends on the shape of the intrinsic stellar profile. The effect of the radial velocity shift on the attenuation factors for model series 1 & 2 with $\log m_o = -4.8$ is shown in Fig. 2 for a range of interstellar column densities. The difference between the two model series is at most 10%. For radial velocities of $\sim 100 \text{ km s}^{-1}$, the effect can result in a $\sim 20\%$ change to the attenuation factor for an interstellar column mass less than 10^{18} cm^{-2} . However, most of the objects considered here have RV's less than 40 km s^{-1} where the difference is only a few percent.

3. Results and discussion

The results based on the four model series are presented in Table 1 with sample profiles given in Fig. 3. For high density model atmospheres (such as that applicable to an active dM1e star like AU Mic, i.e. $\log m_o \sim -4.0$) there is no difference between model atmosphere series 2, 3 & 4, although those calculated for model series 1 show $\sim 10\%$ difference. For example, taking a low interstellar column mass of $\log N_H = 18.00$ gives an attenuation factor of ~ 0.45 in series 2, 3 & 4 while series 1 gives 0.49. For a high interstellar column mass of $\log N_H = 19.00$ the attenuation factors are ~ 0.22 and ~ 0.25 respectively. However, for the 'quiet' stars, say $\log m_o \sim -4.8$, the attenuation factor for a low interstellar column mass of $\log N_H = 18.00$

Table 2. Mean observed Ly α fluxes (in $10^{-14} \text{ erg cm}^{-2} \text{ s}^{-1}$) for a sample of stars from Doyle et al. (1990), Byrne & Doyle (1989,1990) and Landsman & Simon (1993), the $\log N_H$ value, their attenuation factors for the radial velocity from Woolley et al. (1970), the corrected Ly α fluxes (in $10^{-14} \text{ erg cm}^{-2} \text{ s}^{-1}$), the observed H α flux (in $10^{-14} \text{ erg cm}^{-2} \text{ s}^{-1}$) and finally the Ly α to H α ratio. A zero interstellar wind velocity was assumed

Star	RA 2000	DEC 2000	Sp Ty.	$f_{Ly\alpha}$ observed	$\log N_H$	Ly α Att Factor	$F_{Ly\alpha}$ corrected	$F_{H\alpha}$	$F_{Ly\alpha}/F_{H\alpha}$
Gl 54.1	01 12	-17 00	dM5.5e	130	17.90	0.49	265
Gl 65AB	01 39	-17 58	dM5.5e	175	17.40	0.58	300	75	4.0
Gl 182	04 59	+01 47	dM0.5e	250	18.20	0.43	580	70	8.3
Gl 206	05 32	+09 49	dM4e	130	18.20	0.41	320	50	6.4
Gl 234AB	06 29	-02 48	dM4.5e	160	17.90	0.49	325	122	2.7
Gl 278C	07 35	+31 52	dM1.0e	285	18.30	0.37	770	220	3.5
Gl 285	07 45	+03 34	dM4.5e	170	17.90	0.49	345	185	1.9
Gl 380	10 12	+49 28	dM0	210	18.30	0.43	490
Gl 388	10 20	+19 52	dM3.5e	235	17.90	0.48	490	430	1.1
Gl 411	11 03	+36 02	dM1	165	17.90	0.65	255
Gl 494	13 01	+12 23	dM1.5e	130	17.40	0.59	220	145	1.5
Gl 551	14 30	-62 41	dM5.5e	230	17.40	0.59	390	65	6.0
Gl 644AB	16 56	-08 20	dM3.5e	400	18.30	0.37	1080	275	3.9
Gl 729	18 50	-23 50	dM4.5e	140	18.20	0.39	360	50	7.2
Gl 735	18 55	+08 24	dM2e	120	18.50	0.32	375	95	3.9
Gl 784	20 14	-45 10	dM1	95	17.90	0.53	180
Gl 799AB	20 42	-32 25	dM4.5e	410	18.30	0.37	1110	220	5.0
Gl 803	20 45	-31 20	dM2.5e	390	18.40	0.35	1115	340	3.3
Gl 815A	21 00	+40 05	dM3e	110	18.20	0.45	245	80	3.1
Gl 825	21 18	-38 51	dM0	235	17.90	0.55	430
Gl 867A	22 39	-20 37	dM0e	330	18.20	0.39	845	100	8.4
Gl 873	22 47	+44 21	dM4.5e	450	17.40	0.59	760	175	4.3
Gl 876	22 53	-14 15	dM4.5	300	17.90	0.54	555
Gl 887	23 05	-35 52	dM2.5	205	17.80	0.57	360
Gl 900	23 35	+01 36	dM0	100	18.40	0.41	245

is ~ 0.50 in model series 2, 3 & 4, while model series 1 gives 0.57. For a high interstellar column mass of $\log N_H = 19.00$ the attenuation factors are ~ 0.23 and ~ 0.27 respectively. Thus at first sight it may seem that without proper radiative transfer modelling we would be unable to apply this in a general way to the observed Ly α fluxes. However, with the use of the H α flux the calculation of attenuation factors for individual objects is possible.

In Table 2 we tabulate the observed Ly α fluxes for a sample of M dwarfs observed by Doyle et al. (1990), Byrne & Doyle (1989,1990) and Landsman & Simon (1993). For some objects, Landsman & Simon (1993) tabulated several values for the same object. This variation was in some instances a factor of two and could be interpreted as the intrinsic variability in the objects due to changes in magnetic activity. For the objects in common with the above authors, these were generally in good agreement and was certainly less than the reported variability in individual objects. Thus we simply took the mean of these values.

Returning to the present Ly α profile calculations (see also those of Houdebine et al. 1995), the H α line goes into absorption as the log column mass in the transition region goes below $\log m_o \sim -4.3$. Thus for all the dMe stars (i.e. those objects with H α in emission) in Table 2, we took the mean attenuation factor as derived for a transition region log column mass of

$\log m_o = -4.0$. A few of the objects in Table 2 are dM stars (i.e. H α in absorption). These objects are not the ‘inactive’ stars as analyzed by Doyle et al. (1994) but are instead chromospherically active but on a lesser scale than the dMe objects. For these objects we took the mean attenuation factor as derived for a transition region log column mass of $\log m_o = -4.8$. Reference to Table 1 will verify that the probable error in selecting $\log m_o = -4.8$ instead of $\log m_o = -4.6$ is only a few percent and is substantially less than the errors due to the variability in the observed Ly α flux.

Observed H α fluxes were derived from the equivalent widths as given by Linsky et al. (1982), Stauffer & Hartmann (1986), Pettersen (1989) and Mathioudakis & Doyle (1991). Conversion to flux units was via the R magnitude given by Leggett (1992) and the R-band calibration of Bessell (1979). The reported H α equivalent widths from the above sources in individual stars have a variation of the order of 50%, similar to the Ly α variability. The tabulation in Table 2 is therefore based on the mean of these values. For the binaries, the sum of components were included as the Ly α fluxes also refer to the binary system. In an effort to check the above calibration procedure, we compared our derived H α flux with that derived by Pettersen & Hawley (1989). The value for Gl 388 (the system calibrated by Pettersen & Hawley) differed by less than 10%,

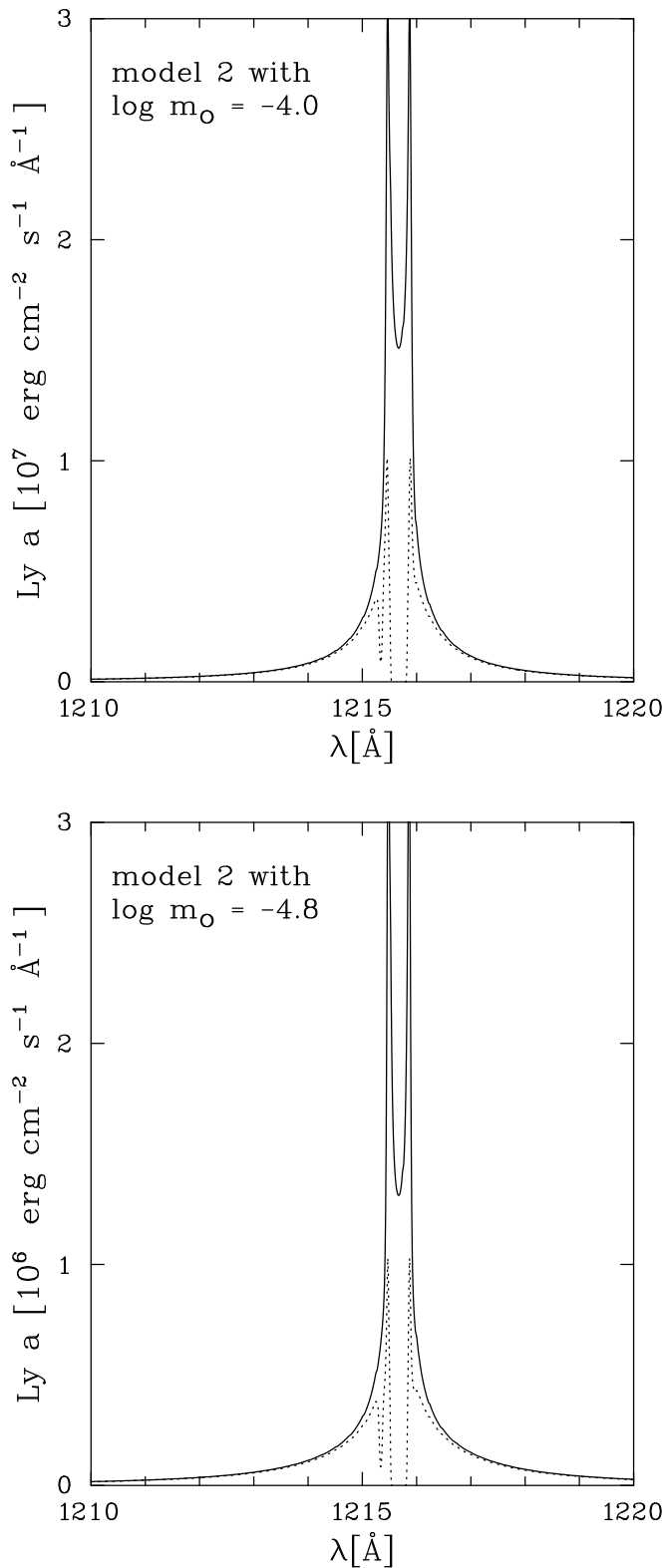


Fig. 3a and b. Ly α profiles before (solid line) and after (dashed line) multiplying by the interstellar attenuation for; **a** model 2 with $\log m_o = -4.0$ (i.e. an active star), **b** model 2 with $\log m_o = -4.8$ (i.e. a 'quiet' star) assuming $\log N_H = 18.00$

which is substantially less than the intrinsic variability in the line. The intrinsic Ly α to H α ratio is also given in Table 2.

All the above calculations assume a zero local interstellar medium velocity, however, recent observations using HST and ground-based data suggest a range of velocities up to $\pm 27 \text{ km s}^{-1}$ (Lallement et al. 1995). However, adopting a non-zero value for the velocity of the interstellar wind would be equivalent to assuming a radial velocity of the same magnitude. Inspection of Fig. 2 shows that there will be no significant effect for N_H greater than $5 \times 10^{18} \text{ cm}^{-2}$, however for $N_H \sim 10^{18} \text{ cm}^{-2}$ the difference may approach a few percent for an interstellar wind velocity of 20 km s^{-1} .

4. Conclusions

The attenuation factors at Ly α were computed for four groups of M dwarf model atmospheres, each with a slightly different structure in the lower atmosphere. The wide range of mass loading (m_o) at the top of the chromosphere explored ranged from very high density model atmospheres representative of the most active flare stars to very low densities applicable for the inactive basal stars. The source of perhaps the largest uncertainty is the interstellar column densities, emphasizing the need for further work in this area and in particular an accurate model of its variation in all directions.

For those M dwarfs with available Ly α data we have used models selected on the basis of H α equivalent widths and computed (i) attenuation factors (ii) intrinsic Ly α fluxes and (iii) the Ly α to H α ratio. Comparing the attenuation factors calculated here with those in common with Landsman & Simon (1993), we find that the present calculations result in attenuation factors which differ by a factor of two. This implies a lower intrinsic Ly α flux than previously calculated. The intrinsic Ly α to H α ratio has a wide range of values, in all cases being greater than unity. The typical value for the active dMe's is $\sim 3 - 5$, although a few have a ratio closer to 2, while others are closer to 8.

Although we have calculated the attenuation factors for a wide range of chromospheric structures, one possible source of error exists in the assumption of complete frequency redistribution. For the Sun, it is known that CRD over-estimates the intensity in the line wings by approximately a factor of five and that profiles calculated with partial redistribution (PRD) fit the observed line much more closely (Vernazza, et al. 1973). However, the density in the atmosphere of an M dwarf is greater than that of the Sun, thus due to an increased number of collisions, the effect of PRD would be much reduced. On the other hand, our M star modelling shows that radiative damping dominates the line broadening throughout the chromosphere where the Ly α wings form, thus suggesting that PRD effects may be important. At present, we do not have Ly α profiles produced using PRD, hence the magnitude of this effect is rather hard to gauge without proper calculations. However, we can say that the present attenuation factors are upper limits which implies a lower limit for the Ly α fluxes in Table 2. This therefore implies that the Ly α to H α ratio in M dwarfs is significantly above unity in all instances.

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