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Electric fields for hydrogen bound-free transitions in magnetic white dwarfs

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Abstract. We present photoionization spectra at optical wavelengths for hydrogen in strong parallel electric and magnetic fields for magnetic white dwarfs obtained with the complex coordinate method. It is shown that spectra calculated for fixed magnetic and electric fields can be averaged over a distribution of electric fields. Such averaged spectra are presented for fixed magnetic field with a Holtsmark distribution of electric fields at wavelengths around $580\,nm$ interesting for a stationary line at $23\,500\,T$.

Key words: atomic data – atomic processes – lines: profiles – stars: magnetic fields – white dwarfs

1. Introduction

Some white dwarf stars possess a strong magnetic field. This was discovered when atomic data on the hydrogen atom in strong magnetic fields became available and the spectrum of these white dwarfs could be explained through stationary transitions of hydrogen over a range of magnetic fields. White dwarfs with magnetic fields up to $100\,000\,T$ have been identified. Absorption features in the spectrum of these stars are caused by hydrogen transitions whose variation of the wavelength with the magnetic field goes through an extremum in the range of fields visible on the star. Without other influences the drop in the absorption spectrum would occur suddenly at the wavelength of the stationary point of the transition. Depending on whether this is a minimum or maximum of the wavelength a dip in the spectrum extends to greater or smaller wavelengths until the wavelength changes so quickly with the magnetic field that no enhanced absorption is visible and the line has become part of the background.

In actual observations though, the onset of the absorption feature is often shifted from the theoretical value of the stationary line by a few Ångström and does occur rather smoothly than all of a sudden. The suspected reason for this are electric fields which shift the energy levels and thus alter the wavelengths. Another problem is that the exact form of the background, on which the absorption feature is superimposed, is not known. Depending on the shape of the background the onset of the absorption feature in the observed spectrum is difficult to locate and can appear shifted from its real value for a constant background.

The background absorption is comprised of bound-bound transitions for non-stationary lines, bound-free transitions and free-free transitions. Photoionization spectra in strong magnetic fields have recently been calculated for bound initial states. Meinhardt (1993) used a direct integration method, Wang and Greene (1991) employed a multichannel R-matrix method, Delande et al.(1991) and Merani et al.(1995) used the complex coordinate method. In the presence of an electric field, all states are coupled to the continuum of the electric field through tunneling and thus become quasi-bound. Tunneling though becomes important only for states near the classical ionization threshold and can be neglected for lower lying states (Seipp et al.1996).

In the atmosphere of a white dwarf an atom is exposed to the electric fields of surrounding atoms, ions and free charges. The motional Stark effect gives an electric field perpendicular to the magnetic one. Thus, the actual field to which each atom is exposed varies statistically (Mathys 1989). The mean values of the electric field felt by each atom in the atmosphere of a magnetic white dwarf are presumed to be of the order 10^6 to $10^7\,V/m$. It has been found that for the same strengths of fields an electric field which is in parallel to an external magnetic field has the strongest influence on the transitions whereas the effect of an electric field perpendicular to the magnetic one is negligible for the field strengths considered (Faßbinder and Schweizer 1996).

Here, we examine the contributions to the background of bound-free transitions in the atmosphere of magnetic white dwarfs. We calculate bound-free transitions from $n = 3$ initial states for hydrogen in a strong magnetic field with electric fields of various strengths parallel to the magnetic field. These are the essential contributions from these transitions to the optical part of the spectrum. But note, that for calculating rigorous total opacities additional transitions from higher n-level initial

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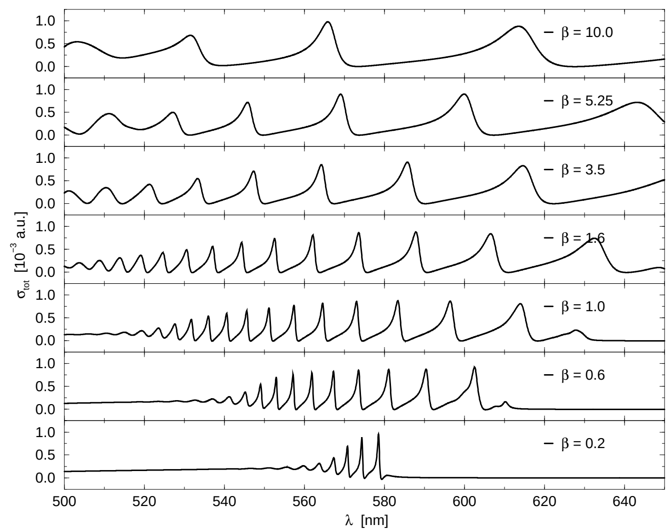

Fig. 1. Photoionization spectra in parallel fields for σ^+ polarization from $n = 3, m = -1$ initial states. The magnetic field is $\gamma = 0.1$ for all spectra and the electric field is given by $F = \beta \cdot F_H$ with $F_H = 10^7\, V/m$.

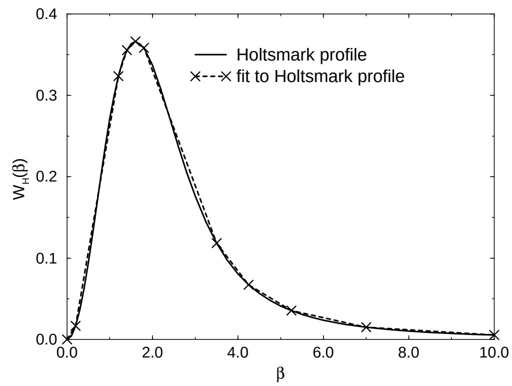

Fig. 2. The Holtsmark distribution of electric fields $W_H(\beta)$. β is defined by F/F_H, where F_H is the mean electric field. The Holtsmark profile is fitted by a choice of 11 points on the distribution and linear interpolation in between these points.

states, as well as bound-bound and free-free transitions have to be taken into account.

The next section gives an overview over the physical problem and its theoretical and numerical treatment used in this work. Sect. 3 describes how to obtain photoionization spectra which have been averaged over an electric field distribution. In Sect. 4 the results of various calculations are presented.

2. Theoretical treatment

The calculation of the discrete eigenstates of the hydrogen atom in strong external fields has become possible through the use of super-computers (Ruder et al.1994). The problem is to solve the Schrödinger equation in the strongly n-mixing regime, n being the principal quantum number, where neither the spherical symmetry of the hydrogen atom nor the cylindrical symmetry of an electron in a magnetic field prevails.

The Hamiltonian for hydrogen in parallel electric and magnetic fields is given by

$$H = -\frac{1}{2}\triangle - \frac{1}{r} + \frac{1}{2}\gamma(l_z + 2s_z) + \frac{1}{8}\gamma^2 r^2 \sin^2\vartheta + Fr\cos\vartheta \quad ,(1)$$

where atomic units and spherical coordinates (r, ϑ, φ) are used. The spin is neglected in actual calculations since it only results in a constant shift of the spectrum and $l \cdot s$-coupling is weak compared to the effect of the external fields. The magnetic field γ is measured in units of $B_0 = \frac{\mu_B B}{8R_\infty} = 2.35 \cdot 10^5\, T$ and the electric field in units of $F_0 = \frac{eEa_o}{2R_\infty} = 5.142 \cdot 10^{11}\, V/m$.

The Hamiltonian (1) is exact only for infinite nuclear mass. At laboratory strength magnetic fields ($\approx 20\,T$ or $10^{-4}\,a.u.$) the effect of the finite proton mass is negligible. For the strong magnetic fields in white dwarfs the finite proton mass is accounted for by a mass scaling factor which is exact for parallel fields (Pavlov-Verevkin and Zhilinskii 1980a, 1980b).

The calculation of continuum states poses additional problems because they are not square-integrable. In recent years though, the complex coordinate method (Reinhardt 1982, Ho 1983) has been successfully applied to these states and photoionization cross-sections have been calculated for strong fields as well as laboratory strength fields (Delande et al.1991, Halley et al.1992, 1993, Seipp and Taylor 1994, Merani et al.1995, Alvarez et al.1991).

In this method the coordinates are transformed by a complex rotation,

$$\boldsymbol{r} \rightarrow \boldsymbol{r}e^{i\theta} \quad . \quad (2)$$

With this transformation the Hamiltonian of the system is continued into the complex plane and becomes non-hermitian. The spectrum of the rotated Hamiltonian consists of real eigenvalues, which are the same as the bound states of the unrotated Hamiltonian, continuum branches, which are rotated about their respective branch points, and discrete complex eigenvalues which are associated with the resonances of the real system. The position of the resonance is given by the real part of the eigenvalue and its FWHM by the imaginary part. The eigenfunctions of the complex eigenvalues are square-integrable.

The Schrödinger equation with the Hamiltonian in (1) rotated by (2) is solved numerically by expanding the wavefunctions over a complete basis set of Sturmians (Clark and Taylor 1982) forming a Hamiltonian matrix. By taking basis functions with real coordinates the expansion coefficients are complex and the Hamiltonian matrix is complex symmetric. The matrix is then diagonalized using a complex implementation written by Delande et al.(1991) of the Spectral Transformation Lanczos algorithm.

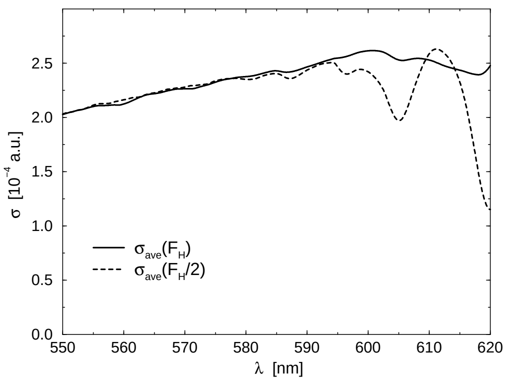

Fig. 3. Two photoionization cross-sections at magnetic field $\gamma = 0.1$ averaged over the Holtsmark distribution of electric fields for mean electric fields $F_H = 10^7\ V/m$ and $5 \cdot 10^6\ V/m$.

Photoionization cross-sections are obtained from the complex eigenvalues E^j and 'rotated' eigenfunctions Ψ^j of the Hamiltonian by (Rescigno and McKoy 1975)

$$\sigma(E) = 4\pi\alpha(E - E_0)\,\mathrm{Im} \sum_j \frac{< \Psi_i|TU^{-1}|\Psi^j >^2}{E^j - E} \quad , \qquad (3)$$

where Ψ_i is the real initial state with energy E_0, T is the dipole operator and U the unitary rotation operator (Reed and Simon 1978), α is the fine-structure constant and E the energy with respect to the field free ionization limit in atomic units. The sum is taken over all eigenstates j of the discretized rotated Hamiltonian.

The electron density distribution at real energy E is given by (Buchleitner et al.1994)

$$\sum_k |\Psi_E^k(\boldsymbol{r})|^2 = \frac{1}{\pi}\,\mathrm{Im} \sum_j \frac{< \boldsymbol{r}|U^{-1}|\Psi^j(\theta) >^2}{E^j(\theta) - E} \quad . \qquad (4)$$

Here, k labels all degenerate states at energy E.

It is thus possible to calculate photoionization spectra from the complex coordinate method where the initial state is disturbed by strong magnetic and electric fields. The line profile of the photoionization spectrum is incorporated in the complex dipole matrix elements $< \Psi_i|TU^{-1}|\Psi^j >$ and the complex energy eigenvalues.

3. Averaging over an electric field distribution

We calculated photoionization spectra in a strong magnetic field with parallel electric fields of various strengths. Fig. 1 shows spectra for a magnetic field of $\gamma = 0.1$ and several electric fields in the optical wavelength range of 500 up to 650 nm. The $3p_{-1}$ and $3d_{-1}$ initial states are included which are the only contributions at this wavelength range from the $n = 2$ and $n = 3$ manifolds, the Balmer and Paschen series. The $3d_{-1}$ initial state only gives contributions for the highest electric fields.

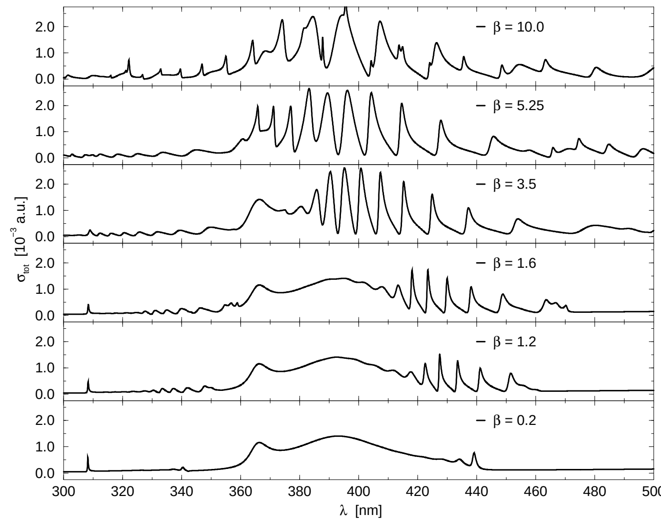

Fig. 4. σ^+ Paschen opacities at $\gamma = 0.1$, electric fields $F = \beta \cdot 10^7\ V/m$ and temperature $T = 15\,000K$.

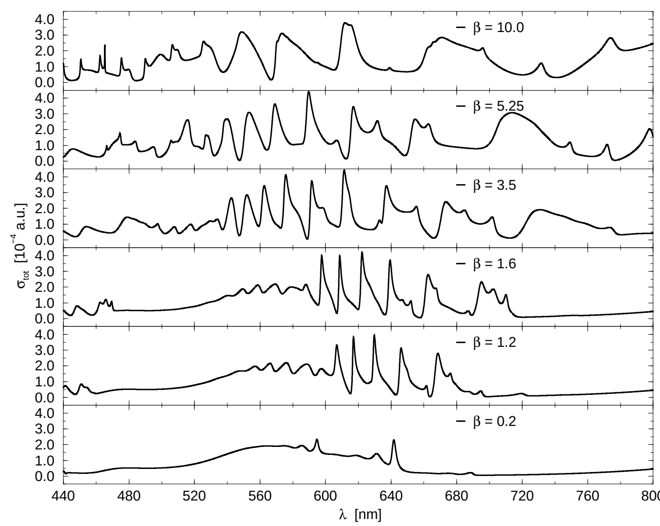

Fig. 5. π Paschen opacities with external fields as in Fig. 4.

The electric field modulates the ionization spectra strongly through an onsetting resonance structure. The resonance features are similar in shape and can be followed moving to lower wavelengths as the electric field is increased. The spectra with electric field are very different from the smooth line near the ionization limit for magnetic field only.

The electric fields which act on each atom in the atmosphere of the white dwarf are distributed statistically. Thus, it is interesting to know the photoionization spectrum averaged over a range of electric fields.

In the absence of an external magnetic field the distribution of electric fields induced by ions was given by Holtsmark (1919). As of now, we do not know of an analytical expression for the distribution in the presence of a strong magnetic field. We thus chose the Holtsmark distribution as a first guess for the electric fields, although an arbitrary distribution can be used with the numerical method employed.

Photoionization spectra were calculated for certain electric fields at constant magnetic field. The electric fields were chosen to give a good approximation to the Holtsmark profile $W_H(\beta)$ when linearly interpolated in between these points. The Holtsmark distribution and a 11 point fit are shown in Fig. 2. The Holtsmark distribution has been tabulated for example by Mozer and Baranger (1960). The factor β is given by F/F_H, where $F_H = 2.61 Ze\, N_i^{2/3}$ is the mean electric field for ion charge Ze and ion density N_i.

Position and width of a resonance are a smooth function of the electric field so that spectra for other electric fields can be interpolated from the ones calculated. The step width of the electric fields only has to be small enough to make identification of particular resonances possible. The averaged photoionization spectrum over electric fields is then obtained by integrating the Holtsmark weighted spectra over the electric fields. The integral is taken as a summation over integrals between electric field values F_k to F_{k+1} , $k = 1, \ldots, f-1$, where f is the number of field values for which spectra have been calculated. The integrals over each electric field step k again are taken as a sum over integrals of smaller field steps $F_{k,i}$ to $F_{k,i+1}$, $i = 1, \ldots, m-1$, with $F_{k,m} = F_{k+1,1} = F_{k+1}$,

$$\sigma_{ave}(\lambda) = \sum_{k=1}^{f-1} \sum_{i=1}^{m-1} \int_{F_{k,i}}^{F_{k,i+1}} W_H(F)\, \sigma(\lambda, F)\, dF \qquad (5)$$

The remaining integral is solved by taking W_H and σ as linear functions over the interval of integration. The photoionization spectrum at electric field $F_{k,i}$ is linearly interpolated from the spectra calculated for F_k and F_{k+1}. The value of the Holtsmark distribution at $F_{k,i}$ is also linearly interpolated from the neighboring fields or the closest values tabulated. The number of interpolations is given by m which has to be big enough to give a smooth curve. For $m = 1$ the calculated spectra are simply summed up and the averaged spectrum is zig-zaggy because the resonances in each spectrum are treated as if they were isolated and not linked to the peaks of other spectra. Through the above interpolation one avoids having to explicitly calculate all the intermediate spectra.

The ionization limit of the pure magnetic field, the first Landau threshold, does not exist quantum-mechanically with a parallel electric field. With increasing electric field resonances rearrange and bound states for wavelengths above the threshold become resonances. Narrow resonances for higher wavelengths, which can still be considered almost bound, have been omitted in the spectra to give the ionization part only. In the complex rotation method where the Hamiltonian is expanded over a basis the resolution near the ionization threshold is limited by the basis size. For example, in the pure magnetic field spectrum the Landau threshold is not completely resolved. But this causes no problem since dense bound states form a quasi-resonance structure when they are not resolved experimentally and the Holtsmark distribution is zero for zero electric field.

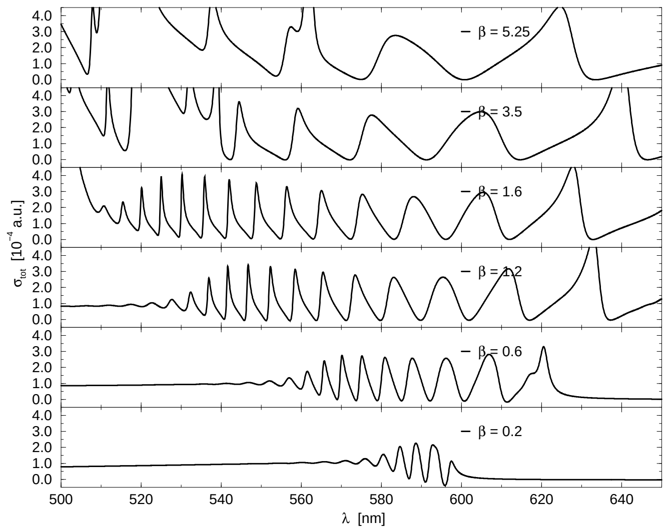

Fig. 6. Unnormalized Paschen opacities for σ^+ transitions for magnetic field $\gamma = 0.08$, electric fields $F = \beta \cdot 10^7\, V/m$ and temperature $T = 15\,000 K$.

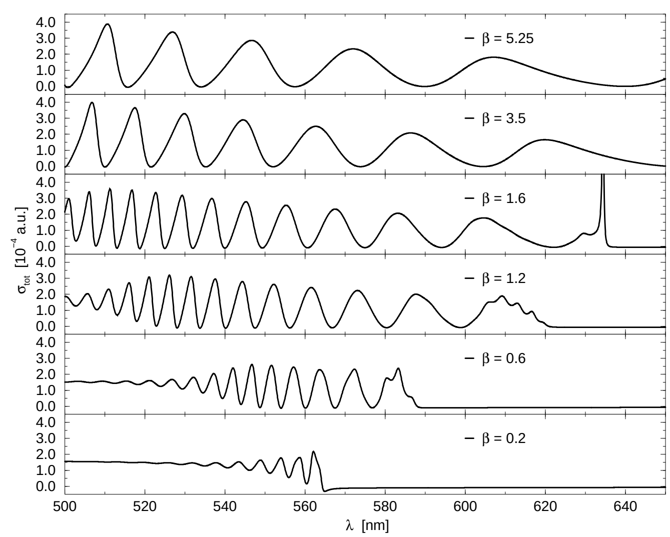

Fig. 7. Same as Fig. 6, but now for $\gamma = 0.12$.

Fig. 3 shows the spectrum at magnetic field $\gamma = 0.1$ averaged over a Holtsmark distribution of electric fields with mean field $F_H = 1 \cdot 10^7\, V/m$ and $5 \cdot 10^6\, V/m$. For this plot spectra at 16 values of the electric field were calculated. The maximum electric field included was $\beta = 15$ at less than 1 per cent of the maximum of the Holtsmark distribution.

4. Results and discussion

In this section we present the results of several calculations on the influence of parallel electric fields on magnetic field Paschen bound-free opacities. Such opacities have recently been calculated by Merani et al.(1995) for strong magnetic fields using the complex coordinate method. In these unnormalized opacities all transitions for a given polarization starting from the $n = 3$ manifold are included. They are calculated as

$$\sigma_{tot}(\lambda, F) = \sum_i \mathrm{e}^{-(E_i - E_{gs})/kT}\, \sigma_i(\lambda, F) \ , \quad (6)$$

where E_i is the energy of the initial state i, E_{gs} is the ground state energy at fields γ and F, k the Boltzmann constant and T the temperature. The temperature gives the absolute value of the opacity and determines the relative strength of each transition. The shape of the total cross-sections does not change dramatically for temperatures varying from 10 000 to 25 000 K, interesting for white dwarfs. For a single transition the Boltzmann factor simply scales the amplitude. A calculation of normalized opacities in LTE involves solving the Saha equation for the magnetic fields in question, a task, which we have not undertaken in this work. Ventura et al.(1992) solved this problem for neutron star magnetic fields.

Figs. 4 and 5 present the Paschen opacities σ_{tot} for a magnetic field of $\gamma = 0.1$ and several electric field strengths at optical wavelengths and for a temperature of 15 000K. Fig. 4 gives the σ^+ polarization whereas Fig. 5 is for π polarization. Note, that Fig. 1 are also σ^+ Paschen opacities for wavelengths above 500 nm. Strong oscillations are induced by the electric field at the thresholds of each transition from the $n = 3$ initial states.

Again, the oscillations from each transition are a smooth function of the electric field and averaging over a distribution of electric fields can be done in the same way as described above. Overlapping resonance structures from different transitions can be treated individually since they are additive. The effect of the electric fields on a single transition can be studied in Fig. 1, where the $3d_{-1}$ contributions to the opacity are negligible.

A widely studied magnetic white dwarf star is Grw+70°8247 (Wickramasinghe and Ferrario 1988, Ruder et al.1994). Its pole strength of the magnetic field is 35 000 $T \approx 0.15$ in atomic units. A dipolar magnetic field varies in strength from the maximum value at the poles to half this value at the equator of the star. Therefore, a range of magnetic fields gives rise to the measured spectrum, depending on the angle of observation with respect to the dipole axis, which changes as the white dwarf rotates. In the case of Grw+70°8247 there is a strong absorption feature in the observed spectrum given by the stationary line of the $2s_0 \rightarrow 3p'_0$ transition at 583 nm and $\gamma = 0.1$. This absorption feature though actually starts at a wavelength between 582 and 582.5 nm. Therefore, interest exists in this region to get a better understanding of the beginning of this line.

In Figs. 6 and 7 the Paschen bound-free opacities for the same range of wavelengths as in Fig. 1 are shown for magnetic fields $\gamma = 0.08$ and 0.12 and various electric fields, respectively.

The maximum strength of the σ^+ Paschen opacities in the optical is larger by roughly one order of magnitude than the maximum strength of the π opacities and two orders of magnitude than the σ^- ones. But the maximum of the π opacities lies within the wavelength range of interest whereas the σ^+ opacities are not maximal therein. Thus, the total cross-sections of the two polarizations are of comparable size for these wavelengths.

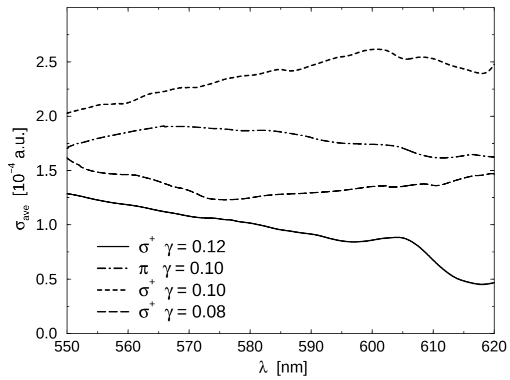

Fig. 8. Paschen opacities for the transitions of the previous figures averaged over a Holtsmark distribution of electric fields with $F_H = 10^7\, V/m$ for wavelengths between 550 and 620 nm. Polarization and magnetic field as labeled, $T = 15\,000K$.

The total cross-sections are averaged over a Holtsmark distribution of electric fields with $F_H = 10^7\, V/m$. The resulting averaged cross-sections σ_{ave} for wavelengths between 550 and 620 nm are plotted in Fig. 8 for the σ^+ transition at magnetic fields $\gamma = 0.08$ and 0.12 and for the π transition at $\gamma = 0.1$. The $\sigma^+, \gamma = 0.10$ cross-section is included for completeness. The upper limit of the electric fields included was taken at a field for which the Holtsmark distribution has dropped to less than 2 per cent of its maximum. In all cases, the strong resonance features for the electric field exhibited by the unaveraged spectra are smeared out. For the $\gamma = 0.08$, 0.12 averaged curves photoionization spectra were calculated at 15 values of the electric field, for the π-transition 13 values were used. The convergence of the single electric field spectra was checked to be sufficient for convergence of the averaged spectra. Typical parameters used in the calculations were 30 000 for the basis size with a maximum of 40 000 , the Sturmians with parameter $\zeta = 2$ and the rotation angle $\theta = 0.1$. Calculation times were up to 1000s on a Cray C90 for a spectrum at fixed fields and each run yielding about 50 eigenvalues.

For a further discussion of the averaged cross-sections we compare them to a photoionization cross-section at a certain electric field in Fig. 9. Two conclusions can be drawn from this figure. Firstly, the resonance structure of the electric field spectrum becomes almost linear when averaged over a broad distribution. Secondly, the spectrum for pure magnetic field can be continued smoothly across the ionization threshold of each transition.

In Fig. 9a for $\gamma = 0.08$ the averaged spectrum diverges at the lower wavelength end from the spectrum for a particular electric field plotted. This is due to the next transition mixing in to this wavelength region at electric fields, which still have considerable contributions in the distribution.

Of course, the averaged spectra depend on the distribution of the electric field. In case of a much narrower distribution, the resonances of the electric field can not be expected to smear out completely. Rather, peaks are to be expected at positions as in the spectrum for the maximum field, but which are broader and flatter depending on the width of the distribution. Also, for a much smaller mean value of the electric field F_H of the Holtsmark distribution the extension of the spectrum across the Landau threshold is not as smooth and does not go as far beyond the threshold because the states close to the threshold play a greater role and eigenstates of the system are still almost bound for higher states closer to the threshold.

The modulations in the averaged spectra calculated are smooth and small compared to the absolute value. The spectra can be approximated by a straight line. This even more so, when one thinks of spectra for slightly different magnetic fields being superposed. When looking on the scale of a few nm around 583 nm the spectra change so little that the shift of the beginning of the absorption line can not be explained by these opacities. On the other hand the stationary transition itself gets shifted towards lower wavelengths with the electric fields. For an electric field of $F = 10^{-4} = 5.14 \cdot 10^7\ V/m$ the shift is already $0.84\ nm$. The coupling to the continuum of the $3p_0'$ state in this transition starts to be non-neglible at an electric field of about $F = 9 \cdot 10^{-4}$. By then the line has moved to $555.36\ nm$ (from $592.99\ nm$ at zero electric field). Therefore, e.g. almost any shift of this stationary line could be explained by an electric field. The appearance of the strong absorption line in the spectrum of the white dwarf limits the statistically relevant electric fields to a few $10^7\ V/m$.

5. Conclusion

We presented hydrogen photoionization cross-sections in external parallel electric and magnetic fields. The spectra were calculated with the complex coordinate method. The field strengths considered in this work are for applications to magnetic white dwarf stars. Transitions for optical wavelengths were investigated.

Examples of unnormalized Paschen opacities have been calculated at certain values of the fields for σ^+ and π polarization. An averaging of the photoionization spectra over a Holtsmark distribution of electric fields at given magnetic field has been performed. Other data than the one presented in this work is available on request.

The electric field has been found to have a strong influence on the spectra of the pure magnetic field. A rich resonance structure is created by the electric field starting from the ionization thresholds of the pure magnetic field. These resonances behave smoothly when the electric field strength is changed.

When the average over a Holtsmark distribution with mean electric field of $10^7\ V/m$ is taken, the resonance structure disappears and the spectra appear as little modulated lines because the resonances move too quickly with change in electric field. The averaged spectrum can be interpreted as the spectrum for pure magnetic field extended across the ionization threshold. It has to be stressed though, that the averaged spectra are dependent on the electric field distribution. Other cases are available on request.

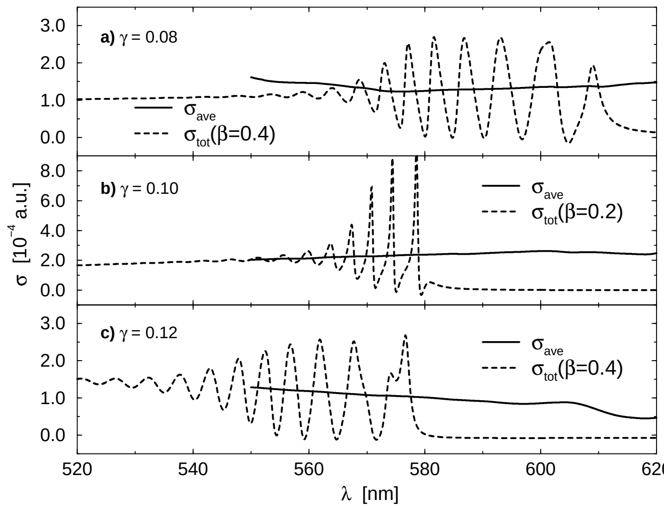

Fig. 9. Electric field averaged cross-sections σ_{ave} for σ^+ polarization at magnetic fields $\gamma = 0.08, 0.10, 0.12$ plotted together with a photoionization spectrum σ_{tot} at the same magnetic field and electric field $F = \beta \cdot 10^7\ V/m$.

The modulations appearing in the electric field averaged opacities are found to be too weak to explain an offset in the absorption profile of a stationary line. Rather, the opacities can be approximated by straight lines over the small wavelength range in question. Transitions from other initial states than the Paschen states considered here must be included in a rigorous treatment but are presumed to have an even smaller effect.

Another interesting feature of the photoionization spectra in parallel fields is the occurrence of very narrow resonances for non-zero electric field. Such narrow resonances also exist in the pure magnetic field case. Finding stationarity of such narrow resonances over a range of magnetic fields as for bound-bound transitions or locality in the field strengths where they exist could make identification of unknown lines and of electric fields existing on the white dwarf possible. Unfortunately though, the narrow resonances which we have investigated are an integral part of the ionization spectrum. Their position strongly depends on the magnetic field strength which determines the Landau threshold. The wavelength of a transition to this narrow resonance thus varies over a wide area for a magnetic field distribution and no absorption feature can be expected to be seen in the observed spectra.

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