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The true shapes of the globular clusters in M 31

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Abstract. The true shapes of the globular clusters (GCs) in M 31 have been derived from observed ellipticities. The non-parametric kernel estimates for the axis ratios distributions in the case of oblate or prolate spheroid hypothesis and the parametric estimates in the case of triaxial ellipsoid hypothesis have been used. The cluster shapes are inconsistent with the oblate/prolate hypothesis and only the shapes of the GCs situated beyond the outermost visible limit of M 31 disk (so called "halo" clusters) are consistent with that hypothesis. We conclude that the GCs in the M 31 are most likely triaxial ellipsoids. The distributions of intrinsic axis ratios derived on the base of triaxial ellipsoid hypothesis have been compared with results published earlier. It has been found out that the clusters in M 31 are more spherical than those in the LMC and in the SMC while the GCs in the Galaxy are rounder than those in M 31.

Key words: globular clusters: general – galaxies: individual: M 31 – galaxies: star clusters

1. Introduction

It is well known that GCs have a nonspherical shape. Studies of their ellipticities $\varepsilon = 1 - b/a$, where a and b are the major and the minor semiaxes of the projected images, indicate that the apparent shapes vary from cluster to cluster and from one galaxy to another. The most probable cause of this behavior are some differences in the dynamical evolution of GCs and parents galaxies as a whole.

Unfortunately, the projected shapes we may determine from the observations could not provide directly information about their current intrinsic shape – oblate/prolate spheroids or triaxial ellipsoids. Therefore, the estimation of the average true shapes of GCs is an important astronomical problem.

Several methods of investigation of the distributions of true axial ratios for galaxies, clusters and clusters of galaxies have been proposed. Most of them are based on the integral equations, first used by Hubble (1926), which relate the probability densities of the true and apparent axial ratios in a population of randomly oriented oblate or prolate spheroids of revolution – the only hypothesis ensuring some information about the intrinsic shape (see also Sandage et al. 1970; Binney 1978).

Within the framework of the parametric function estimation an analytical solution of this problem has been proposed by Sandage, Freeman & Stokes (1970) and Fall & Frenk (1983). Di Fazio & Flin (1988) have employed an analytical approximation of the observed axial ratio distribution and a numerical integration to obtain the distribution of apparent axial ratios. Another procedure has been applied from Plionis, Barrow & Frenk (1991) in the case of revolution spheroids. They have chosen an analytical distribution as a model of the true axial ratios of galaxy clusters. Varying the parameters of this distribution and integrating numerically they attain a fit to the apparent axial ratio distribution.

Within the framework of the nonparametric approach the rectification iterative algorithm developed by Lucy (1974) has been used from Noerdlinger (1979) and Binney & de Vaucouleurs (1981) to estimate the true galaxy shape as a function of their morphological type. Fasano & Vio (1991) have applied the same method to the new measurements of the flattening of elliptical galaxies. Han & Ryden (1994) and Ryden (1995) have taken advantage of the nonparametric kernel estimators to investigate the intrinsic shapes of GCs in the Galaxy, M 31, LMC and SMC, of dwarf elliptical galaxies in the Virgo cluster, of ordinary elliptical galaxies, of brightest cluster galaxies and of clusters of galaxies.

The purpose of this paper is to examine the intrinsic axial ratio distribution of GCs in Andromeda galaxy on the base of the relatively large sample of ellipticities measured by Staneva, Spassova & Golev (1996). The result is compared to the similarly achieved distributions for GCs in M 31 (Han & Ryden 1994).

2. Deprojection

2.1. Inversion formulae

As a result of deprojection we hope to get some notion about the intrinsic form of the GCs but this is possible only after the sup-

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positions that the globulars are randomly oriented with respect to us and that they are the oblate or prolate spheroids. In principal, for the nonspherical shapes of the GCs several effects could be responsible. Among them the internal rotation and velocity anisotropy are the most probable causes of the globular cluster's flattening (Fall & Frenk 1983; Han & Ryden 1994). Each of them works at a different evolution stage. The rotation plays the major role in relaxed clusters and the velocity anisotropy gives a significant contribution to the shape formation in the early cluster evolution.

For the GCs having intrinsic shapes mainly supported by an internal rotation we can adopt the oblate spheroid model, whereas the presence of velocity anisotropy requires a triaxial ellipsoid model. Using the commonly adopted notation in the last case we have $x^2+y^2/\beta^2+z^2/\gamma^2 = a^2$, where a, b and c are the semiaxes of the ellipsoid, $\beta = b/a, \quad \gamma = c/a, \quad 0 \leq \gamma \leq \beta \leq 1$. The projected shape can be written as $x^2 + y^2/q^2 = \overline{a}^2$, where $q = \overline{b}/\overline{a}$ is the apparent axial ratio, $\overline{a}$ and $\overline{b}$ are the semiaxes of the ellipse and $0 \leq q \leq 1$.

Let us suppose the GCs are oblate spheroids ($\beta = 1$). The probability densities of the intrinsic $\Psi_o(\gamma)$ and apparent $\Phi(q)$ axial ratios in a population of randomly oriented objects are related as follows (Sandage et al. 1970; Binney 1978):

$$\Phi(q) = q \int_0^q \frac{\Psi_o(\gamma)}{\sqrt{1-\gamma^2}\sqrt{q^2-\gamma^2}} d\gamma, \tag{1}$$

or after inversion (Fall & Frenk 1983)

$$\Psi_o(\gamma) = \frac{2}{\pi}\sqrt{1-\gamma^2}\frac{d}{d\gamma}\int_0^\gamma \Phi(q)\frac{dq}{\sqrt{\gamma^2-q^2}}. \tag{2}$$

Hereafter we assume $\Phi(0) = 0$ – a condition undoubtedly satisfied for the GCs provides, by the way, unique inversion for the oblate case Eq. (2) (Plionis, Barrow & Frenk 1991). An integration by parts leads to a suitable form for numerical evaluations

$$\Psi_o(\gamma) = \frac{2}{\pi}\frac{\sqrt{1-\gamma^2}}{\gamma}\int_0^\gamma q\frac{d}{dq}\Phi(q)\frac{dq}{\sqrt{\gamma^2-q^2}}. \tag{3}$$

In the case of prolate spheroids the integral equation

$$\Phi(q) = \frac{1}{q^2}\int_0^q \frac{\gamma^2\Psi_p(\gamma)}{\sqrt{1-\gamma^2}\sqrt{q^2-\gamma^2}} d\gamma \tag{4}$$

gives the relation under consideration and after inversion unique without any conditions we have

$$\Psi_p(\gamma) = \frac{2}{\pi}\frac{\sqrt{1-\gamma^2}}{\gamma^2}\frac{d}{d\gamma}\int_0^\gamma q^3\Phi(q)\frac{dq}{\sqrt{\gamma^2-q^2}}. \tag{5}$$

The respective numerical evaluation formula takes the form:

$$\Psi_p(\gamma) = \frac{2}{\pi}\frac{\sqrt{1-\gamma^2}}{\gamma^3}\int_0^\gamma q\frac{d}{dq}\left[q^3\Phi(q)\right]\frac{dq}{\sqrt{\gamma^2-q^2}}. \tag{6}$$

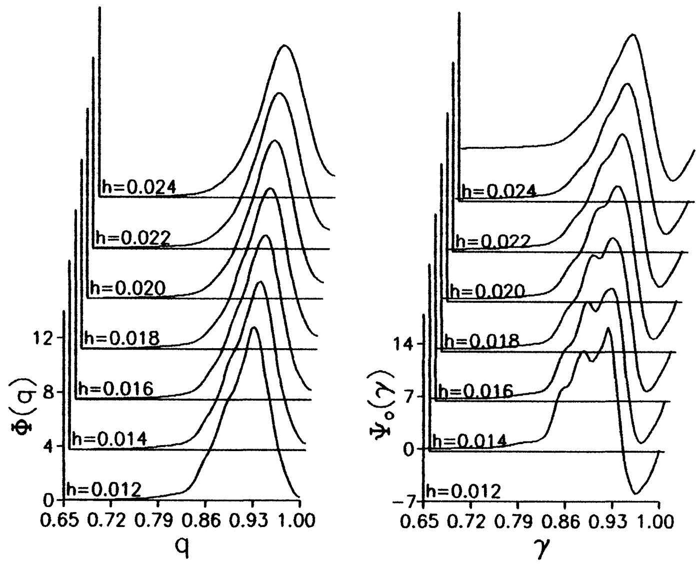

Fig. 1. The set of estimations to the distributions $\Phi(q)$ and $\Psi_o(\gamma)$ that serve us for determination of the optimal smoothing parameter h. For the considered sample of 172 GCs in M 31 (Staneva, Spassova & Golev 1996) we receive h = 0.019.

Although we expect an oblate spheroid form according to evolutionary ideas, there is no way to receive an unequivocal notion about the population composition of both oblate and prolate spheroids. Indeed, if the integral equations (1) and (4) have unique inversions the probability distribution of apparent axial ratios of a oblate-prolate mixture does not necessarily have a unique inversion (see Theorem in Plionis, Barrow & Frenk 1991).

2.2. Nonparametric approach

The nonparametric approach to the estimation for unknown density distribution $\Phi(q)$ is necessary if there are not any assumptions about the shape of distribution function. In this subsection we use the density of smoothed empirical distribution – so called Parzen-Rosenblatt density estimate (Parzen 1962; Rosenblatt 1956) or kernel estimate (Borovkov 1984; Silverman 1986). Its usual form is

$$\widehat{\Phi}_n(q) = \frac{1}{n\,h_n}\sum_{i=1}^n K\left(\frac{q-q_i}{h_n}\right), \tag{7}$$

where we denote the sample of measured axial ratios as $\{q_i\}_1^n$, $K(q)$ – the kernel function normalized to unity and h_n – smoothing parameter. The kernel estimate $\widehat{\Phi}_n(q)$ approximates the density distribution $\Phi(q)$ well if $K(q)$ is sufficiently smooth and bounded function (Borovkov 1984). The optimal choice of $K(q)$ and h_n is embarrassed by reason of their dependence on the smoothness of the unknown function $\Phi(q)$. Let us assume about the functions $\Phi(q)$ and $K(q)$:

a) an existence and continuity of $\Phi''(q)$;
b) $d^2 \equiv \int K^2(q)\, dq < \infty$;

c) $\int qK(q)\,dq = 0$ (this condition is always fulfilled for symmetric kernels $K(q)$);
d) $D^2 \equiv \int q^2 K(q)\,dq < \infty$.

Then we can derived the optimal value of h_n that minimizes the integrated mean square error $I_n \equiv \int E\left[\widehat{\Phi}_n(q) - \Phi(q)\right]^2 dq$ asymptotically (Borovkov 1984)

$$h_n^{opt} = \left[d^2/(nD^4\varphi)\right]^{\frac{1}{5}} \quad (8)$$

and we get the minimum value of I_n

$$I_n^{\min} = 5/4\varphi^{\frac{1}{5}} \left(D^2 d^4\right)^{\frac{2}{5}} n^{-\frac{4}{5}} + o\left(n^{-\frac{4}{5}}\right), \quad (9)$$

where $\varphi \equiv \int \left[\Phi''(q)\right]^2 dq$. The Eq. (9) allows us to determine the optimal function $K(q)$ that minimizes D^2d^4 on condition that $\int K(q)\,dq = 1$ and $\int qK(q)\,dq = 0$. Such function is the Epanechnikov kernel $K(q) = 3(1-q^2)/4, |q| < 1$.

As you can see [Eqs. (3) and (6)] to obtain the intrinsic distributions $\Psi(\gamma)$ we need a proper approximation of the first derivative of $\Phi(q)$. Using the first derivative of $\widehat{\Phi}_n(q)$ as such estimator and supposing instead of a) and b)

$a')$ an existence and continuity of $\Phi'''(q)$;
$b')$ $e^2 \equiv \int \left[K'(q)\right]^2 dq < \infty$,

we are able to determine the optimal value of h_n for this case and the corresponding minimum of $J_n \equiv \int E\left[\widehat{\Phi}'_n(q) - \Phi'(q)\right]^2 dq$:

$$h_n^{opt} = \left[e^2/(nD^4\psi)\right]^{\frac{1}{7}}, \quad (10)$$

$$J_n^{\min} = 7/4\psi^{\frac{3}{7}} \left(D^6 e^4\right)^{\frac{2}{7}} n^{-\frac{4}{7}} + o\left(n^{-\frac{4}{7}}\right), \quad (11)$$

where $\psi \equiv \frac{1}{3}\int \left[\Phi'''(q)\right]^2 dq$. The optimal kernel $K(q)$ now has to ensure the minimization of D^6e^4. Instead of solving this problem one can choose a specific kernel function. As we have mentioned above we need a sufficiently smooth and bounded kernel. A possible and often used kernel function (Silverman 1986; Merritt & Tremblay 1994; Ryden 1995) is the normal kernel

$$K(q) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{q^2}{2}\right). \quad (12)$$

In the present work we employ another kernel function that possesses the requisite properties

$$K(q;\mu) = \frac{1}{\Gamma(1/2\mu)} \exp\left[-\left(\frac{q}{\mu}\right)^{2\mu}\right], \quad (13)$$

where $\Gamma(x)$ is the Euler gamma function. The parameter μ is chosen to minimize D^2d^4 or D^6e^4, respectively. In both cases the kernel $K(q;\mu)$ furnishes fewer values of $I_n^{\min}$ and $J_n^{\min}$ than the normal kernel Eq. (12). If we want to estimate the density distribution $\Phi(q)$ we have to use $\mu \approx 2.0963$ and the estimation of $\Phi'(q)$ needs $\mu \approx 1.4633$, the case that interests us.

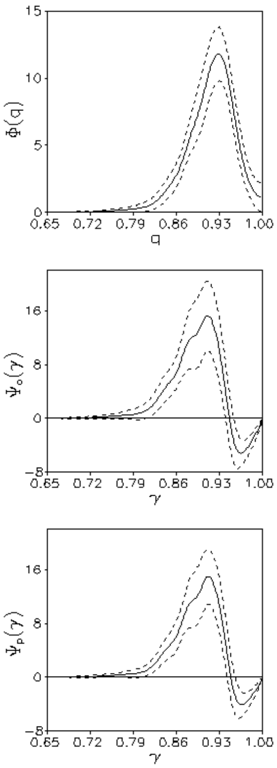

Fig. 2. The nonparametric kernel estimate of the distribution of apparent axial ratios Φ and the distributions of intrinsic axial ratios Ψ_o and Ψ_p for the sample of 172 GCs in M 31 (Staneva, Spassova & Golev 1996). The dashed lines are the 98% confidence bands. The smoothing parameter is $h = 0.019$; the observational noise error – $\delta\varepsilon_b = \pm 0.02$.

The last step before applying the nonparametric estimate consists of determination of the optimal smoothing parameter h_n^{opt}. The main obstacle is the lack of knowledge of the smoothness of searching distribution function and therefore we are not able to use Eqs. (8) and (10). Usually iterative data-based schemes are employed but the smoothing parameters derived produce the noisy density distributions. We accept the interactive method of smoothing level determination (see Merritt & Tremblay 1994) after which the wanted value h_n gives the smoothest curve that keeps in average the features of the distributions obtained with smaller smoothing parameters.

In all nonparametric estimations presented below we utilize the reflective boundary condition at $q = 0$ and $q = 1$ (see Silverman 1986; Ryden 1995) and "adaptive kernel" procedure (Abramson 1982; Merritt & Tremblay 1994). We made use of a fixed value of the kernel width h_n and other possibilities to treating the boundaries – the transformation $q' = -\ln(1-q)$ (Tremblay & Merritt 1995) and an absence of any condition (Vio et al. 1994), but the results received are not essentially different. The absence of the "adaptive kernel" procedure is also not a crucial for our following conclusions.

We intend to apply the nonparametric approach to testing the hypothesis that the GCs in the sample of 172 systems in M 31 (Staneva et al. 1996; we exclude G196 because it is not

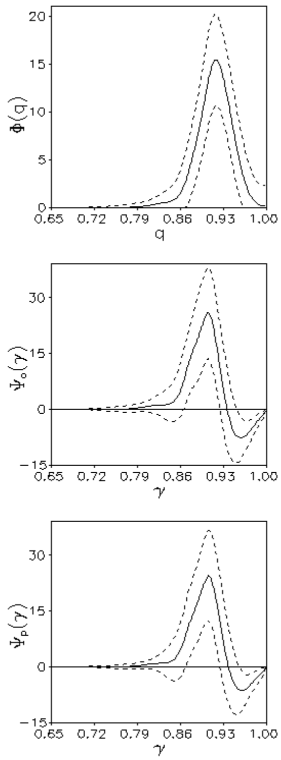

Fig. 3. The same as Fig. 2 but for the 17 "disk" GCs. The smoothing parameter is h = 0.021; the observational noise error – $\delta\varepsilon_b = \pm 0.02$.

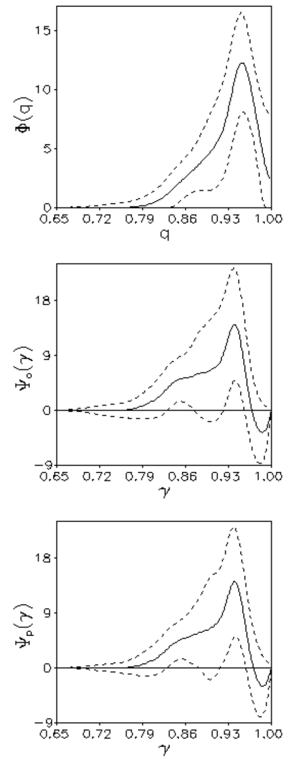

Fig. 4. The same as Fig. 2 but for the 17 "halo" GCs. The smoothing parameter is h = 0.021; the observational noise error – $\delta\varepsilon_b = \pm 0.02$.

a globular cluster) are all oblate or prolate randomly oriented spheroids. With the set of estimates shown in Fig. 1 we are able to determine the optimal smoothing parameter in the sense of the mentioned above. The value of the kernel width accepted is h = 0.019. In Fig. 2 we present the nonparametric kernel estimate of the distribution of apparent axial ratios Φ and the distributions of intrinsic axial ratios if the GCs are oblate spheroids Ψ_o and prolate spheroids Ψ_p [Eqs. (3) and (6)]. The negative part of intrinsic distributions Ψ_o and Ψ_p is physically nonsensical and to draw a strict conclusion we need only the confidence bands that give an account of the possible sources of error in the estimates.

Staneva, Spassova & Golev (1996) have analyzed the errors admitted in the ellipticity determination. The authors have not found significant systematic errors and have given root mean squares of the errors of each ellipticity estimation $\{\delta\varepsilon_i\}_1^n$. Other errors are those attributed to the observational noise and their maximum value $\delta\varepsilon_b$ does not exceed ± 0.02 for spherical GCs. From the initial sample of measured axial ratios $\{q_i\}_1^n$ we derive a large number $\left(10^3\right)$ of new random samples

$$\left\{q_j + \left(|\delta\varepsilon_j| + |\delta\varepsilon_b|\right)\left(2r_j - 1\right)\right\}_1^n, \tag{14}$$

where $r_j \in [0, 1]$ are uniformly distributed random numbers and n = 172. We place the confidence bands to comprise 98% of the computed estimations of Φ, Ψ_o and Ψ_p for each new set. The lack of nearly circular GCs and the oblate/prolate spheroid hypothesis led the estimates for Ψ_o and Ψ_p to become negative. Since the upper confident limits for intrinsic distributions Ψ_o and Ψ_p have negative part we categorically (at the 99% one-sided confidence level) reject the hypothesis that all GCs are oblate or prolate randomly oriented spheroids.

It is interesting to see whether the above conclusion remains in force if we separate the sample of globulars into "halo" and "disk" clusters. The "halo" clusters are those beyond an ellipse of semimajor axis $\sim 2°$, marking the outermost visible limit of the M 31 disk (Reed, Harris & Harris 1992). There are 17 "halo" clusters in the sample considered. The same number of globulars is located in an ellipse of semimajor axis $\sim 16'$ and we refer to them conditionally as "disk" clusters. Using the interactive procedure described above we determine the kernel width h = 0.021 and compute the estimates for the apparent and intrinsic axial ratio distributions with the confidence bands (Figs. 3 and 4). For the "disk" clusters (Fig. 3), the negative parts of the upper confident limits for the intrinsic distributions Ψ_o and Ψ_p again enable us to reject the assumption that they are oblate or prolate spheroids. Concerning on the "halo" clusters (Fig. 4), although the estimates for the intrinsic distributions also have negative parts, we cannot reject the oblate/prolate hypothesis at the 99% one-sided confidence level because the upper confident limits are entirely positive. The one-sided confidence levels at which we can decline the oblate/prolate spheroid hypothesis are for the Ψ_o – 69% and for the Ψ_p – 76%.

We dispose of the mean isophotal flattenings of another relatively rich sample consisting of 48 GCs in M 31 based on

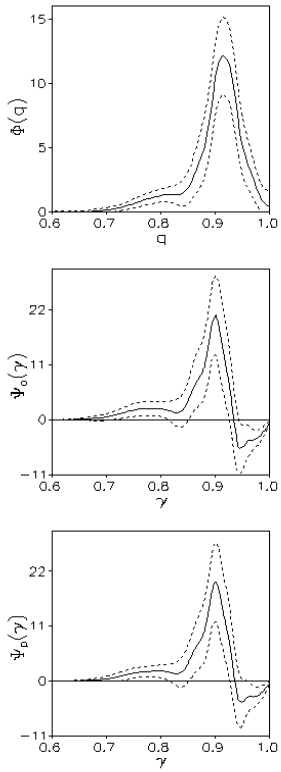

Fig. 5. The same as Fig. 2 but for the sample of 48 GCs in M 31 (D'Onofrio et al. 1994). The smoothing parameter is $h = 0.021$; the observational noise error – $\delta\varepsilon_b = \pm 0.025$; the typical errors – $\{\delta\varepsilon_i = \pm 0.015\}_1^n$.

CCD images obtained with four telescopes (D'Onofrio et al. 1994). The nonparametric estimates for this sample have been derived by kernel width $h = 0.021$, observational noise error $\delta\varepsilon_b = 0.025$ and typical error $\{\delta\varepsilon_i = 0.015\}_1^n$, and are shown in Fig. 5. As in the whole sample of Staneva, Spassova & Golev (1996) the GC shapes for the sample of D'Onofrio et al. (1994) are inconsistent with the oblate/prolate spheroid hypothesis.

2.3. Parametric approach

The same result, even thought not so weighty one, can be received in terms of parametric function estimations. An flexible way to deproject measured axial ratios consists of a numerical evaluation of the integral (1) after suitable choice of the intrinsic axial ratio distribution $\Psi(\gamma)$ (Plionis, Barrow & Frenk 1991). The parameters of the distribution $\Psi(\gamma)$ can be determined by a fitting procedure. As a model of $\Psi(\gamma)$ we take a Gaussian distribution

$$\Psi(\gamma) \sim \exp\left[-\left(\gamma - \gamma_0\right)^2 / \left(2\sigma_0^2\right)\right]. \qquad (15)$$

The result of the deprojection is shown in Fig. 6. The intrinsic axial ratios according to Eq. (15) (oblate spheroid case – curve 2; prolate spheroid case – curve 3) and their projection after Eq. (1) (curve 1 – the same for oblate and prolate spheroids) are plotted. It can be seen that the best fit of histogram is unacceptable and

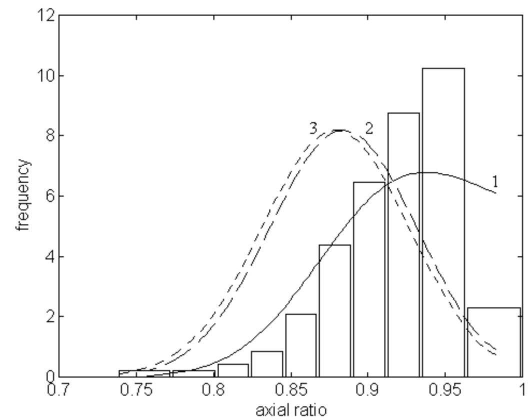

Fig. 6. The distribution of observed axial ratios (histogram) and the projection (curve 1) of the Gaussian distribution models for the intrinsic axial ratios: oblate spheroids (curve 2, long dashes) – $\gamma_0 = 0.88 \pm 0.02$, $\sigma_0 = 0.044 \pm 0.008$; prolate spheroids (curve 3, short dashes) – $\gamma_0 = 0.87 \pm 0.02$, $\sigma_0 = 0.047 \pm 0.009$.

we can reject the oblate/prolate spheroid hypothesis on the base of the χ^2-test ($\chi^2 = 37.7$ whereas the 5% critical region for the test corresponds to $\chi^2_{cr} = 18.3$).

After discarding the oblate/prolate spheroid hypothesis remains nothing but suppose the GCs in M 31 are triaxial ellipsoids keeping the only possible assumption for their random orientation of the axes. We are constrained to use parametric representation of the intrinsic distribution remembering that there is not a unique inversion from the apparent distribution of the axis ratios. To derive the true shapes from measured axial ratios we have applied the approach of Han & Ryden (1994). Supposing a Gaussian distribution of the true axial ratios

$$\Psi(\beta, \gamma) \sim \exp\left\{-\left[(\beta - \beta_0)^2 + (\gamma - \gamma_0)^2\right] / \left(2\sigma_0^2\right)\right\}, \qquad (16)$$

we can generate a great number $\left(\sim 10^5\right)$ of randomly oriented GCs and project them (see Binney 1985) to receive the distribution of the apparent axial ratios. In the case of an oblate Gaussian distribution model ($\beta = \beta_0 = 1$) this method is similar to the mentioned above and the deprojection gives approximately the same result: $\gamma_0 = 0.87 \pm 0.01$ and $\sigma_0 = 0.036 \pm 0.004$.

The observed cumulative distribution and the best fitting curves according to the Kolmogorov-Smirnov (hereafter K-S) criterion (in the cases of the oblate spheroid model and the triaxial ellipsoid model) are shown in Fig. 7. Since the 5% critical value of K-S test is $D_{5\%} = 0.103$ only the triaxial ellipsoid hypothesis can be adopted. The best fit distribution $\Psi(\beta, \gamma)$ has $\beta_0 = 0.932 \pm 0.007$, $\gamma_0 = 0.852 \pm 0.009$, $\sigma_0 = 0.022 \pm 0.008$. The standard deviations of the parameters β_0, γ_0, σ_0 and of the K-S test D are computed using the set of random samples generated by means of Eq. (14).

3. Discussion

Except the two samples of ellipticities considered above (Staneva et al. 1996; D'Onofrio et al. 1994) there is another

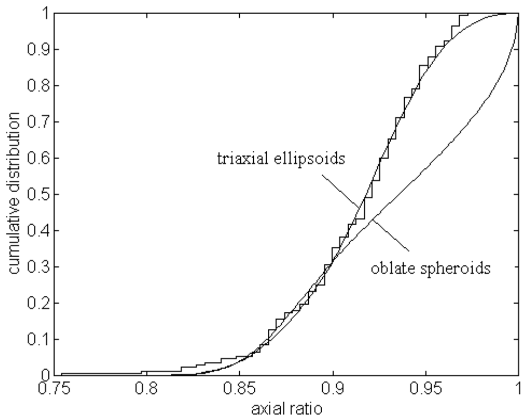

Fig. 7. Cumulative distributions: the observed axial ratios (histogram) and the apparent axial ratios of the test globular clusters (solid lines). The distributions have the parameters: triaxial hypothesis – $\beta_0 = 0.932 \pm 0.007$, $\gamma_0 = 0.852 \pm 0.009$, $\sigma_0 = 0.022 \pm 0.008$, $D = 0.054 \pm 0.012$; oblate hypothesis – $\gamma_0 = 0.864 \pm 0.013$, $\sigma_0 = 0.021 \pm 0.004$, $D = 0.271 \div 0.009$; $D_{5\%} = 0.103$ in both cases.

Table 1. Comparison of the best-fit parameters

source	object	β_0	γ_0	σ_0	model
Han & Ryden	Galaxy	–	0.955	0.130	oblate
(1994)	M 31	–	0.930	0.110	oblate
	LMC	0.856	0.742	0.005	triaxial
	SMC	0.917	0.717	0.000	triaxial
this work	M 31	0.932	0.852	0.022	triaxial

one consisting of only 18 GCs (Lupton 1989). If we attempt to estimate nonparametrically the axial ratio distributions from so poor sample we inevitably get the oversmoothed estimations because of the dependence of the kernel width on the sample length [see Eqs. (8) and (10)]. Probably that is the reason of the conclusions of Han & Ryden (1994) maintaining that the intrinsic shapes of the globulars in M 31are statistically indistinguishable from those of the Galaxy and that the GCs in M 31 are all oblate spheroids. The kernel estimates for the distribution of apparent axis ratios $\Phi(q)$ derived by Han & Ryden (1994) for the Galaxy sample (White & Shawl 1987) increase monotonically with $q \rightarrow 1$, whereas the estimations received with the samples of Staneva et al. (1996) and D'Onofrio et al. (1994) have the maximum at q = 0.93 and q = 0.92 respectively (see Figs. 2 and 5). By the way, one can see that the employment of the two other boundary conditions above mentioned instead of the reflective one leads to the similar maximum within the range $(1 - h, 1]$, where the reflective boundary condition distorts the kernel estimates. Naturally it can be guessed that the discrepancy is due to the systematical differences between the ellipticity determination by Luton (1989) and by Staneva et al. (1996) but the Wilcoxon test shows insignificant difference for the samples of 16 common GCs with 95% confidence level. In additional confirmation of above assertion we refer to the Fig. 7. in the previous paper (Staneva et al. 1996) where the common ellipticities are plotted versus the corresponding Lupton's ellipticities.

Table 1 gives the parameters of the best fits in Han & Ryden (1994) related to the Galaxy, LMC (Large Magellanic Cloud) and SMC (Small Magellanic Cloud) and ours. The comparison of the axial ratios β_0 and γ_0 and dispersion σ_0 shows that the clusters in M 31 cover the intermediate region of the shapes:

a) The lack of round clusters in the M 31 sample, similarly to the LMC and SMC cluster samples, shows the necessity of triaxial ellipsoid model for intrinsic axial ratios (see Han & Ryden 1994).
b) The GCs in M 31 are appreciably more spherical than LMC and SMC ones.
c) The GCs in M 31 are less round than the Galaxy clusters, nevertheless the observed ellipticities cover the same range.

A similar conclusion on the intermediate state of the M 31 clusters can be made on the base of the nonparametric estimates for the apparent distributions: there is no maximum for the Galaxy and the maximum shifts from q = 0.93 for M 31 to q = 0.86 for LMC (see Ryden 1995).

The dynamical evolution is the most probable reason of the shape differences of the GCs. According to the Fall & Frenk's investigation (1985) the initial velocity dispersion anisotropy at the cluster formation must decay with the time and the rotation becomes the major shape formation factor. The triaxial ellipsoids supported by the velocity anisotropy at the time of formation evolve to oblate spheroids. Simultaneously, the loss of angular momentum, carried out from the high velocity stars, makes the clusters rounder. If all assumptions, made or accepted, are correct we may deduce that the M 31 GCs are dynamically younger than the Galaxy clusters nevertheless overall similarities between the two systems. Stronger conclusions can be made after a comparison of the cluster relaxation times in both galaxies.

The different shapes of the "halo" and "disk" clusters suggest either an existence of two spatially different cluster populations or a presence of some systematic uncertainty in the ellipticity determination due to the background discrepancy within the domain of the "halo" and "disk" GCs. The first circumstance may indicate the influence of the tidal distortions on the dynamical cluster evolution near the galactic disk. The second one may lead to the sizable unnatural absence of round GCs ($\varepsilon < 0.03$) in the whole sample, but Staneva, Spassova & Golev (1996) have not found significant systematic errors. This result is only preliminary because the "halo" GCs under consideration are a small part of the whole sample.

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