

# The Blazhko effect on line profiles in the variable star RR Lyrae<sup>\*</sup>

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**Abstract.** It is shown that the metallic line doubling phenomenon is well visible when the Blazhko effect is maximum while only a line broadening takes place when it is minimum. At this last phase, the hydrogen emission is very weak (under the continuum) but its intensity increases progressively towards the Blazhko maximum. The hydrogen line doubling does not present a noticeable Blazhko phase variation. The wavelength position of the metallic double line system attests that the ballistic motion of the atmosphere has a weaker amplitude when the Blazhko effect is minimum than at its maximum intensity. We detect a variation of the  $\gamma$ -velocity deduced from metallic absorption lines during the Blazhko period but additional accurate observations are required to determine its amplitude. The acceleration of the shock wave, inducing line doubling phenomena, is extremely large at all Blazhko phases (from a Mach number around 5 to 25) between the metallic and hydrogen formation regions. Thus a high hypersonic regime always exists in the high atmosphere of RR Lyr. These observational results show that the radial pulsation is manifestly maximum at the maximum Blazhko effect and its intensity is more and more reduced up to its minimum. Finally it emerges that high quality line profiles, well distributed over the Blazhko period, can provide some decisive tests to determine the relevant mechanism at the origin of the Blazhko effect.

**Key words:** hydrodynamics – shock waves – magnetic field – stars: pulsation – stars: variables: RR Lyrae – stars: individual: RR Lyr

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## 1. Introduction

About one third of the observed RR Lyrae stars present a long-term modulation of their pulsation. More exactly, this consists in quasi-periodic changes of the amplitude and shape of the luminosity and radial velocity curves while the pulsation period remains constant. Here “quasi-periodic” means that none of the RR Lyrae stars with Blazhko effect exhibit strictly periodic pulsation contrary to single periodic RR Lyrae stars which repeat their light- and velocity-curves from cycle to cycle with amazing

regularity. The modulation period ranges from several tens to several hundred times the pulsation period. This modulation is called the *Blazhko effect* because Blazhko (1907) was the first to report it on RW Dra about ninety years ago. It is not observed in first overtone RR Lyrae stars but only in fundamental pulsators. This effect is a unique phenomenon among the radially pulsating variable stars and its physical origin is not yet well known. A general review has been recently given by Smith (1995).

From photometric observations, Shapley (1916) noticed that RR Lyrae itself shows a Blazhko modulation over a cycle of about 41 days. Because RR Lyr is the brightest of the RR Lyrae stars, extensive series of spectroscopic observations including the effect of the modulation were carried out by some investigators. Struve (1947) discovered the occurrence of transitory H emission and Struve and Blaauw (1948) showed that its intensity varies in the 41-day cycle. Preston, Smak and Paczynski (1965) found that the visibility of doubling of the H absorption components is correlated with the strength of the H emission in the 41-day cycle and also that the doubling of the higher members of the Balmer series always precedes that of the lower ones. From the data of these two last studies, it appears that the so-called  $\gamma$ -velocity i.e., the observed average radial velocity over one pulsation period, is also a function of the 41-day cycle. Preston, Smak and Paczynski (1965) interpreted these phenomena as a consequence of a variation of the critical level of the shock-wave formation region during the 41-day cycle.

The doubling phenomenon of the metallic absorption lines has been only clearly demonstrated recently by Chadid and Gillet (1996a, hereafter Paper I). It is also expected to be the predominant contribution of the considerable and very narrow FWHM increase observed in RR Lyr (Chadid and Gillet 1996b, Paper II). One of the aims of this paper is to determine the dependence of doubling visibility vs. the 41-day cycle. The observation of a short-lived doubling of metallic lines means that a strong shock wave already exists just above the photosphere contrary to the conclusions of the most elaborate theoretical models (Fokin 1992). Thus the presence or the absence of this doubling during the different Blazhko phases constitutes basic information for future models.

Although, from all the observational results above, it appears that the Blazhko effect has a strong influence on the dynamics state of the atmosphere, its physical origin is not yet completely

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<sup>\*</sup> Based on observations obtained at the Observatoire de Haute-Provence (France).

clear. In 1975, Stellingwerf listed six plausible explanations including tidal effects, internal resonances between two or more pulsation modes or the effect of a magnetic field on radial oscillations. Up to now, the two last suggestions are the most popular. Kovács (1995a) examines the present theoretical status of radial and non-radial mode interactions together with the effect of convection or noise. It appears that the present hydrodynamical codes are indecisive and need further improvement such as a better treatment of shocks and convection. Moreover observations using modern detectors are absolutely necessary to allow a pertinent comparison with models. Cousens (1983) showed that a combination of the effects of fundamental radial pulsation, magnetic field and rotation can quantitatively explain the Blazhko cycle. Recently, Takata and Shibahashi (1996) present an oblique pulsator model which predicts the dependence of the Blazhko amplitude upon the strength of the magnetic field. These two last models need a magnetic field of the order of one kgauss to explain the observed Blazhko effect. First Babcock (1958) and more recently Romanov, Udovichenko and Frolov (1987, 1994) put into evidence that the magnetic field of RR Lyr varies with a period equal to the pulsating one and an amplitude around 1.5 kgauss and that the mean magnetic field strength presents a periodic variation with the phase of the Blazhko effect. These results seriously point out that the Blazhko effect may be associated with the magnetic activity of the star.

In Sect. 2 we briefly describe the observations and their phasing. The variation of the  $\gamma$ -velocity is discussed in Sect. 3 and the effect of the long-term modulation of the pulsation on line profiles is presented in Sect. 4. Section 5 is devoted to the discussion of the obtained results together with the present status of the interpretation of the Blazhko effect. Finally some concluding remarks are given in Sect. 6.

## 2. The observations and their phasing

The spectroscopic observations were obtained with the ELODIE spectrograph at the 1.93-m telescope at Observatoire de Haute-Provence during 6 nights (3 to 5 August 1994, 8 August 1995, 5 September 1995 and 17 October 1995). This instrument (Baranne et al. 1996) covers a 3000 Å spectral range from the near ultraviolet to the near infrared with a resolving power  $R \simeq 42,000$ , a signal-to-noise ratio  $S/N$  around 50 for an exposure time between 5-10 mn giving a time resolution around 1% of the pulsation period (13 h 36 mn). The description of the data reduction can be found in Paper I.

The phasing of the observations can be only approximative because we have not simultaneous photometry. In Paper I, we have adopted the ephemeris used by Gillet and Crowe (1988). The Blazhko cycle is defined as a 41-day variation of the amplitude of the *light curve*. Hereafter the zero phase corresponds to the *maximum* observed light amplitude. This latter is not constant. More exactly, the amplitude of the Blazhko effect itself varies over approximately a 4-year cycle (3.8 to 4.8 years, Detre and Szedl 1972). The beginning of this very long cycle is a *minimum* of the Blazhko amplitude. The extrapolation of the epoch of amplitude maximum of the light curve over a long time

interval can be hazardous because at the beginning of each 4-year cycle a significant *phase* discontinuity ( $0.1 \leq \Delta\psi \leq 0.5$ ) is observed in the 41-day cycle (Detre and Szedl 1972). Nevertheless and except during the strongest amplitude 4-year cycle, this discontinuity  $\Delta\psi$  remains constant during the whole following cycle. From Dalmazio's investigation (1992), based on 1082 visual magnitudes of RR Lyr in 1991, we determine the 1991-Blazhko ephemeris:

$$\text{HJD}(\text{max. light ampl.}) = 2, 448, 549.296 + 40.8E. \quad (1)$$

More recently, Dalmazio (1995) obtains 950 visual magnitudes of RR Lyr in 1994 from which we deduce the 1994-Blazhko ephemeris:

$$\text{HJD}(\text{max. light ampl.}) = 2, 449, 631.312 + 40.8E. \quad (2)$$

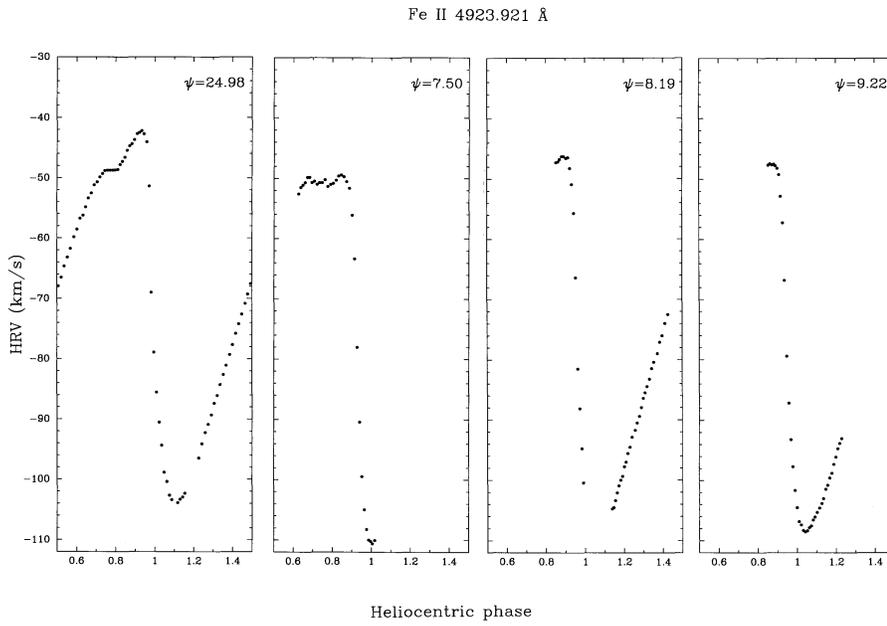
The above maximum occurs at October 5, 1994 i.e., that is well consistent with our three 1995 observational runs. Dalmazio expects that it corresponds to the beginning of a new 4-year cycle. Consequently the Blazhko's phase of our three 1995 runs are  $\psi = 7.50, 8.19$  and  $9.22$  respectively while one of 1994 is  $\psi = 24.98$ . Dalmazio (1995) shows that a phase discontinuity of 0.31 must be expected between these two 4-year cycles. From his 19 visual maxima in 1994, Dalmazio (1995) confirms the validity of the Rocznik's ephemeris (Zakrzewski 1996) which gives the pulsation phase:

$$\text{HJD}(\text{max. light}) = 2, 446, 654.368 + 0.566839E. \quad (3)$$

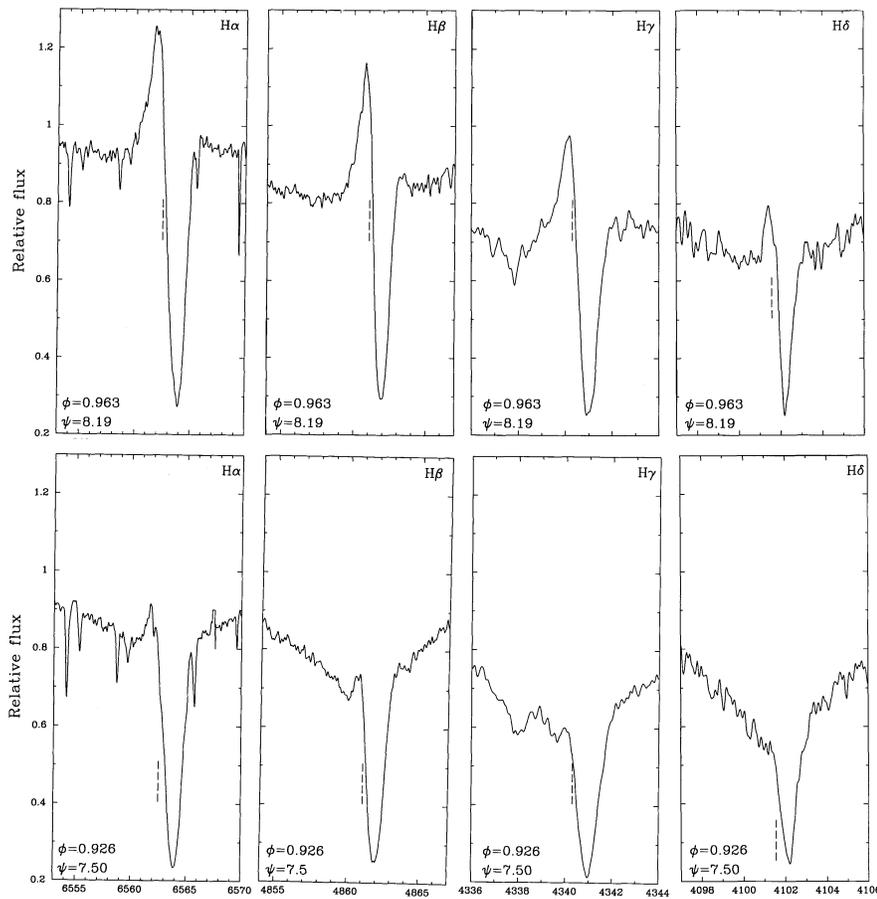
In this paper, we have used these three ephemeris. The pulsation phase is  $\phi$  and the Blazhko phase is  $\psi$ .

## 3. The $\gamma$ -velocity variation

Fig. 1 shows the radial velocity curves of the  $\lambda 4923.921$  Fe II absorption lines vs. the Blazhko phase  $\psi$ . Note that the velocity curve at  $\psi = 24.98$  is composed of observations from three consecutive nights contrary to the three other curves. Moreover the jump around phase 1.0 in the velocity curve at  $\psi = 8.19$  is due to a technical problem occurred during the observations. The  $\gamma$ -velocity can be only correctly determined for  $\psi = 24.98$  because we have a complete pulsating period. A Fourier fit gives a  $\gamma$ -velocity of  $-71.1 \pm 0.1$  km/s. Nevertheless a simple visual inspection of the velocity curves at  $\psi = 24.98$  and  $7.50$  clearly suggests that a variation of the  $\gamma$ -velocity must be present. This is consistent with the Fig. 2 of Struve and Blaauw (1948) which put into evidence a variation of the  $\gamma$ -velocity with a period of approximately 42 days. Unfortunately the radial velocities of Struve and Blaauw (1948) and Preston, Smak and Paczynski (1965), which were deduced from 22 metallic absorption lines plus 9 hydrogen lines (H9 to H21) and from 9 metallic lines respectively, have not a large enough quality (photographic plates) to determine the exact shape and amplitude of the  $\gamma$ -velocity curve. Thus additional observations over a few complete pulsating periods are strongly required and may help to distinguish among the various models for the Blazhko effect.



**Fig. 1.** Variation of the radial velocity during the rising light for different Blazhko phases  $\psi$ . The pulsation phase is given in abscissa velocities refer to the single ionized metallic absorption line  $\lambda 4923.921$  Fe II.



**Fig. 2.** Line profiles of the four first members of the Balmer series at two different Blazhko phases  $\psi$ . The pulsation phase  $\phi$  is given just above the Blazhko phase. The maximum observed emission occurs at  $\psi = 8.19$  while the minimum one at  $\psi = 7.50$ . The laboratory wavelength is marked by a small vertical dashed line.

## 4. Line profile variations

### 4.1. Hydrogen lines

The maximum hydrogen emission is observed at the Blazhko phase  $\psi = 8.19$  (Fig. 2a) and for  $H\alpha$  the relative flux reaches about 1.25. Later, at phases 9.22 and 24.98, it is near 1.1. The minimum emission intensity, which is under the continuum (0.92 for  $H\alpha$ , Fig. 2b), occurs at  $\psi = 7.50$ . The emission always decreases from  $H\alpha$  to  $H\delta$ . From our observations, it appears that the maximum emission intensity occurring at the Blazhko phase  $\psi = 24.98$  is smaller than that observed at  $\psi = 9.22$  in the following 4-year cycle. This is consistent with the expected increasing amplitude at the beginning of this new cycle but as shown by spectra at  $\psi = 8.19$ , this does not seem a strict rule.

The two absorption components, which are well visible after the disappearance of the emission, represent in the framework of the Schwarzschild's explanation (1952) two layers in opposite motion. The component separation, which is caused by the presence of an upward shock wave (see Fokin and Gillet 1997), is about 110-120 km/s and is called the shock amplitude (velocity difference between pre- and postshock velocities). We have not clearly detected a variation of this amplitude with the Blazhko phase. We can only say that the shock velocity in the layers where the hydrogen absorption lines are formed is very large (around 160 km/s).

### 4.2. Metallic lines

How does the metallic line doubling phenomenon vary during the Blazhko cycle? Figs. 3 and 4 show the evolution of the Fe II  $\lambda 4923.921$  and Mg I  $\lambda 5183.6042$  line profiles during the doubling phenomenon. From our observations, it seems that the doubling visibility is variable from one Blazhko cycle to the following ( $\psi = 8.19$  and 9.22). This is not the consequence of the  $S/N$  ratio because it is better at  $\psi = 9.22$  than at  $\psi = 8.19$  and the integration time is the same (5 min). In fact, because the visibility of the components is noticeably dependent on the distribution of the spectra during the doubling phenomenon (the doubling only occurs over 4 or 5 spectra), it sometimes appears that the components are less marked but the line broadening remains the same.

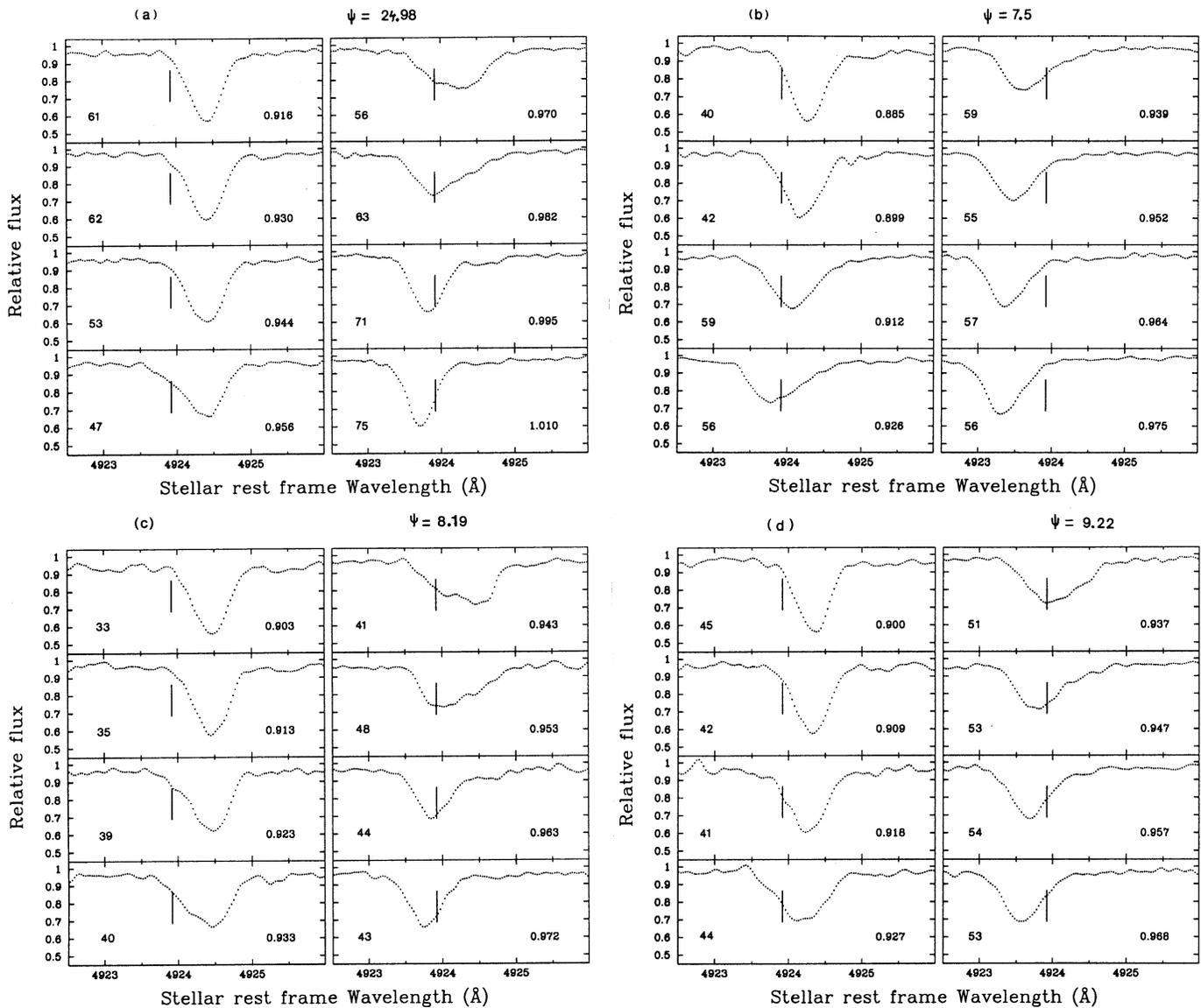
Just at the Blazhko minimum ( $\psi = 7.50$ ), the line doubling is not detected: only a broadening can be observed (Figs. 3b and 4b). This is well consistent with the very weak hydrogen emission at this phase. The separation of the two expected line components is around 21 km/s while it reaches 27 km/s at the three other Blazhko phases. This means that a shock amplitude larger than 30 km/s is necessary to put into evidence the two absorption components or that the shock must have a Mach number approximately larger than 5. Moreover, at these phases, the shortest wavelength component is redshifted until the maximum line doubling, while it is already blueshifted when only a broadening takes place ( $\psi = 7.50$ ). In the framework of the interpretation proposed in Paper I, this means that the velocity of the upward shock traversing the metallic layers is larger than the velocity of the infalling atmosphere when the doubling is

not visible. On the other hand when the doubling is observed i.e., when the shock amplitude is large, the infalling velocity of the highest part of the atmosphere must be larger than the post-shock velocity. In other words, the shock is first receding for an observer at rest in front of the star (Eulerian frame) although it has an upward motion in mass zones (Lagrangian frame). Finally this indicates that the ballistic motion of the atmosphere has a larger amplitude when the Blazhko effect is maximum than minimum.

## 5. Discussion

The component separation increases with the shock amplitude in agreement with the Schwarzschild's explanation. Consequently, we must expect the shock front velocity to be quite small just above the photosphere, where the metallic absorption lines are formed, than in the high atmosphere in which the hydrogen lines are formed. Indeed, the observed component separation is only around 24 km/s for metallic lines but 115 km/s for  $H\alpha$ . This means that the shock front is strongly accelerated in the atmosphere because its velocity increases by a factor 5. The fast density decrease with the altitude must explain this effect but we can also expect that the shock radiative losses certainly remain moderate between these two formation regions. Thus we get through a supersonic hydrodynamic regime (Mach number  $M < 5$ ) to a high hypersonic one ( $M \simeq 25$ ).

The observational results presented above show that the *radial* pulsation amplitude appears maximum at the zero Blazhko phase while it more and more decreases up to a minimum at  $\psi = 0.50$ . If we accept the observations of Babcock (1958) and Romanov, Udovichenko and Frolov (1987, 1994), namely a variable and periodic (13.6 h and 40.8 d) magnetic field with an intensity up to 1.5 kgauss, then the oscillating oblique magnetic rotator model (OOMR, Cousens 1983) is quite plausible. The work of Biront et al. (1982) has shown that a pulsating magnetic star always has significant non-radial motions at its *surface* while the pulsation remains purely radial in the absence of the field. In fact, the radial motion excited by the  $\kappa$ -mechanism is mainly perturbed by the Lorentz force to induce non-radial components whose the symmetry axis coincides with the magnetic axis. Thus, as the star rotates, the aspect angle of the non-radial components changes with respect to the observer and consequently, a light/radial velocity long-term modulation must be expected. The two other plausible explanations of the Blazhko effect discussed by Kovács (1995b) would not be relevant here because they do not explain a modulation of the magnetic field. As first suggested by Detre and Szeidl (1973), the 41-day cycle would be equal to the rotation period of the star while the 4-year cycle would be interpreted as the magnetic cycle of RR Lyr. This would be the analog of the 11-year solar magnetic cycle ( $P/2$ ) which has observed field strengths of 0.1-1 kgauss. Stothers (1980) expects that the abrupt changes (discontinuities) of the Blazhko period at the beginning of a new 4-year cycle corresponds to the sudden generation or destruction of magnetic fields. Indeed, because active regions are not randomly distributed on the solar surface but rather occur, in addition to

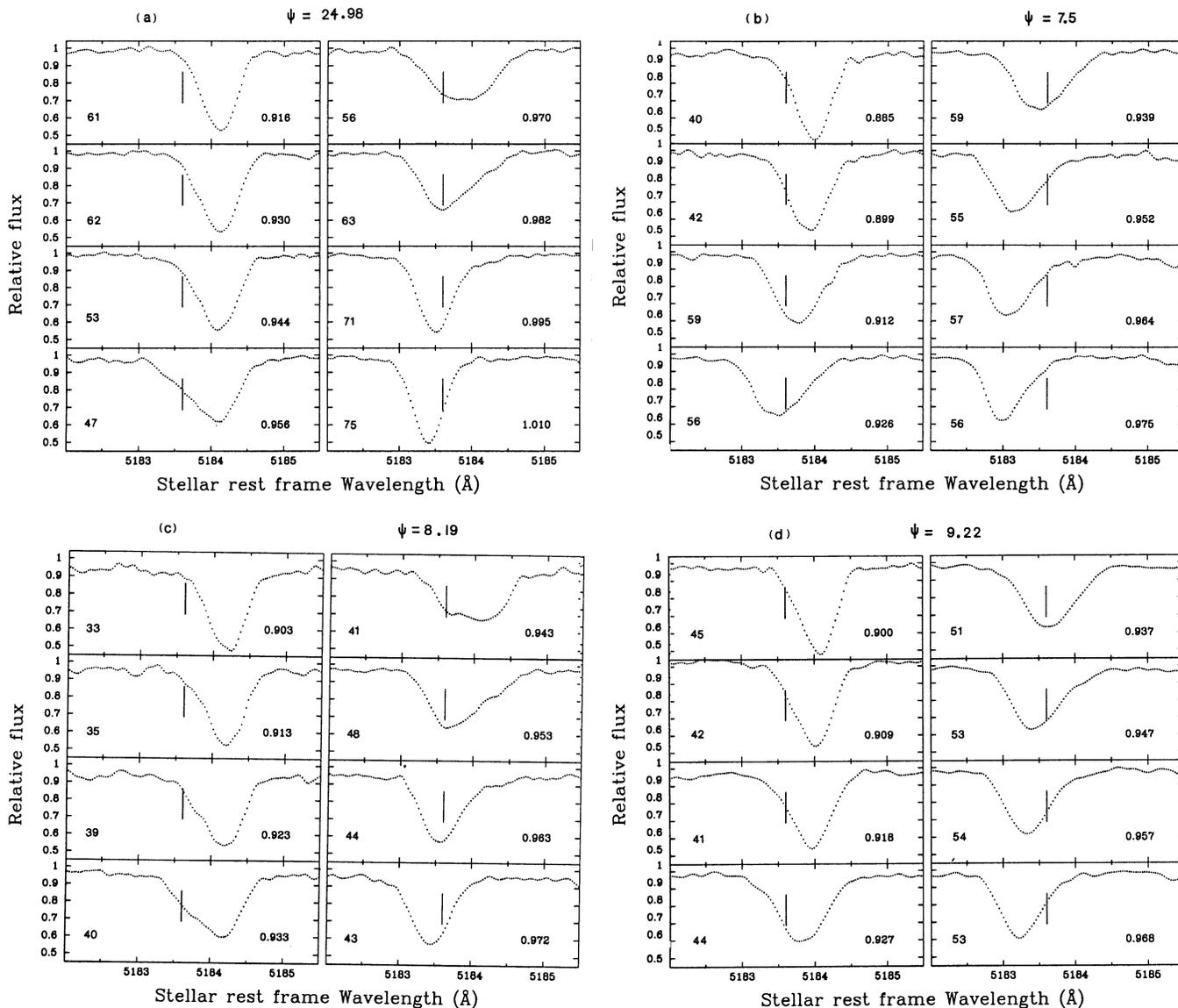


**Fig. 3.** Variation of the single ionized metallic absorption line  $\lambda 4923.921$  Fe II at different Blazhko phases  $\psi$  during the line doubling phenomenon. The pulsation phase is given in each lower right corner while the  $S/N$  ratio is given in the lower left corner. The small vertical line marks the position of the laboratory wavelength. Points are pixel values.

the preferred latitudes of emergence (butterfly diagram), at  $180^\circ$  separations in opposite hemispheres (*active* longitudes, see for instance Wilson 1994), it is possible that when a new 4-year cycle begins, the magnetic intensity increases at a different longitude with respect to that of the end of the previous cycle. Thus the 41-day period discontinuity would be the consequence of a longitude shift (Detre and Szeidl 1973).

As noted in Sect. I, the existence of an observable magnetic field in RR Lyr has been detected several times. The first measures of Babcock in the mid-fifties were performed during a maximum of a 4-year cycle but Preston (1967) did not detect magnetic fields in 1963 i.e., during a minimum 4-year cycle. Later, Romanov, Udovichenko and Frolov (1987, 1994), using a 6-m telescope, found again a variable magnetic field during

(1983) or just before (1978, 1982) or after (1984) two 4-year minima. It is expected that if the intensity of the magnetic field decreases below 300 gauss approximately, the magnetic effect will be very difficult to detect, such as the Blazhko one (Takata and Shibahashi 1996). Stothers (1980) suggests that the apparent absence of the Blazhko effect would be due to the analog of the long minimum of the much less regular 80-year cycle of the sun or by the even more extreme prolonged minima observed in the solar activity. He also advances that the RR Lyrae stars without a Blazhko effect could be understood as a temporary cessation of a strong magnetic activity. Recall also that the existence of a low frequency modulation in the light and velocity curves needs, in the OOMR model, a star rotating around an axis not aligned with the magnetic axis of symmetry ( $\beta \neq 0$ ).



**Fig. 4.** Variation of the neutral metallic absorption line  $\lambda 5183.6042$  Mg I at different Blazhko phases  $\psi$  during the line doubling phenomenon. The pulsation phase is given in each lower right corner while the  $S/N$  ratio is given in the lower left corner. The small vertical line marks the position of the laboratory wavelength. Points are pixel values.

From the observations of Preston, Smak and Paczynski (1965), Cousens (1983), comparing observed and theoretical light and radial velocity curves, found that the best “fit” seemed to occur for the angle  $i = 45^\circ$  between the line-of-sight and the axis of rotation and the angle  $\beta = 30^\circ$  between the magnetic axis of symmetry and the axis of rotation. An inspection of their curves, clearly reveals that the use of high quality spectra of ELODIE, will provide the possibility of a very fine test to the OOMR model. Moreover, it would be possible to check the different assumptions considered by the model and to know the contribution of “magnetic” terms to the line broadening or to the line profile deformation by comparing with our observations.

Is there any non-radial component in the pulsation of RR Lyr? This is a primary question and constitutes, with new high

quality observations of magnetic field variations and Fourier analyses of accurate light curves (Takata and Shibahashi 1996), the basic future step in the research of the physical origin of the Blazhko effect. To detect non-radial components in line-profile variations, one of the best tools is certainly the moment method (see e.g. Aerts, De Pauw and Waelkens 1992). Blazhko RR Lyrae stars are expected to be purely radial pulsator (as such non-Blazhko RR Lyrae variables) at the zero Blazhko phase  $\psi$  while the amplitude of the supposed non-radial components must be maximum at  $\psi = 0.5$  i.e., when the light amplitude is minimum. In the framework of the OOMR model, the magnetic field would affect the radial component in favour of non-radial ones to give the observed long-term modulation. This is consistent with the fact that on the period-amplitude diagram, the

highest light amplitude of the Blazhko stars always fits with the period-amplitude relation of the singly periodic RR Lyrae stars (Szeidl 1988). Thus high spectral resolution observations, both at maximum and minimum Blazhko effects, will be able to detect and to identify the non-radial components.

Kovács (1995b) discussed the three possible models of the amplitude modulation. They all need non-radial pulsation components. The OOMR model requires a direct resonant interaction between the radial fundamental and the necessary low-order non-radial mode (Takata and Shibahashi 1996) to obtain the observed single period pulsation. The two other models concern a resonant pulsation between the radial fundamental and a non-radial modes but in the framework of a dynamical (i.e., between two modes with nearly equal frequencies) or stationary interaction respectively. The amplitude of the modulation of the last model, together with the OOMR one, depends on the aspect angle. If we assume a random distribution, the modulation amplitudes of all known Blazhko stars do not correspond to this distribution. Thus, although the second model appears presently more viable to Kovács, he concludes that they all are still possible and new observations are highly indispensable to find out the right physical mechanism at the origin of the Blazhko effect.

## 6. Conclusions

The observations reported in this paper well confirm the line doubling of metallic absorption lines which was put into evidence for the first time by Chadid and Gillet (1996a). Its intensity i.e., the component separation, depends of the Blazhko phase. It is minimum at the minimum light amplitude and maximum at the maximum light amplitude. From the metallic and hydrogen line separations, we show that the shock undergoes a large acceleration during its propagation between these two formation regions. This implies that shock radiative losses are not enough important to decrease the shock energy and consequently to induce a visible shock weakening. During the minimum light amplitude ( $\psi = 7.50$ ), the hydrogen emission is very weak but it increases more and more with the light amplitude. It is also shown from the wavelength position of the metallic double line profile that the amplitude of the atmospheric ballistic motion strongly depends on the Blazhko phase: it is considerable when the Blazhko effect is maximum and vice versa. All these results indicate that the Blazhko effect decreases the magnitude of the radial component.

The high quality observations are not yet sufficient to finally determine the physical origin of the Blazhko effect. A special observational effort would be decisive. Moreover, the line profiles presented in this paper well show that it is now possible to realize a very fine comparison with theoretical models. From the three most plausible models, the OOMR one seems a natural way to explain the observed magnetic variations reported a few times but the reality of these observations seems to be underestimated. Moreover the explanation of the 4-year cycle in connexion with the magnetic activity of the star requires a real attention.

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*Note.* Together with Chadid & Gillet 1996 a and b (Papers I and II), this paper constitutes the main part of the Ph. D. thesis' work of M. Chadid under the scientific direction of D. Gillet (University of Toulouse, 1996).

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