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Research Note

A possible observational test of the global inhomogeneity

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Abstract. Lauer and Postman (1994) announced that the direction of the peculiar motion of Local Group with respect to the Abell clusters is not identical to the maximum of dipole anisotropy of cosmic microwave background radiation. This fact may be explained either standardly by a bulk stream of these clusters or by a global inhomogeneity of the temperature of microwave background across the whole Hubble radius. The second explanation is in principle allowed, e.g., in inflationary scenarios. It is shown that the careful measurement of the quadrupole anisotropy of cosmic microwave background may be used as an observational test between these two alternatives. Predictions of the expected values of the quadrupole components of sizes $\simeq (1 - 15)\mu K$ are presented. Unfortunately, the COBE data are still ambiguous to do any detailed comparison of these theoretical predictions with the observational data.

Key words: cosmic microwave background – large-scale structure of the universe

1. Introduction

The observed (Smoot et al. 1992; Bennett et al. 1992; Bennett et al. 1994) dipole anisotropy of the cosmic microwave background radiation (hereafter CMBR) is standardly interpreted as a Doppler shift caused by the peculiar motion of Local Group with velocity $\boldsymbol{v}_C$ toward l_C = 271^o, b_C = +29^o, where $|\boldsymbol{v}_C| = v_C \simeq 620$ km s^{-1} (cf. Peebles 1993). The Local Group moves with respect to the so called "preferred frame" (hereafter PF), in which CMBR is maximally symmetric; i.e. it is homogeneous and isotropic (Peebles 1993). If this standard interpetation is correct, then the direction defined by l_C and b_C, and the peculiar velocity $\boldsymbol{v}_L$ of Local Group, determined from observations of the clusters of galaxies, must be identical.

A study (Lauer & Postman 1994; hereafter LP) of the distribution of clusters of galaxies being nearer than $150h^{-1}$ Mpc (H = 100 h km s^{-1} Mpc^{-1} is the Hubble-parameter; $0.5 \lesssim h \lesssim 1.0$) shows that the velocity of Local Group $\boldsymbol{v}_L$ - with respect to the "Abell cluster inertial frame" (hereafter ACIF) - is toward l_L = 220^o, b_L = -28^o, where $|\boldsymbol{v}_L| = v_L \simeq 561$ km s^{-1}. It is confirmed with a high degree of confidence (LP) that ACIF is not identical to PF; i.e. $\boldsymbol{v}_L \neq \boldsymbol{v}_C$.

There may be, in principle, four possibilities. 1. The observations of LP are incorrect, and one may further assume that $\boldsymbol{v}_L = \boldsymbol{v}_C$. 2. The measurement of LP is correct and one has $\boldsymbol{v}_L \neq \boldsymbol{v}_C$. Then one assumes that ACIF moves - with respect to PF - with a velocity $\boldsymbol{v}_F$ toward l_F = 343^o, b_F = +52^o, where $|\boldsymbol{v}_F| = v_F \simeq 689$ km s^{-1}. This motion of ACIF should be caused by an unknown mass concentration toward (l_F, b_F) being at distance $\gtrsim 100h^{-1}$ Mpc (LP). Then the actual velocity of Local Group - with respect to PF - is $\boldsymbol{v}_C = (\boldsymbol{v}_L + \boldsymbol{v}_F)$. In this case the standard Friedmannian model further holds. (More exactly it holds up to the precision $\sim 10^{-5}$ (Peebles 1993), where there will surely exist departures from this model.) 3. The measurement of LP is correct and one has $\boldsymbol{v}_L \neq \boldsymbol{v}_C$. But ACIF is identical to PF, and in PF CMBR is not homogeneous and isotropic; the departure from the maximal symmetry are of order $\sim 10^{-3}$. In other words, it is assumed that there exist global intrinsic inhomogeneities of order 10^{-3} of CMBR across the whole Hubble radius. (We will call this situation as "non-Friedmannian possibility"; although in this case the Friedmannian model is also fulfilled up to the precision 10^{-3}. In other words, the terms "Friedmannian model" and "non-Friedmannian model" are taken fully for the convenience only. In both cases the Friedmannian model holds for a high accuracy, and the difference is only quantitative; for the possibility No.2 this accuracy is of order 10^{-5}, for the possibility No.3 this accuracy is of order 10^{-3}.) 4. The measurement of LP is correct and one has $\boldsymbol{v}_L \neq \boldsymbol{v}_C$. ACIF is identical to PF, and in PF CMBR is further homogeneous and isotropic.

Concerning the first possibility the situation is ambiguous. Riess et al. (1995) queries the results of LP. On the other hand, Graham (1996) again confirms the conclusion of LP. All this means that no unambiguous rejection of the conclusion of LP on observational ground exists yet. Hence, the investigation of possibilities 2. - 4. is highly topical.

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Even in LP the explanations 2. and 3. are mentioned. The possibility 2. is the standard interpretation expected in Friedmannian cosmology. But it needs a huge and a not observed mass concentration in direction defined by $\boldsymbol{v}_F$. Hence, the non-Friedmannian possibility No.3 must also be taken seriously.

Several papers discuss the possible origin of the global $\sim 10^{-3}$ inhomogeneity of CMBR. A decade ago the authors studied a model of the observable part of the Universe, in which both the matter and the temperature of CMBR at any subhorizon and superhorizon scales are not homogeneous, and there is a monotonous gradient of both quantities (Mészáros 1986; Mészáros & Vanýsek 1988). This gradient should be caused by the fact that the part of Universe inside the horizon is in fact a small part of a much larger inhomogeneous spherical region with a well defined center and edge. In this spherical region there should be a monotonous decreasing of density, temperature of CMBR, etc... The edge of this spherical region is beyond the present horizon. Practically the same idea was proposed by Gunn (1988), Paczyński & Piran (1990) and Turner (1991). The difference concerns only the exterior beyond the edge of spherical region; we assume the existence of the standard Schwarzschildian vacuum, while other authors suggest the false vacuum following from the inflationary scenario. But in any case, if this is the situation, then in the frame coupled with the Local Group these intrinsic anisotropies of order 10^{-3} will also be measurable, but there is a mixture of these terms and the Doppler shift coming from the motion of this system with respect to ACIF.

In principle, also the possibility 4. is allowed. Nevertheless, as shown in Mészáros & Molnár (1996), this possibility is excluded and need not be discussed later.

The straightforward observational decision between the possibilities 2. and 3. may be done from the detailed surveys of spatial distribution of galaxies. These procedures should either to reject the claim $\boldsymbol{v}_C \neq \boldsymbol{v}_L$ (confirmation of possibility No.1) or to find the required mass concentration in the direction $(\boldsymbol{v}_C - \boldsymbol{v}_L)$, if these two vectors are actually different (confirmation of possibility No.2). If neither of these possibilities occur, then the possibility No.3 should be accepted. This is a time consuming procedure, and, in addition, it may give only an indirect support for the third possibility. Hence, trivially, further observational tests are highly required.

In this paper we ad hoc reject the possibility No.1. Thus, we assume that the data of LP are correct, i.e. $(\boldsymbol{v}_C - \boldsymbol{v}_L) \neq 0$. We develop a new observational test, allowing a decision between the Friedmannian and non-Friedmannian alternatives. We will ad hoc assume, as a working hypothesis, that no mass concentration exists toward $(l_F,\ b_F)$ at distance $\gtrsim 100h^{-1}$ Mpc, and the intrinsic anisotropy of CMBR is caused by global inhomogeneity across the whole Hubble radius. Then we obtain a measurable effect occuring only in the case, when the possibility No.3 holds, and not existing in the case, when the possibility No.2 occurs.

2. Global anisotropy

Both in the Friedmannian and in the non-Friedmannian case one has (Peebles 1993; equs.6.38 and 6.47)

$$T' = \frac{T}{\sqrt{1 - V_L^2}}(1 + V_L \cos\vartheta_2), \tag{1}$$

where T (T') is the temperature of CMBR in PF (in the frame coupled with the Local Group), and where ϑ_2 defines the angle between an arbitrary direction and the direction of vector $\boldsymbol{V}_L = \boldsymbol{v}_L/c$. (In this paper the angles, temperatures, etc... in PF are unprimed; the same quantities seen by an observer moving with the Local Group are primed; $V_R = v_R/c$, where c is the light velocity, and $R = L, C, F$.) The key difference between the two alternatives is given by the fact that $T = const.$ for the possibility No.2, but $T \neq const$ for the possibility No.3.

If the possibility No.3 occurs one will have

$$T = T_o(1 + a\cos\vartheta_1 + a^2 x \cos^2\vartheta_1). \tag{2}$$

In Eq. (2) one has $T_o = 2.73$K and a, x are real dimensionless numbers, where $a \sim 10^{-3}$, $|x| \sim 1$, and $a > 0$ (x may also be negative). ϑ_1 defines the angle between an arbitrary direction and the direction toward the center of spherical region. The choice (2) was suggested first by Mészáros (1986), and was precised by Paczyński & Piran (1990). In these two articles it is shown that the existence of the intrinsic dipole and quadrupole anisotropies follows from the assumption of spherical symmetry. The concrete value of a is free, but the intrinsic quadrupole term must be of order $\sim a^2$. Hence, x must be of order unity. Recently this choice was queried by Jaroszyński & Paczyński (1995), but again supported by Langlois & Piran (1995). Thus, we will ad hoc assume the fulfilment of Eq. (2).

In order to be in accordance with the measurements of the dipole anisotropy of CMBR and the data of LP one must have (LP)

$$a = V_F, \tag{3}$$

and $\vartheta_1 = 0$ must be at the direction defined by vector $(\boldsymbol{V}_C - \boldsymbol{V}_L) = \boldsymbol{V}_F$. Here $\boldsymbol{V}_F$ does not define a motion; but, contrary, $(l_F,\ b_F)$ define the direction and V_F the size of intrinsic dipole anisotropy of CMBR. The situation is illustrated in Fig 1., where point X is fully arbitrary.

If one uses the relation (3) and omits any terms smaller than $\sim 10^{-6}$ (i.e. any terms $V_R^n;\ n \geq 3,\ R = C, F, L$; are always omitted), the substitution of relation (2) into Eq. (1) gives (we used $(1 - V_L^2)^{-1/2} = 1 + (V_L^2/2) + ...$)

$$\begin{aligned}\frac{T'}{T_o} &= 1 + \frac{V_L^2}{2} + V_L\cos\vartheta_2 + V_F\cos\vartheta_1 \\ &+ V_F V_L \cos\vartheta_1\cos\vartheta_2 + V_F^2 x\cos^2\vartheta_1. \end{aligned} \tag{4}$$

Considering also the abberation Eq. (4) gives (see also Appendix)

$$\frac{T'}{T_o} = \left[1 - \frac{V_L^2}{2} - V_L V_F \cos\omega'\right]$$

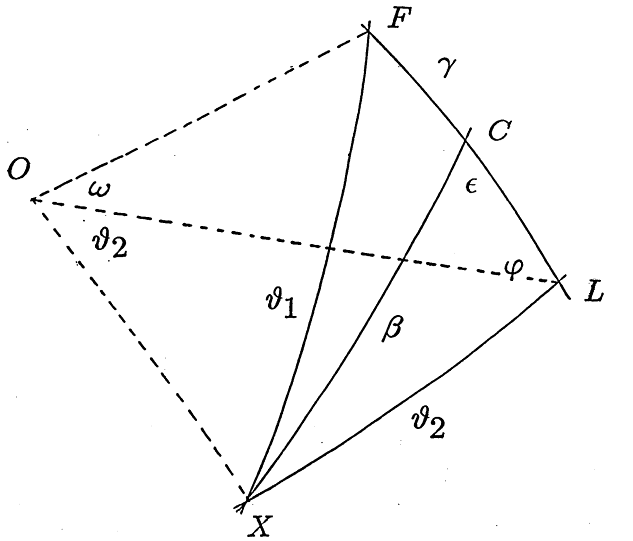

Fig. 1. Illustration of the intrinsic global anisotropy of CMBR. F defines the direction of the maximum of the global intrinsic dipole and quadrupole anisotropy of CMBR seen by the observer O being in rest at ACIF = PF. L denotes the direction of the motion of Local Group with respect to ACIF = PF; C defines the measured direction of the maximum of dipole anisotropy of CMBR as is seen by an observer coupled with Local Group. X is fully arbitrary; this may be chosen at any direction of sky. The triangles FXC, CXL and FXL are common spherical triangles. Their sides are: $\vartheta_1, \beta, \gamma$; $\beta, \vartheta_2, (\omega - \gamma)$; $\vartheta_1, \vartheta_2, \omega$; respectively. ϵ and φ are the angles of the spherical triangle CXL.

$$\begin{aligned}
&+ [V_L(1 - V_F)\cos\vartheta_2' + V_F(1 + V_L\cos\omega')\cos\vartheta_1'] \\
&+ \left[V_L^2\cos^2\vartheta_2' + 2V_F V_L\cos\vartheta_1'\cos\vartheta_2' + V_F^2 x\cos^2\vartheta_1'\right] \\
&= \left[1 - \frac{V_L^2}{2} - V_L V_F\cos\omega'\right] \\
&+ \left[V_C\cos\beta' + V_L V_F(-\cos\vartheta_2' + \cos\omega'\cos\vartheta_1')\right] \\
&+ [V_C^2\cos^2\beta'] + [V_F^2(x-1)\cos^2\vartheta_1']. \qquad (5)
\end{aligned}$$

The physical significance of terms at the right-hand-side of Eq. (5) is straightforward. At the first square-bracket the numbers of order $\sim 10^{-6}$ give small non-measurable corrections to one. The first term $V_C\cos\beta'$ in the second bracket is the ordinary dipole term of size $\sim 10^{-3}$ being identical to the case, when vector $\boldsymbol{V}_C$ indeed describes the motion of Local Group. The remaining terms in this bracket are at $\sim 10^{-3}$ times smaller dipole terms correcting with such a small value the direction and size of dipole anisotropy. Clearly, these corrections are unimportant. Hence, we have reproduced the result of LP: the dipole anisotropy here is identical to the standard case, when $\boldsymbol{V}_C$ actually describes the motion of Local Group (up to $\sim 10^{-3}$ not important corrections). In other words, the study of dipole anisotropy itself cannot decide the question that the term $V_F\cos\vartheta_1'$ either describes an actual motion of ACIF or is an intrinsic dipole anisotropy of CMBR. The third term is "the kinematic quadrupole anisotropy term" of CMBR, if $\boldsymbol{V}_C$ defines an actual motion of Local Group (Peebles 1993). Thus, this term will exist both in the standard case, when $\boldsymbol{V}_C$ defines a real motion, and also in the case discussed here. It is an axially symmetric term with axis defined by $\boldsymbol{V}_C$. Only the fourth term is new not existing in the Friedmannian case. This is an axially symmetric term with respect to the direction of vector $\boldsymbol{V}_F$. In other words, in Eq. (2) the intrinsic quadrupole term is in principle measurable in the frame coupled with Local Group (except for the case $x = 1$). This effect ("non-kinematic quadrupole anisotropy") will be discussed at the next Section.

3. Non-kinematic quadrupole anisotropy

The non-kinematic quadrupole anisotropy defined by

$$\delta T = T_o(x-1)V_F^2\cos^2\vartheta_1' \qquad (6)$$

may be written as follows. If the galactic coordinates of X are $(l,\ b)$, one obtains

$$\begin{aligned}
\cos\vartheta_1' &= \sin b\sin b_F + \cos b\cos b_F\cos l\cos l_F \\
&+ \cos b\cos b_F\sin l\sin l_F, \qquad (7)
\end{aligned}$$

and substituting this into Eq. (6) one obtains

$$\begin{aligned}
\delta T &= T_o(x-1)V_F^2\Big[\frac{1}{3} + \left(\frac{2}{3} - \cos^2 b_F\right)\frac{3\sin^2 b - 1}{2} \\
&+ \frac{1}{2}\sin 2b_F\cos l_F\sin 2b\cos l \\
&+ \frac{1}{2}\sin 2b_F\sin l_F\sin 2b\sin l \\
&+ \frac{1}{2}\cos^2 b_F\cos 2l_F\cos^2 b\cos 2l \\
&+ \frac{1}{2}\cos^2 b_F\sin 2l_F\cos^2 b\sin 2l\Big] \\
&= T_o(x-1)\frac{V_F^2}{3} + Q_1\frac{3\sin^2 b - 1}{2} + Q_2\sin 2b\cos l \\
&+ Q_3\sin 2b\sin l + Q_4\cos^2 b\cos 2l + Q_5\cos^2 b\sin 2l. \qquad (8)
\end{aligned}$$

The first term - not depending on b and l - gives a non-essential $\sim 10^{-6}$ order correction to T_o. The remaining five terms define the five components of the intrinsic non-kinematic quadrupole anisotropy of CMBR. Numerically these values are:

$$\begin{aligned}
Q_1 &= T_o(x-1)V_F^2\left(\frac{2}{3} - \cos^2 b_F\right) = 4.1(x-1)\mu K, \\
Q_2 &= T_o(x-1)\frac{V_F^2}{2}\sin 2b_F\cos l_F = 13.4(x-1)\mu K, \\
Q_3 &= T_o(x-1)\frac{V_F^2}{2}\sin 2b_F\sin l_F = -4.1(x-1)\mu K, \\
Q_4 &= T_o(x-1)\frac{V_F^2}{2}\cos^2 b_F\cos 2l_F = -2.9(x-1)\mu K, \\
Q_5 &= T_o(x-1)\frac{V_F^2}{2}\cos^2 b_F\sin 2l_F = 1.9(x-1)\mu K. \qquad (9)
\end{aligned}$$

These values are the theoretical predictions of non-kinematic quadrupole anisotropy, not existing in case No.2.

4. Discussion and conclusion

The predictions (Eq. (9)) should be compared with the observational data. Here we shortly discuss this question.

First, one has to note that the theoretical predictions have a great uncertainty. They are as precise as the coordinates $(b_F,\ l_F)$ themselves. This problem is intensively discussed by LP, and therefore no further discussion is needed here. Second, from Eq. (5) it follows that the term defined by Eq. (6) is coupled with the term $V_C^2 \cos^2\beta'$, and these two terms have the same order ($\sim 10^{-6}$). Of course, the standard kinematic quadrupole anisotropy may also be decomposed into its five components (cf. Smoot et al. 1992), and hence the predicted non-standard values are accompanied by the standard terms. Fortunately, if the angles γ', ω', and the values V_C, V_F, V_L are known, these two terms are separable. Third, further quadrupole anisotropy terms of CMBR may be generated at any time during and after the recombination. The quadrupole anisotropies of order $\sim 10^{-6}$ expected to be generated during the recombination due to the Sachs-Wolfe effect are often discussed and should have fully different behaviour than the intrinsic quadrupole term discussed in this paper. Hence, fortunately, they should well be separable from other terms (see, e.g., Bennett et al. 1992 for details). The quadrupole anisotropies generated at the post-recombination era should not play any essential role (Mészáros & Molnár 1996). Hence, the mixture of terms given by Eq. (9) with the other sources of quadrupole anisotropy should not lead to unsolvable problems.

From the observational point of view, after the launch of COBE, there is a good hope that anisotropies of the required size will be measurable. The first COBE data (Smoot et al. 1992, Bennett et al. 1992) announced the measurement of quadrupole components just of the predicted sizes; nevertheless, the newer interpretation of data (Gould 1993; Banday et al. 1994; Bennett et al. 1994; Tegmark & Bunn 1995) highly queries the discovery of any quadrupole terms. Hence, unfortunately, any detailed comparison of the theoretical predictions with observations is still premature. It remains to hope that in the near future the new instrumentation will also allow us to do this comparison.

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Appendix A: abberation and the sum of two dipole terms

Between the angles $\varphi, \omega, \vartheta_1$ and ϑ_2 (Fig.1) the following relation holds

$$\cos\vartheta_1 = \cos\vartheta_2 \cos\omega + \sin\vartheta_2 \sin\omega \cos\varphi. \tag{A1}$$

Due to the aberration the primed angles are given by

$$\begin{aligned}
\cos\vartheta_2 &= \frac{\cos\vartheta_2' - V_L}{1 - V_L \cos\vartheta_2'} = \cos\vartheta_2' - V_L \sin^2\vartheta_2' + \ldots, \\
\cos\omega &= \cos\omega' - V_L \sin^2\omega' + \ldots, \\
\varphi &= \varphi'.
\end{aligned} \tag{A2}$$

ϑ_1' may be defined by Eq. (A.1), if in the right-hand-side the primed angles are used.

The sum of two dipole anisotropies defines one single dipole anisotropy via:

$$\begin{aligned}
& V_L \cos\vartheta_2' + V_F \cos\vartheta_1' \\
&= V_L(\cos(\omega' - \gamma') \cos\beta' + \sin(\omega' - \gamma') \sin\beta' \cos\epsilon') \\
&+ V_F(\cos\gamma' \cos\beta' - \sin\gamma' \sin\beta' \cos\epsilon') = V_C \cos\beta', \\
V_C &= V_L \cos(\omega' - \gamma') + V_F \cos\gamma', \\
V_L &= V_F \sin\gamma' / \sin(\omega' - \gamma').
\end{aligned} \tag{A3}$$

It is an elementary mathematics that, because between the vectors $\boldsymbol{V}_L$ and $\boldsymbol{V}_F$ the angle is ω', and between the vectors $\boldsymbol{V}_C$ and $\boldsymbol{V}_F$ the angle is γ', Eq. (A3) gives $\boldsymbol{V}_C = (\boldsymbol{V}_L + \boldsymbol{V}_F)$.

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