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Kinematics of the local universe

IV. Type dependence in the diameter Tully-Fisher relation and implications on the mass-luminosity structure

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Abstract. There is a type dependence in the diameter (Tully-Fisher) TF relation in the spiral galaxy range $1 \leq T \leq 8$. This dependence appears as a zero-point shift from one type to another, while the slope seems to remain constant, close to 0.5 (inverse relation) as expected from our simple disc + bulge + dark halo model. The model also predicts well the amount of the shifts, when reasonable values of the mass-to-luminosity ratios of the disc and bulge components are used. A method to study the dark mass fraction β in different galaxy types, based on the model, is introduced. First applications suggest that in the range $2 \leq T \leq 7$ the value of $\beta \simeq 0.5 - 0.8$ within the radius gr_0 that the TF-measurements refer to. There are also zero-point shifts in the B-magnitude TF-relation (the inverse slope ~ 0.1 according to our work), though smaller than in the diameter relation, as expected from the model. The found type dependencies will diminish the scatter in the TF-relations and facilitate their use in our KLUN programme for measuring the kinematics of the local galaxy universe.

Key words: galaxies: spiral – galaxies: photometry – galaxies: distances and redshifts – galaxies: structure – astronomical data bases: miscellaneous

1. Introduction

The present study prepares the way for our attempt to measure the kinematics of the local galaxy universe (Paturel et al. 1990) using the diameter Tully-Fisher (TF) relation for a sample of 5171 galaxies. This sample was intended to be complete down to the apparent diameter of D_{25} = 1.6 arcmin (corresponding to $\log D_{25}$ = 1.2 where D_{25} is given in 0.1 arcmin). The completeness of the sample has been discussed by Paturel et al. (1994); see also Bottinelli et al. (1995) for a discussion of the inclination correction based on this sample.

Though generally having larger scatter than the magnitude relation, the diameter TF-relation has some advantages as a distance indicator that may be mentioned here. 1) The sample can be made large, thanks to the establishment of a homogeneous system of diameters D_{25}, originally coming from different sources (Paturel et al. 1991). 2) The large sample also allows a detailed examination of the properties of the TF-relation. 3) Practically no inclination correction is needed (Bottinelli et al. 1995). However, the scatter in the relation is rather large, and it is important to try to reduce this before attempting to measure distances, e.g. by investigating any possible dependence on galaxy type, which is the central topic of the present paper. We also ask whether the found systematic changes could be interpreted in terms of the mass-to-luminosity ratios of the disc and bulge components, together with a dark halo, making some steps towards a better understanding of the physics of the TF relation.

M. Roberts first noticed in 1978 that for samples of the same absolute magnitude, late type galaxies have systematically smaller intrinsic line widths than early type systems. This type dependence, corresponding to a zero-point difference in the magnitude relation, was also suggested by Rubin et al. (1985) from direct measurements of rotation curves.

The TF-relation based on the diameter measured at the standard B-magnitude isophote of 25 mag (arcsec)$^{-2}$ is from general considerations expected to depend on type. A simple example explains this best: let us assume that all the galaxies with the same isophotal diameter, have identical discs, on which the diameter is measured. Depending on type, there is in addition a bulge component, providing a fraction of the total mass. Then one expects that going from late (disc-) types towards types with significant bulges, the maximum rotational velocity increases for a fixed isophotal diameter correctly determined on

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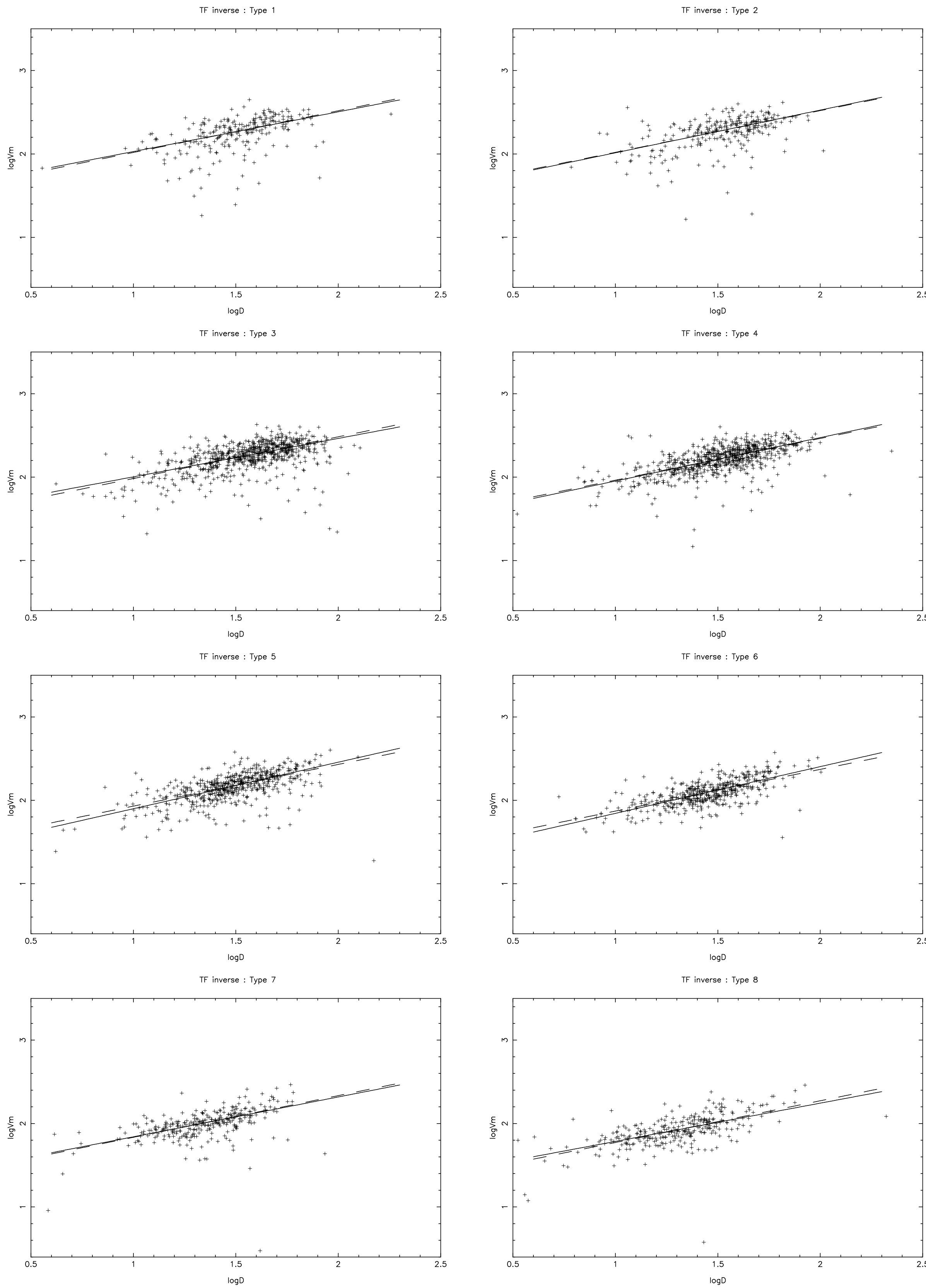

Fig. 1. Inverse Tully-Fisher relation for diameters. Visualisation of the different regression lines for the different morphological types (full line). The dashed line corresponds to a "forced" slope a' = 0.5.

Table 1. slope and zero-points for the direct (a,b) and the inverse (a',b') diameter TF-relations

type	1	2	3	4	5	6	7	8
a	1.04 ± 0.03							
b	-1.007	-1.033	-0.945	-0.899	-0.870	-0.826	-0.863	-0.914
n	28	31	84	83	104	89	51	30
a'	0.5 ± 0.015							
b'	1.518	1.520	1.483	1.464	1.430	1.371	1.335	1.273
n	231	256	685	701	525	458	292	302

Table 2. slope and zero-points for inverse magnitude TF-relation

type	1	2	3	4	5	6	7	8
a'	0.1 ± 0.03							
b'	0.182	0.197	0.171	0.141	0.107	0.063	0.038	-0.022
n	201	254	679	687	511	447	264	224

the disc component. One also expects that at a constant absolute magnitude there is a similar, though smaller effect.

This simple model naturally leads one to study the type dependence using in the first place the inverse TF-relation (to see what happens with $\log V_m$ at fixed linear diameter). The inverse relation (hereafter iTF) has also the very useful property that it may be constructed in an unbiased manner for a large number of galaxies if a sufficiently good velocity field model is available (kinematic distances) and if certain general conditions are fulfilled. Especially there should be no selection according to $\log V_m$; this seems to be the case considering the very good detection rates in all HI surveys.

In this study we use the same velocity field model as in the inclination correction study of Bottinelli et al. (1995), in order to derive kinematic distances. In this Virgo centric infall model (Peebles's 1976 model) the velocity of the Virgo cluster is 980 km s^{-1} and our infall velocity 150 km s^{-1} (the Virgo distance is taken to be 16.5 Mpc, corresponding to H_0 = 68.5 km s^{-1} Mpc^{-1}). Because this kinematic model is necessarily simplified and does not take into account larger scale streamings (like the Great Attractor) or random velocities, we make checks on whether the model has a significant influence on the present results. Errors due to the kinematical model tend to make the slope of the inverse TF-relation too shallow. However, experiments with synthetic data and the Tolman-Bondi velocity field have shown that the inverse relation may be derived using the Peebles's model relatively accurately when the random velocity dispersion is not larger than 100 km s^{-1} (Ekholm, 1995).

2. The sample

The sample is basically the same as in Bottinelli et al. (1995) with the exception that now we include $Sa-$ and $Sm-$galaxies which usually are not utilized in TF-distance determinations because of their large scatter. However, when trying to find and understand possible type dependencies, it is important to have the extreme classes in the sample.

During this study it was noticed that the way of determining galaxy type has some influence on the detected type dependence and this led to a revision in how the types are assigned to the galaxies in the LEDA data bank, from which the present galaxies are extracted. We describe the question of types in the future data paper.

As in Bottinelli et al. (1995), we exclude very face-on and edge-on galaxies, allowing the inclination range $0.07 \leq \log R_{25} \leq 0.80$. Galactic latitudes $|b| < 15°$ are omitted and galactic extinction corrections for the diameters are made according to RC2 (de Vaucouleurs et al. 1976). The inclination correction was derived in the above paper: $\Delta logD = 0.04 \log R_{25}$. As we rely on the Virgo-centric velocity field model to derive the kinematical distances, galaxies closer than about 25° from Virgo cannot be used. The final sample contains 3560 galaxies.

3. Type dependence as revealed by the inverse diameter relation

As noted in the Introduction, we study primarily the type dependence using the inverse TF-relation ($\log V_m = a' logD + b'$), i.e. presenting the data and calculating the regression lines as $\log V_m$ against $logD$ where D is the linear diameter calculated from the kinematical distance and the angular size D_{25}. The panels of Fig. 1 show the inverse relations for the types T = 1 to 8. It is seen even by eye that there are systematic differences along the Hubble sequence. Note also the large scatter for Sa galaxies and the occasional outliers in every type. It may be of some significance that in the early types there is a slight curving down at small diameters, possibly due to the influence of the bulge profile on D_{25} (Sect. 6). In Fig. 2 we give the slopes and the zero-points, respectively, for each type, together with the error bars. The mean value of the slopes is 0.503 . We checked this value by combining all the types in one $logD - \log V_m$ diagram and shifting them to have the same $\langle logD \rangle$. Such a normalization is necessary in order to overcome the bias due to the particular locations of the clouds of data points for different

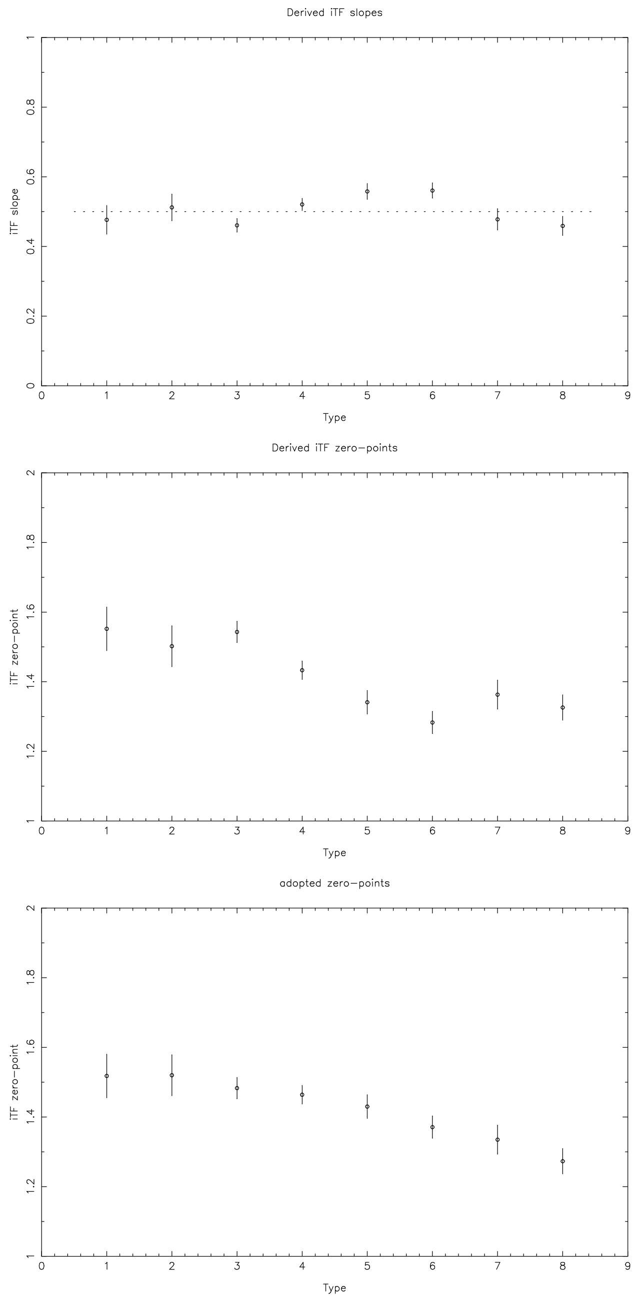

Fig. 2. From top to bottom: the slopes, zero-points, and zero-points when using a fixed common slope a'=0.5, of inverse diameter relation, plotted against the morphological Hubble type of galaxies.

types. This resulted in the slope $a' = 0.504 \pm 0.01$. From the small scatter of slopes in Fig. 2 we conclude that one may use the same slope for all types, and adopt $a' = 0.5$. This slope is shown as the dashed lines in Fig. 1. The middle panel of Fig. 2 gives the progression of the zero-points b' according to type T. There is a clear decrease towards the later types. The fluctuations around the general trend are clearly due to the fluctuations in the slopes of the regression lines. When we fix the slope to $a' = 0.5$, we get the final zero-points in the bottom panel of Fig. 2. The numerical values are listed in Table 1.

In the panels of Fig. 3 we give average surface brightness vs. residual from the iTF relation. Each type reveals a significant dependence between these quantities: positive $\log V_m$ residuals go together with a brighter surface of the galaxy. This indicates that the scatter in the diameter iTF relation is not due to errors in $logD$ (nor in $\log V_m$), but is mostly intrinsic. Additional support for the small errors in $logD$ (coming e.g. from errors in kinematical distances) is obtained from our study of the M vs. $logD$ relations which for late types give the slope 5 as expected from the pure disc model, but significantly different slopes for the early types. That the errors in $logD$ do not dominate, gives further significance to the slope 0.5 derived above.

4. Type dependence in the inverse magnitude TF relation

As will be discussed in Sect. 7.4, the type effect in the diameter TF relation implies also a type effect in the magnitude relation, though with a decreased amplitude. We checked this prediction with those 4311 galaxies of our sample which also have B-magnitude; with the other restrictions (see Sect. 2) this number becomes 3214. The slope and the zero-points corresponding to different types were derived similarly as for the diameter relation in Sect. 3.

For the common slope the value of 0.1 was adopted (Fig. 4, top panel). Bottom panels in Fig. 4 show the zero-points for the separate regressions and for the fixed slope. The latter ones we take as giving the actual zero-point shifts. It is satisfying to see that these, while following the shifts in the diameter relation, have a smaller amplitude as expected from the simple model of Sect. 7.4. We shall discuss the magnitude relation (both direct and inverse) in more detail elsewhere. Here it is sufficient to note that the slope 0.1 is also expected from the disc+bulge model similarly as the slope 0.5 for the diameter iTF relation.

The discussion above utilized galaxies at very different distances (radial velocities). It is interesting to see if the type effect is equally well seen for different radial velocities. To this end we have calculated the cumulative averages of $\log V_m{}'$ for each type in restricted radial velocity ranges, where $\log V_m{}'$ is normalized using the inverse slope as follows (D and M are calculated linear diameters and absolute magnitudes):

$$\log V_m{}' = \log V_m - 0.5(logD - 1.5),$$

for diameters,

$$\log V_m{}' = \log V_m + 0.1(M + 20.0),$$

for magnitudes

The cumulative averages $\langle \log V_m{}' \rangle$ plotted in Fig. 5 reveal clearly the zero-point shifts in the inverse relations all along the velocity axis (note that the ordinate has been put to zero for the complete sample for $T = 8$). At small radial velocities the scatter naturally increases due to the small number of galaxies there, though the types keep well separated almost down to 1000 km s^{-1}. The fact that there is no clear trend away from horizontality

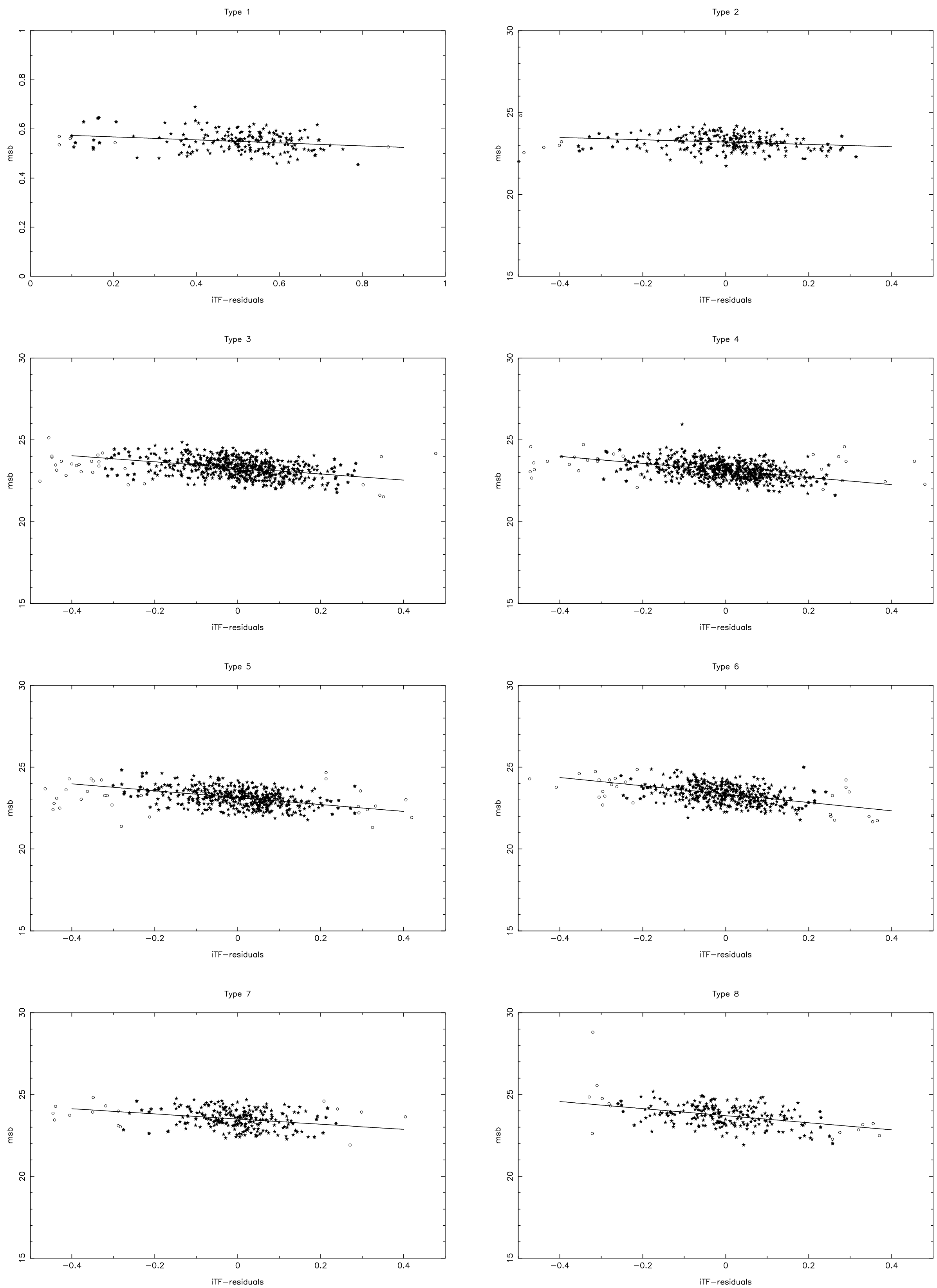

Fig. 3. Correlation between mean surface brightness and iTF residuals in $\log V_m$, for the different types.

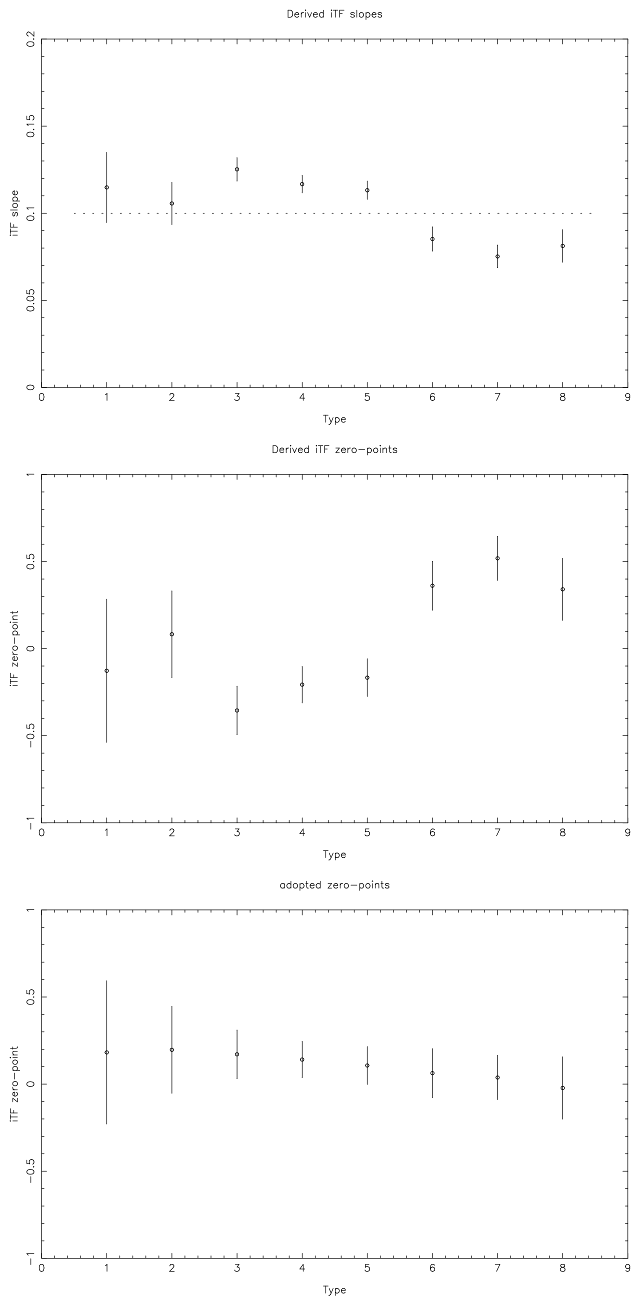

Fig. 4. Same as Fig. 2, now for inverse magnitude relation. A fixed common slope a'=0.1 was used in the bottom panel.

in the diagrams also lends support to the slope adopted. This pair of diagrams also shows quite nicely the expected smaller type effect for magnitudes.

5. Type dependence in the direct relation

The direct TF-relation is naturally expected also to show a type dependence. However, special care is needed in the analysis, because especially the zero-point shifts may be here deformed by the Malmquist bias. Our tool, as in our several previous studies, is the method of normalized distances. Details may be found in Bottinelli et al. (1995). Because we now know that there are zero-point differences, these must be incorporated in the formula for normalized distances as an additional factor $10^{-[b(T)-b(8)]}$. A few iterations are needed when one calculates from the unbiased "plateau" the regression lines for each type. The procedure is facilitated by our assumption that the slopes of the lines are identical. Note that this is not necessarily true, because intrinsic variations of $\log V_m$ at a constant $logD$ or M may depend on type.

In order to derive the slope, we made several experiments with different normalized distance limits defining the plateau. It was found that the slope does not change when different reasonable plateau-limits are used. This is as expected from the simulations performed by Ekholm (1996). For the diameter relation, we finally adopted the slope a = 1.04, common for all types. Unfortunately, even with our large sample, the unbiased subsample defined by the plateau is reduced to a few hundreds of objects. Then, only types 3 to 7 still contain a sufficient number of points (see Table 1). Despite the large error bars, we observe again for this type range that the TF slope remains quite stable, and that the zero-points seem to shift progressively as expected from Sect. 3 (see Fig. 6 and top panel of Fig. 7). We emphasize that our models in Sect. 7 are based on the type effect revealed by the inverse relations where it is naturally expected and more easily detected.

Bottom panels of Fig. 7 show the zero-points from the regressions giving the slopes in the top panel, and after fixing the slope to 1.04. The zero-points for the constant slope are given in Table 1. It should be noted that the zero-points in Table 1 are not calibrated in the usual sense, but correspond to the adopted kinematical distance scale.

6. Photometric diameter D_{25} in the presence of disc + bulge

In trying to understand the type effect, it is best first to turn one's attention to the photometric diameter D_{25}, because this diameter may change when on the disc component one adds the bulge. Although this cannot explain the type effect, it is important to know when the diameter no more reflects only the underlying disc, but is also partially determined by the bulge.

6.1. Relations for the exponential surface brightness law

We first collect several useful results from the exponential surface brightness law of de Vaucouleurs (1959):

$$\ln\left[\frac{\sigma(r)}{\sigma(0)}\right] = -\left(\frac{r}{r_0}\right)^{1/\beta} \quad (1)$$

where $\sigma(r)$ is the surface brightness at radius r and r_0 is the scale factor. For a disc the parameter β is equal to 1 and for a bulge it is equal to 4. E.g., putting $\sigma(r)$ equal to the value corresponding to 25 mag (arcsec)$^{-2}$, one obtains for either of these components the isophotal linear size D_{25}. In magnitude units one may write:

$$\mu_d(r) = \mu_d(0) + 1.086(r/r_0) \quad \text{(disc)} \quad (2)$$

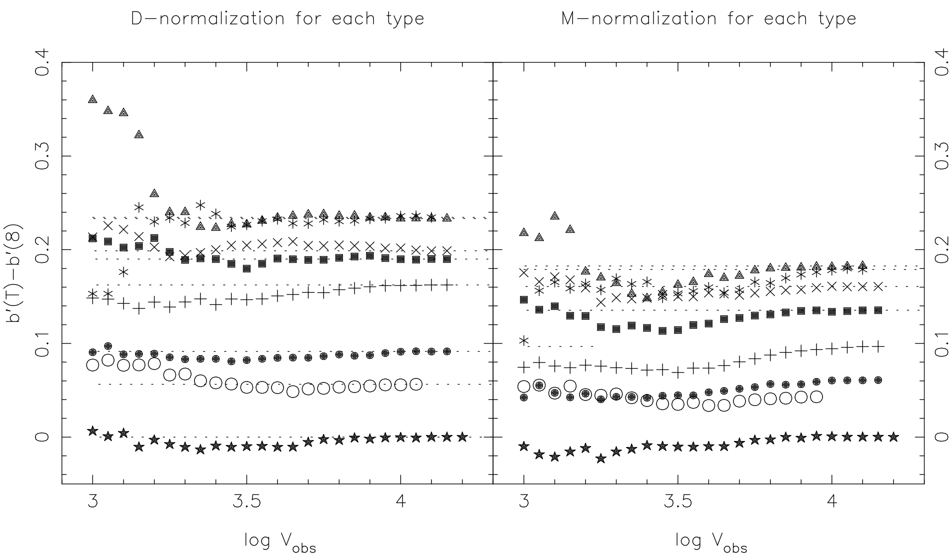

Fig. 5. The type effect as seen in the $\langle logV'_m \rangle$ (i.e. relative zero-point shift $b'(T) - b'(8)$) vs. radial velocity $logV_0$ diagram for both diameter and magnitude inverse relations. Different symbols refer to types ranging from 1 to 8: $*$ for T = 1, black triangle, $\times$, black square, +, $\bullet$, $\circ$ and $\star$ for T = 2–8 respectively.

$$\mu_b(r) = \mu_b(r_e) + 8.325[\left(\frac{r}{r_e}\right)^{\frac{1}{4}} - 1] \quad \text{(bulge)} \tag{3}$$

Here r_e is the effective radius, with one half of the luminosity coming from inside r_e. μ is in units of $M_\odot$ (arcsec)$^{-2}$ and r in pc. For such profiles the total luminosities are given by :

$$L_d = 2\pi\sigma_d(0)r_0^2 \tag{4}$$

$$L_b = 7.215\pi\sigma_b(r_e)r_e^2 \tag{5}$$

Using the above relations and transforming the surface brightnesses into mag arcsec^{-2}, one gets the following useful relations, connecting the isophotal size and the total luminosity to each other:

$$r = 0.367L_d^{\frac{1}{2}}10^{0.2[\mu_d(0)-26.98]}[\mu_d(r) - \mu_d(0)] \tag{6}$$

$$r = 0.21L_b^{\frac{1}{2}}10^{0.2[\mu_b(0)-26.98]}\left[\frac{\mu_b(r) - \mu_b(r_e)}{8.325} + 1\right]^{\frac{1}{4}} \tag{7}$$

Here the other units are 1 pc and 1 $L_\odot$. An important observation, and also essential for much of the present study, is the small range of $\mu_d(0)$ for real discs (= 21.67 ± 0.3 Freeman 1970; Bosma & Freeman 1993). From this one obtains $r_{25} \simeq 3.2r_0$.

6.2. *Shift of* r_{25} *with increasing bulge fraction*

We shall first have a general look at the importance of the bulge in galaxies of different types and with different sizes at the isophotal distance of r_{25} from the centre. To this end the photometric data given by Kent (1985), Simien & de Vaucouleurs (1986), and Kodaira et al. (1986) were inspected.

Fig. 8 gives the difference in surface brightnesses ($\mu_b - \mu_d$) of the bulge and disc components for different types as calculated at the radius r_{25}, according to the data in the mentioned catalogues. Though there are be some differences in the different collections of data, it seems clear that for $T > 1$ the disc component dominates at r_{25} . In the case of Sa, the bulge may become important. Below we show that even there it is important only for quite small sizes.

It is important to know how within a fixed type, the contributions from the bulge and the disc at r_{25} depend on this size, as especially this may influence the type effect in the TF-relation. Here one needs more data than provided by the above references and we try to use our own sample in the following manner: For each galaxy we can calculate its absolute magnitude using the kinematic distance and its apparent B-magnitude corrected for the inclination effect using the recipes in Bottinelli et al. (1995). The absolute magnitude may then be divided into luminosities L_b and L_d, using the average fraction $k = L_b/L_{tot}$, given by Simien & de Vaucouleurs (1986): k = 0.4, 0.32, 0.24, 0.16, 0.09, 0.05, 0.02, 0.01 for T =1,..,8, respectively. The photometric profile (scale length r_0) for the disc may be easily obtained putting $\mu_d(0)$ = 21.5, corresponding to the strict proportionality $r_0 \propto L_d^{\frac{1}{2}}$. Unfortunately, for the bulge one cannot assume a similar regularity. However, it is natural to think that there is still some simple relation between the effective radius r_e and the luminosity L_b, and inspection of Kent's (1985) data (Fig. 9) led us to write

$$r_e = 0.867L_b^{0.35} \tag{8}$$

For the derivation of this relation from Kent's r-data, we have used the colour $B-r$ =1.3 for the bulge. We also confirmed (Fig. 10) the value of $\mu_d(0)$ = 21.5 from Kent's data, using now the colour $B - r$ = 0.7 for the disc alone (Sect. 7.1). The relation (8) differs from the steeper one derived by Kormendy (1977), but we note that he used early type spirals within a narrow absolute magnitude range for the bulges.

From L_b, relation (8), and Eq. (5) we derive r_e and $\sigma_b(r_e)$ for the bulge. Having now in hand the disc and bulge profiles $\sigma_d(r)$ and $\sigma_b(r)$ (in a statistical sense) for the given galaxy, we

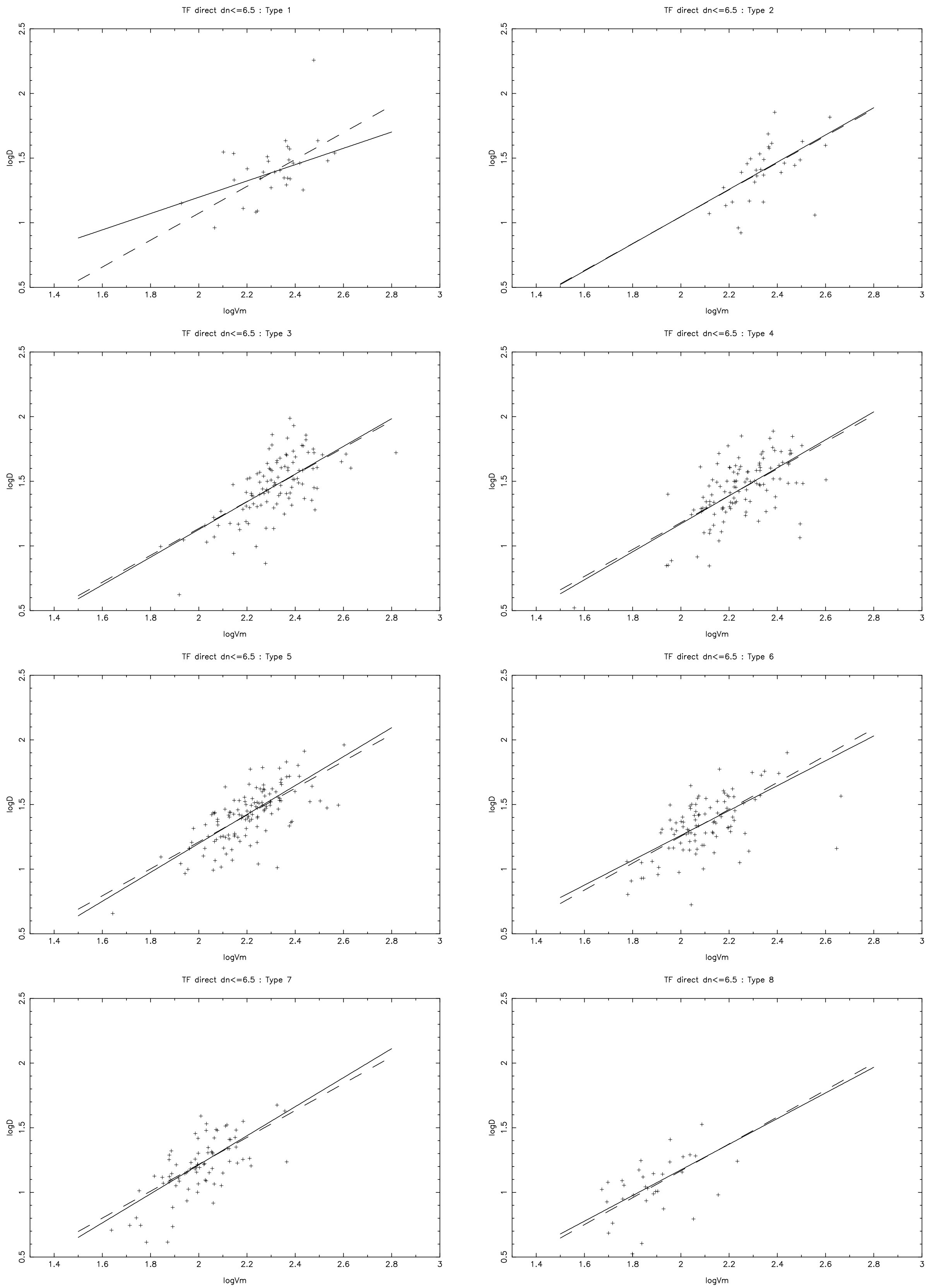

Fig. 6. Direct Tully-Fisher relation in diameter obtained with datas from the unbiased plateau ($d_n \leq 6.5$). Visualisation of the different regression lines corresponding to the different morphological types (full line). The dashed line corresponds to a ”forced” slope a=1.04 .

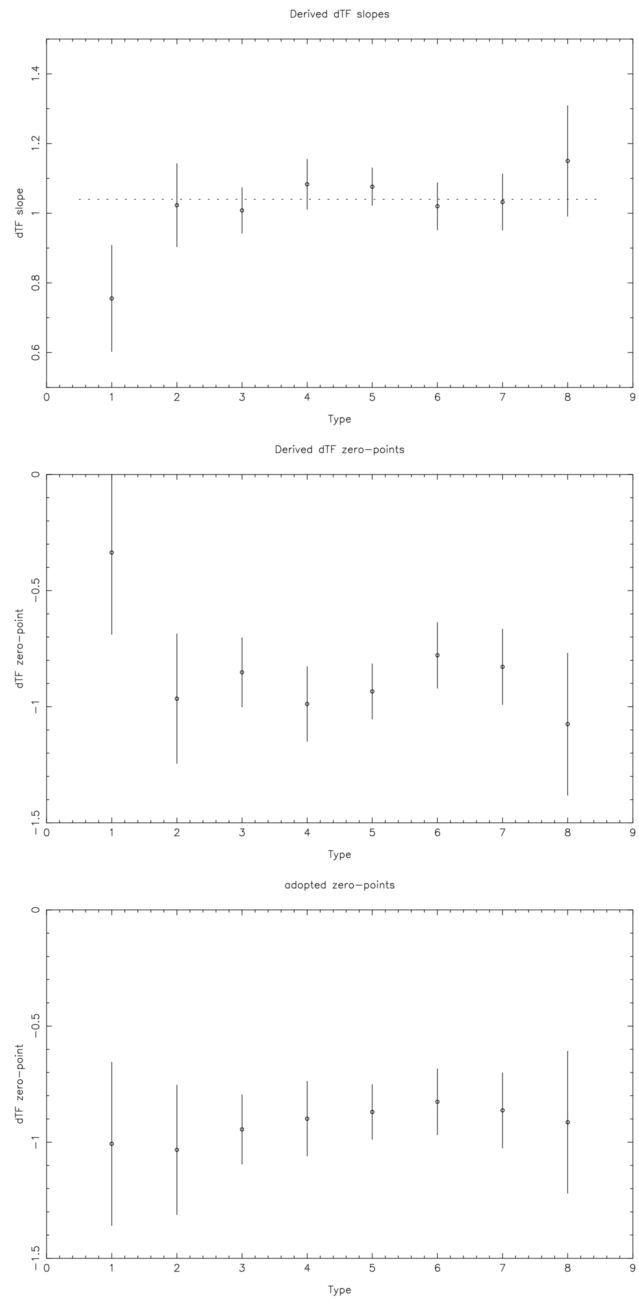

Fig. 7. Same as Figs. 2 and 4, now for *direct* diameter relation. A fixed common slope a=1.04 was used in the bottom panel.

find the radius r where the sum of the profiles add to the surface brightness of 25 mag (arcsec)$^{-2}$ by solving the equation

$$\sigma(r_{25}) = \sigma_d(r_{25}) + \sigma_b(r_{25}),$$

or

$$10^{-0.4\mu(r_{25})} = 10^{-0.4\mu_d(r_{25})} + 10^{-0.4\mu_b(r_{25})} \tag{9}$$

At this r_{25}, the surface brightness difference $-(\mu_b - \mu_d)$ is shown in Fig. 11 for different types and diameters. The different panels suggest that the bulge contribution is important only for quite small galaxies even for types 1 and 2, and almost always the r_{25} radius is determined by the disc.

The above results make it unlikely that the bulge + disc composition could significantly contribute to any type dependence simply through the shift in r_{25} caused by the bulge. However, some signs of this effect might be seen in the diameter TF relation for early types and small sizes. Together with the small dispersion in $\mu_d(0)$, this also implies that fixing the linear isophotal size $logD$, gives us about identical discs.

We have confirmed those conclusions by calculations based on the analytical expressions for the profiles. A bulge is added on a disc and it is looked how r_{25}, originally measured on the pure disc, changes. This problem again requires solving Eq. (9) similarly as above. Our calculations show that the shift caused by the bulge, even for early types (large k) and small disc sizes, is quite small. For example, when adding an Sa-bulge (defined by k = 0.4) to a small disc ($logD_{disc}$=1.2) or to a large one ($logD_{disc}$=1.8), one obtains respectively an increase of 2.65% or 1.2% for the measured diameter.

7. Interpretation of the type effect in terms of the mass- luminosity structure

Why should $logV_m$ depend on the galaxy type at a constant $logD$? We give a simple explanation which presently satisfies us, though it clearly needs more elaboration. As was seen in Sect. 6, the size D_{25} is determined exclusively by the disc, excepting small galaxies, and fixing $logD$ keeps the discs approximately identical. Then what changes, when one goes from late to early types, is the increasing contribution to light and mass by the bulge. Though in light this effect does not seem so strong (relatively small k mostly), the larger mass-to-luminosity ratio of the bulge enhances its importance. Things are complicated by the fact that the discs of different types may have different M/L ratios, especially when one considers only the stellar discs. Kennicutt et al. (1994) have suggested that the star formation is going on more strongly in the discs of the late types which (the stellar discs) should then have smaller (M/L_B) ratios. We give first the simple explanation which assumes that the discs (stars + gas) have similar M/L independent of type.

7.1. Type effect at constant logD: a simple interpretation

The maximum rotational velocity V_{max} is determined by the total mass inside the radius R_{max} (M_d, M_b, M_h, respectively, for disc, bulge and dark halo mass):

$$M_{tot} = M_d + M_b + M_h \tag{10}$$

We make the assumption that at these radii beyond r_{25} where $\log V_m$ is effectively measured, the dark mass is a constant fraction β of the total mass, i.e. proportional to the total luminous mass of the galaxy. This assumption, which seems to be the simplest possible, has the advantage that the proportionality constant β drops off from our final result. In addition, in Sect. 7.3 we find that this assumption is needed to understand in the simplest manner the slope of the inverse TF relation, i.e. when one

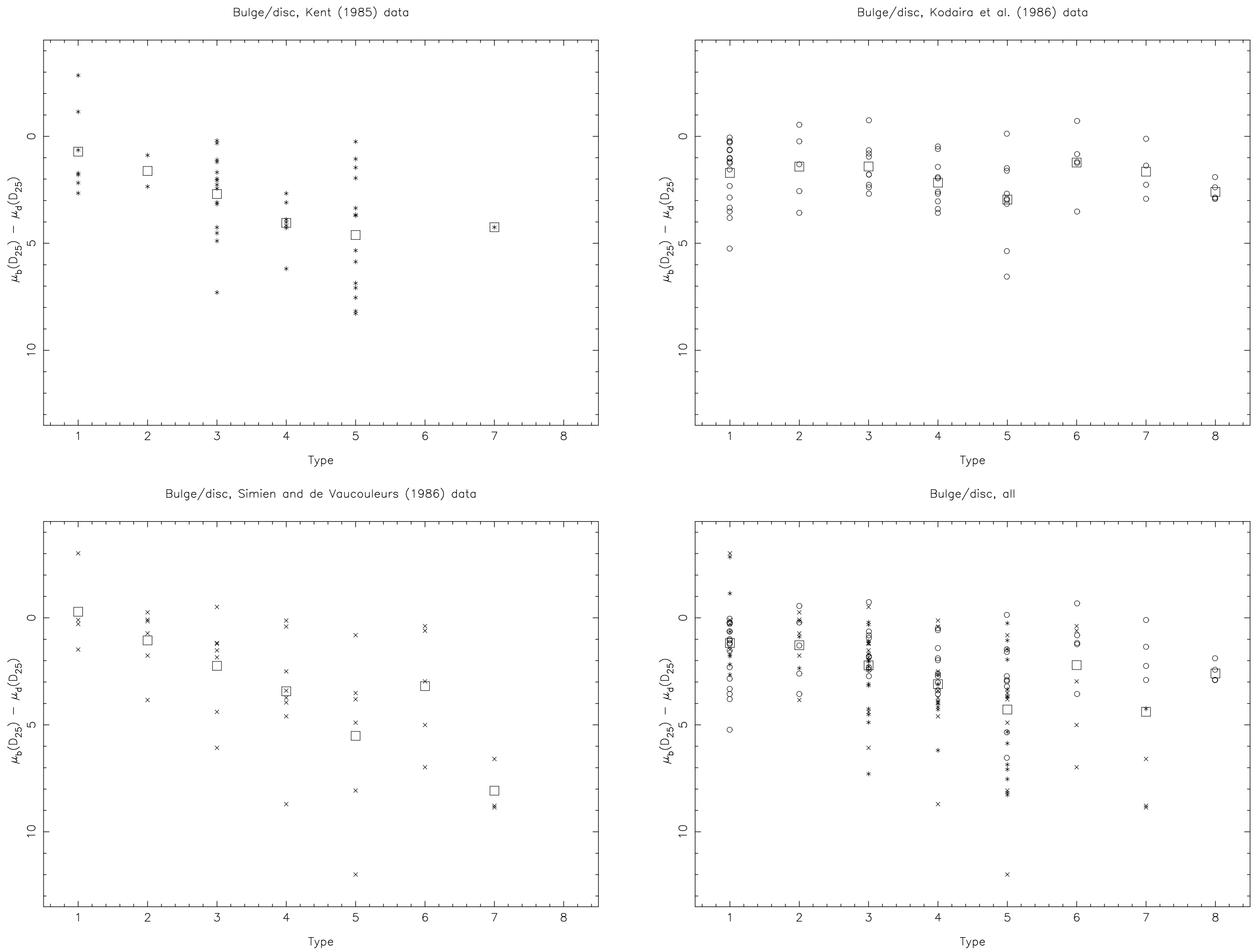

Fig. 8. Relative influence of the bulge and disc components on the photometric profile inside the 25 mag isophote. Surface brightness differences $\mu_b(r_{25}) - \mu_d(r_{25})$ v.s. morphological type for different samples. We used Kent 1985 data in panel 1, Kodaira et al. 1986 data in panel 2, Simien and de Vaucouleurs 1986 data in panel 3; panel 4 shows the result with the three samples mixed.

looks at galaxies of different sizes instead of looking at one fixed $logD$. Hence we have, at a constant disc size:

$$M_h = \beta M_{tot} \tag{11}$$

Let us assume that the $\log V_m$ measurement refers to a radius which is gr_0 (g is roughly constant, independent of type). This radius being quite probably well beyond the optical edge, we may write for the mass inside gr_0:

$$M(gr_0) = (M/L)_{disc} L_d (1 + \alpha K)/(1 - \beta) \tag{12}$$

Here

$$K = k/(1 - k) \tag{13}$$

$$\alpha = \frac{(M/L)_{bulge}}{(M/L)_{disc}} \tag{14}$$

the latter assumed to be constant, as well as the M/L ratios themselves. Now from the dynamical law together with the proportionality $L_d \propto r_0^2$, follows the simple result that

$$\Delta \log V_m = 0.5 \Delta \log (1 + \alpha K) \tag{15}$$

Fig. 12 shows the observed zero-point shifts against the predictions of Eq. (15), with α put equal to 2.5. A nice agreement is seen, especially in the type range 2 - 7. Apparently, the simple model explains the zero-point shifts for different types in this T-range. Such values of α have appeared in the discussions of the disc-bulge-halo model of galaxies (Whitmore & Kirshner 1981), though this question has not been widely discussed. Kent (1986) when decomposing the disc and the bulge and fitting mass-models on rotation curves, derives also M/L ratios for the two components. Depending on the assumptions in the solutions, different ratios were obtained. For Sb galaxies the three solutions gave average values of α of about 1.03, 1.85 and 1.73 (negative and one excessively large α omitted from the data given by Kent's Tables III, IV, and V). However, these results refer to the r-band, and need a correction to the B-band. For a very rough correction, let us assume that for the bulge $B - r$ = 1.3, for spirals typically (disc + bulge) $B - r$ = 0.9 (Kent 1984) , and k = 0.24. This would require for the disc alone $B - r \simeq 0.7$ and a correction factor to α (in r) of about 1.7 in order to get α in

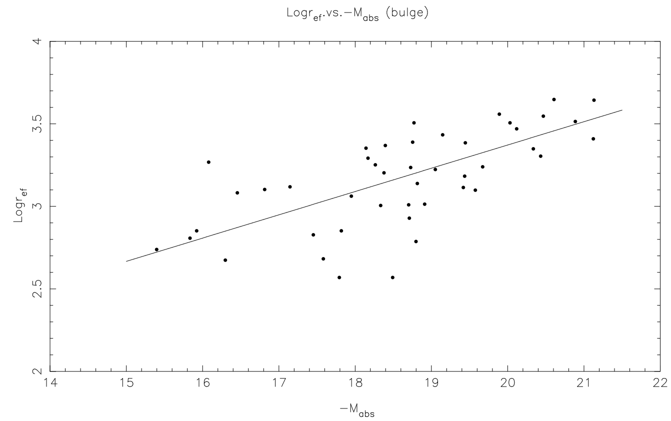

Fig. 9. Correlation between the effective radius of a bulge and its absolute magnitude (datas in r-band from Kent 1985). Relation r_e = $0.867L_B^{0.35}$ obtained from Kent's datas corrected to the B-band (46 spiral galaxies).

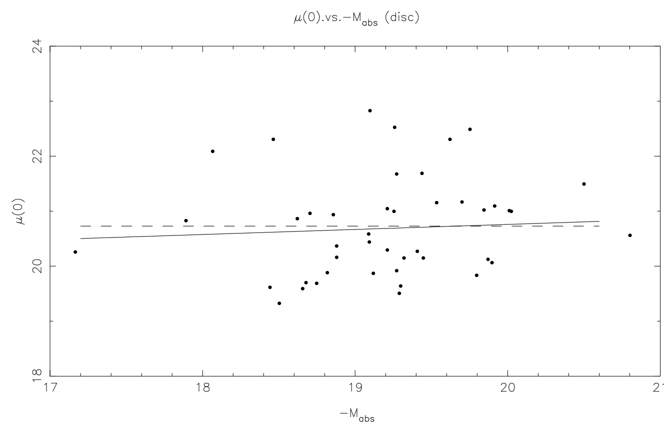

Fig. 10. Central surface brightness of a disc against its absolute magnitude (datas in r-band from Kent 1985). The constant value $\mu_d(0)$ = 21.5 ± 0.3 (B-band) is confirmed.

the B-band. Then the above values of α become 1.75, 3.15, and 2.94, and one may say that Kent's study is not in disagreement with the value of 2.5 that was needed above.

7.2. Discs with different $B-V$ colours

According to Casertano & van Albada (1990) M/L is smaller for late types, because these are presently producing more young stars per unit mass. This interpretation is on line with the view that the systematic photometric changes along the Hubble sequence are basically due to different star formation rates (SFR) (Kennicutt et al. 1994). This might be seen as a corresponding change in the disc's total (M/L_B) ratio (it certainly should be seen in the stellar disc's M/L).

Now the luminous mass is proportional to

$$(M/L)(T)L_d + K(M/L)_bL_d \qquad (16)$$

where $(M/L)(T)$ for the disc depends on the type, on the average, and one may decide to keep the bulge mass-to-light ratio $(M/L)_b$ constant. In order to play with some numbers, we take the "S86" relation between the $B-V$ colour and M/L for an evolving disc population, from Table 1 of Kennicutt et al. (1994) and adopt the average $(B-V)$ colours for types, obtained from our sample, as given in Table 3.

According to this model M/L = 3.72 for the case of very small SFR, while = 1.22 for the colour corresponding to Sc. However, we need typical colours for the discs, and these may be derived using the above total colours, the colour for the bulge (= 0.90), and the bulge-to-total light ratio k. The result of this calculation is also shown above as well as the corresponding $(M/L)_{B,disc}$ from Kennicutt et al. (1994).

In addition to stars we must here take into account the gas mass. Let y_{HI} be the fraction of the total mass of HI and y_{H2} the corresponding number for the molecular gas. We take representative figures from Roberts & Haynes (1994) for y_{HI} and add to these 0.02 (0.0 to types 7,8) to account for the molecular component. Because y_{HI} is measured relative to the total mass, now the dark mass component enters the end result (factor β). The total mass within the measuring radius gr_0 becomes:

$$M(gr_0) = [(M/L)_d + K(M/L)_b]\frac{L_d}{(1-y_{tot}-\beta)}, \qquad (17)$$

where $(M/L)_d$ is taken from Table 3 and $(M/L)_b$ = 3.7. The zero- point shifts in $\log V_m$ are again 0.5 $\Delta \log M(gr_0)$.

The thus predicted zero-point shifts for the different types are given in Table 4 with the observed ones, using the dark mass fraction β = 0.75 or 0.80 . The last value gives good agreement with the observed zero-point shifts for types 2 to 7. From Table 4 one may see that adopting for T=8 the total colour 0.39 ("T = $8'$") or, especially, taking β equal to 0.75, while keeping it equal to 0.8 for the other types, gives a better agreement for this very late galaxy type (see Fig. 12). In view of the uncertainties in the various steps leading to these predictions, we do not put very much importance on such deviations nor on the exact value of β. However, it may be asked whether such a large β is believable.

From the model that we have used above, one may easily calculate for each type the M/L ratios referring to inside the radius gr_0. These values are given in Table 3 for the case β = 0.8 . They follow well the behaviour of the averages in the M/L vs. type diagram of Roberts & Haynes (1994), though are shifted upwards by a factor of about 1.8 (when H_0 = 70 km s^{-1} Mpc^{-1}). As those authors calculated the mass using D_{25} as the size $2R$ in the formula RV_{rot}^2, that factor would in principle give the value of gr_0, i.e. where $\log V_m$ is typically measured.

One possibility to decrease the value of β, is to increase the total gas mass fraction y_{tot} (Eq. 17). E.g., with β = 0.7, we get equally good predictions for the zero-point shifts, if we increase y_{tot} by a factor of 1.5 . Another set of y_{tot} may be obtained from the ratios of molecular to atomic gas derived by Young & Knezek (1989) from a large sample of spiral galaxies. Multiplying y_{HI} by 1 + $M(H2)/M(HI)$ (from their Table 1),one gets y'_{tot} in Table 3. Then a good agreement with the zero-point shifts is given by β=0.5-0.55 for the types 2-7 and β=0.1 for the late type

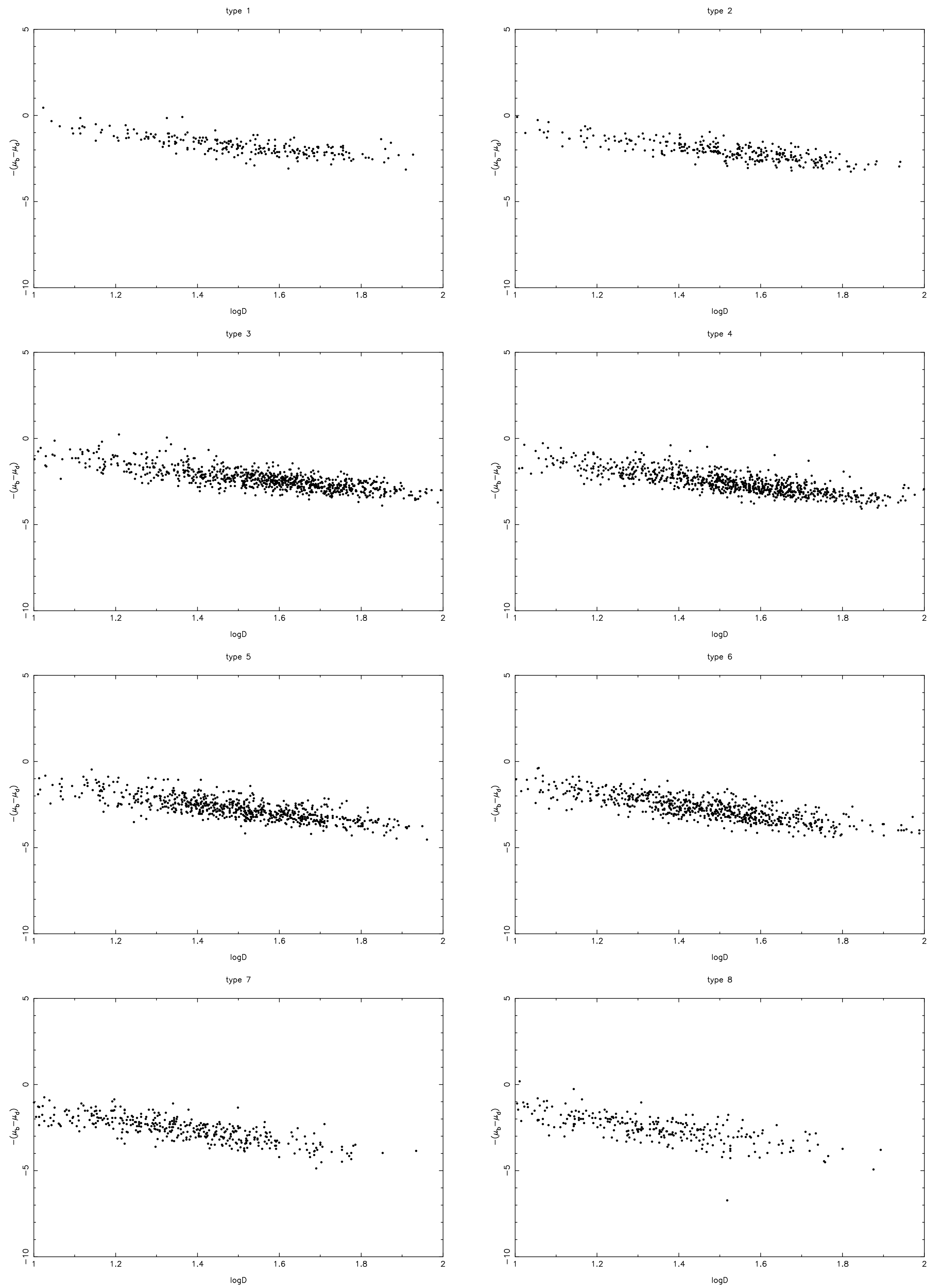

Fig. 11. Relative influence of the bulge and disc components on the photometric profile inside the 25 mag isophotote. Surface brightness differencies $\mu_b(r_{25}) - \mu_d(r_{25})$ v.s. intrinsic diameter; diagrams obtained for each type from a sample of 1606 spirals having both B-magnitudes and apparent diameters.

Table 3. column 1: morphological type (LEDA);
column 2: average total colour index (using LEDA);
column 3: disc colour index (this paper);
column 4: disc mass-to-light ratio (using Kennicutt et al., 1994);
columns 5,6: HI and total gas masses (Roberts, Haynes, 1994);
column 7: mass-to-luminosity ratio (this paper);
column 8: total gas mass (this paper, using Young and Knezek, 1989).

T	$(B-V)_0^T$	$(B-V)_{disc}$	$(M/L_B)_{disc}$	y_{HI}	y_{tot}	$M/L(gr_0)$ (β = 0.8)	y'_{tot}
1	0.72	0.57	1.44	0.03	0.05	15.8	0.15
2	0.68	0.56	1.37	0.04	0.06	15.2	0.13
3	0.61	0.50	1.08	0.06	0.08	14.3	0.17
4	0.55	0.47	0.96	0.07	0.09	12.7	0.17
5	0.50	0.45	0.87	0.09	0.11	12.6	0.16
6	0.41	0.38	0.64	0.10	0.12	9.9	0.13
7	0.39	0.38	0.64	0.12	0.12	8.8	0.14
8	0.43	0.42	0.76	0.14	0.14	13.2	0.17

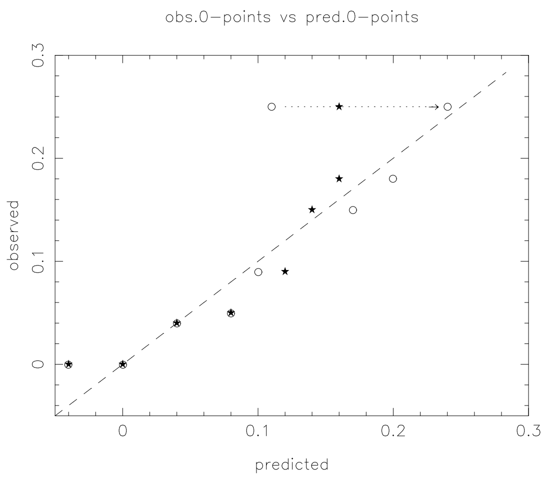

Fig. 12. Comparison between observed zero-points and predicted ones using the simple model described by Eq. 12 (stars) and the more accurate one described by Eq. 17 (open circles). For type 8, we obtained a better agreement with β=0.75 instead of 0.8 .

Table 4. Predicted and observed zero-point shifts.
column 1: morphological type (LEDA);
columns 2,3: predicted zero-point shifts using the second model;
column 4: predicted zero-point shifts using the first simple model;
column 5: observed zero-point shifts

Type	β = 0.75	β = 0.8	Simple model	Observed zero-point shifts
1	-0.04	-0.04	-0.04	0.00
2	0.00	0.00	0.00	0.00
3	0.05	0.04	0.04	0.04
4	0.10	0.08	0.08	0.05
5	0.13	0.10	0.12	0.09
6	0.20	0.17	0.14	0.15
7	0.24	0.20	0.16	0.18
8	0.18	0.11	0.16	0.25
(8’)		(0.24)		(0.25)

8. Clearly the present method has potentials for investigating the value of β in different galaxy types, though the above examples show the importance of a good knowledge of the trends in the gas contents.

7.3. Slope of the inverse diameter TF-relation

Above we have assumed, supported by the observations, that there is a common slope of about 0.5 for all types. This value is expected for a pure disc with the total mass proportional to r_{25}^2. Having in mind the TF-relation where log V_m measures the maximum rotation velocity, we may calculate the radius r_{max} where that maximum is reached. For this we need the relation giving the luminosity inside radius r:

$$L_d(r) = 2\pi\sigma_d(0)r_0[r_0 - \exp\left(-\frac{r}{r_0}\right)(r + r_0)] \tag{18}$$

If the mass follows the luminosity, then $V^2(r)$ is proportional to $L_d(r)/r$, an expression seen to contain r everywhere as r/r_0. Differentiation hence gives the maximum rotation at $r_{max} = gr_0$ where g = 1.8 . Then it is seen that V_{max}^2 is proportional to r_0 and to r_{25}. This means that for a pure disc the slope of the inverse diameter TF- relation is expected to be 0.5 . This result is, of course, not new, but is given here in order to emphasize the remarkable thing that the observations give this same slope for a range of galaxies, containing in addition to the disc, a bulge and dark matter.

What implication the slope of 0.5 has on these components? If we fix the bulge-to-total ratio k, consider galaxies of different scale lengths r_0, assume that log V_m is measured around gr_0 (g is assumed roughly constant regarding to the general shape of spiral rotation curves), and require that the dark mass inside gr_0

is $M_h(gr_0)$ which may depend in some special way on gr_0, then one may write for the rotational velocity at gr_0:

$$gV^2 \simeq (M/L)_d 2\pi\sigma_d(0)(1+\alpha K)\frac{r_0}{g} + \frac{M_h(gr_0)}{gr_0} \quad (19)$$

where $r_0 \simeq r_{25}/3.2$ (see Sect. 6.1)

If $logV$ is strictly proportional to $0.5 \log r_{25}$, then it is clear that $M_h(gr_0)/gr_0$ must be proportional to r_{25}, and hence the dark mass inside gr_0 must be proportional to M_{disc} and to M_{tot}. In this way, considering galaxies of a fixed type but different sizes, we arrive at the same assumption that above was found useful while interpreting the zero-point shifts for different types with fixed size.

7.4. Expected zero-point shifts for the magnitude inverse TF relation

In the magnitude inverse TF diagram, the total magnitude is kept fixed. Then for a given k, the constant luminosity L_{tot} is related to the disc luminosity as $L_{tot} = L_d/(1-k)$. Above it was shown that at a constant size, i.e. at a constant L_d the zero-point shift is given by Eq. (15). Evidently, at a constant L_{tot}, L_d changes with k as $(1-k)$, hence r_0 as $(1-k)^{1/2} L_{tot}^{1/2}$ (instead of being constant as in the diameter iTF). Then, an additional term is needed to Eq. (15), resulting in the total shift (in the simple model of Sect. 6.2) :

$$\Delta \log V_m = 0.5\Delta \log(1+\alpha K) + 0.25\Delta \log(1-k) \quad (20)$$

The additional term makes the shifts smaller than in the diameter inverse TF relation. Do we see this in the observations? Fig. 13 shows the zero-point shifts for the diameter (open circles) and magnitude (stars) iTF relations, normalized to 0.0 at T = 8 where the correction term is practically zero. The shifts in the magnitude relation are smaller, as expected. The broken line shows what happens when the correction term is substracted from the magnitude relation shifts: the agreement with the observed zero-point shifts in the diameter relation is good.

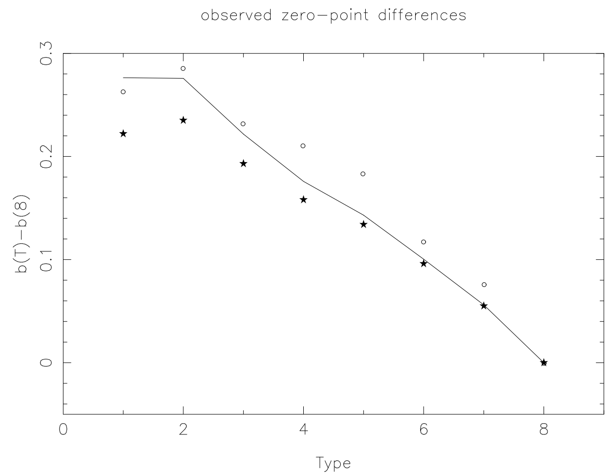

Fig. 13. Observed zero-point shifts for the inverse Tully-Fisher relation in magnitude (stars) and in diameter (open circles). The broken line shows the expected values for the diameter relation when the zero-point shifts in magnitude relation are known (see Eq. 15 and Eq. 17).

8. Discussion and conclusions

In the present paper we have shown that there is a type dependence in the zero-points of the diameter and also B- magnitude TF-relations. The new knowledge of the zero-point differences will have direct impact on our future attempt to use the TF-relation in the study of the kinematics of the local galaxy universe. The type dependences in both the diameter and magnitude relations were shown to be expected from simple disc + bulge + dark halo models, where the de Vaucouleurs disc determines the photometric diameter. This discussion of the origin of the type dependence should form a useful starting point for more detailed investigations of such models, including the fraction of dark mass needed, using large galaxy samples and the TF-relations. It is also clear that improved knowledge of the total gas fractions is important for the present kind of analysis of the dark mass fraction β within the radius gr_0 that usual TF-relations refer to. Our present calculations gave for β values in the range 0.5-0.8 .

A concise summary of the main conclusions:

* the type dependence may be seen as a zero-point shift, while the slope remains the same in the range $1 \leq T \leq 8$, and especially in the range $2 \leq T \leq 6$, to be used in the KLUN analysis
* the slope of the inverse diameter TF-relation is quite close to 0.5 as expected from the simple model, where the dark mass fraction β is a constant; the slope of the inverse magnitude TF-relation is close to 0.1, also as expected from the same model
* as the main ingredient of the model is the de Vaucouleurs exponential disc, we confirm that the bulge component does not generally influence the determination of the photometric diameter at the 25 mag $(\mathrm{arcsec})^{-2}$ level
* it is concluded that the kinematical distance scale used to calculate linear diameters (and absolute magnitudes) is sufficiently good so that at constant calculated diameter the intrinsic variations in $logV_m$ (mass) dominate
* we do not see evidence for a curvature in the TF-relations
* the zero-point shifts may be understood in terms of the disc + bulge + dark halo model, with reasonable M/L ratios for the disc and bulge components

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