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ASTRONOMY AND ASTROPHYSICS

The level of agreement between theoretical and observed globular cluster luminosity functions

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Abstract. Luminosity functions from theoretical stellar evolution calculations are compared with observed ones of several galactic globular clusters (M30, M92, M68, NGC6397, M4, M80, NGC6352, NGC1851). Contrary to earlier results of Faulkner & Swenson (1993) and Bolte (1994) we find no significant discrepancy that could indicate the neglect of important physical effects in the models. However, it is confirmed that the subgiant branch is the most sensitive part and shows the largest deviations in the luminosity function comparison, if parameters are unappropriate. We also find that the main sequence is suited less than the Red Giant Branch for the calibration of theoretical luminosity functions, mainly because of apparent completeness problems. While for individual clusters different changes in the model assumptions might resolve mismatches, there is no systematic trend visible. It rather appears that the quality of the luminosity function in the subgiant part is insufficient and that improved observations of this particular region are necessary for a better comparison. At the present quality of luminosity functions theory is in agreement with observations and a postulation of WIMPs acting in stellar cores does not seem to be justified. However, we conclude that improved data for the main sequence and subgiant branch are clearly needed to exploit the potential of luminosity functions as a diagnostic means for stellar evolution theory.

Key words: stars: evolution – stars: luminosity function – globular clusters: general

1. Introduction

Globular clusters are compared with the theory of stellar structure and evolution by use of two functions: the colour-magnitude-diagram (CMD) displaying the surface properties of the stars and the luminosity function (LF) measuring the initial mass function (IMF) for the unevolved lower main sequence and the speed of evolution after the turn-off. According to Faulkner & Swenson (1993, hereafter "FS") the agreement with the observed LF of the globular clusters M30, M92, M68 and NGC6397 is sufficiently bad and unresolvable with standard stellar structure physics that an additional physical mechanism acting during the main-sequence evolution has to be postulated. In particular, they found that the observed LFs show an excess of stars on the subgiant branch, which is interpreted as a prolonged subgiant phase resulting from a higher central hydrogen abundance at the turn-off and an initially broader hydrogen burning shell, which is narrowing during the subgiant phase. The shell, developing at the end of the main sequence phase in the standard case, can be made more extended if some part of the stellar core would be isothermal. In fact, such models have the maximum of the energy generation in the center until they are approximately halfway between the turn-off and the base of the red giant branch. Faulkner & Swenson (1988, 1993) investigated the evolution of low-mass stars on the main sequence and the subsequent subgiant and giant branch under the assumption of such an isothermal core extending over the innermost 10% of the stellar mass and found a better agreement between theoretical and observed LF. Bolte (1994) confirmed these results for M30 using an improved V-band LF but the same theoretical models (Bergbusch & VandenBerg 1992). As a side-effect of the isothermal cores, globular clusters would be younger by about 20% than usually thought (FS).

The origin of the efficient energy transport needed in the stellar center was suggested to be found in the presence of WIMPs accreted by the star during its main-sequence phase (FS). Bolte (1994) already mentioned that only a very small region in the WIMPs' mass–cross-section–phasespace is left over from various experiments and theoretical expectations. In addition, solar models without an isothermal core better fit the helioseismological data (Cox, Guzik & Raby 1990; Kaplan et al. 1991). A similar result is reported by Basu & Thompson (1996) for a solar seismic model with a reduced central temperature simulating the effect of some additional energy transport by WIMPs.

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These and additional arguments led Bolte to the conclusion that WIMPs are unlikely to be a major constituent of halo dark matter accreted by stars and affecting their core structure. Since his paper the parameter space for WIMPs – when assumed that they are neutralinos – has constantly been shrinking (Fornengo 1994; Mignola & Berezinsky, private communication); thus his conclusion is more justified than ever. However, depending on new ideas the nature of WIMPs might be different and they possibly might exist and act in stars.

Independent of this, the result of FS and Bolte (1994) remains: that there was an apparent mismatch between theoretical and observed GC luminosity functions, which can be reconciled by an isothermal core. The reason for the isothermality would remain unclear. Since this discrepancy is a severe challenge to stellar structure theory, we re-investigated the quality of both theoretical and observational LFs. In Sect. 2 we will discuss the method of obtaining a theoretical LF and problems and errors associated with the observed LFs. In the following section, we will perform standard comparisons with the Faulkner & Swenson and other clusters and discuss the results. In Sect. 4 we will investigate variations in the standard physics and the effect of isothermal cores. At last, our conclusions follow in Sect. 5.

2. Luminosity functions: calculated and observed

To compare theoretical luminosity functions with the observed ones of different globular clusters we need isochrones for various ages, chemical compositions and initial mass functions (IMF). With these quantities we obtain the number of stars in any given interval of visual magnitude by

$$\frac{dn}{dM_V} = \frac{dn}{dm} \cdot \frac{dm}{dM_V}.$$

For the IMF we use the classical power law form:

$$\frac{dn}{dm} = m^{-s},$$

with m being the stellar mass. To find the best agreement with observational results we tried to fit the data of each cluster with several luminosity functions for different ages, metallicities and exponents for the power law of the IMF (we recall that the IMF affects the MS only); these parameters are either taken from the literature (metallicity, distance modulus) or, in the case of cluster age, have been determined with our own isochrones. The best fit for each cluster will be shown in the next section. All LFs are normalized to the total number of stars on the RGB. We have checked that the normalization does not depend on the brightness range used. As will be demonstrated in Sect. 4, the RGB-part of the LF is not at all influenced by any assumption we have tested, including that of an isothermal main-sequence core. In contrast, the main-sequence part depends strongly on the IMF and – though less – on the helium content. It is therefore natural to prefer the RGB for the normalization of the LFs. Since the distance modulus is not known exactly, the LFs might be shifted by up to $\approx 0\overset{m}{.}1$, which certainly is within the errors. This way, no unique "best fit" is possible, but a small uncertainty in metallicity or age remains, because small changes in these quantities do not change the shape of the LF.

The helium mass fractions of all isochrones is $Y = 0.23$ or $Y = 0.24$. This small difference does not influence the LFs (Sect. 4.3 and Ratcliff 1987) The range of age and metallicity covered by the adopted isochrones are $10 \div 20$ Gyr and $Z = 0.0001 \div 0.006$, resp. We only used solar metal ratios within Z. Salaris et al. (1993) have demonstrated that for metal poor stars only the total or *global* metallicity is important for evolution, isochrones and therefore LFs. This has been confirmed by Salaris et al. (1996). We thus can savely ignore α-element metal ratios, as long as the *global* or total metallicity is correct.

All evolutionary calculations have been made with the Frascati Raphson Newton Evolutionary Code (FRANEC) whose general features and physical inputs have already been described in previous papers (see e.g. Chieffi & Straniero 1989). For all metallicities except $Z = 0.0002$ we adopted the isochrones of Chieffi & Straniero (1989) and Straniero and Chieffi (1991) with radiative opacity coefficients from the Los Alamos opacity library (Huebner et al. 1977; Ross & Aller 1976 solar metal ratios), combined with the Cox & Tabor (1976) opacities in the low temperature region (below $10^4\,K$). For $Z = 0.0002$ we built isochrones with the latest OPAL opacity tables (Rogers, private communication, and Rogers & Iglesias 1992; Grevesse & Noels 1993 solar metal ratios) combined with the molecular opacities of Alexander & Fergusson (1994). The different choices for the opacity tables is not relevant because, as we will discuss in Sect. 4, the LFs are almost completely unaffected by the adopted opacity coefficients. For the equation of state (EOS) we considered two separate regions: an high-temperature region ($T > 10^6$ K), where matter can be assumed to be completely ionized and where we adopted the EOS of Straniero (1988) and a low temperature region ($T < 10^6$ K) where partial ionization takes place. In this last region the thermodynamical properties of partially ionized matter are derived from the Saha equation as described in Chieffi & Straniero (1989); the pressure ionization is included according to the method described by Ratcliff (1987). The colour transformation of Kurucz (1992) was used to transform from the theoretical temperatures and luminosities to colours and visual brightness.

For a comprehensive discussion about the comparison between theoretical and observational luminosity functions, we refer to Ratcliff (1987) and references therein. Here we just wish to note that, despite of some evident advantages of LFs such as the fact that they are almost independent of the unknown details of model envelope structure, the most constraining disadvantage of this method is the requirement of a complete count of stars down to very faint magnitudes ($M_V \geq 20$). This is the main reason why luminosity functions are not used very frequently. During the last years, the situation has been improved with the availabily of CCDs and related software packages; however, it still is not possible to claim that the main problems have been solved completely.

The difficulties in building observational luminosity functions include: the problem of lack of completeness at low magnitudes (even if modern techniques are available to make a quantitative evaluation of the completeness, see e.g. Bolte 1989), the proper normalization between various data sets to build the total luminosity function of a cluster, the removal of background and foreground objects, crowding, and possible systematic errors which could occur during the process of data reduction. It is also important to mention statistical noise: to find all possible features in the subgiant region one needs bins as narrow as $0\overset{m}{.}2$ with a sufficiently large number of stars such that the stochastic variations become smaller than about 10% (see e.g. Chieffi & Gratton 1986). At present, at least to our knowledge, there are few cluster data available which fulfill these requirements. Usually observational data are presented with the statistical error only, but the real errors could be higher. In all cases we are using data already prepared for LFs, i.e. we use the number of stars in brightness bins, where some corrections for completeness had been applied by the observers.

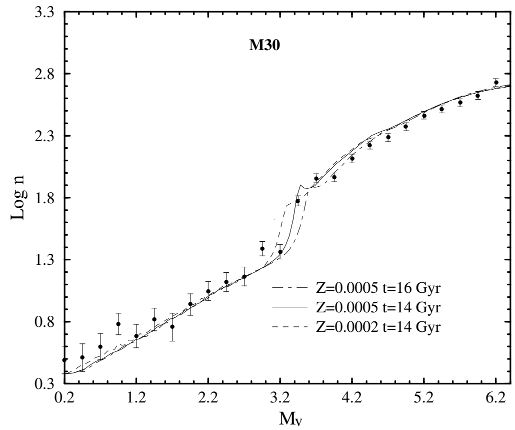

Fig. 1. The observed luminosity function (LF) for M30 (Bolte 1994; symbols) and theoretical LFs for ages of 14 and 16 Gyr and two metallicities. The uncertainty assigned to each bin in the LF is a combination of Poisson noise and the uncertainty in calculating the correction for incompleteness as discussed in Bolte (1989)

3. Standard luminosity functions of individual globular clusters

3.1. M30

We compared the observational LF of M30 taken from Bolte (1994) with our theoretical standard LF. The composition was Y = 0.23 and Z = 0.0005, where Z is the total metallicity equivalent to that of the Bergbusch & VandenBerg (1992) evolutionary tracks with $[Fe/H] = -2.03$ and $[O/Fe]$ = +0.7. We chose this composition, because it is the one used by Bolte (1994) and we wanted to compare our results with his. For the IMF we chose s = 2 and for the distance modulus δm = 14.65, again in agreement with Bolte (1994).

The choice of the cluster age is more complicated. We could either use the same age as Bolte (1994), which was based on the Bergbusch & VandenBerg (1992) isochrones, or use the age our own calculations involve. To approach the problem of the age determination for M30 let us first recall a well-known fact: the group of very metal-poor galactic globulars (M30, M15, M68, NGC6397, M92) has been shown to have indistinguishable observational CM diagrams and thus appears to be coeval within less than 1 Gyr (see e.g. VandenBerg et al. 1990; Walker 1994; Stetson et al. 1996). In spite of such a direct evidence, however, the difference in luminosity between the Horizontal Branch (HB) and the TO (a parameter used to infer the age of the cluster, see Sect. 4.4) reported in the literature differs appreciable (see e.g. Buonanno, Corsi & Fusi Pecci 1989 or Chaboyer & Kim 1995). For example, Chaboyer & Kim (1995) use different ΔV(TO-HB) values for M68 and M92 and determine ages of 13.30 $\pm$ 1.4 Gyr resp. 17.15 $\pm$ 2.1 Gyr. This can be interpreted as an evidence for the present uncertainties in the determination of the absolute cluster ages, due to the difficulties to define precisely the TO and HB luminosity. This is particularly true for M30, M92, and NGC6397, which show very blue HBs with very few stars in the RR Lyrae region, in a way that the determination of the HB luminosity is a very difficult task. Thus we decided to determine only the age of M68, which shows a well defined HB in the RR Lyrae region and for which accurate photometry is available, and assume all other clusters of this group to be coeval with M68. From ΔV(TO-HB) as given by Walker (1994), we found with our isochrones and HB models an age of about 14 Gyr for the adopted metallicity of Z=0.0005. We compare this with the age determined for M30 using the distance modulus of 14.65 ± 0.15 (obtained by Bolte 1994 with the subdwarf fit) and fitting the TO luminosity, which is $\approx$ 13 Gyr. Therefore 14 Gyr is a reasonable choice taking into account the errors in determining cluster ages, which are at least 2 Gyr. We checked that an age variation of this magnitude does not affect the shape of the LF, but results only in a brightness shift, which can be compensated by changes of $0\overset{m}{.}1$ in the distance modulus, which is within the errors. Finally we note that the age for M30 determined with our models using the ΔV(TO-HB) given by Chaboyer & Kim (1995) gives 16 Gyr, which is definitely higher, but still within the general uncertainties of cluster age determinations.

We also constructed a LF with a lower metallicity of Z = 0.0002, which is obtained if oxygen- resp. α-enhancement is ignored. Actually, Djorgovski (1993) gives $Z = 10^{-4}$ (without α-enhancement) based on results by Zinn & West (1984) and Armandroff & Zinn (1988). For this metallicity, which is within the possible range for M68, the age of M68 determined by the ΔV(TO-HB) method is $\approx$ 14 Gyr. If we use Bolte's distance modulus and fit the TO luminosity for M30, we obtain $\approx$ 15 Gyr. Therefore, within the uncertainties, the age of M30 is the same for both possible metallicities.

The result of our LF fits is shown in Fig. 1, where we have normalized to the RGB. We display LFs of both ages for Z = 0.0005. Apart from a small – and unobservable – difference in the subgiant bump maximum brightness, the two LFs are equivalent except for a shift in M_V by ≈ 0.1. In addition we show the LF for Z = 0.0002 and 14 Gyr, which we consider to be the best fit. All our LFs agree with the observed values at all points within 2σ, in contrast to the results of FS. With the same

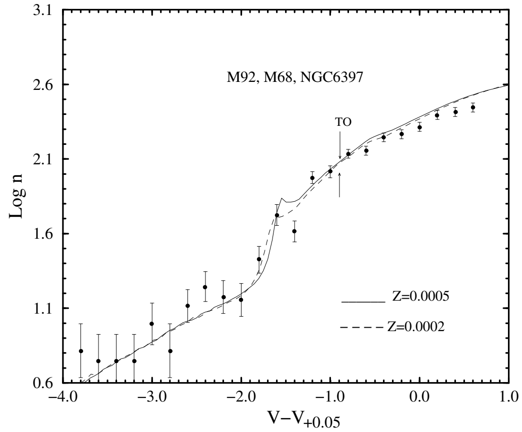

Fig. 2. The combined observed LFs of M92, M68 and NGC6397 (Stetson 1991) and two LFs for age 14 Gyr and s = 2. Errorbars reflect purely statistical errors. As described in Stetson (1991) the origin of magnitudes has been arbitrarily shifted to the point on the upper main sequence which is 0$^{\rm m}$05 redder than the bluest colour at the turn off (indicated by TO)

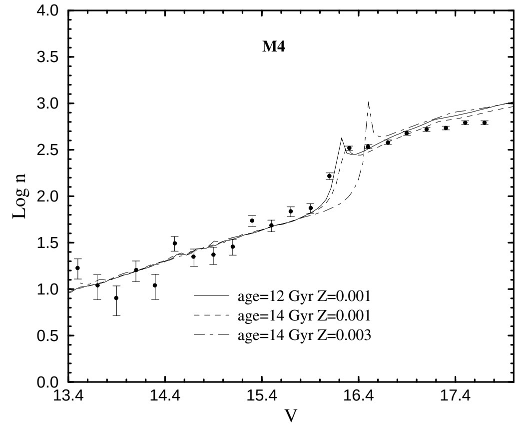

Fig. 3. The observed LF of M4 (Piotto & Saviane, in prep.) and two theoretical LFs for δm = 12.75, Z = 10^{-3} and age 12 (solid) and 14 (dashed) Gyr, the latter one shifted by 0$^{\rm m}$1. Also shown is a LF of Z = 3 10^{-3}. Errors are purely statistical

normalization method, IMF slope and chemical composition, Bolte (1994) finds a systematic underabundance of observed main sequence stars extending up to the TO, while our LFs are only slightly too rich in main sequence stars. Our result for M30 already indicates that the basis for postulating non-standard physics for low-mass star evolution might not exist.

3.2. M92, M68, NGC6397

As in Stetson (1991) we created a composed LF of the three globular clusters M92, M68, NGC6397 (Fig. 2), which show very similar CMDs. A necessary condition for this procedure is that the clusters have nearly identical metallicity, reddening, age and IMF (here s = 2). Then, one can assume that at a given point (Stetson chose the one being 0$^{\rm m}$05 redder than the bluest colour at the turn off) they also have the same absolute brightness and their CMDs can be superimposed. The same procedure applies for the theoretical isochrone. Although we have followed Stetson (1991), we think that such a combination of LFs is not advisable. Again, we compared the observations with our standard LFs for two metallicities, Z = 0.0005 and 0.0002 (for both isochrones the zero points were determined individually). This range encloses the determined metallicities for these clusters including α-enhancement (Salaris & Cassisi 1996). The higher value, as well as all other parameters is in agreement with Stetson (1991). As discussed in the last subsection, the age of M68 was determined to be close to 14 Gyr for both metallicities, and therefore we used this value for this group of clusters, postulated to be coeval. Within 2σ (statistical errors only) theory matches observations at all points except the one at $V - V_{+0.05} = -1.4$ (here within 3σ); the fit employing the lower metallicity (the value probably closer to the metallicity of M92 and M68) is slightly better. As in the case of M30 we cannot detect any significant discrepancy, but the main sequence is slightly underabundant in stars again.

3.3. M4 and NGC1851

For these and the following clusters the data are from Piotto & Saviane (in prep.) and the distance modulus (here: δm = 12.75) from Djorgovski (1993). Alternative values are consistent with this, e.g. Kanatas et al. (1994), who give 12.84 $\pm$ 0.19. For M4 we found the best agreement for the parameters Z = 10^{-3} (9 10^{-4} in Djorgovski 1993), s = 1, and t = 14 Gyr (shifted by 0$^{\rm m}$1). The age is the result of the ΔV(TO-HB) method using the observational value of 3.55 $\pm$ 0.16 (Buonanno et al. 1989) and agrees with that of Chaboyer & Kim (1995). The observational data are well reproduced by this theoretical LF (Fig. 3) and a second one with a lower age of 12 Gyr (unshifted). The 12 Gyr LF fits better to the subgiant region, the 14 Gyr one to the main sequence. For comparison we also show the LF for an metallicity of 3 10^{-3}, which would result if strong α-enrichment were present in M4 (cf. Salaris & Cassisi 1996). Although this LF has been shifted by 0$^{\rm m}$1, the fit is bad.

NGC1851 (Fig. 4) can be fitted by our theoretical LF with the parameters δm = 15.46, Z = 10^{-3} (Djorgovski 1993) and an age between 12 and 14 Gyr. Using the ΔV(TO-HB) of Chaboyer & Kim (1995), we determine the age to be $\approx$ 13 Gyr, in agreement with Chaboyer & Kim, who give 12.6 $\pm$ 1.6 Gyr. The distance modulus is in agreement with those of Alcaino (1976) and Stetson (1981) within 0$^{\rm m}$05. There is no information about possible α-enhancement available. The data are given as absolute visual magnitudes. As in the preceeding cases, there are too few main sequence stars observed (but the counts are still within 3σ), which we suggest to result from uncorrected incompleteness. Otherwise, the data are fitted within 2σ.

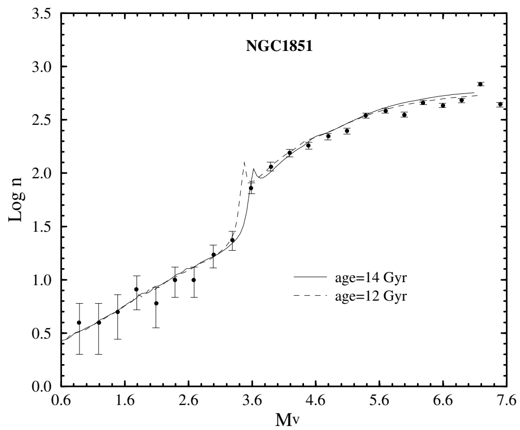

Fig. 4. The observed LF of NGC1851 (Piotto & Saviane, in prep.) and our theoretical LFs for s = 2.35, δm = 15.46, Z = 10^{-3} and ages 12 and 14 Gyr. Errors are purely statistical

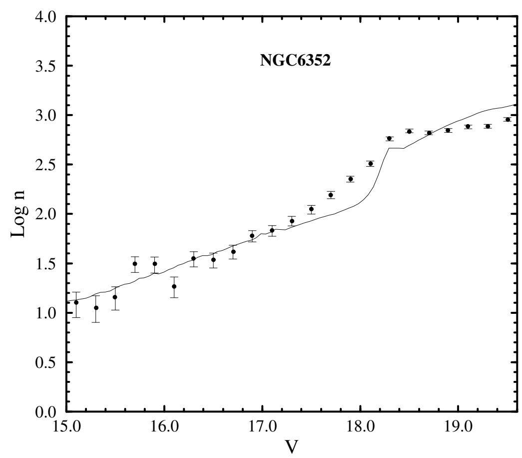

Fig. 5. The observed LF of NGC6352 (Piotto & Saviane, in prep.) and our "best" theoretical LF for δm = 14.52, s = 1, Z = 10^{-3} and age 18 Gyr (statistical errors only)

3.4. NGC6352

For this cluster we could not obtain a satisfying fit. Even our best case, shown in Fig. 5, is comparably poor for both the subgiant and main sequence region, where we have an over- resp. underdensity of observed stars. The slope of the subgiant to giant branch is smaller than theoretically predicted. According to the dependencies investigated by Ratcliff (1987), such a shape of the LF could indicate a very low metallicity. However, the metallicity used is Z = 0.001, which is already lower than Djorgovski's (1993) estimate of 0.006. Using this value results in a definitely worse LF fit. We have tested very low metallicities, but the fit does not improve significantly; the same being true for age variations. We conclude that this cluster needs substantially better observations and an accurate determination of its metallicity before one can compare it with theoretical LFs.

3.5. M80

Our last cluster is M80. For this cluster, no reliable age determination is possible because of its paucity of HB stars and its RGB, which is redder than that of other globular clusters supposed to have the same metallicity. Therefore, neither the ΔV(TO-HB) method nor the superposition of CMDs can be applied savely. Following Brocato et al. (1996) we assume that M80 is coeval with M3 (see this paper for arguments supporting this). For M3 we determine an age of 16 $\pm$ 1.5 Gyr based on $\Delta V(\mathrm{TO}-\mathrm{HB})$ = 3.52 $\pm$ 0.09 (Buonanno et al. 1989), which is in close agreement with Chaboyer & Kim (1995), who give 3.54 $\pm$ 0.09. The other parameters were Y = 0.24, s = 2, Z = 2 10^{-4} or 5 10^{-4} (Djorgovski 1993 gave Z = 4 10^{-4}, but Brocato et al. 1996 argue for the lower value as well; there is no information about a possible α-enhancement for this cluster); for the distance modulus we initially took δm = 15.24 (Djorgovski 1993) and obtained a bad fit with a severe underdensity of observed subgiants and main-sequence stars (Fig. 6). However, the structure of the observed LF indicates that the bump at $V \approx 19.2$ is the true location of the subgiants, indicating that the distance modulus might be incorrect. We therefore have alternatively used δm = 15.57 $\pm$ 0.10 (Brocato et al. 1996; see also Scotti 1995). The resulting LF is shown in Fig. 7. Evidently, it fits much better. This example illustrates how a "discrepancy" may arise from a wrong distance modulus. There still is an overdensity of main-sequence stars visible in Fig. 7. Recently, Brocato et al. (1996) have compared their own photometry with our theoretical luminosity functions and find for identical parameters a good fit *including* the main sequence. We therefore are sure that our main sequence underdensity results from a completeness problem in the Piotto & Saviane (in prep.) data.

4. Luminosity functions with varying assumptions

Up to now our theoretical luminosity functions have been compared with the observed ones for several globular clusters, discussing the possible sources of uncertainty related to the observed LFs. However, to get an idea of how reliable such a comparison is, one must also analyse the possible uncertainties in the theoretical LFs. For this purpose we tested the influence of variations in the main physical input quantities (opacities, chemical composition, age, equation of state, etc.) on the luminosity functions, but also that of an hypothesized isothermal core as suggested by FS and Bolte (1994). Some of these parameters have already been discussed in several papers (see e.g. Chieffi & Gratton 1986, Ratcliff 1987 and references therein); in these cases we only summarize the results, since the basic picture has not changed by the recent improvements in opacities and the EOS. A compilation of these tests can be found in Leone (1995).

4.1. Input physics variations

To test the influence of the different calculations of radiative *opacity* coefficients on LFs we compared LFs constructed with all the (four) possible combinations of choosing the Los Alamos

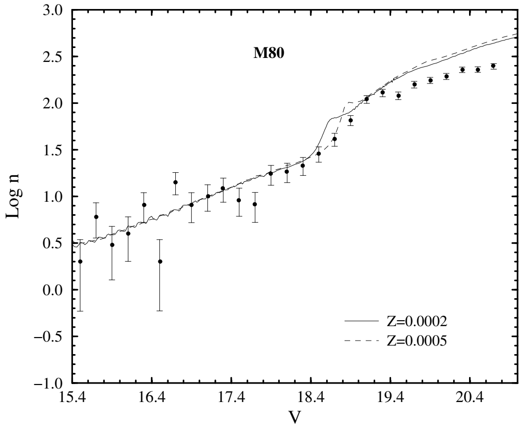

Fig. 6. The observed LF of M80 (Piotto & Saviane, in prep.) and theoretical LFs for δm = 15.24, two metallicities and age 16 Gyr. Notice the discrepancy at $V \approx 18.8$, which arises from the short distance modulus used (see text). Errors are statistical ones only

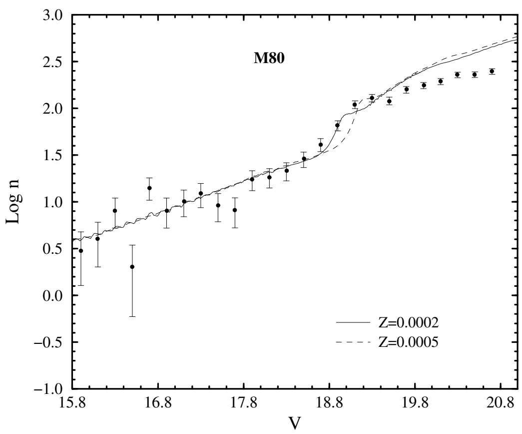

Fig. 7. The same as Fig. 6 (solid line), but with a corrected distance modulus of δm = 15.57 (Brocato et al. 1996)

opacity tables (Huebner et al. 1977) or the updated OPAL opacities (Rogers & Iglesias 1992; Rogers 1994, private communication) for the interior opacities (for temperatures above 10^4 K) and of choosing Cox & Tabor (1976) or the new Alexander & Fergusson (1994) molecular opacity tables for the cool external regions. It is well known from e.g. solar models and homology considerations that opacity influences the stellar luminosity of main sequence models and thereby their lifetimes; however, this results only in a global shift of the LF; a change in the luminosity function's shape is not to be expected, since the shell luminosity depends only weakly on the opacity. Our tests revealed that for $Z < 10^{-3}$ there are no appreciable differences at all. For higher metallicity, the subgiant bump varies by 0.08 dex or less, which is approx. equivalent to a change of 0.15 in $[Fe/H]$, i.e. within the observational errors.

We also compared LFs constructed by using the EOS of Straniero (1988) with those where the updated Livermore EOS (Rogers 1994; Rogers et al. 1996) had been used. The latter one is based on an approach which avoids an ad hoc treatment of the pressure ionization and includes also subtle quantum effects in the corrections for Coulomb forces. Again, the influence is negligible. Note, however, that the new OPAL EOS leads to a reduction of GC ages (Chaboyer & Kim 1995; Salaris et al. 1996) and therefore also influences LFs indirectly.

One physical parameter which may influence the stellar evolution is the efficiency of the $nuclear\ reaction\ rates$. The rate of the proton-proton reaction is too low to be directly measured in the laboratory and it can be determined only theoretically, while for most of the other important hydrogen burning reactions the adopted astrophysical factors are extrapolations of experimental data taken at energies higher than those relevant for stellar interiors (see e.g. Rolfs & Rodney 1988). The uncertainties usually claimed for the rate of the reactions which drive the H burning are below 5% for the reactions of the pp chain (somewhat higher for ^{7}Be + p) and smaller than about 15% for the reactions of the CNO bi-cycle. A variation in the efficiency of these reactions leads to a minor change in the CNO-burning temperature but does not influence the stellar luminosity or evolutionary timescales as can be inferred from homology considerations of shell burning stars. We constructed LFs where the reaction rates for the p+p, ^{3}He+^{3}He and ^{14}N+p, resp., were changed by a constant factor beyond the experimental errors. These LFs remained almost unaffected. Only in the case of the ^{14}N+p reaction modified by a huge (hypothetical) factor of 5, we found a shift in the luminosity of the LF, which, however, is still within the range of the globular distance modulus uncertainty. Since a global change in the evolutionary speed cannot be discriminated from a distance variation, we had chosen this reaction for this test, because the CNO-cycle becomes important only when the hydrogen shell is developing, such that the influence is larger after the turn-off than on the main sequence. Nevertheless, the change is too small to be important.

4.2. Influence of chemical composition

Ratcliff (1987) already pointed out that the dependence of the LFs on the $helium\ content$ is not high and cannot be separated cleanly from that of the unknown IMF. Here we checked that a change in Y of $\pm$ 0.01 w.r.t. a central value of Y = 0.23 (which reflects the uncertainty in the helium content of galactic globulars) has no detectable influence at all on the LFs.

It is well known (see e.g. Paczynski 1984, Ratcliff 1987) that the $metallicity\ content$ strongly affects either the position or the slope of the subgiant branch of the luminosity function. We convinced ourselves that a change in $[Fe/H]$ of $\pm$ 0.2 dex, which is much larger than the average spread in metallicity of any given cluster (see, e.g. Suntzeff 1993 and references therein), gives an effect which can be corrected by shifting the LF by less than $0\overset{m}{.}1$, which in turn is less than the uncertainty in the distance modulus.

All our LFs have a scaled solar $metal\ composition$, while a growing amount of observational data shows that low metallicity globular clusters are very probably enhanced in all of the α-elements (see e.g. Lambert 1989). Salaris et al. (1993) demonstrated that the isochrones for α-enhanced composition are the same as scaled solar LFs with the corresponding global metallicity. This has been confirmed recently by Salaris et al. (1996). In addition, metal ratios affect evolutionary speed – if at all – only via the CNO-cycle efficiency, similar to the correspondig reaction rates (see above). Therefore, no influence on LFs is expected and solar-scaled metal mixtures can be used savely. We have checked and confirmed this point with additional α-enhanced LFs.

We finally add that Proffit & VandenBerg (1991) pointed out that differences in the shapes of the luminosity functions between canonical models and those which include *helium diffusion* are too small to be observationally detectable, if the small corresponding age reduction of $\approx 10\%$ is taken into account. If this effect is neglected, the LFs are fainter by $\approx 0\overset{m}{.}1$.

4.3. Age, IMF, mixing length and bolometric correction

The latter three of these parameters have been discussed extensively by Ratcliff (1987) and Paczynski (1984). They noted that the assumed initial mass function affects only the main sequence part of the LFs, while the LFs are almost independent of the adopted treatment of convection, in particular of the choice of the mixing length parameter (because convection only influences surface temperature as do low-temperature opacities).

On the contrary, the position of the subgiant region of the LF is strongly dependent on age; it could therefore be used as an independent way of determining GC ages or to check that ages determined by use of CMDs result in consistent LFs. In all our cases we indeed have used ages consistent with determined values. Additionally, we checked that a variation in age of $\pm$ 2 Gyr, that is within the claimed uncertainty of determined globular cluster ages, can again be compensated by a shift in luminosity of about $0\overset{m}{.}1$, which is well within the distance uncertainty (see, e.g. Fig. 3).

Following a suggestion of our referee, we also studied the influence of different bolometric corrections on the LFs. We examined B.C.s of VandenBerg & Bell (1985), Buser & Kurucz (1978 & 1992) and Kurucz (1992). The resulting LFs (for $Y = 0.23$, $Z = 2\,10^{-4}$, age 15 Gyr) are extremely similar but show a systematic shift in M_V, which is largest between Kurucz and VandenBerg & Bell ($0\overset{m}{.}18$). Buser & Kurucz is in between those two. This difference is rather large and lies at the edge of the quoted mean error of distance moduli. Although we use the Kurucz (1992) transformations, which yield the brightest LFs, we do not find that a systematic shift to lower brightness is required for our fits. The known problems in the colours obtained from these transformations do not influence our LFs at all.

We can thus conclude that the shape of the LFs is well defined theoretically, being little sensitive to the usual uncertainties in the physical inputs used in the theoretical models of globular cluster stars. In particular we emphasize that the luminosity functions are not affected at all by the uncertainties in mixing length parameter and complicated low temperature opacities which constitute one of the main problem in the calculation of theoretical CMDs. On the other hand this implies that any significant discrepancy, which cannot be ascribed to observational uncertainties as the incompleteness of the main sequence LF, is hard to reconcile within standard stellar structure theory.

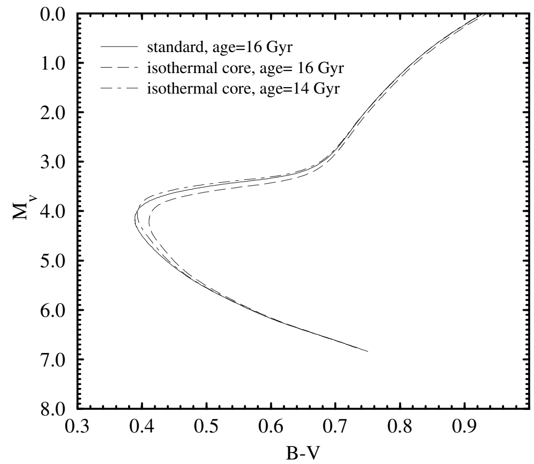

Fig. 8. Comparison between a standard isochrone of 16 Gyr and two isochrones (of the indicated ages) obtained from models with an isothermal core in the innermost 10% in mass (Z=0.0002, Y=0.23)

4.4. Additional energy transport in core

Within the standard scenario we found, as already discussed in Sect. 3, a good fit for most clusters we examined except for NGC6352, whose LF shows a very different behaviour with respect to the theoretical one both in the subgiant and main sequence region. For all other clusters, we could not find any evidence for a significant discrepancy; in fact, we would claim that the theoretical LFs fit the observations very well. Since FS and Bolte (1994) found that their fits improved for stellar models with an isothermal core in the innermost 10% in mass, we also investigated this non-standard change in the theoretical models. To this end, we closely followed the procedure of FS and evolved stellar models under the assumption of a strongly enhanced energy transport in the central 10% of their mass. We achieved this by reducing the radiative opacity by a factor of 10^{-4} artificially. Otherwise, the assumptions, parameters and the procedure for creating the LFs is the same as in Sect. 3.

Before we turn to the LFs, we wish to address an additional point made by FS, who stated that in the "isothermal core" case an age reduction of the globular clusters by about 20% would result. This conclusion was based on the comparison of the turn-off (TO) age of two models of $M = 0.80 M_\odot$ (standard) and of $M = 0.8185 M_\odot$ (with isothermal core) which have the same TO temperature. We decided to calculate both standard and isothermal core isochrones (Fig. 8). If the age is the same for both (16 Gyr), they have the same TO visual magnitude and a sightly different (B-V) by about $0\overset{m}{.}02$. On the other hand, an isothermal core isochrone of 14 Gyr has the same TO colour as the standard one, in agreement with FS, but a different visual magnitude ($\Delta M_V \approx 0\overset{m}{.}1$).

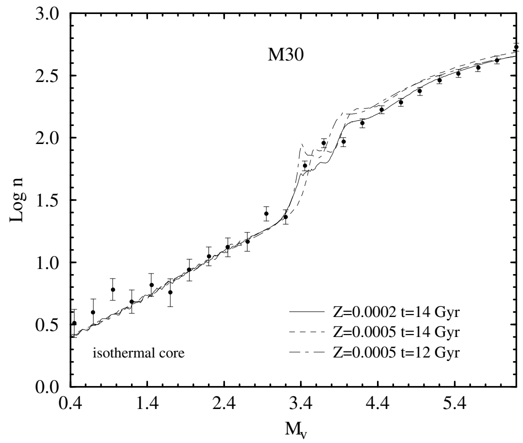

Fig. 9. Comparison between the observed luminosity function of M30 (as in Fig. 1) and our "isothermal" LFs for Z = 0.0005 (short dashed line for 14 Gyr; long-short dashed for 12 Gyr) resp. 0.0002 (solid). Note the additional bump at $M_V \approx 4.2$. All other parameters are as in Fig. 1

There are two main procedures to determine the age of galactic globulars (see e.g. Demarque et al. 1991; Salaris et al. 1993 and references therein): the ΔV(HB-TO) method, which uses the M_V difference between the TO and the HB at the level of the RR Lyrae stars, and the Δ(B-V) method, which uses the (B-V) colour difference between the TO and the base of the RGB. Both these procedures are claimed to have an internal error of about $\pm$ 2.5 Gyr (see e.g. Salaris et al. 1993). Depending on what method one prefers, isochrones with identical TO luminosity resp. effective temperature have to be compared, and therefore an age reduction will or will not be found for the isothermal case. Since the TO luminosity is not affected by an isothermal core, a possible age change in the case of the ΔV(TO-HB)-method depends on the reaction of HB models. Since we impose isothermality on the main sequence cores without a specific physical reason (e.g. a physical model for WIMPs), we do not know what the consistent structure and brightness change on the HB will be. As long as one uses only the TO luminosity (e.g. if the distance has been determined by subdwarf fitting), there will be no age change due to the isothermal cores.

It is evident from Fig. 8 that both the 14 and the 16 Gyr isothermal isochrones fit the standard one within the estimated uncertainties. However, if forced to choose, we would prefer the one of higher age, i.e. the one with the same TO luminosity. The reason is that the determination of the TO colour is a difficult procedure affected by our poor knowledge of stellar envelope models (e.g. mixing length formulation of convective energy transport, $T_{\rm eff}$-(V-B) relation, low temperature opacities).

We now compare our "isothermal" LFs with the observed ones; for each cluster the chemical composition, the IMF and the distance modulus are as in Sect. 3. Fig. 9 shows our comparison for M30 for two ages and both metallicities. The lower age of 12 Gyr results from the determined age in the standard case plus the possible age reduction due to isothermality discussed above. In disagreement with FS and Bolte (1994), even these "best" fits do not appear to be an improvement over the standard ones (Fig. 1). However, the Z = 0.0002, 14 Gyr LF equals our standard best fit in quality (but note that the age reduction has not been included here!). The Z = 0.0005, 12 Gyr LF, which is the one most consistent with our standard parameters, fits within 2σ with the exception of the data point at M_V = 3.9 due to a second bump, which is typical for the isothermal LFs. The LF for the lower metallicity and lower age (not shown in Fig. 9) fits even worse.

Fig. 10 shows the comparison between observational data and isothermal LFs for the composed LF of M92, M68, NGC6397. Age and metallicities are chosen as in the standard case. Neither in this case do the fits improve over the standard one, even if a lower age is allowed.

For all other clusters, the isothermal LFs are worse than the standard ones, although within 3σ. Neither can the fit to NGC6352 be improved by the assumed energy transport in the stellar interior. We therefore are again in disagreement with FS: We cannot agree that standard LFs show a clear discrepancy and that "isothermal" LFs do remove this. However, we also have to submit that the "isothermal" LFs can neither be excluded, although we think they fit slightly worse, mainly due to the appearance of a second bump at the turn-off.

We finally wish to illustrate in the case of M80 that the choice of normalization and different LF parameters is very important for the conclusions about fit quality. In Fig. 11 we show fits obtained by normalizing to the main sequence, i.e. assuming that there is no completeness problem. The dotted line is the standard case with parameters as in Fig. 6, i.e. with the smaller distance modulus. Around the TO, there appears to be a deficit in the theoretical LF. The solid line is a fit with an isothermal core of the same age, which – when shifted – already improves the fit. At last, a fit with an isothermal core and an age of only 14 Gyr (dashed) produces a very good fit, and one would conclude from this that both an additional energy transport in the core (by WIMPs) and a younger age are indicated or even necessary. However, we have shown in Sect. 3 that a sufficiently good fit is also obtained by a standard LF and a larger distance modulus, which gives a consistent subgiant bump location as well. The apparently good isothermal-core fit therefore has three problems: first, it seems problematic to use the shorter distance modulus and a lower age at the same time; second, the LF shows two bumps, while the oberserved one has only one, which must correspond to the subgiant region; and finally, we already have argued that the normalization to the red giant branch is preferable. To repeat ourselves: we think that the longer distance is the correct one, for which we obtain an acceptable fit for the standard LF (Fig 6). However, we demonstrated as well, that with the shorter distance, a lower age, and a normalization to the main sequence the isothermal LFs could yield an equally good fit. For the larger distance, a satisfying fit could not be obtained with the isothermal core models.

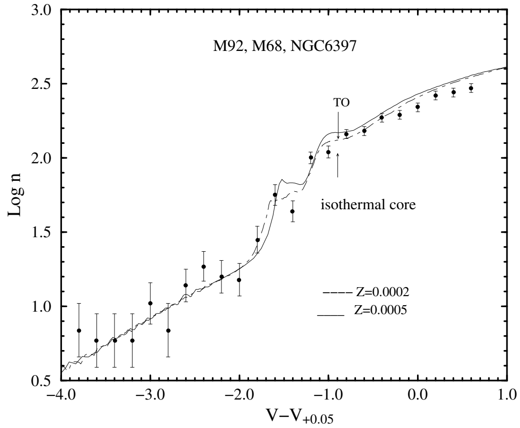

Fig. 10. Comparison between the combined observed luminosity function of M92, M68 and NGC6397 (as in Fig. 2) and our "isothermal" LFs for Z = 0.0005 (solid) and 0.0002 (dashed)

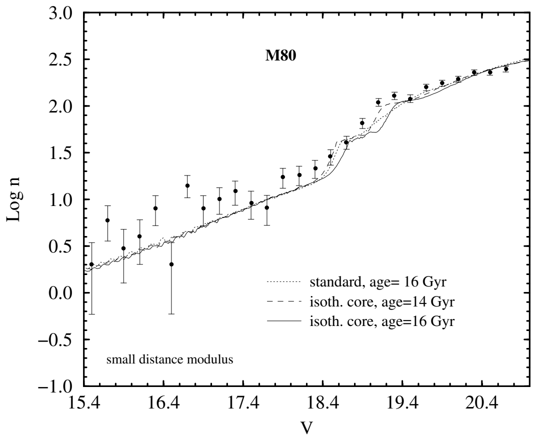

Fig. 11. Comparison between the observed luminosity function of M80 and our theoretical ones, when normalized to the main sequence and when using the small distance modulus of δm = 15.24. Shown are the standard case (dotted) and two "isothermal" LFs of 16 (solid) and 14 Gyr (dashed); other parameters are as in Fig. 6 (Z = 0.0002)

5. Discussion and conclusions

The prime intention of this paper has been to independently investigate whether there indeed exists a systematic and significant discrepancy between observed and theoretical luminosity functions of Globular Clusters as claimed by Faulkner & Swenson (1993) and Bolte (1994). We have demonstrated in Sect. 3 that for the clusters used in their papers and for additional data by Piotto & Saviane (in prep.) the agreement between our standard luminosity functions and the observations is good for reasonable assumptions about cluster composition, distance and age. Actually, we have used values from the literature for these parameters, but exploited the range of uncertainties to find the best fit. All data points can be fitted within 3σ of the statistical errors, except for the main sequences. To illustrate the influence of pure number statistics we have performed the following test:

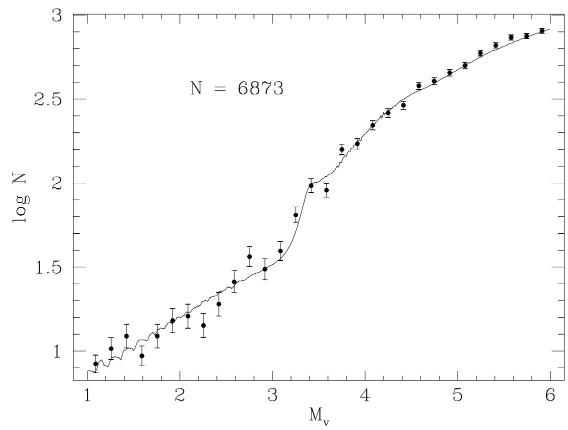

Fig. 12. Theoretical and synthetic luminosity functions for standard parameters (16 Gyr; s = 2, Z = 2 10^{-4}). 26000 random points in the N-M_V plane were chosen, out of which 6873 fell below the theoretical probability function and were accepted. The number of brightness bins is 30. 1σ errors are indicated

we took one of our standard LFs (16 Gyr; s = 2, Z = 2 10^{-4}) and constructed a synthesized LF by the rejection method to construct random deviates. Fig. 12 shows the comparison between the synthetic and the theoretical LF for a total number of stars and a number of brightness bins comparable to the cases discussed in this paper. The similarity, for example with Fig. 1, demonstrates that our fits are perfect within the statistical errors, and that deviations like those at M_V = 3.6 or 3.8 are to expected. In fact, some of our cases show a better agreement than would be consistent with the statistics. In these cases, the fit parameters might be overdetermined. Of course, the systematic deviations on the main sequences are a clear indication of non-statistical errors. Since all clusters by Piotto & Saviane (in prep.) are more deficient with respect to the main-sequence parts of the theoretical LFs, while those used by FS are to a smaller extend, we ascribe this to an underestimate in the completeness correction applied by the observers. This is consistent with the fact that Bolte (1994) used the most sophisticated completeness correction for M30 and this cluster shows the smallest deviation. Clearly, there is a need for a further improvement both in the data and in the completeness corrections of the main sequence LF. Bolte (1994) for M30 and Brocato et al. (1996) for M80 have already demonstrated that such an improvement is achievable.

Alternatively, one could speculate that the IMF for these clusters is flatter than assumed. This uncertainty – and the additional one of the helium content – prevented us from using the main sequence for the normalization of the LFs, in spite of the much smaller statistical errors. Instead, we used the RGB for normalization, whose LF-slope is an extremely stable quantity independent of all input variables. In the case of M80 we demonstrated that the use of the main sequence would lead to a "discrepancy" in the standard LF and its resolution by an isothermal core, just as was the case in the papers triggering the present work.

The main sequence being inadequate for a detailed comparison and the red giant branch being robust against model changes, the subgiant branch is left to reflect model differences, parameter influences and potential problems. However, the present observations do not resolve the LF in this region sufficiently well and with adequate accuracy. We propose that in the future observations aiming at obtaining the LFs of GCs should concentrate exactly on this region.

We found one cluster (NGC6352), which withstands all attempts to fit a theoretical LF. The shape of the observed one is in fact very strange and looks like that of a very low-metallicity system in terms of the missing subgiant break, and like one of high helium content with respect to the slope at the TO (Ratcliff 1987). However, within the parameter range investigated by us, we could not obtain any good fit. We rather suspect that the subgiant break was not resolved properly, and this should be checked by further observations.

Of all quantities we have tested for their influence on the LFs, we found that metallicity and distance (see the example of M80; Figs. 6 and 7) have the largest influence. Age variations of up to 2 Gyr can be compensated by a luminosity shift smaller than the quoted distance uncertainty (because old clusters change their TO-luminosity hardly with age). Discrepancies in the LF should therefore first raise the question of correct distance or metallicity determinations. In this context information about α-element enrichment is important, too.

In Sect. 4 we repeated the LF-calculations, but with a very efficient energy transport in the innermost 10% of our models (following again FS and Bolte 1995). Our results show that the fits are not improved significantly. Since we found no evidence for a LF-discrepancy when using standard physics, it is not necessary to discuss properties of hypothetical WIMPs for additional energy transport in the core of main-sequence stars.

To summarize, we have shown that the agreement between observed and theoretical luminosity functions appears to be very good, with problems only arising on the lower main sequence (corrections for completeness might have to be improved) and for the exact shape of the subgiant bump and break (resolution in brightness), which have the potential to yield information about metallicity, age and distance of the cluster. However, to exploit this potential diagnostic value, a definite improvement in the observation of both branches is required.

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