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Nonlinear galactic dynamo models with magnetic-supported interstellar gas-density stratification

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Abstract. We demonstrate the ability of a highly simplified galactic model to generate mean-field magnetic fields. The generated large-scale magnetic field feeds back to the vertical density stratification which itself is one of the essentials of the basic α-effect. The non-magnetic part of the stratification is due to the gravity potential of the stellar component of the galactic disk. The rotation law of the gas also results from the stellar-disk potential. A *uniform* turbulence field with *uniform* correlation time of order of 10 Myr is completing the model. The high nonlinearity in the theory determines the amplitude of the generated magnetic field. If only the conventional α-quenching is involved in the code, then the magnetic amplitude would always exceeds 10 μG in contrast to the observations (Table 4). This value is only reduced to the observed numbers of a few μG if the complete magnetic feedback is considered also in the Reynolds equation (Table 3).

The magnetic field configuration is always even with respect to the equator (quadrupolar) and all models with correlation times longer than 10 Myr are oscillating.

Surprisingly enough, the magnetic influence on the vertical stratification does *not* lead to a thicker disk. The magnetic fields in our models in general lead to a flattening of the disk.

Key words: magnetohydrodynamics (MHD) – turbulence – interstellar medium: magnetic fields – galaxies: magnetic fields

1. Introduction

Dynamo theory considers rotating stratified turbulences as able to induce large-scale magnetic fields. Only a few very elementary constituents are necessary for this phenomenon, so that an interesting example of self-organization is formed. Geometry and time-dependence of the self-excited field can already be computed from a linear theory (cf. Stix 1975, Parker 1971, 1979, Stepinski & Levy 1988, Krasheninnikova et al. 1990, Meinel et al. 1990, Brandenburg et al. 1990, Schmitt 1990, Donner & Brandenburg 1990, Mestel & Subramanian 1991, Tosa & Chiba 1990, Elstner et al. 1992, Moss & Brandenburg 1992, Camenzind & Lesch 1994). The linear theory provides a critical eigenvalue. If it is exceeded by the real 'material' then the magnetic field grows exponentially. It grows as long as the magnetic feedback is weak. The resulting magnetic field amplitude, therefore, contains information about the dominating nonlinearity.

If, e.g., the observed field is of the order of the 'equipartition field',

$$B \simeq B_{\rm eq} \equiv \sqrt{4\pi\,\rho\langle \boldsymbol{u}_0^2\rangle}, \qquad (1)$$

then one can be sure that the prevailing feedback of the magnetic field has to do with the 'microscopic' turbulence[1].

If Eq. (1) is not justified by the observations, attention must be drawn to the other major feedback possibility of the system, i.e. the action of the large-scale Lorentz force on macroscopic structures. This effect is called as the Malkus-Proctor effect after their paper from 1975. In general, a highly complicated flow system results to prevent the exponential growth of the magnetic field (Brandenburg et al. 1991, Barker & Moss 1994, Fuchs et al. 1995). For galaxies in their simplified nearly 1D geometry there is a special situation. Stationarity adopted one finds for the vertical component of the Reynolds equation

$$\bar{u}_z \frac{d\bar{u}_z}{dz} + \frac{1}{\rho}\frac{dp^{\rm tot}}{dz} = -k_z, \qquad (2)$$

with $p^{\rm tot}$ as the total pressure, i.e. the sum of turbulence pressure, magnetic pressure and the pressure by the cosmic ray. Gas pressure can be ignored. Mass conservation requires

$$\frac{d}{dz}(\rho\bar{u}_z) = 0, \qquad (3)$$

so that with the symmetry condition $\bar{u}_z(0) = 0$ the vertical flow completely vanishes and (2) becomes simply

$$\frac{1}{\rho}\frac{dp^{\rm tot}}{dz} = -k_z. \qquad (4)$$

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[1] The so-called $\eta_{\rm T}$-quenching is excluded in this consideration.

Here k_z is the gravitation by the stars in the stellar disk. Eq. (4) for given turbulence determines the vertical density profile of the galactic gas (cf. Dobler et al. 1996).

2. Basic equations

2.1. The turbulent EMF

The evolution of the mean magnetic field $\bar{\boldsymbol{B}}$ is governed by the dynamo equation

$$\frac{\partial \bar{\boldsymbol{B}}}{\partial t} = \text{rot}\,(\bar{\boldsymbol{u}} \times \bar{\boldsymbol{B}} + \mathscr{E}), \tag{5}$$

where $\mathscr{E}$ is the turbulent electromotive force (EMF), $\mathscr{E} = \langle \boldsymbol{u}' \times \boldsymbol{B}' \rangle$, and $\bar{\boldsymbol{u}}$ the mean velocity (Parker 1979, Krause & Rädler 1980). We employ cylindrical polar coordinates, (ϖ, ϕ, z), and assume the solution as axisymmetric, $\partial/\partial\phi = 0$.

We further assume approximate scale separation and write

$$\mathscr{E}_i = \alpha_{ij}\bar{B}_j + \eta_{ijk}\bar{B}_{j,k}. \tag{6}$$

The tensors α and η must be specified on the basis of a suitable model for the rotating interstellar turbulence.

2.2. The α-tensor

If only the density stratification produces the α-effect the tensor takes the form

$$\alpha_{ij} = -\alpha_{ij}^{(\rho)}\,\frac{d\log\rho}{dz}\,\langle \boldsymbol{u}_0^2\rangle\,\tau_{\text{corr}} \tag{7}$$

with

$$\alpha_{ij}^{(\rho)} = c_\alpha \begin{pmatrix} \frac{2}{5}\Omega^*\tilde{\psi} & -U^{\text{buo}} & 0 \\ U^{\text{buo}} & \frac{2}{5}\Omega^*\tilde{\psi} & 0 \\ 0 & 0 & -\frac{2}{15}\Omega^*\tilde{\psi}_z \end{pmatrix} \tag{8}$$

with

$$\tilde{\psi} = \left(\psi + \frac{15}{8}\frac{\bar{B}_z^2}{\bar{B}^2}\psi_1\right) \tag{9}$$

and

$$\tilde{\psi}_z = \left(\psi_z - \frac{15}{16}\frac{\bar{B}_z^2}{\bar{B}^2}\psi_1\right). \tag{10}$$

The α-quenching functions ψ have been derived in Rüdiger & Kitchatinov (1993). U^{buo} is a magnetic-advection velocity often called the magnetic buoyancy,

$$U^{\text{buo}} = \frac{1}{8\beta^2}\left(-\frac{3+5\beta^2}{(1+\beta^2)^2} + \frac{3}{\beta}\arctan\beta\right) \tag{11}$$

(Kitchatinov & Rüdiger 1992). It starts with $\beta^2/5$ for weak fields and vanishes like $3\pi/(16\beta^3)$ for strong fields. It is $\beta = |\bar{B}|/B_{\text{eq}}$ and Ω^* is the Coriolis number

$$\Omega^* = 2\tau_{\text{corr}}\Omega. \tag{12}$$

A constant and uniform correlation time is assumed. A possible dependence of the correlation time on the magnetic field, e.g. via Alfvèn wave interaction, would introduce a new magnetic feedback mechanism and is excluded here. The equipartition value of the turbulence, B_{eq}, is defined by Eq. (1). The contribution of the velocity gradient to the α-effect proves to be zero in our model (see Eq. (28) below).

2.3. The η-tensor

For slow rotation and weak magnetic field the eddy diffusivity tensor takes the simple form

$$\eta_{ijk} = \eta_{\text{T}}\epsilon_{ijk}, \tag{13}$$

so that

$$\mathscr{E} = -\eta_{\text{T}}\,\text{rot}\,\bar{\boldsymbol{B}}. \tag{14}$$

The eddy diffusivity, however, is only a simple tensor unless the magnetic field feeds back. Then the η-tensor becomes much more complex:

$$\eta_{ijk} = \eta_{\text{T}}(\bar{B})\,\epsilon_{ijk} + \hat{\eta}(\bar{B})\,\epsilon_{ilk}\bar{B}_j\bar{B}_l + ... \tag{15}$$

The reference value for the eddy diffusivity η_{T} is

$$\eta_0 = c_\eta\langle \boldsymbol{u}_0^2\rangle\tau_{\text{corr}}. \tag{16}$$

The free parameter c_η should not exceed unity. We shall work with the conventional value $c_\eta = 0.33$. In Rüdiger et al. (1994) first consequences of the magnetic η-quenching, (15), are demonstrated. In the present paper the effect, however, had to be ignored, simply for numerical reasons.

2.4. Vertical stratification

The turbulence pressure plays an important role in the vertical momentum conservation

$$\frac{d}{dz}\left(\rho\langle u_z'^2\rangle + \frac{\bar{B}_x^2 + \bar{B}_y^2 - \bar{B}_z^2}{8\pi} + p_{\text{CR}}\right) = -\rho\,k_z. \tag{17}$$

Only the intensity of the vertical turbulence contributes. The pressure due to cosmic ray is p_{CR} (cf. Parker 1992). The potential is due to stars,

$$k_z = \frac{\sigma^2(r)}{z_0}\left[\tanh\left(\frac{z}{z_0}\right) + \varepsilon(r)\frac{z}{z_0}\right], \tag{18}$$

with

$$\sigma^2 = (15.4\ \text{km/s})^2 \cdot e^{-\frac{r-r_\odot}{0.44 r_\odot}} \tag{19}$$

and $z_0 = 250$ pc, $r_\odot = 8.5$ kpc. ε reflects the contribution of the dark matter (Fröhlich & Schultz 1996).

Self-gravitation only plays a minor role. Eq. (18) for prescribed turbulence and magnetic field gives the density profile $\rho = \rho(z)$. With the surface density, Σ, it results from

$$\rho = \frac{\partial\Sigma}{\partial z}. \tag{20}$$

The boundary conditions are simply

$$\Sigma(z = 0) = 0, \quad \Sigma(z = \infty) = \Sigma_0. \tag{21}$$

The influence of the magnetic field on the turbulence pressure is involved in a general theory of the magnetic influence on the Reynolds stress (Moffatt 1966, Rüdiger 1974, Rädler 1974, Roberts & Soward 1975). It can be summarized in the general formulation

$$\langle u_i' u_j' \rangle = \langle \boldsymbol{u}_0^2 \rangle \left(\Psi(\beta)\delta_{ij} + \Psi_1(\beta) \frac{\bar{B}_i \bar{B}_j}{\bar{B}^2} \right), \tag{22}$$

with

$$\Psi = \frac{1}{8\beta^2} \left(\frac{\beta^2 - 1}{\beta^2 + 1} + \frac{\beta^2 + 1}{\beta} \arctan\beta \right) \tag{23}$$

and

$$\Psi_1 = \frac{1}{8\beta^2} \left(\frac{\beta^2 + 3}{\beta^2 + 1} + \frac{\beta^2 - 3}{\beta} \arctan\beta \right). \tag{24}$$

In particular, for the vertical intensity is

$$\langle u_z'^2 \rangle = \langle \boldsymbol{u}_0^2 \rangle \left(\Psi + \Psi_1 \bar{B}_z^2 / \bar{B}^2 \right) \tag{25}$$

(Kitchatinov 1994).

3. The dynamo model

Let us start with a very simple model with a minimum of input parameters. We have only to specify the rotation law and the turbulence model. In the last analysis even the rotation law is also given by the stellar component of the galaxy, so that in general only the turbulence must be specified. If indeed the explosions of supernova and/or super-bubbles are driving the interstellar turbulence, then everything is yielded for the generation of magnetic fields by the stellar component of the galaxies.

The law of rotation is known in general: Beyond a rigidly rotating core with $r = r_\Omega$ the angular velocity is inversely proportional to r so that

$$\Omega = \Omega_0 \begin{cases} 1 & \text{if } r < r_\Omega \\ r_\Omega / r & \text{if } r > r_\Omega \end{cases}. \tag{26}$$

Thus the velocity of the outer part is uniform,

$$V = r_\Omega \Omega_0. \tag{27}$$

For our models the linear velocity is taken as $V = 220$ km/s. The turnover radius r_Ω is the only length-scale in the theory, if – as we shall do – the turbulence is assumed as homogeneous in the whole space, hence

$$\langle \boldsymbol{u}_0^2 \rangle = \text{const.}, \qquad \tau_{\text{corr}} = \text{const.} \tag{28}$$

Then the equipartition field strength only runs with the density,

$$B_{\text{eq}}^2 = 4\pi\rho(z)\langle \boldsymbol{u}_0^2 \rangle. \tag{29}$$

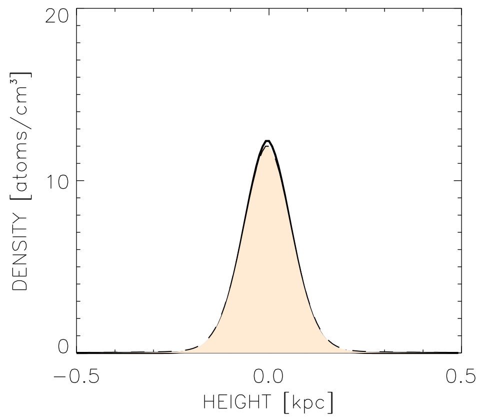

Fig. 1. The vertical density stratification after Eq. (17) without and with magnetic field (grey). Turbulence intensity is 10 km/s, correlation time $\tau_{\text{corr}} = 10$ Myr

It is rather low in the halo region of the galaxy. The turnover radius for the models is always $r_\Omega = 2$ kpc.

So the system is defined as extremely informal: turbulence, rotation and gravitation. As we shall demonstrate, generation of large-scale magnetic fields can be the consequence of this configuration. Both the input parameters (28) are playing very different roles in the frame of the resulting self-organization.

4. Results for models with turbulence-quenching

First we are dealing only with conservation models, i.e. the η-quenching is ignored as well as the magnetic quenching of the turbulent pressure. From numerical reasons we start with models for α-parameters $c_\alpha = 0.1$. The code is based on the concept by Elstner et al. (1990) and Rüdiger et al. (1993).

4.1. Models with α-quenching

Magnetic fields are generated if the turbulence intensity

$$u_{\text{T}} = \sqrt{\langle \boldsymbol{u}_0^2 \rangle} \tag{30}$$

is not too high. A critical value for u_{T} is about 40 km/s. No magnetic field exists for higher turbulence intensity. For lower intensities, however, the induced magnetic strength is practically independent of the turbulence intensity (Schultz et al. 1994). There is no possibility to relate the magnetic field strength to the turbulence intensity (Table 1). There is only a slight dependence of the magnetic field strength on the correlation time. The longer the correlation time the weaker the field. More dramatic, however, is that for longer correlation times the stationary solutions are changing to oscillating solutions – which effect has certainly consequences (cf. Elstner et al. 1996). The formal reason for this phenomenon is that increasing correlation times produce stronger α-effect so that the regime moves towards the α^2-dynamo regime. The large observed pitch angles of the magnetic fields of galaxies indeed suggest the invalidity of a simple $\alpha\Omega$-dynamo mechanism. The consequences of the

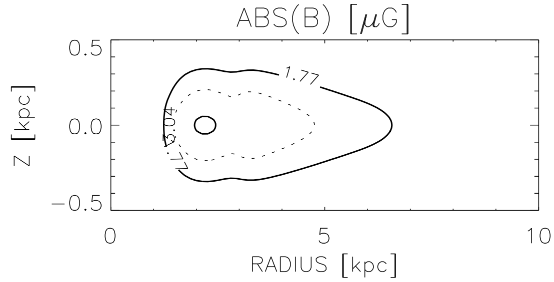

Fig. 2. Isocontours of the total amplitude of the magnetic field in radius r and in z. Turbulence is the same as in Fig. 1

Table 1. Characteristic magnetic fields in μG for dynamo models with density profiles derived from the vertical stratification. η-quenching is ignored, the turbulence intensity is given in km/s. The sign $\sim$ indicates oscillating solutions, the following number is the cycle time. All times in Myr, all solutions showing quadrupole symmetry. The α-parameter c_α is 0.1

$\tau_{\rm corr}$	$u_{\rm T}$ = 10	$u_{\rm T}$ = 15	$u_{\rm T}$ = 20
10	4.8	4.8	4.4
20	4.7	4.6	3.8
30	4.3	4.3	1.1...1.3 ($\sim$, 100)
40	4.0	4.0	2...2.5 ($\sim$, 30)

Table 2. Magnetic fields in μG for dynamo models with extra magnetic influence on the turbulence-pressure. The α-parameter c_α is 0.1. Further notation as in Table 1

$\tau_{\rm corr}$	$u_{\rm T}$ = 10	$u_{\rm T}$ = 15	$u_{\rm T}$ = 20
10	3.2	3.3	3.4
20	3.2	3.2	3.0
30	3.1	3.0	1.0...1.3 ($\sim$, 70)
40	3.0	2.6 ($\sim$, 150)	1.5...2.2 ($\sim$, 60)

Table 3. The same as in Table 2 but for c_α = 1.

$\tau_{\rm corr}$	$u_{\rm T}$ = 10	$u_{\rm T}$ = 15	$u_{\rm T}$ = 20
10	7.1	8.3	9.4
20	7.0 ($\sim$, 350)	7.8 ($\sim$, 180)	8.8 ($\sim$, 250)
30	6.5 ($\sim$, 90)	7.7 ($\sim$, 60)	7...9 ($\sim$, 50)
40	6...10 ($\sim$, 200)	7...8 ($\sim$, 30)	8...12 ($\sim$, 50)

Table 4. Reference values to Table 3. The only nonlinearity involved is α-quenching, density profile is fixed.

$\tau_{\rm corr}$	$u_{\rm T}$ = 10	$u_{\rm T}$ = 15	$u_{\rm T}$ = 20
10	13	15	17
20	13	15	16 ($\sim$, 270)
30	13	14...16 ($\sim$, 260)	11...15 ($\sim$, 300)
40	13 ($\sim$, 150)	15...20 ($\sim$, 170)	14...16 ($\sim$, 800)

saturated magnetic field for the density stratification are demonstrated in Fig. 1. As was to be expected, the gas is smoothed out, the concentration to the midplane is reduced by the magnetic pressure. *The thickness, however, of the gaseous disk practically remained unchanged.* In Fig. 2 the isolines of the total amplitude of the magnetic field for the turbulence model with the lowest parameters, i.e. $u_{\rm T}$ = 10 km/s and $\tau_{\rm corr}$= 10 Myr, are given. We find the maximum amplitudes close to the turnover point but the field geometry is characterized by a large-scale pattern in radius as well as in the vertical. All the fields – even the oscillating – exhibit a quadrupolar parity, i.e. symmetry with respect to the midplane.

4.2. Models with extra turbulence-pressure quenching

Now also the influence of the induced magnetic field on the turbulent pressure in the equation for the vertical stratification, (17), is taken into account. The corresponding turbulence-quenching function is given in Eq. (25). The magnetic influence in (17) is thus twice: magnetic pressure (direct) and magnetic quenching of the turbulence pressure (indirect). The system is thus highly nonlinear. The only fixed quantity remains the eddy diffusivity.

The results are presented in Table 2. All the various models produce nearly the same amplitude of the magnetic field, i.e. about 3 μG. The field is smaller than in Table 1. In Fig. 6 is shown that – surprisingly enough – the galactic disk *becomes flatter by the dynamo action*. There is another striking property of our models. They prove to be rather insensitive against another choice of the free parameter c_α. Increasing of this number from 0.1 to unity increases the amplitude of the field only by a factor of 2 or 3. This is in high contrast to the models of Sect. 4.1, i.e. without extra turbulence-pressure quenching. They are not yielding a finite solution for c_α = 1. This problem completely disappears if the quenching (25) is also involved. Just for the c_α = 1 the computed magnetic amplitudes comply with the observations (Table 3). Figs. 3 and 4 demonstrate the character of the oscillations which are concentrated to the inner 1 or 2 kpc. In opposition, the outer differential-rotation domain, with r exceeding r_Ω, is always in an almost stationary regime. This is a basic property for galactic dynamos. As shown in Fig. 5 it even exists for the simple α-quenching models of Tab. 4. In order to find the influence of Eq. (17) on the dynamo model it is necessary to compare the numbers of Table 3 with numbers for a model with pure α-quenching. Those numbers are given in Table 4. They are very characteristic for traditional dynamo models. The strength of the α-effect compared with the differential rotation grows both with the turbulence intensity and the correlation time. Hence, in these cases the α-effect dominates and the magnetic fields are oscillating. Steady solutions are only in the upper left. The temporal behavior of the models is striking. For correlation times of 20 Myr or more there are always oscillating parts in the magnetic field distribution. An example is given in Fig. 3. The inner part of the solution is oscillating while its outer part exhibits practically stationary behavior. In

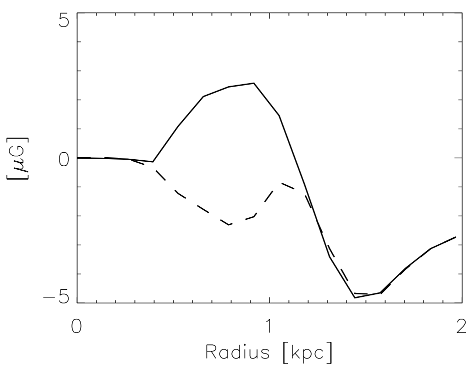

Fig. 3. The oscillating radial magnetic field in the galactic midplane at different cycle phases. Turbulence intensity is 10 km/s, correlation time 20 Myr

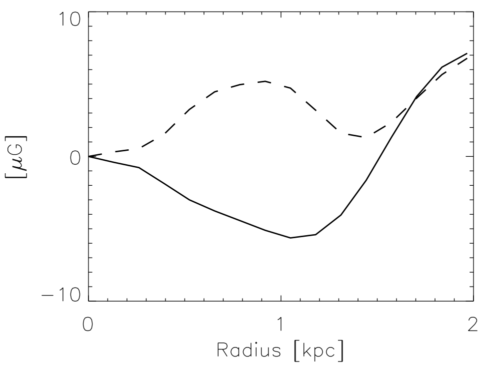

Fig. 4. The oscillating radial magnetic field in the galactic midplane at different cycle phases. Turbulence intensity is 20 km/s, same correlation time as in Fig. 3

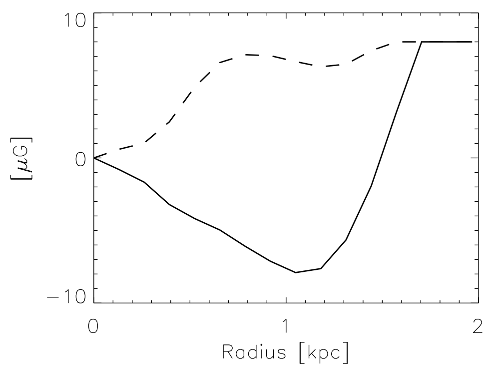

Fig. 5. The oscillating radial magnetic field in the galactic midplane at different cycle phases for the simplified α-quenching model of Table 4. Turbulence intensity is 20 km/s, same correlation time as in Fig. 3

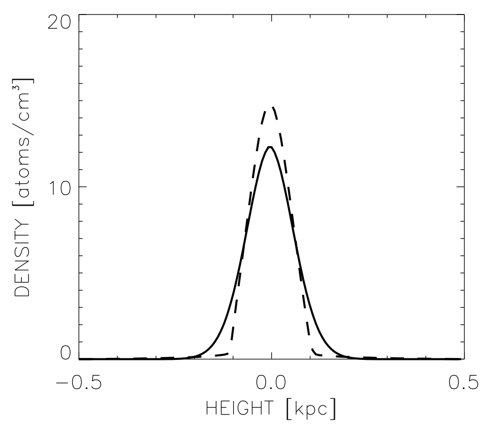

Fig. 6. The vertical density stratification after Eq. (17) without (solid) and with (dashed) magnetic field. The complete magnetic influence on the turbulence is considered. Turbulence intensity is 10 km/s, correlation time 10 Myr

the inner part the non-uniformity of the rotation does not play any role.

We have already found oscillating galactic dynamo fields in previous models (cf. Elstner et al. 1996). By increase of the correlation time stationary quadrupoles have changed to oscillating dipoles. Now, however, the equatorial parity remains quadrupolar and some part of the solution starts to vary.

Another striking property of the highly nonlinear model is the behavior of the density stratification. Only η-quenching is still ignored for numerical reasons. As plotted in Fig. 6 the influence of the magnetic field is flattening the disk. The complicated nonlinear interaction of magnetic field and turbulence in our models is producing a more steep density profile but with very similar maxima.

5. Results for models without any turbulence-quenching

An interesting question arises whether our 2D models also work without any micro-quenching. The idea is to check whether the system is able to adjust itself without any magnetic influence upon the microscale.

The magnetic field flattens the density profile so that the α-effect is reduced and a balance finally arises. Whether this is possible is the first question. The second concerns the magnitude of the induced magnetic field. The equipartition value $B_{\rm eq}$, of course, does no longer play any role. It is an open question which characteristic value of the magnetic amplitude will result. No previous dynamo model has attacked this question. As we know from the investigation of simple models with various α-quenching laws, such as

$$\alpha \propto \frac{1}{1+\beta^n}, \tag{31}$$

the magnetic amplitudes are running with

$$B_{\rm max} \propto B_{\rm eq} \mathscr{D}^m \tag{32}$$

with $\mathscr{D}$ as the dynamo number. The exponent m is a (smooth) function of n (Fig. 7). Needless to say that relations such as (32) are completely loosing their meaning if only the structure of the galaxy determines the magnetic field amplitude.

The results of the calculations are given in Table 5. Again only fields are excited with quadrupolar geometry. A striking

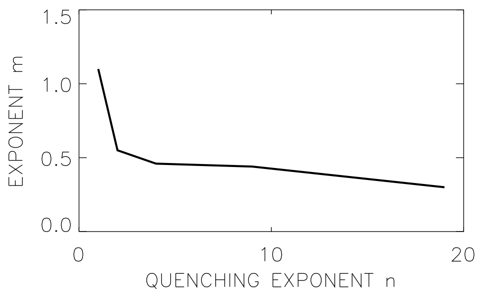

Fig. 7. The form of the α-quenching function (31) determines the magnetic field amplitude (32), here taken from a simple zeroth-order dynamo model simulation by A. Brandenburg

Table 5. The same as in Table 1 for dynamo models without any turbulence quenching. c_α = 0.1.

	u_T = 10	u_T = 15	u_T = 20
10	27	25	17
20	inst.	25	15
30	inst.	24	4...4.5 ($\sim$, 60)
40	inst.	inst.	10...25 ($\sim$, 60)

result is that not for all parameters stable solutions have been found. Again the magnetic fields are oscillating for high turbulence intensities.

Most interesting, however, are the numbers concerning the magnetic field amplitudes. The fields are remarkably stronger (by a factor of 5) if α-quenching is absent. As the observed magnetic fields are of order of 5 μG the argumentation is challenging that a system without micro-quenching yields too strong magnetic fields. It is, however, highly interesting in the frame of the dynamo theory that the system is able by adjusting its structure to limit the magnetic field generation. The resulting density profiles, however, are extremely flat and far from the reality (Fig. 8). Either our models are exhibiting instability or the stable configurations are unlike galaxies. For strong fields it is thus not allowed to ignore the magnetic influence upon the structure and evolution of the considered object.

6. Discussion

We are constructing mean field galactic dynamos with only a very few ingredients. A given flat stellar disk simultaneously leads to a vertical stratification of the interstellar gas and the characteristic shear of the angular velocity of its rotation. The second component of the model is a given field of turbulence with uniform intensity. The immediate consequence is the generation of a turbulent large-scale electromotive force and, further, the operation of a mean-field dynamo. The resulting magnetic pressure is influencing the (vertical) structure of the galaxy. The new density gradient, on the other hand, gives rise to a modified α-effect with consequences for the induced field.

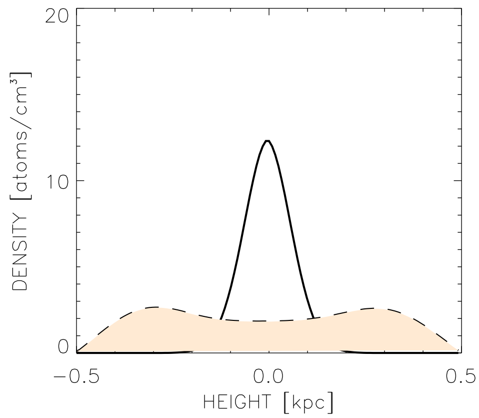

Fig. 8. The vertical density stratification after Eq. (17) without and with magnetic field (grey). No magnetic influence on the turbulence is considered. Turbulence intensity is 10 km/s, correlation time $\tau_{\rm corr}$ = 10 Myr

The described nonlinear system is asked for its stability. Table 5 presents the results. We only find unstable solutions or solutions with an extremely thick disk structure. The field amplitudes in the latter case are exceeding the observed values by a factor of 5.

As a consequence, the galactic dynamo needs further nonlinearities. The main point is that the micro-scale motion, i.e. the turbulence, is quenched by the magnetic field. Here in particular two different possibilities exist. First, the turbulent EMF will be quenched by the magnetic field and, second, the turbulence pressure in the vertical stratification equation also depends on the magnetic field.

In the present paper we are still ignoring the effect of the η-quenching, i.e. the magnetic suppression and modification of the eddy diffusivity tensor. There are serious indications that the effect modifies the findings and predictions of the dynamo theory but in the present state such solutions are not yet available. The general rule in Rüdiger et al. (1994) was that oscillating solutions are only slightly modified by the η-quenching concept in opposition to the stationary modes.

In Table 3 the results of our simulations are given. They are very clear. The *maximal* amplitudes of the magnetic fields are with ≤ 10 μG close to the observations. The fields are stationary for short correlation times ($\leq$ 10 Myr) and they have time-dependent character for longer correlation times. Such findings, however, are valid in the frame of an axisymmetric ($\partial/\partial\phi = 0$) code. With these restrictions in mind it is no problem to produce in relatively short real times (< 1...2 Gyr) with very few simple assumptions the observed magnetic amplitudes by means of a galactic dynamo.

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