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On the masses and luminosities of double-mode Cepheids

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Abstract. Linear adiabatic models of double-mode or beat Cepheids (DMC) have been constructed for testing the effects of different chemical compositions and corresponding mass-luminosity relations on the period ratios, in order to determine the $M-L$ relations which better represents double-mode Cepheids in Galaxy and LMC. The study has not given a clearcut result, and indicates that generally DMC with low metallicity ($Z \lesssim 0.016$) cannot be used for discriminating reliably between standard and nonstandard relation. The latter gives slightly better results in some cases, but it is confirmed that it is not able to reproduce the galactic second overtone to first overtone pulsator (CO Aur), unless using unrealistic Z values.

It is shown that a modified standard relation, with a decreased sensitivity to the metallicity, is able to give a satisfactory representation of all the DMC. Finally, the sensitivity of P_2/P_1 ratio to the T_e of the models is pointed out.

Key words: stars: Cepheids – stars: oscillations

1. Introduction

Christensen-Dalsgaard & Petersen (1995) have studied linear adiabatic models of double-mode or beat Cepheids (DMC) with the aim of making a comparison between stars in the Galaxy and those recently observed in LMC (Alcock et al. 1995). They conclude that period ratios for DMC models computed with OPAL opacities (Iglesias et al., 1992), and for masses corresponding to evolution calculations, are broadly consistent with the observed values. They attempted to derive also some conclusions on the mass - luminosity ($M-L$) relation valid for DMC, by comparing the predictions of a standard relation and that for models with a moderate amount of overshooting from convective cores. They find that a standard relation and Z = 0.01 provide an excellent fit to the observed location of the LMC stars in the (logP_0, P_1/P_0) diagram, but on the other hand the same relation appears clearly inconsistent with the results for P_2/P_1, for realistic values of Z. An overshooting-type relation appears to give better results for LMC stars, but fails completely if we try to reproduce P_2/P_1 of the galactic DMC CO Aur (Antonello et al., 1996). It is important to remark that Christensen-Dalsgaard & Petersen used the same $M-L$ relation for different Z values, while for example Becker et al. (1977; hereinafter BIT) report different, even if approximated, $M-L$ relations according to the different chemical composition. The purpose of the present paper is to discuss the effects of this dependence.

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2. Models

The linear adiabatic models were constructed with different $M-L$ relations of the type

$$\log L/L_\odot = a \log M/M_\odot + b. \qquad (1)$$

In the nonstandard case, the coefficients a and b were obtained taking into account the available evolutionary tracks reported by Bressan et al. (1993) for Y = 0.28, Z = 0.02, and Fagotto et al. (1994) for Y = 0.24, Z = 0.004 and Y = 0.25, Z = 0.008. These evolutionary models are for mild overshoot ($\Lambda_c \sim 0.5$) and are constructed with OPAL opacities. Approximated $M-L$ relations were derived from the second and third crossing of the instability strip performed by the evolutionary models with masses larger than 4 $M_\odot$. We found that, in the mass range of interest (3 - 6 $M_\odot$), the difference between relations with different composition is not very large. For a given mass the log L difference is about 0.1, and the corresponding effects on our pulsation models are rather small. In other words, in our context the dependence of the overshoot relation from Z is negligible. For this reason we decided to use the same relation (COV) for all the pulsation models.

As regards the standard relations, we considered their dependence on Z derived by BIT from the *second crossing* of the instability strip by the evolutionary models. We remark that this dependence is much larger than that derived from the so-called *blue point*. For the hydrogen content we adopted X = 0.7. The coefficients a and b of the relations are reported in Table 1.

The mass range of the pulsational models was $\sim 2.8-6M_\odot$, and the T_e range was 5800 - 6300 K for $1O/F$ (first overtone and fundamental mode pulsators) and 6200 - 6500 K for $2O/1O$ (second overtone and first overtone mode pulsators). The T_e range for $1O/F$ is based on the observational results of galactic

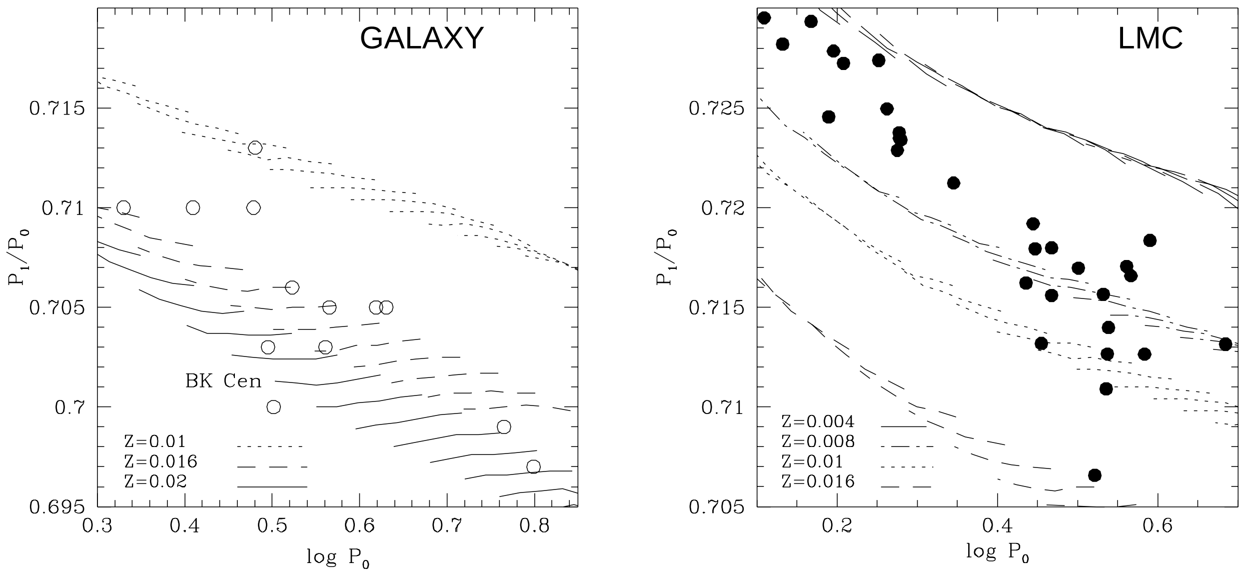

Fig. 1. Period ratio vs. period of $1O/F$ DMC in the Galaxy (left panel) and LMC (right panel), compared with model results for a nonstandard (overshoot) $M-L$ relation. The lines connect models with the same mass

Table 1. Adopted Mass-Luminosity relations (see text)

	Z	a	b
COV		3.52	0.91
BIT	0.020	3.680	0.460
	0.010	3.426	0.936
	0.004	3.275	1.220

DMC. The T_e range for $2O/1O$ is based on the color of the galactic Cepheid CO Aur which indicates a high T_e, and on the fact that the mode stability obtained with specific linear nonadiabatic calculations suggestes that the second overtone cannot be excited if the T_e is too large. The comparison of the models with LMC DMC reported in the next sections shows indeed that the possible T_e range for $2O/1O$ DMC should be narrow.

3. Results and discussion

3.1. $1O/F$ DMC

The models are compared with the observed DMC in the Petersen diagrams shown in Fig. 1 and 2. The nonstandard relation is able to reproduce nicely the galactic and LMC Cepheids; only the galactic DMC BK Cen is slightly 'out'. It is interesting to remark that, among the DMC analyzed by Andrievsky et al. (1994), BK Cen is the star with the largest $[Fe/H]$ value, with a corresponding Z close to 0.02. On the whole the results obtained with the nonstandard relation for galactic DMC is broadly in agreement with those obtained by Simon and Kanbur (1994).

Turning to the standard relation models, there are some apparent difficulties in accommodating both Galaxy and LMC DMC in the same nice way of nonstandard relations. What we learn from Fig. 1 and 2 is that the real $M-L$ relation cannot be as largely dependent on Z as indicated by BIT (see Stothers & Chin, 1991).

Another consequence of Fig. 1 and 2 is that the period ratios of $1O/F$ DMC are explained essentially only by the different metallicities, and, within the present uncertainties, are less sensitive to the adopted $M-L$ relation or to other parameters. A support to this view is given by a simple exercise. Taking into account the observed spread of period ratios (~ 0.013) and $[Fe/H]$ values (~ 0.4) of galactic DMC with P_0 between 3 and 4 d, and the fact that in this range $[Fe/H]$ roughly increases when P_1/P_0 decreases, it is possible to deduce by linear extrapolation that the observed period ratios of LMC DMC and their spread imply metallicity values which are not only lower than those of galactic DMC, but are also in rough agreement with the expected values for LMC.

Finally, one should note how the observed large spread of period ratios at long periods, compared with the spread at short periods, is reproduced nicely by the models.

A comparison with Christensen-Dalsgaard & Petersen work shows some differences concerning mainly galactic DMC with low period ratio. The difference could be explained by the fact that they used envelope models with convection included, treated in the mixing-length approximation (α = 2), while we used purely radiative envelope models. Possible differences in the adopted T_e appear of minor importance in the case of $1O/F$ DMC.

3.2. $2O/1O$ DMC

Fig. 3 shows new interesting features with respect to the work of Christensen-Dalsgaard & Petersen. The nonstandard relation is able to reproduce the DMC of LMC with some difficulty for the star with the highest period ratio. The standard relations are not able to reproduce the stars with shortest period. A comparison

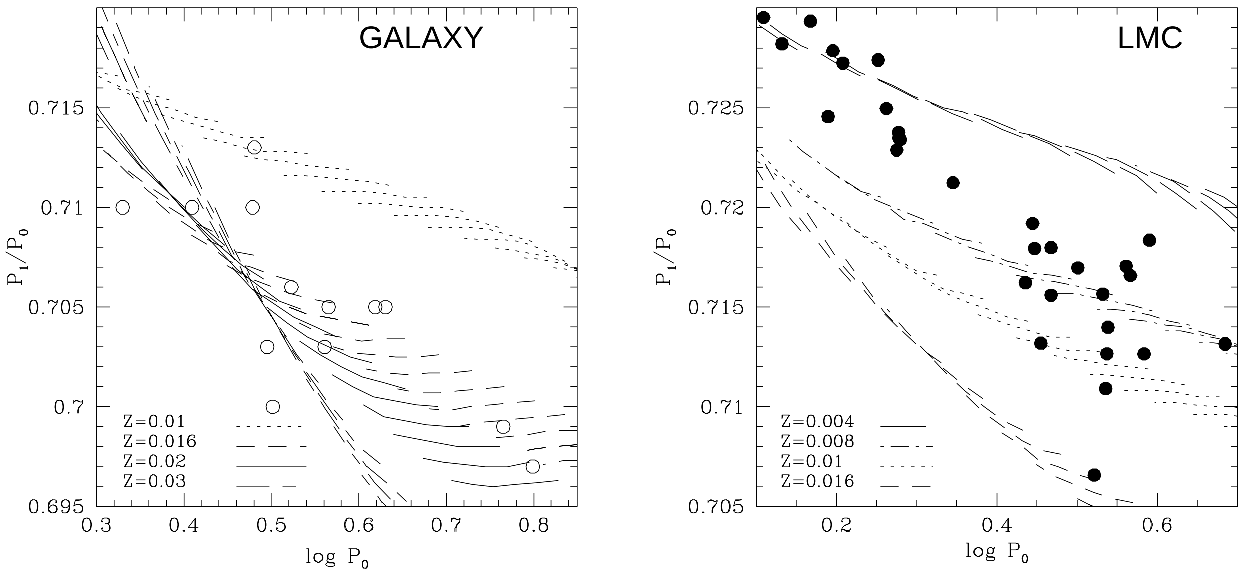

Fig. 2. Period ratio vs. period of $1O/F$ DMC in the Galaxy (left panel) and LMC (right panel), compared with model results for standard $M-L$ relations. The lines connect models with the same mass

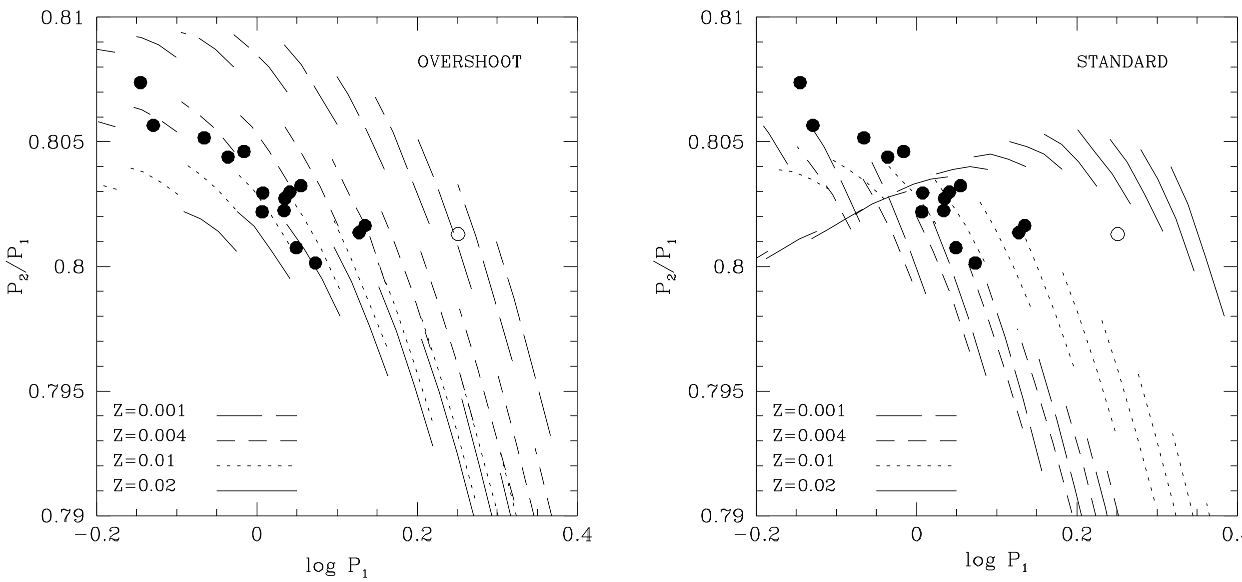

Fig. 3. Period ratio vs. period of $2O/1O$ DMC in LMC (*dots*) and Galaxy (*circle*), compared with model results. The lines connect models with the same mass

of the two panels of Fig. 3 suggests that the reason for the problems of the standard models probably could be the same as that discussed in the previous section.

An interesting feature to be noted is the sensitivity of the period ratios to the T_e of the models. The segments shown in Fig. 3 correspond to a T_e range of only 300 K. We have tried to get some indications on the T_e of LMC DMC from their observed colors, but these are not reliably transformed into the standard system. What is possible to say is that both $1O/F$ and $2O/1O$ DMC in LMC appear to have a large $V-R$ range, which could imply a T_e range larger than 500 K. If true, this would have an evident effect on the theoretical P_2/P_1 vs P_1 diagram: the allowed P_2/P_1 range (mainly for long period models) would be much larger than that shown in Fig. 3, and the difficulty above mentioned would be in part avoided; on the other hand, the effect on the P_1/P_0 vs. P_0 diagram would be much smaller.

The case of the galactic Cepheid CO Aur is different. The nonstandard relation is not able to give a reasonable result for a reasonable value of Z, whereas a standard relation for Z between 0.01 and 0.02 yields a reasonable result. Therefore, it is clear that the $M-L$ relation depends at some level on Z. This conclusion, however, apparently is at variance with the discussion in the previous section.

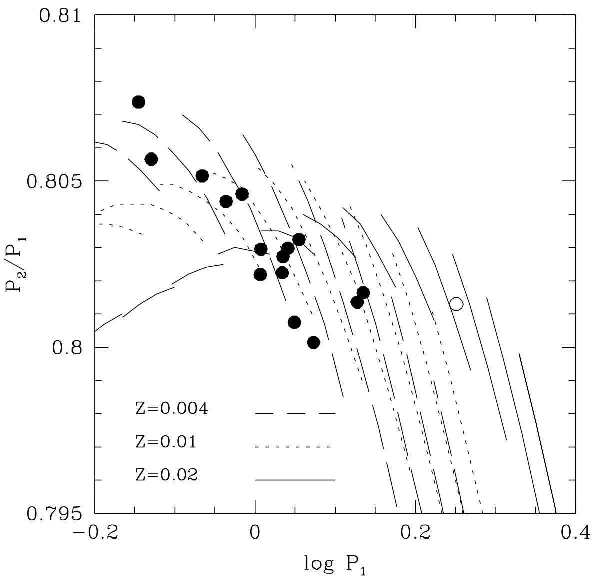

Fig. 4. Period ratio vs. period of $2O/1O$ DMC in LMC (*dots*) and Galaxy (*circle*), compared with model results for modified $M-L$ relations and large T_e range (see text)

4. A possible modification?

From the previous sections we got some general indications on how the theoretical $M-L$ relation should be modified in order to match satisfactorily the observations. The indication from $1O/F$ DMC was apparently in opposite direction of that of $2O/1O$ DMC. A compromise between the two could be the use of a standard relation which depends on Z less strongly than that suggested by BIT. In order to check this point, we have made the following test. Assuming for CO Aur $Z \sim 0.02$, we considered a relation with a=3.68 (in our context the coefficient a is of minor importance) and b=0.57, which reproduces the periods of this star; we recall once more that the CO Aur model fix a point of the 'true' $M-L$ relation owing to its low sensitivity to the Z value, provided that Z is within a reasonable range. Then we assumed for Z=0.01 a $M-L$ approximately similar to that given by BIT, with b = 0.79. For log $M \sim 0.65$ this implies

$$\Delta \log L \sim 22\Delta Z, \quad (2)$$

instead of $\Delta \log L \sim 31\Delta Z$ given by the original BIT relations. Taking into account relation (2) we derived an $M-L$ relation also for Z = 0.004 (b = 0.92). The effects of our assumptions on the models are the following: 1) the problems mentioned in Sect. 3.1 for galactic $1O/F$ DMC are reasonably solved; 2) there are completely negligible differences for $1O/F$ DMC of LMC between models for nonstandard and modified relations; 3) the short period $2O/1O$ DMC in LMC are satisfactorily well reproduced.

Fig. 4 shows the P_2/P_1 vs P_1 diagram obtained with the modified relations, and an adopted T_e range of the envelope models of LMC DMC between 6200 and 6800 K; a higher T_e gives a larger P_2/P_1.

5. Conclusion

Linear adiabatic pulsation models of DMC have been compared with observed DMC in Galaxy and LMC in order to identify the $M-L$ relation valid for Cepheids. The comparison between standard and nonstandard relations has not given clearcut results. On the one hand, models constructed with a nonstandard relation are able to reproduce $1O/F$ DMC characteristics both in Galaxy and LMC slightly better than standard models, and the same occurs for $2O/1O$ DMC in LMC. On the other hand, it is confirmed that CO Aur, the galactic $2O/1O$ pulsator, cannot be reproduced with a nonstandard $M-L$ relation, unless using unrealistic Z values. The study has shown that the original $M-L$ relation for standard models is exceedingly dependent on Z, and it is possible to obtain a satisfactory result for all the DMC by reducing this dependence. Moreover, it has shown the sensitivity of $2O/1O$ period ratios to the T_e, which must be taken into account when making this kind of comparisons. It must be remarked that the present study is based on linear models, therefore nonlinear and nonadiabatic effects could affect the present scenario. Convection was not included, but we think that its effects on short period Cepheids would be small.

It is important to note that the differences between standard and nonstandard relations become negligible for $Z \lesssim 0.016$, therefore DMC in galaxies with such metallicities cannot be used for discriminating reliably between the relations. More interesting results should be obtained by observing $2O/1O$ DMC in external galaxies with $Z \geq 0.02$. Furthermore, since a large weight is given to the CO Aur case, it is essential to make an accurate determination of its metallicity. One could ask if the modified relations here introduced could be applicable to single mode Cepheids, or if there is a discrepancy between single and double-mode Cepheids; in fact, fundamental and first overtone mode Cepheids show resonance effects between pulsation modes which require models constructed using a strictly nonstandard $M-L$ relation. We note that, if there was not CO Aur, our conclusion would be that a nonstandard relation must be preferred and there are no discrepancies with single mode Cepheids.

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