

Mass-luminosity relation of low mass stars^{*}

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Received 4 October 1994 / Accepted 12 September 1996

Abstract. The data on dynamic masses and multicolor photometry of 56 M-type components of binary/multiple systems was collected. Critical evaluation of late type stars bolometric correction scales have been performed. Our refined and reduced data is compared with published empirical and theoretical mass-luminosity relations. Our data does not exclude the existence of a step-like feature at $M_V = 12^m$. The best agreement between observations and theoretical models is found for recent calculations of D’Antona & Mazzitelli (1994) with Alexander opacities. We conclude that present-day knowledge of the mass-luminosity relation at faintest magnitudes is not sufficient for making definite conclusions on the initial mass function of low mass stars.

Key words: low mass stars – stars: fundamental parameters – stars: luminosity function, mass function

1. Introduction

The relation between the mass of a star and its luminosity is a fundamental law that is used in various fields of astrophysics. It is especially important for construction of the initial mass function from the luminosity function of stars. The most complicated situation presently exists at the low mass stars domain because of scarcity of observations in this mass range and the complexity in understanding the nature of such objects. This is especially important since recent confirmations of microlensing effects (Alcock et al. 1993; Aubourg et al. 1993) as well as deep sky counts (Tinney, Reid & Mould 1993) show a large low mass population and its importance in galactic evolution. Furthermore, knowledge of characteristics of red and brown dwarfs is needed for galactic model constructions (Bahcall & Soneira 1980; Robin & Cr ez e 1986). Lastly we should mention that the problem of local missing mass is still open (Ashman 1992) and low mass stars and substellar objects are among the best candidates for its source.

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^{*} Tables 2 and 4 are only available in electronic form at CDS via ftp 130.79.128.5 or <http://cdsweb.u-strasbg.fr/abstract.html>

The investigation of mass-luminosity relation (MLR) of faint stars is topical from different points of view. Progress in development of observational tools provides astrometric, photometric and spectral data of high accuracy. A lot of new data on masses and luminosities of low mass stars has been published during the last years. On the other hand, new calculations of M- and brown dwarfs evolution recently appeared (D’Antona & Mazzitelli 1994, Baraffe et al. 1995).

In the course of the present study, we collect data from literature on “dynamical” masses and photometric parameters of M-type components of binary/multiple stars and create the most extended sample published so far to our knowledge. In order to use all available photometric data on these objects, we investigate relations between different photometric systems for cool stars. The complete analysis of published empirical and theoretical MLRs and a number of published BC scales for cool main sequence stars is also carried out.

In Sect. 2 we describe briefly the principles of observational data collection, main sources and general characteristics of the data. A description of the data reduction is given in Sect. 3. We present some results of comparison of both empirical and theoretical MLRs with observational data in Sect. 4 and, finally, in Sect. 5 we draw conclusions.

2. Data collection

Independent stellar mass and luminosity determination is possible only for components of some types of binary/multiple systems. Resolved astrometric binaries with trigonometric parallax and photocentric orbit provide masses of components. Other sources are eclipsing double-lined spectroscopic binaries and resolved spectroscopic binaries. Systems with a higher degree of multiplicity can provide us, in principle, with information for at least some of the components.

To construct the MLR, empirical data for 56 stars in 30 low mass multiple systems, main sequence components of spectral class M0 and later were collected. Preferring to examine extra stars rather than missing some, we collected data for those stars that were classified as M0 or later by at least one source. We

Table 1. Cross-identification

GJ/Other	GSC	HIC	IRAS PSC	Name	Components
22	402700349	2552AB	00295+6657	BD+66°34	A, B, C
34	366302669	A = 3821A	00461+5732	η Cas	B
65	585500280 + 585502032	—	01365–1812	L 726-8 + UV Cet	A, B
67	282302455	7918	01387+4221		B
98	004200911	11452AB	02251+0412	ADS 1865	A, B
166	531300997	A = 19849A	04136–0742	40 Eri	C
234	478900966	30920	06266–0246	Ross 614	A, B
278	245702407	AB = 36850AB	07314+3159	EF = YY Gem	E ^a , F ^a
352	547200348	46706AB	09289–1316		A, B
379	454200134	49868AB	—	Kui 47	A, B
473	087400306	—	—	Wolf 424	A, B
508	346002162	65026AB	13175+4802	ADS 8862	B
570	610001225	B = 73182B	14545–2112		B, C ^a
623	349500601	80346	—		A, B
630.1	388101071	—	—	AB = CM Dra	A ^a , B ^a
644	564201503	82817AB	16527–0815	Wolf 630	A
661	350101952	84140AB	17106+4543		A, B
677	208700451	85582AB	—	ADS 10585	A, B
704	262102683	B = 88745B	18051+3033	99 Her	B
725	393000364	91768A + 91772B	18421+5934	Σ 2398	A, B
795	051900866	101955B	20371+0447	Kui 99	B
815	317201090	103655AB	—	Kui 103	A, B, C ^a
860	399100092	110893AB	22263+5727	Krü 60	A, B
863.1	398301132	111293AB	—	Kui 112	A, B
896	172300023	116132AB	23293+1939	EQ Peg	A, B
1005	583900513	1242	—	LHS 1047	A, B
1245	314901636	—	—	G 208-44	A, C ^a
ADS 8048	492401114	54155	11019-0359		B, C
ADS 9352	AB = 148000302	AB = 71914	—	Wo 9490	A, B
Kui 84	042500802	87991AB	—	BD+4°3562	A, B, C

^a Components' designations do not correspond to GJ nomenclature.

expect that all of our stars are Pop I ones, unless any evidence of Pop II membership is met (see discussion below).

We considered only those systems where determination of dynamical masses of components is possible, that is, where masses are derived directly from celestial mechanical considerations rather than from relations between mass and some observational parameter.

The complete list of selected stars along with a cross-identification with the Guide Star Catalogue, Hipparcos Input Catalog and IRAS Point Source Catalog (if possible) performed in the course of the present study is presented in Table 1. The Preliminary Version of the Third Catalogue of Nearby Stars by Gliese & Jahreiss, hereafter GJ (Jahreiss 1993), nomenclature of the systems is given in column 1 for all objects except of three systems that have not GJ names (ADS and Kuiper numbers are given for them here and in other tables). The second column contains the GSC identifier, formed by combining the number of the corresponding GSC-file (4 leading digits) with the number of the object in the file (5 digits). HIC and IRAS PSC identifications are given in the third and fourth columns respectively. If the components of a system have separate entries in the GSC or HIC, both names are given in the corresponding column joined by “+” sign. The cases where an identifier is

assigned only to a part of a system are indicated by “=” sign in the corresponding column. The fifth column contains the most common name of the system used in literature. The list of components used in the present work (according to GJ designation) is placed in the sixth column (except for GJ 570 C, GJ 815 C and GJ 1245 C, which do not have specific identifications in GJ, and are assigned the letter “C” by us). Three other exceptions are Castor, GJ 630.1 and Kui 84. For the sextuple Castor system we indicate Castor A (SB2 system, GJ 278 A) as A and B components, Castor B (SB2 system, GJ 278 B) as C and D components, and remote eclipsing binary YY Gem (GJ 278 C) as E and F components. For GJ 630.1 we designate components of the eclipsing binary CM Dra as A and B, and distant white dwarf as C. One component of visual pair Kui 84 is a spectroscopic binary. We indicate the close system as B and C components, and more distant star as A. All these exceptions are indicated in Table 1.

To construct the MLR we have to know the following parameters: mass (in accordance with conditions described above), parallax (trigonometric only, to avoid use of additional calibration scales), apparent magnitude (all available photometry in order to provide a set of colors to determine bolometric corrections and to increase data accuracy).

Mean weighted values of the period, semimajor axis and fractional mass were determined for each system. The weights were assigned in accordance with the following considerations. For period and semimajor axis, a weight was taken being proportional to observed portion of the orbit (strictly saying, to "observational time / period" ratio) and being equal to 1 when more than one period was observed. In accordance with Tokovinin's (1996) considerations, the weight for speckle-interferometric determinations of semimajor axis and period was taken higher by a factor of two than for astrometric ones. We did not take into account rather seldom cases when published values of a period or a semimajor axis were very different from more recent measurements. When no (even formal) error was provided in literature, we estimated its value from a weight, that was assigned in accordance with considerations mentioned above and was based on a whole set of observational data.

The above procedure was not applied for the following stars. Fractional masses for spectroscopic binaries GJ 570 BC, GJ 815 AC and Kui 84 BC are calculated in another way. Mass ratio for these pairs is obtained from spectroscopic orbit. GJ 815 AC and Kui 84 BC have remote components, and, in both cases, total mass for components of close pair is calculated from the orbit of the wide pair. For the components of the eclipsing and spectroscopic binaries GJ 278 EF (YY Gem) and GJ 630.1 AB (CM Dra) mass values were used according to Leung & Schneider (1978) and Metcalfe et al. (1995), correspondingly.

The collected averaged systems data is presented in Table 2. This list includes names, parallaxes, periods, semimajor axis and fractional masses with their calculated errors, and references to observational data. Parallaxes were always taken from the van Altena et al. (1991) catalog. The columns containing period, semimajor axis and fractional mass are empty for some spectral pairs discussed above.

It seems necessary to discuss some specific objects from our sample in more detail.

GJ 22 B

Visual magnitude $12^m.4$ given by Gliese (1969) seems to be too bright, comparatively to the observed value $13^m.78 - 13^m.65$. Photometric data of Eggen (1968) shows significant excess in blue part of spectrum and probably was observed during the flare. Only photometric data for bands V and redder is used in our work (see Sect. 3 for photometric reductions).

GJ 166 C and GJ 863.1

The kinematics of GJ 863.1 suggests that the system belongs to a halo. Eggen (1968) indicated a space velocity of this system with components (U, V, W) of $(+187, -55, -136)$ km s⁻¹. These values are much greater than is usual for a disk population star. W component is especially large. Such velocities are conventional rather for Population II. Different UVW-values, of about 1.5-2 times less, are given in the GJ catalog. The Population II membership could be also suspected for the triple system

GJ 166. Young et al. (1987) give the following space velocity of this system: (U, V, W) of $(-101, -10, -34)$ km s⁻¹. We should mention, however, that a high proper motion alone is not a definitive indicator of age.

GJ 379

Heintz (1987a) indicated that the orbit is provisional, and the value of a^3/P^2 and respectively the total mass cannot be considered well known because of the weak separability of a and $\cos i$.

GJ 473

GJ 473 A and B are suggested as candidates for being brown dwarfs. According to Henry et al. (1992a) the orbit is quite uncertain, but Heintz (1993a) insists on its high accuracy.

GJ 630.1

The triple system GJ 630.1 includes spectroscopic red dwarf binary dM4e CM Dra AB (GJ 630.1 A, LP 101-15) and common proper motion white dwarf GJ 630.1 B (LP 101-16). GJ 630.1 A was found to be binary flare star by Eggen & Sandage (1967) with adopted preliminary period of $0^d.63398$. Martins (1975) found a more precise period ($0^d.63430$) and found a very shallow (about $0^m.04$) secondary minimum at phase about 0.5. He interpreted the secondary minimum as evidence of the presence of a very low mass component in the system. But the detailed study of Lacy (1977) including construction of radial velocity curve has shown that this assumption is incorrect, and that CM Dra system has a period of $1^d.26838965$ and includes two nearly identical components. In the recent investigation of Metcalfe et al. (1995) the very close result was obtained for the period: $1^d.2683909$. So this binary is remarkably similar to the known eclipsing system GJ 278 C (YY Gem), but with masses and radii smaller by a factor of about 3. It should be noted that Lacy (1977) wrongly identified system CM Dra as GJ 630.1 (instead of 630.1 A). The physical characteristics obtained by Martins (1975) were not taken into account in the present study.

There is an indication, that this binary belongs to Pop II objects: Lacy (1977) gives for the CM Dra system (U, V, W) = $(+105 \pm 7, -118 \pm 8, -38 \pm 2)$ km s⁻¹. Metcalfe et al. (1995) mention unusually high values of the helium abundance for the components.

GJ 704 B

Habets & Heintze (1981) assign a spectral type M0 for GJ 704 B, while photometric data ($M_V, B - V$) indicates K spectral type for this star. Our data is confirmed by Henry & McCarthy (1993). As Feierman (1971) pointed out, considerable difference in the brightness of GJ 704 components can lead to sufficient uncertainty of mass determination. The system demonstrates a lower metallicity (Bikmaev 1991).

GJ 725

Keeping in mind the assumption (van de Kamp et al. 1968, Baize 1976, Heintz 1978) of the presence of a third body in the system, we can not regard the values of masses as very reliable. More recent investigations (Hershey 1982, Heintz 1987) show that presence of the third component in the system can not be confirmed. Hershey (1982) indicates that any discovery of supposed planetary object (10-15 Jup.) require either observations for much greater interval of time or instrumentation of prolonged accuracy free of systematic errors at the 0".001.

GJ 815

This star, discovered to be triple, is described as "overmassive" (Russell & Gatewood 1980). Blazit et al. (1987) noted the discrepancy between the calculated orbit of wide pair (AC — B) and observations. Both components of the wide pair are BY Dra type variables (spotted stars).

GJ 896

The difference between V-magnitudes of the components, Δm_V , is very uncertain. Heintz & Borgman (1984) indicate that the system is difficult to measure, and the orbit is provisional. A higher orbit curvature is required (see discussion in Heintz 1984a).

GJ 1005

Henry & McCarthy (1993) indicate that accurate mass calculation for this system has proven problematic because of the small separation of the pair (never measured more than 0".4), resulting in astrometric solutions with large errors in the semimajor axis and fractional masses. Hershey & Taff (1993) using the results of HST observations report that the orbit needs correction, and the semimajor axis may be about 20% larger than the computed value. For the V magnitudes of GJ 1005, different observers give very different values, which produce a large error for the average value.

ADS 8048 BC

The distance to the system is high. Heintz (1986a) characterizes it as "faint and very difficult".

Kui 84

This system was discovered by Kuiper as visual binary and one of it's components was recently found (Tokovinin, 1994) to be a spectroscopic binary. Orbital parallax, found by Tokovinin (1996), is different from trigonometric one, used in the paper. Tokovinin (1994) does not exclude that the system is younger than the majority of dwarfs in the solar neighbourhood.

Table 3. BC scales

Reference	T_{eff}	Method
Johnson (1965)	2660 - 4410	IR-photometry of M-dwarfs
Johnson (1966)	2750 - 26500	IR-photometry
Greenstein et al. (1970)	2250 - 3500	IR-photometry of M-dwarfs
Veeder (1974)	2450 - 4300	IR-photometry of M-dwarfs
Traat (1976)	2200 - 3870	IR-photometry of M-stars
Hayes (1978)	2620 - 47000	IR-photometry
Carney & Aaronson (1979)	4600 - 6300	IR-photometry of dwarfs and subdwarfs binary systems data
Habets & Heintze (1981)	2875 - 35500	optical and IR-photometry of dwarfs
Reid & Gilmore (1984)	2600 - 4300	optical and IR-photometry of late-M dwarfs
Bessell (1991)	2450 - 4000	optical and IR-photometry of late-M dwarfs

3. Bolometric correction scales and photometric reductions

To compare the theoretical relations between mass and luminosity with empirical data we need bolometric corrections (BC)

$$M_{\text{bol}} = M_V + BC,$$

where M_{bol} is the bolometric absolute magnitude, and M_V is the visual absolute magnitude. The bolometric corrections depend on the physical conditions in stellar outer layers, and are poorly known for late type objects.

There is a number of BC scales published in the literature (see Table 3). As it can be seen from Fig. 1, similar results are quoted in most of the papers. All the scales from Fig. 1 except Habets & Heintze (1981) for $T_{\text{eff}} < 3800\text{K}$ and Bessell (1991) for $T_{\text{eff}} < 3000\text{K}$ are in good agreement. The typical width of a band in Fig. 1 (which could be treated as BC scale accuracy) where almost all published BC scales lie is about 0.^m5. As for the data from Table 3 which is not displayed in Fig. 1, the BC scale of Reid & Gilmore (1984) is in good agreement with the Traat (1976) scale. Carney & Aaronson (1979) affirm that their scale differs from the one of Johnson by not more than 0.^m05; Veeder's data also agrees with the latter.

To construct the BC scale, Habets & Heintze (1981) used bolometric magnitudes that had been calculated from radii and temperatures of binary stars. Only five stars from their set have $T_{\text{eff}} < 3650\text{K}$. Moreover, they utilize only linear (and, consequently, rough) approximation for the MLR and other relations used in their study. Haywood (1993) also mentions their overestimation of low mass stellar radii.

The Bessell (1991) scale also does not agree with most of the scales under comparison, but if we compare V-R versus BC or V-K versus BC relations of Bessell (1991) and Traat (1976),

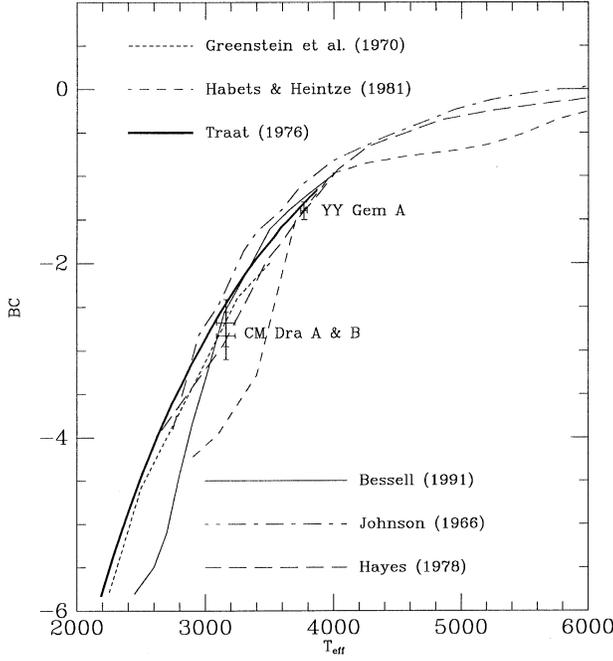


Fig. 1. BC scales

the agreement is excellent. This means that the two scales have different temperature calibrations. Fundamental temperatures based on two pairs of eclipsing binaries (CM Dra and YY Gem) are only known for the spectral range. Unfortunately, large errors in observational fundamental values of T_{eff} for these systems do not allow us to make definite conclusions on the advantage of any scale. Nevertheless, to make a definitive choice we use available photometric data for the binaries. We regard only Johnson photometry for uniformity. Observational colors were taken from Leung & Shneider (1978), Gershberg (1978) for YY Gem A and from Popper (1980) for CM Dra. T_{eff} was estimated from a number of color indices with the help of both scales. Resulting values were then compared with observational fundamental data. Both Traat's and Bessell's temperatures are close to each other and to observational values for YY Gem A (Leung & Shneider 1978). Photometric observations of the second component of YY Gem are too poor to be useful for T_{eff} estimations. We did find that for the CM Dra Bessell's scale gives slightly underestimated (about 150 degrees) values for the temperature, in comparison with the fundamental one (Lacy 1977). We do have to note that both CM Dra and YY Gem systems exhibit emission-line spectra, UV Ceti type flare activity and BY Dra type variability. Besides there is some evidence of possible Pop II nature of CM Dra in the literature (see the discussion in Sect. 2). Observational points for YY Gem and CM Dra taken from the Catalogue of astrophysical parameters of binary systems (Malkov 1993) are also plotted on Fig. 1.

Both scales are identical with respect to YY Gem A and CM Dra colors, but we have chosen the Traat scale for further calculations because it is in good agreement with the majority of other independent scales, and extends to the low temperature region farther.

Once the BC scale is chosen, there are two possibilities for comparison of observations with theoretical data: either to convert observational magnitudes of stars to bolometric ones, or to transform theoretical MLRs to visual magnitudes. Both approaches are used in the present study.

The collected data was obtained in different photometric systems. The Traat (1976, 1990) scales are realized in the multicolor broad-band Johnson UBVRIJHKL system, which includes the systems UBV (Johnson & Morgan 1951), RI-JKLMNQ (Johnson 1965; Morrison & Simon 1973), H (Mendoza 1967). Therefore, it was necessary to transform all the collected photometric data to the broad-band multicolor Johnson system, and hence to search for preliminary transformation relations for this system using other cool star photometric systems. This question was discussed in Straizys (1977), but newer data has appeared. Besides, only photometric systems for cool stars were the subject of the current study. Therefore, we made reductions to the Johnson system from the following photometric systems:

1. Kron RI: introduced by Kron & Smith (1951). Relation to the Johnson system:

$$(R - I)_J = 1.25(R - I)_K + 0.0625$$

for late stars;

$$R_J = R_K - 0.38,$$

$$(R - I)_K > 0.4$$

$$R_J = R_K + 0.18 - 0.5(R - I)_K,$$

$$(R - I)_K < 0.4$$

The relations were published by Eggen (1971). The Eggen-Kron version of this system described by Eggen (1975) is nearly identical with the original one, as shown by Eggen (1968).

2. Cousins VRI (Kron-Cape, Kron-Cousins): introduced by Cousins (1976). We derived relations for the Johnson system for cool stars using Bessell & Weis (1987) relations between the Cousins, and RI Kron plus V Johnson systems:

$$V_C \approx V_J,$$

$$(V - R)_J = 0.137 + 0.943(V - R)_C + 0.622(V - R)_C^2 - 0.208(V - R)_C^3,$$

$$(R - I)_J = -0.197 + 1.644(R - I)_C - 0.560(R - I)_C^2 + 0.156(R - I)_C^3$$

3. Kron-Mayall PVI: introduced by Kron & Mayall (1960). They also give the relations to Johnson system:

$$V_J = V_{KM} - 0.075(P - V)_{KM} - 0.027,$$

$$-0.4 < (P - V)_{KM} < 1.0$$

$$(B - V)_J = 0.10 + 0.96(P - V)_{KM},$$

$$-0.4 < (P - V)_{KM} < 1.0$$

$$(B - V)_J = 0.00 + 1.06(P - V)_{KM},$$

$$1.0 < (P - V)_{KM} < 1.5$$

According to Kron & Mayall (1960) the I magnitude is the same as in RI Kron system.

4. Eggen (P, V)_E: introduced by Eggen (1955). He also gives the relations between (P, V)_E and Johnson system:

$$(B - V)_J = 0.9638(P - V)_E + 0.1208$$

$$V_J = 0.0015(P - V)_E + V_E$$

It must be mentioned that in the last relation the error of the factor at $(P - V)_E$ is equal to ± 0.024 , which is 1600%. That is why practically we consider $V_J \approx V_E \pm 0.024$.

The V magnitude for every star was determined as a weighted average from the observational V values converted to the Johnson system. Exceptions are GJ 22 A, GJ 22 C and GJ 67 B: for these stars only JHK photometry is available. It should be noted that determination of luminosity from the $(J - H)$, $(H - K)$ and $(J - K)$ color indices has low confidence. Using relations from Henry & McCarthy (1993) between J, H, K magnitudes and color indices $(V - J)$, $(V - H)$ and $(V - K)$ respectively, V and BC values were obtained. Beside these three stars, GJ 234 AB, GJ 508 B, GJ 570 BC, GJ 623 AB and GJ 1245 AC are not clearly resolved at visible wavelengths; it leads to less precise V and M_V values.

Using parallax values, the absolute visual magnitudes were obtained. Interstellar absorption was suggested to be unimportant because of the nearness of the studied objects. Next, the absolute bolometric stellar magnitude corresponding to every available color was determined with the help of Traat (1976, 1990) relations. These relations include bolometric corrections, stellar magnitudes, effective temperatures and color indices for different subclasses of spectral class M (and earlier in Traat 1990). Mainly, the Traat (1976) data was used, where results for normal and for flare or emission M dwarfs are presented separately. This paper includes data for the $(U - V)$, $(B - V)$, $(V - R)$, $(V - I)$, $(V - H)$, $(V - K)$, $(V - L)$ color indices. In Traat (1990) only averaged data for color indices $(U - V)$, $(B - V)$, $(V - I)$, $(V - J)$, $(V - K)$, $(V - L)$ is presented. $(V - J)$ was recalculated for both of the relations of Traat (1976) by interpolating data from Traat (1990). Since data in Traat (1990) is not restricted by spectral class M, these relations were used for stars on the boundary between spectral classes K and M.

If necessary, required color indices were computed by combining data from different sources (for instance, V from one source, R and $(R - I)$ from another one give $(V - R)$ and $(V - I)$ values). If only color indices which are not included in Traat tables were known, for instance, $(R - I)$, then from $(V - R)$ and $(V - I)$ the $(R - I)$ relation was also interpolated. There is a number of stars for which only the V value is known. In this case, bolometric magnitude determination is impossible by the method used here which uses color information.

The final bolometric magnitude was derived as a weighted average. The bolometric luminosity was obtained according to $\log L/L_\odot = 0.4(4.69 - M_{\text{bol}})$,

where 4.69 is the solar bolometric magnitude value. Table 4 includes derived parameters of 56 objects: adopted V-magnitudes

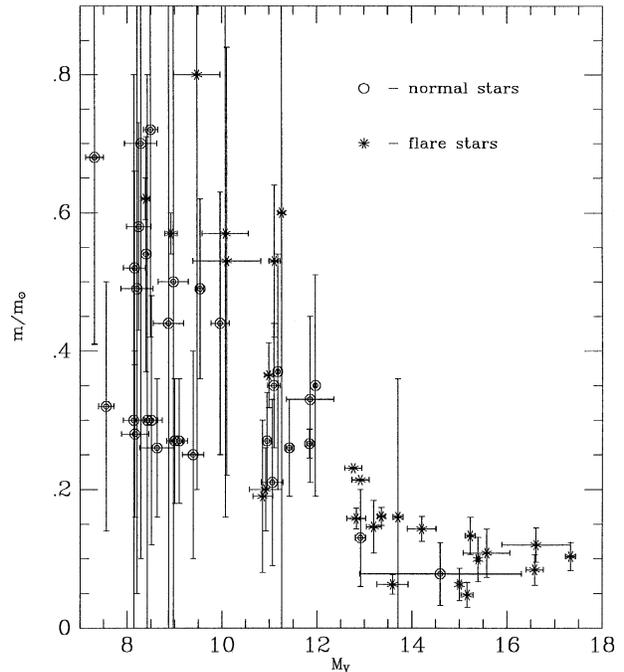


Fig. 2. All collected stars

(see Table 2 for references), bolometric corrections (if calculated), absolute visual values, luminosities (if calculated) and masses with their standard deviations. The same data is presented in Figs. 2 and 3. The typical standard deviation for photometric data in Table 4 is about $0^{\text{m}}.25$ in V. Our sample contains at least 26 objects classified as flare stars (they are indicated by an asterisk in the V-column); some other stars are suspected to be flares. Stellar masses fall into the range between 0.048 and 0.8 solar masses; the typical standard deviation is about 50% in mass. Note, that for 17 stars with masses lower than 0.2 solar mass only components of GJ 1005 were not recognized as flaring ones (see Table 4).

4. Discussion of results

As it can be seen from the plots and previous discussion, our data has non-uniform quality. To test theoretical and empirical relations, we used only the better observational points. Objects suspected to belong to Pop II (GJ 166 C, GJ 630.1, GJ 863.1) and ones having dubious data (GJ 379, GJ 815, GJ 896, GJ 1005), discussed in Sect. 2; as well as other objects having the standard error in the mass more than 50% were excluded from the analysis.

30 selected objects are indicated in the last column of Table 4. Note that no stars (except of GJ 623 B) among the selected objects have standard error in absolute magnitude more than $0^{\text{m}}.5$.

4.1. Comparison with empirical MLRs

It is reasonable to compare our observational data with the empirical MLRs previously published by Scalo (1986), Kroupa et

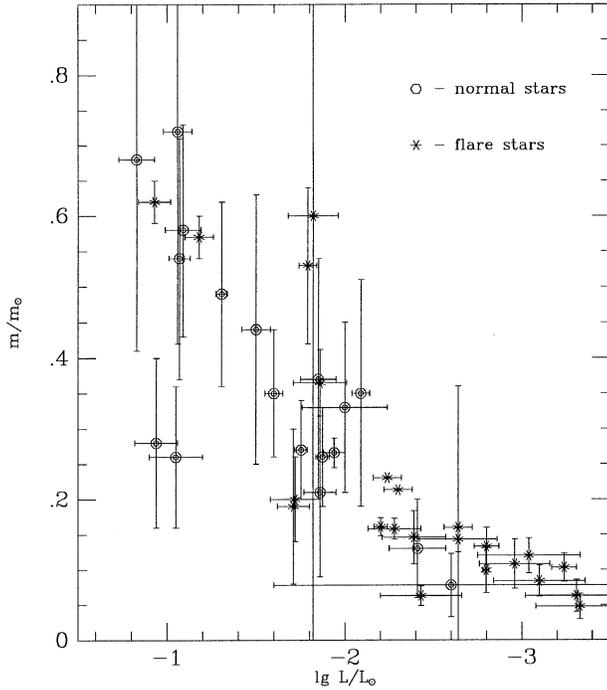


Fig. 3. All collected stars with known L/L_{\odot}

al. (1993) and Henry & McCarthy (1993). They are shown in Fig. 4. As can be seen, our data is in reasonable agreement with all the relations. The agreement for faintest stars ($M_V > 12^m$), having the most accurate data, is good. But it seems, the data suggests even more flat relation, than it was derived previously. For brighter stars the relations constitute an upper boundary for the observed data points. A considerable spread of empirical data points can be observed in this part of the mass — M_V diagram. This spread can be attributed to higher absolute errors in the mass, and also to the existence of a couple of “overluminous” stars with $m = 0.2 \div 0.3m_{\odot}$. Since there is a number of causes which could produce this overluminosity effect (as discussed below), and only one reason responsible for “underluminous” stars in the mass — M_V plane (unresolved multiplicity, discussed in the Appendix), we regard the agreement of our brighter star data with empirical MLRs as a satisfactory one.

It should be noted that present data at least does not reject a hypothesis on existence of step-like feature of the MLR at $M_V = 12^m$ introduced first by Kroupa et al. (1993). Remarkably, that the Henry & McCarthy (1993) MLR reproduces this “step” even better (see Fig. 4). Unfortunately the data scarcity, insufficient accuracy of masses of brighter stars, and relatively narrow mass interval involved in present study prevent us to make more definite conclusion. Nevertheless we pay attention to this feature due to its importance in the MLR applications. As was shown by Malkov (1987), the behaviour of the MLR’s derivative plays an important role in the process of initial mass function (IMF) determination from luminosity function (LF). The feature at $M_V = 12^m$ leads, in particular, to the appearance of the maximum on the LF, while the IMF continues to increase

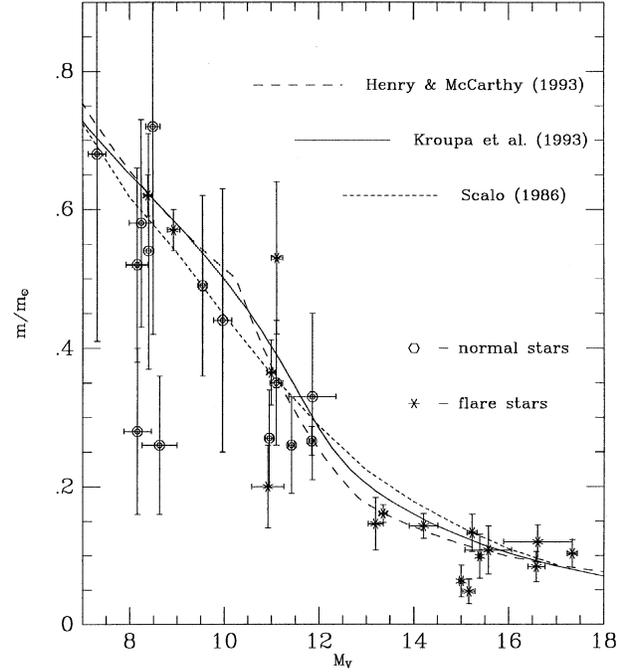


Fig. 4. Empirical MLRs vs selected observational data

towards lower masses. Possible physical reasons for this behaviour of the MLR are proposed by Kroupa et al. (1993).

Let us come back to overluminous stars (GJ 677 A, B). It seems that their deviation (of order of 3 sigma in mass) from average strip could not be attributed to random errors only. Taking in the account mass uncertainty we estimate the luminosity excess of order of 2^m to 4^m in M_V . Too high luminosities of GJ 677 A and B were pointed out also by Lippincott (1982), but not explained. There is a number of possible causes that can lead to a real or apparent displacement of a star in the mass — luminosity plane.

The following types of objects can display extra luminosity for a given mass:

- Underevolved (still contracting to the ZAMS) stars, which can display a luminosity excess of up to 5^m (taking into account the earliest stages of evolution). The probability to meet such stars increases as the mass decreases.
- Stars with metal deficiency (Pop II objects) can exceed luminosity of their Pop I counterparts by up to 1^m ;
- Flaring stars observed during a flare (up to about 2^m at the flare peak).

Besides (this note concerns mass — $\log L/L_{\odot}$ diagram), it is possible to make a mistake when determining the bolometric stellar magnitude, due to the use of unproper bolometric correction scale if we consider the flare star as an ordinary one. The possible luminosity error can (for the smallest masses) be up to 0.2 dex in $\log L/L_{\odot}$.

Since there is no indication of flare activity for overluminous stars GJ 677 A and B, and due to low probability to observe both of them in the maximum of a flare, we are forced to attribute the extra luminosities to their evolutionary status. Another reason

Table 5. Theoretical MLRs

Reference	Masses, m_{\odot}	Y	Z
Copeland et al. (1970)	0.250 - 2.00	0.37	0.03
	0.250 - 2.00	0.395	0.005
	0.250 - 2.50	0.27	0.03
	0.250 - 2.00	0.28	0.02
	0.250 - 2.50	0.095	0.005
	0.250 - 2.90	0.099	0.001
Hoxie (1970)	0.102 - 0.20	0.24	0.021
Grossman et al. (1974)	0.085 - 0.50	0.29	0.03
D'Antona & Mazzitelli (1982)	0.070 - 0.20	0.40	0.02
	0.090 - 0.20	0.25	0.02
Sienkiewicz (1982)	0.085 - 0.20	0.25	0.001
	0.063 - 0.30	0.26	0.04
	0.065 - 0.30	0.27	0.03
	0.074 - 0.30	0.295	0.005
Vandenberg et al. (1983)	0.092 - 0.30	0.299	0.001
	0.100 - 0.75	0.25	0.02
	0.100 - 0.75	0.25	0.01
	0.150 - 0.75	0.25	0.001
	0.150 - 0.75	0.25	0.0001
Neece (1984)	0.150 - 0.75	0.25	0.00001
	0.150 - 0.75	0.20	0.001
Burrows et al. (1989)	0.150 - 0.55	0.29	0.03
	0.030 - 0.20	0.22	0.02
Dorman et al. (1989)	0.030 - 0.20	0.25	0.02
	0.076 - 0.55	0.28	0.02
Kroupa et al. (1990)	0.093 - 1.00	0.28	0.02
D'Antona & Mazzitelli (1994)	0.020 - 2.50	0.28	0.019
	0.078 - 0.60	0.275	0.02
Baraffe et al. (1995)	0.080 - 0.40	0.275	0.006
	0.085 - 0.20	0.275	0.0006

that the objects are too bright for the fits is that the parallaxes are wrong. Both components of GJ 677 are the furthest off the fits, and the system is among the most distant one in the sample.

Now we can return to the “peculiar” stars of Sect. 2 to see whether they also decline from our best-star MLR. In fact, GJ 815 A and GJ 896 A are shifted to lower luminosities. This can indicate that the multiplicity of such stars is more than it was considered. The fact that GJ 863.1 is overluminous matches the hypothesis that this star belongs to Population II.

4.2. Comparison with theoretical MLRs

Theoretical MLRs (see Table 5) based on evolutionary stellar models concern different ranges within the low mass region. Relations of D'Antona & Mazzitelli (1982), Sienkiewicz (1982), Burrows et al. (1989) and Hoxie (1970) refer only to the lowest mass range. The wider mass range is considered by Grossman et al. (1974), Vandenberg et al. (1983), Dorman et al. (1989), Kroupa et al. (1990), D'Antona & Mazzitelli (1994), and Baraffe et al. (1995).

Recent calculations of D'Antona & Mazzitelli (1994) were carried out for Kurucz and Alexander low temperature opacities and two different models of overadiabatic convection (the

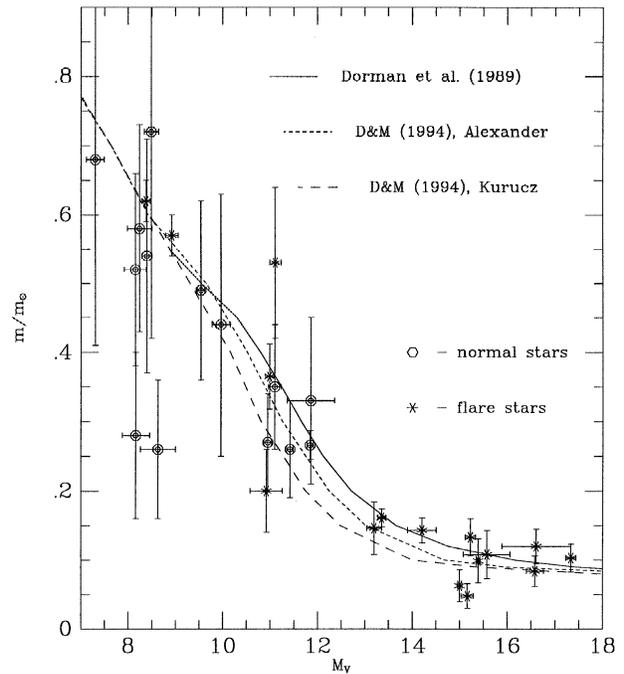


Fig. 5. Better theoretical MLRs vs selected observational data. Visual magnitudes

mixing-length theory and Canuto & Mazzitelli model). We have compared the ZAMS models of D'Antona & Mazzitelli in the mass — $\log L/L_{\odot}$, mass — $\log T_{\text{eff}}$, and $\log L/L_{\odot}$ — $\log T_{\text{eff}}$ planes. As the comparison has shown, the convection treatment is not important in the case of M stars MLR, which depends mainly on the adopted opacity. From this point on, we use tracks with Canuto-Mazzitelli convection treatment and both opacity sets. Kurucz opacity models have higher temperatures than those of Alexander, that leads to lower BCs, and consequently brighter absolute magnitudes for the same bolometric luminosities.

We compared the MLRs with observations in both the mass — $\log L/L_{\odot}$ plane and the mass — M_V plane.

The comparison of all relations from Table 5 with empirical data shows the following: the majority of theoretical curves is displaced with respect to observational points towards higher masses and lower luminosities. It should be noted that generally the newer the models, the better the agreement with observations. The models published since the late eighties display the best agreement. The most recent calculations of Baraffe et al. (1995), which have been carried out with non-gray atmospheres, show good general agreement with our data, but for lower metallicity models ($Z=0.006$, $[M/H]=-0.5$) only. The theoretical MLR of Baraffe et al. of solar metallicity lies beyond the empirical sequence for $M_V < 15^m$, being displaced to fainter magnitudes and higher masses. Since it is hardly to believe, that the bulk of our stars has metallicity typical for thick disk population, we attribute the disagreement to the models. There is an evidence (Chabrier et al., 1996), that new generation synthetic atmospheres, incorporated into evolutionary models remove the disagreement. But since the models of Chabrier et al. are not so

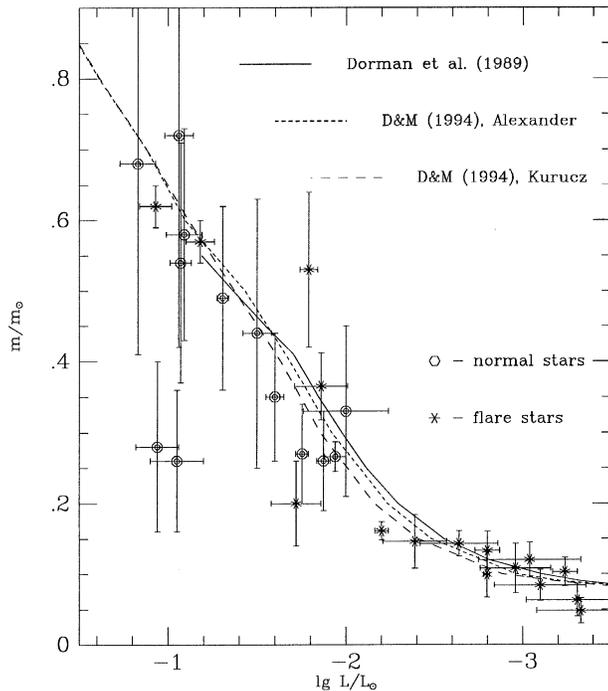


Fig. 6. Better theoretical MLRs vs selected observational data. Bolometric luminosities

far available to us we exclude them from the following discussion.

Careful comparison of modern calculations with our data shows that the most adequate MLRs are deduced from Dorman et al. (1989) and from D’Antona & Mazzitelli (1994) evolutionary tracks (see Figs 5 and 6). We should mention the advantages of a comparison of theoretical and observational data in the plane mass — M_V rather than plane mass — $\log L/L_\odot$. In the former case, one should take into consideration not only the model luminosities, but also their temperatures, necessary for bolometric corrections calculation. This involves into comparison the complete set of stellar parameters ($\log L/L_\odot$, $\log T_{\text{eff}}$, and mass) instead of a pair ($\log L/L_\odot$, mass), which should be used in the latter case. This is especially important in the light of temperature dependence of D’Antona & Mazzitelli models for different opacity tables has been mentioned above. As a result, the best agreement was found for D’Antona & Mazzitelli set corresponding to Alexander opacities. In Table 6 we display this relation both in bolometric luminosities and in visual magnitudes converted by us from D’Antona & Mazzitelli ZAMS (the last model is slightly underevolved) with help of Traat (1976, 1990) BC — T_{eff} scale.

4.3. Application of the initial mass function

One of the most important applications of the MLR is the transformation of observational LF to the IMF. If one knows the empirical LF, one can restore the MLR and compare it with observational data, supposing that the IMF has power-law or lognormal shape. To do this, we have to solve a differential

Table 6. The best theoretical MLR converted to visual magnitudes

Mass	$\lg L/L_\odot$	M_V	Mass	$\lg L/L_\odot$	M_V
1.00	-0.170	5.36	0.40	-1.684	10.52
0.90	-0.377	5.95	0.30	-1.919	11.27
0.80	-0.612	6.71	0.20	-2.235	12.25
0.70	-0.881	7.64	0.15	-2.482	13.02
0.60	-1.096	8.42	0.10	-2.961	14.68*
0.50	-1.432	9.65			

* slightly underevolved.

equation for the MLR. Setting boundary conditions, we can solve the Cauchy problem. This method was described in more detail by Malkov (1989, 1990).

First of all, we must know the LF of field stars. During the past decade the well known Wielen et al. (1983) LF (hereafter WJK) is widely cited. As Jahreiss (1994) informed, a new calculation of the local LF was carried out with the GJ catalog, and WJK LF is still valid. There are no significant changes, especially for the faint end ($M_V > 10^m$).

Some other LFs are discussed in a recent review by Stringfellow & Bessell (1994). Choosing a LF for our purposes, we compare Kroupa et al. (1993) LF with that of Jarrett et al. (1994) — hereafter respectively KTG and JDH — and with WJK LF. KTG slightly correct WJK LF and extend it up to $M_V = 17^m$. Meanwhile, JDH implement a new method of constructing the field star LF: they make photometry of stars against some highly obscured, nearby dark clouds. They found a strong increase of their LF at $M_V > 16^m$: such a feature can be imperceptible on “astrometric” WJK and KTG LFs that should be incomplete at the faintest magnitudes. The problem of the LF determination is also complicated by presence in the sample of unresolved binaries (see e.g. Piskunov and Malkov, 1991). We nevertheless will use KTG LF for further calculations, keeping in mind the fact that the LF of faint field stars is currently unconstrained.

Some results are shown in Fig. 7. One can see that the observed MLR can be reproduced from the empirical LF with the help of both power-law and lognormal monotonous IMFs. It should be kept in mind that the closer we are to the red-brown dwarfs border, the higher the influence of stellar evolution (the larger Kelvin time), the lower the confidence of our calculations. So we need to make some additional assumptions on the behaviour of the star formation rate for less massive stars (Malkov & Piskunov 1988).

Note that our goal was not to find the best MLR, satisfying both the IMFs and the binary stars’ data. We also did not consider in detail the reasons responsible for the MLR fluctuations. But we should stress that even the best present-day observational data on the MLR and local LF do not exclude both lognormal and Salpeter (1955) IMFs.

5. Conclusions

We can draw the following conclusions.

For construction of the MLR, a detailed analysis of available data is necessary: only 30 of 56 M-type components of

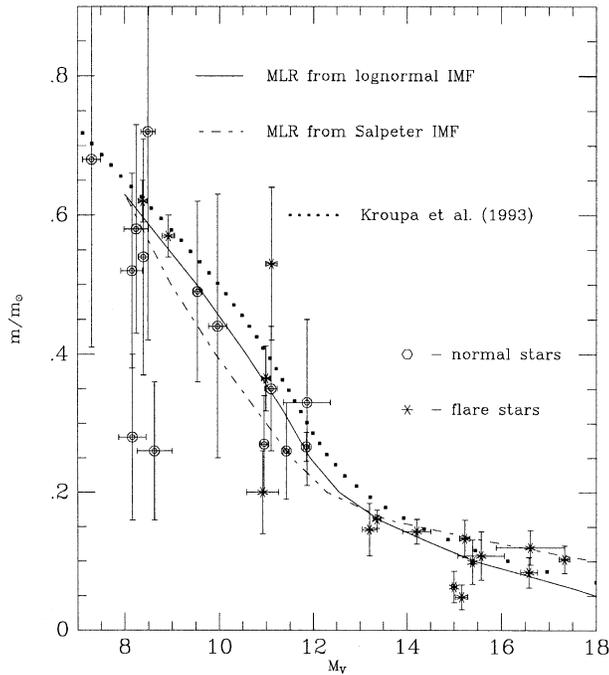


Fig. 7. MLRs calculated from smooth IMFs. Kroupa e.a. (1993) empirical MLR is shown for comparison.

binary/multiple stars were recognized as of high enough quality for this purpose. Our data is in reasonable agreement with recently published empirical MLRs. It should be especially noted that it does not exclude the existence of a step-like feature at $M_V = 12^m$ for the first time detected by Kroupa et al. (1993). Present-day theoretical models of low mass stars show good overall agreement with empirical data. The best agreement between observations and theoretical models is found for recent calculations of D’Antona & Mazzitelli (1994) with Alexander opacities. We found the convection treatment is not important in this mass range for the MLR construction problem. It was also found that the most efficient way to compare theoretical relations with observational data is to use the mass — M_V diagram. We provide this diagram for the best D’Antona & Mazzitelli (1994) theoretical ZAMS. It should be stated that present-day knowledge of the MLR and LF at the faintest magnitudes is not sufficient for making definite conclusions on the IMF behaviour at low masses.

We attempted to understand the nature of strong deviation of some stars from the average relation in the mass — luminosity plane. Several explanations of such deviations were proposed.

Acknowledgements. This research has made use of the Simbad database, operated at CDS, Strasbourg, France. AP is grateful to the CDS administration and staff for their hospitality and for providing tools and resources necessary for the study. We would also like to thank Dr. I. Mazzitelli, who kindly provided us with the results of his calculation of theoretical tracks. It is a pleasure to acknowledge helpful conversations with O. M. Smirnov. The comments of the anonymous referee produced significant improvement of the paper. We are also grateful to Drs. M. A. Smirnov and A. A. Tokovinin for very useful

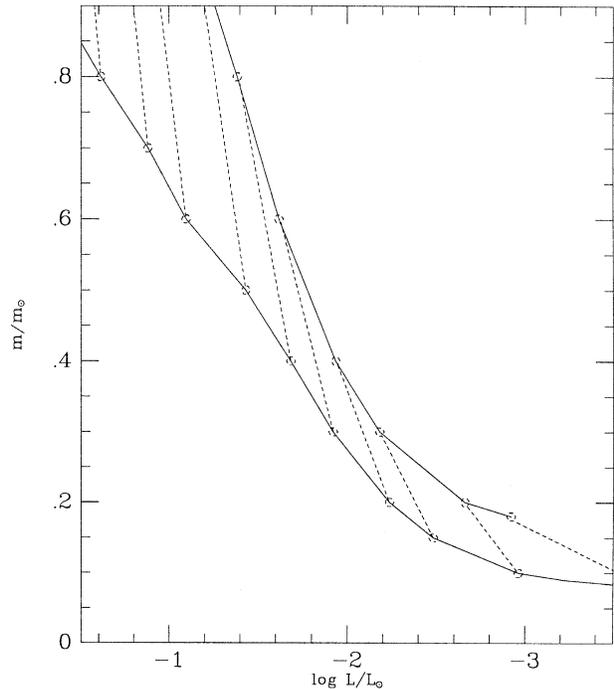


Fig. 8. Unresolved binaries and MLR

discussions. This research was partly supported by the Russian Foundation of Fundamental Research (grant 95-02-04260).

Appendix A: effect of unresolved binaries in the mass – luminosity plane

The effect of unresolved binarity is one of the most important causes which is able to shift a star from its “normal” position in the mass-luminosity plane. If two stars are seen as a single one, the dynamically determined mass of this object equals to the sum of the component masses, while its luminosity is the combined one.

Due to the non-linearity of the MLR, the resulting object appears to have too little luminosity for its mass (or, vice versa, an excess mass for its luminosity). It can be shown that the shift is more when the component masses are similar, and is at a maximum when the components are of equal masses. One can also point out the area above the “normal” MLR in the mass-luminosity diagram where unresolved binaries can be located. The effect is illustrated in Fig. 8 for the theoretical MLR from Table 6, in the case when both components are of equal masses. Dotted lines connect the positions of unresolved binaries with the positions of their components. The upper solid line in Fig. 8 is determined as

$$m = 2f(\log(L/2)),$$

where $f(\log L)$ is the MLR from Table 6.

Unresolved binaries with unequal components lie between the two solid lines in Fig. 8. In turn, if some object lies above the upper solid line, its position can be explained in the same

fashion by its higher multiplicity (we did not regard this case here).

The multiplicity of an object can be estimated from a detailed analysis of its color indices that are affected by unresolved binarity. Subtraction of the primary star spectrum from the composite (observational) one (as was recently shown in Kirkpatrick & McCarthy 1994) could be mentioned as another method.

References

- Alcock C., Akerlof C. W., Allsman R. A., et al., 1993, *Nature* 365, 621
- Ashman K. M., 1992, *PASP* 104, 1109
- Arend S., 1949, *Ann. O. Belg.* 4, 187
- Aubourg E., Bareyre P., Bréhin S., et al., 1993, *Nature* 365, 623
- Bahcall J. N. & Soneira R. M., 1980, *ApJS* 44, 73
- Baize P., 1944, *l'Astronomie* 58, 62
- Baize P., 1946, *AJ* 51, 221
- Baize P., 1949a, *J.d'Obs.* 32, 50
- Baize P., 1949b, *J.d'Obs.* 32, 60
- Baize P., 1950, *J.d'Obs.* 33, 1
- Baize P., 1951, *Ann. Astrophys.* 14, 85
- Baize P., 1955, *J.d'Obs.* 38, 37
- Baize P., 1957, *J.d'Obs.* 40, 17
- Baize P., 1966, *J.d'Obs.* 49, 1
- Baize P., 1976, *A&ApS* 26, 177
- Baize P., 1985, *Circ. Inf. UAI (Comm. 26)* 67
- Baize P. & Petit M., 1989, *A&ApS* 77, 497
- Baraffe I., Chabrier G., Allard F., Hauschildt P. H., 1995, *ApJ* 446, L35
- Bessell M. S., 1990, *A&ApS* 83, 357
- Bessell M. S., 1991, *AJ* 101, 662
- Bessell M. S. & Weis E. W., 1987, *PASP* 99, 642
- Bikmaev I. E., 1991, *Bull. Spec. Astrophys. Obs. - North Caucasus* 25, 1
- Blazit A., Bonneau D., Foy R., 1987, *A&ApS* 71, 57
- Burrows A., Hubbard W. B., Lunine J. I., 1989, *ApJ* 345, 939
- Carney B. W. & Aaronson M., 1979, *AJ* 84, 867
- Chabrier G., Baraffe I., Plez B., 1996, *ApJL* 459, L91
- Copeland H., Jensen J. O., Jorgensen H. E., 1970, *A&A* 5, 12
- Coppenbarger D. S., Henry T. J., McCarthy D. W. Jr., 1994, *AJ* 107, 1551
- Cousins A. W. J., 1976, *Mem.R.A.S.* 81, 25
- D'Antona F. & Mazzitelli I., 1982, *A&A* 113, 303
- D'Antona F. & Mazzitelli I., 1994, *ApJS* 90, 467
- Dorman B., Nelson L. A., Chau W. Y., 1989, *ApJ* 342, 1003
- Duquennoy A. & Mayor M., 1988, *A&Ap* 200, 135
- Dzervitis U. K. 1973, *Photometric study of red stars*, Riga, p. 131
- Eggen O. J., 1955, *AJ* 60, 65
- Eggen O. J., 1956a, *AJ* 61, 405
- Eggen O. J., 1956b, *AJ* 61, 462
- Eggen O. J., 1965a, *AJ* 70, 69
- Eggen O. J., 1965b, *AJ* 70, 19
- Eggen O. J., 1967, *ARA&Ap* 5, 105
- Eggen O. J., 1968, *ApJS* 16, 49
- Eggen O. J., 1971, *ApJS* 22, 389
- Eggen O. J., 1974, *PASP* 86, 697
- Eggen O. J., 1975, *PASP* 87, 107
- Eggen O. J. & Sandage A., 1967, *ApJ* 148, 911
- Eichhorn H. & Alden H.L., 1960, *AJ* 65, 148
- Erro B. I., 1971, *Bol. Obs. Tonantzintla y Tacubaya* 6, 143
- Feierman B. H., 1971, *AJ* 76, 73
- Fekel T. Jr., Bopp B. W., Lacy C. H., 1978, *AJ* 83, 1445
- Fleischer R., 1957, *AJ* 62, 379
- Gatewood G., 1973, *AJ* 78, 777
- Gershberg R. E., 1978, *Low mass flare stars*, Nauka, Moscow, p.111
- Geyer D. W., Harrington R. S., Worley Ch. E., 1988, *AJ* 95, 1841
- Gliese W., 1969, *Veröff. Astron. Rechen-Inst. Heidelberg* 22
- Greenstein J. L., Neugebauer G., Becklin E. E., 1970, *ApJ* 161, 519
- Grossman A. S., Hays D., Graboske H. C., 1974, *A&A* 30, 95
- Habets C. M. H. J. & Heintze J. R. W., 1981, *A&AS* 46, 193
- Hall R. G., 1952, *AJ* 57, 47
- Harrington R. S., 1990, *AJ* 100, 559
- Harrington R. S. & Behall A. L., 1973, *AJ* 78, 1096
- Harris D. E. & Johnson H. M., 1985, *ApJ* 294, 649
- Hartkopf W.I. & McAlister H.A., 1984, *PASP* 96, 105
- Hayes D. S. 1978, in: *IAU Simp. 80*, eds. A. G. Davis Philip, D. S. Hayes, p. 65
- Haywood M. 1993, *A&A*, 282, 444
- Heintz W. D., 1962, *Veröff. München* 5, 143
- Heintz W. D., 1969, *AJ* 74, 768
- Heintz W. D., 1972, *AJ* 77, 160
- Heintz W. D., 1974, *AJ* 79, 819
- Heintz W. D., 1975, *ApJS* 29, 315
- Heintz W. D., 1976, *ApJ* 208, 474
- Heintz W. D., 1978, *ApJ* 220, 931
- Heintz W. D., 1979, *AJ* 84, 1223
- Heintz W. D., 1980, *ApJS* 44, 111
- Heintz W. D., 1984a, *AJ* 89, 1063
- Heintz W. D., 1984b, *PASP* 96, 439
- Heintz W. D., 1984c, *A&ApS* 56, 5
- Heintz W. D., 1985, *Circ. Inf. UAI (comm. 26)* 96/97
- Heintz W. D., 1986a, *A&ApS* 64, 1
- Heintz W. D., 1986b, *AJ* 92, 446
- Heintz W. D., 1986c, *A&ApS* 65, 411
- Heintz W. D., 1987a, *AJ* 94, 1077
- Heintz W. D., 1987b, *PASP* 99, 1084
- Heintz W. D., 1988, *AJ* 96, 1072
- Heintz W. D., 1989a, *A&Ap* 211, 156
- Heintz W. D., 1989b, *A&Ap* 217, 145
- Heintz W. D., 1990a, *Observatory* 110, 131
- Heintz W. D., 1990b, *AJ* 99, 420
- Heintz W. D., 1991a, *A&ApS* 90, 311
- Heintz W. D., 1991b, *AJ* 101, 1071
- Heintz W. D., 1993a, *A&Ap* 277, 452
- Heintz W. D., 1993b, *AJ* 105, 1188
- Heintz W. D., 1993c, *PASP* 105, 44
- Heintz W. D., 1993d, *A&ApS* 98, 209
- Heintz W. D. & Borgman E.R., 1984, *AJ* 89, 1068
- Henry T. J. & McCarthy D. W. Jr., 1993, *AJ* 106, 773
- Henry T. J., Johnson D. S., McCarthy D. W. Jr., Kirkpatrick J. D., 1992a, *A&A* 254, 116
- Henry T. J., McCarthy D. W. Jr., Freeman J., 1992b, *AJ* 103, 1369
- Hershey J. L., 1972, *AJ* 77, 251
- Hershey J. L., 1973, *AJ* 78, 935
- Hershey J. L., 1982, *AJ* 87, 145
- Hershey J. L., Taff L.G., 1993, *BAAS* 25, 1426
- Hoxie D. T., 1970, *ApJ* 161, 1083
- Ianna P. A., 1979, *AJ* 84, 127
- Ianna P. A., Rohde J. R., McCarthy D. W. Jr., 1988, *AJ* 95, 1226
- Jahreiss H. 1993, private communication
- Jahreiss H. 1994, private communication
- Jarrett T. H., Dickman R. L., Herbst W., 1994, *ApJ* 424, 852

- Johnson H. L., 1965, *ApJ* 141, 170
 Johnson H. L., 1966, *ARA&A* 4, 193
 Johnson H. L. & Morgan W. W., 1951, *ApJ* 114, 522
 Joy A. H. & Abt H. A., 1974, *ApJS* 28, 1
 Kamper K.W., 1966, *AJ* 71, 389
 Kamper K.W., 1976, *PASP* 88, 444
 Kamper K. W. & Beardsley W. R., 1986, *AJ* 91, 419
 Kirkpatrick J. D. & McCarthy D. W. Jr., 1994, *AJ* 107, 333
 Kisselev A.A. & Kiyaveva O.V. *Ap&Space Sci.* 142, 181
 Kron G. E. & Mayall M. V., 1960, *AJ* 65, 581
 Kron G. E. & Smith J. L., 1951, *ApJ* 113, 324
 Kron G. E., Gascoigne S. C. B., White H. S., 1957, *AJ* 62, 205
 Kroupa P., Tout C. A., Gilmore G., 1990, *MNRAS* 244, 76
 Kroupa P., Tout C. A., Gilmore G., 1993, *MNRAS* 262, 545
 Kuiper G. P., 1943, *ApJ* 97, 275
 Lacy C. H., 1977, *ApJ* 218, 444
 Leung K.-C. & Shneider D. P., 1978, *AJ* 83, 618
 Lindsay V., Marcy G. W., Wilson K., Moore D., 1987, *BAAS* 19, 714
 Lippincott S. L., 1953, *AJ* 58, 135
 Lippincott S. L., 1955, *AJ* 60, 379
 Lippincott S. L., 1958, *AJ* 63, 314
 Lippincott S. L., 1960, *AJ* 65, 383
 Lippincott S. L., 1975, *AJ* 80, 831
 Lippincott S. L., 1982, *AJ* 87, 1237
 Lippincott S. L. & Borgman E. R., 1978, *PASP* 90, 226
 Lippincott S. L. & Hershey J. L., 1972, *AJ* 77, 679
 Lippincott S. L. & McDowall R.J., 1979, *PASP* 91, 471
 Lippincott S. L., Brawn D., McCarthy D. W. Jr., 1983, *PASP* 95, 271
 Luyten W. J., 1956, *PASP* 68, 258
 Malkov O. Yu., 1987, *Astrophysics* 26, 288
 Malkov O. Yu., 1989, *Nauchnye Informatsii* 67, 63
 Malkov O. Yu. 1990, in: *Proc. Internat. Workshop, Errors, Bias and Uncertainties in Astronomy*, eds. Jaschek C., Murtagh F., Strasbourg, Sept. 1989, Cambridge Univ. Press, p. 373
 Malkov O. Yu., 1993, *Bull. Inf. CDS* 42, 27
 Malkov O. Yu. & Piskunov A. E. 1988, *Astrophysics*, 29, 720
 Marcy G. W. & Benitz K.J., 1989, *ApJ* 344, 441
 Marcy G. W. & Moore D., 1989, *ApJ* 341, 961
 Mariotti J.-M., Perrier C., Duquennoy A., Duhoux P., 1990, *A&Ap* 230, 77
 Martin E. L., 1949, *Publ. Obs. Trieste* 231
 Martins D. H., 1975, *PASP* 87, 163
 McCarthy D. W. Jr., 1983, in: *IAU Coll. 76*, eds. A. G. D. Philip, A. R. Upgren, L. Davis Press, Schenectady, p. 107
 McCarthy D. W. Jr. & Henry T. J., 1987, *ApJL* 319, L93
 McCarthy D. W. Jr., Henry T. J., Fleming T. A., et al., 1988, *ApJ* 333, 943
 McCarthy D. W. Jr., Henry T. J., McLeod B., Christou J. C., 1991, *AJ* 101, 214
 Mendoza E. E., 1967, *Bol. Obs. Tonantzintla y Tacubaya* 4, 114
 Metcalfe T. S., Mathieu R. D., Latham D. W., Torres G., 1995, *Wisconsin Astrophysics preprint N 575*
 Morel P. J., 1969, *AJ* 74, 245
 Morrison D. & Simon Th., 1973, *ApJ* 186, 193
 Muller P., 1951, *Bull. Astron.* 16, 208
 Neece C. D., 1984, *ApJ* 277, 738
 Piskunov A. E. & Malkov O.Yu., 1991, *A&A* 247, 87
 Plaut L., 1953, *Pub. Kapteyn Astron. Labor. Groningen* 55, 1
 Popper D. M., 1980, *ARA&Ap* 18, 115
 Probst R. G., 1977, *AJ* 82, 656
 Protitch M. B., 1955, *Bull. Obs. Belgrade* 19, 11
 Reid N. & Gilmore G., 1984, *MNRAS* 206, 19
 Robin A. & Cr ez  M., 1986, *A&A* 157, 71
 Rodgers A. W. & Eggen O. J., 1974, *PASP* 86, 742
 Rodono M., 1978, *A&A* 66, 175
 Russell J. & Gatewood G., 1980, *AJ* 85, 1270
 Salpeter E. E., 1955, *ApJ* 121, 161
 Scalo J. M., 1986, *Fund. Cosm. Phys.* 11, 1
 Sienkiewicz R., 1982, *Acta Astron.* 32, 275
 Strai yys V. L., 1977, *Stellar multicolor photometry*, Vilnius, p. 19
 Strand K. Aa., 1969, *AJ* 74, 760
 Stringfellow G. S. & Bessell M. S., 1994, *ARAA* 31, 433
 Tinney C. G., Reid I. N., Mould J. R., 1993, *ApJ* 414, 254
 Tift W.G., 1955, *AJ* 60, 144
 Tokovinin A. A., 1994, *Pis'ma Astron. Zh.* 20, 368
 Tokovinin A. A., 1996, private communication
 Traat P. A., 1976, *Publ. Tart. astrophys. obs.* 44, 282
 Traat P. A., 1990, Ph. D. Thesis, Tartu
 Upgren A. R. & Mesrobian W. S., 1971, *AJ* 76, 78
 Vaiana G. S., Cassinelli J. P., Fabbiano G., et al., 1981, *ApJ* 245, 163
 van Altena W.F., Lee J.T., Hoffleit D., 1991, *The General Catalogue of Trigonometric Stellar Parallaxes, Preliminary Version*, Yale University Observatory
 van Biesbroeck G., 1947, *AJ* 53, 23
 van Biesbroeck G., 1949, *AJ* 54, 163
 van de Kamp P., 1938, *AJ* 47, 1
 van de Kamp P., 1959, *AJ* 64, 236
 van de Kamp P., 1971, *ARA&Ap* 9, 107
 van de Kamp P. & Flather E., 1955, *AJ* 60, 448
 van de Kamp P. & Worth M.D., 1971, *AJ* 76, 1129
 van de Kamp P., G kkaya N.G., Heintz W.D., 1968, *AJ* 73, 361
 van den Bos W.H., 1959, *ApJS* 4, 45, 1959
 Vandenberg D. A., Hartwick F. D. A., Dawson P., Alexander D. R., 1983, *ApJ* 266, 747
 Veeder G. J., 1974, *AJ* 79, 1056
 Wallenquist A., 1981, *Nova Acta Reg. Soc. Scien. Upsaliensis Ser. VA*, 4
 Wanner J. F., 1969, *AJ* 74, 229
 Weis E. W., 1982, *AJ* 87, 152
 Weis E. W., 1993, *AJ* 105, 1962
 Wielen R., Jahreiss H., Kr ger R., 1983, in: *IAU Coll.76*, eds. A.G.D.Philip, A.R.Upgren, L.Davis Press, Schenectady, p. 163
 Wierzbinski St., 1958, *Acta Astr.* 8, 183
 Wieth-Knudsen N., 1953, *Lund. Obs. Ann.* 12, A39
 Wilson R.H. Jr., 1954, *AJ* 59, 256
 Wooley R., Epps E. A., Penston M. J., Poccock S. B., 1970, *Roy. Obs. Ann.* 5
 Worley C. E., 1969, *AJ* 74, 764
 Worley C. E. & Behall A.L., 1973, *AJ* 78, 650
 Worley C. E. & Heintz W. D., 1983, *Fourth Catalog of Orbits of Visual Binary Stars*, Pub. of the USNO, vol. 24, part 3
 Young A., Sadjadi S., Harlan E., 1987, *ApJ* 314, 272
 Zimmermann G., 1939, *Astr. Nachr.* 268, 157