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Research Note

Analytical fits to theoretical spectra of X-ray burst sources: a hint on the surface gravity

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Abstract. Simple analytic formulae have been developed to fit the existing grid of X-ray spectra of very hot neutron star atmospheres. Model atmospheres analysed here were computed in both the radiative and hydrostatic equilibrium, and included precisely the effects of relativistic Compton scattering, The grid of fitted models covers the range of $T_{\rm eff}$ from 8×10^6 K up to 3×10^7 K, and $\log g = 15.0$ down to the Eddington limit, but was restricted to fully ionized hydrogen/helium gas with the He number abundance $N_{\rm He}/N_{\rm H} = 0.11$. Both $T_{\rm eff}$ and $\log g$ in the above models are defined on the surface of a neutron star.

Fitting parameters given in the paper define the relation $T_{\rm c} - T_{\rm eff}$ between the observed color temperature of a general X-ray burst and the effective temperature, and depend also on the surface gravity $\log g$ and the general relativistic redshift. Results of this paper can be used for the estimates of both $T_{\rm eff}$ and $\log g$ directly from the observed spectra of X-ray bursts, as long as the composition of accreted matter is restricted to H and He alone, and the redshift correction is negligible or at least can be guessed.

Key words: X-rays: general – stars: neutron – X-rays: bursts

1. Introduction

X-ray burst sources form a subclass of low-mass X-ray binary systems, in which accretion of gas from a secondary star onto the neutron star causes both persistent emission and recurrent bursts of X-rays. Most of the objects are Type I X-ray bursters, in which recurrent thermonuclear flashes develop in the neutron star envelope and raise the effective temperature $T_{\rm eff}$ above $2 - 3 \times 10^7$ K, which is the cause of eruptive X-ray emission. Observational properties and the nature of these phenomena have been discussed in extensive review articles (Lewin & Joss 1981, 1983; Joss & Rappaport 1984; van Paradijs & Lewin 1988; Lewin, van Paradijs & Taam 1993).

X-ray burst spectra contain some information concerning basic parameters of the event. If we assume, that the X-ray emission is generated in a very hot neutron star atmosphere, then the analysis of a burst spectrum should yield values of the effective temperature $T_{\rm eff}$ and surface gravity in the stellar photosphere $\log g$, with perhaps some hints concerning the chemical composition of matter. The analysis of model atmospheres was extensively described in Mihalas (1978). At the $T_{\rm eff}$ exceeding several MK and photon energies relevant to X-rays, all the classical equations and techniques presented there have to be enhanced by terms describing Compton scattering of photons by a very hot free electron gas (cf. Pomraning 1973; Rybicki & Lightman 1979). Model atmosphere computations including Compton scattering, and the relevant spectral analysis are very complex tasks (eg. London et al. 1986; Ebisuzaki 1987; Babul & Paczyński 1987; Madej 1989, 1991; Titarchuk 1994).

Due to the very limited spectral resolution of archival X-ray burst observations, which is of order 25 %, an interpretation of observational data requires fitting of the observed counts by predicted (or just assumed) theoretical spectra expressed by simple and compact formulae. At present, observational data are routinely fitted by a blackbody spectrum, which yields the observed color temperature $T_{\rm c}$. There exist several methods for the determination of the effective temperature $T_{\rm eff}$ of a burst at infinity, if the $T_{\rm c}$ is known, which were recently reviewed by Lewin et al. (1993). Unfortunately, there exists no useful relation which determines gravity.

In the present paper we attempt to revise previous $T_{\rm c} - T_{\rm eff}$ scales, and to give a method for the estimate of $\log g$ in the photosphere of a neutron star at various phases of a burst. Let us temporarily ignore general relativistic effects (redshift factor), then all theoretical spectra, $T_{\rm eff}$, and $\log g$, can be set to the values observed at infinity. The following considerations are restricted to the arbitrary case in which X-ray emitting gas is a mixture of hydrogen and helium alone.

2. Source grid of theoretical spectra

In this paper we analyse the set of model atmospheres of hot neutron stars, which was described and published in Madej (1991),

Table 1. Listing of source model atmospheres

$T_{\rm eff}$	8×10^6	1.257×10^7	2×10^7	3×10^7
$\log g$	15.0	15.0	15.0	15.0
	14.5	14.5	14.5	14.8
	14.0	14.0	14.2	14.7
	13.5	13.5	14.1	
	13.0	13.3	14.0	
	12.5	13.2		
$^a \log g_{cr}$	12.396	13.177	13.964	14.643

^{a} Critical gravities taken from Paper I (cgs units)

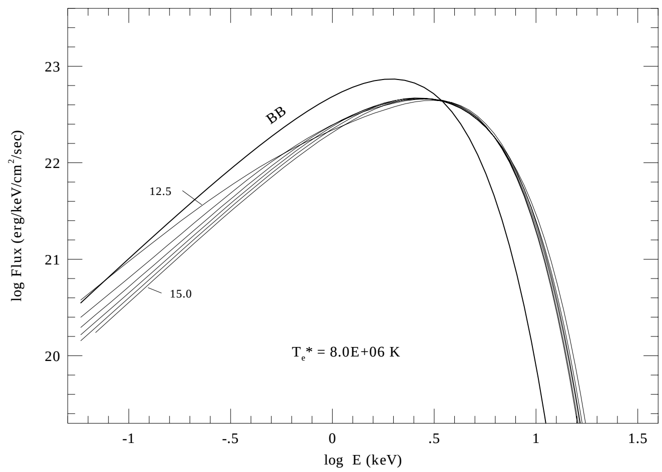

Fig. 1. X-ray spectra of hydrogen/helium models with $T_{\rm eff} = 8 \times 10^6$ K $= 0.689$ keV and various surface gravities. Solid curve (BB) denotes blackbody spectrum of the $T_{\rm eff}$. Spectra of model atmospheres evolve with changing $\log g$, which is mostly pronounced in low energy tail. Decrease of gravity causes increase of soft X-ray flux, and slight flattening around the peak flux

hereafter Paper I. All the models were computed assuming plane-parallel geometry, hydrostatic and radiative equilibrium and an equation of state for an ideal gas consisting of perfectly ionized hydrogen and helium of the solar number abundance, $N_{\rm He}/N_{\rm H} = 0.11$. No heavier elements were included in computations. A total of 20 model atmospheres were computed, and their parameters ($T_{\rm eff}$ and surface $\log g$) are listed in Table 1 (cgs units).

Table 1 lists also logarithms of the critical gravities, at which radiation pressure gradient precisely compensates the gradient of gas pressure at some level in H/He atmosphere, thus limiting $\log g$ of hydrostatic models. Values of $\log g_{cr}$ were obtained by extrapolation of non-grey hydrostatic models.

The models in Paper I included very careful treatment of Compton scattering, which is the dominant source of opacity in almost all X-ray bursting neutron stars. The presence of very strong scattering terms in the source functions generally causes deviations from the Planckian shape and introduces non-local coupling in the atmosphere, typical to NLTE models (Mihalas 1978). Both the equation of transfer and the equation of radiative equilibrium included Compton scattering frequency redistribution profiles (Pomraning 1973), which allow us to trace in detail the transfer of photons in frequency space also in cases that the photon energy is *not* much less than the electron rest mass, $mc^2 = 511$ keV. In fact, the hottest models in Table 1 ($T_{\rm eff} = 3 \times 10^7$) were computed with the mesh of discrete photon energies exceeding 140 keV. Such computations cannot be accurately done with a simplified Kompaneets equation, which is valid in the range $h\nu \ll mc^2$ (Rybicki & Lightman 1979). Moreover, the Kompaneets equation ignores the finite width of the Compton scattering profile and its asymmetry.

Figs. 1 and 2 display some of the X-ray spectra presented in Paper I, computed for extreme values of $T_{\rm eff}$, together with the Planck function corresponding to that $T_{\rm eff}$. It is evident, that all theoretical spectra are significantly harder, than the blackbody curve. Moreover, the theoretical X-ray spectra clearly exhibit a dependence on the surface gravity $\log g$ of a neutron star. In case of the coolest models at $T_{\rm eff} = 8 \times 10^6$ K (Fig. 1), a decrease of the surface gravity $\log g$ from 15.0 down to 12.5 (cgs units) causes distinct rise of the low-energy branch of X-ray spectra and a slight decrease around flux maximum (both effects were also discussed by Lewin et al. 1993). For the hottest models ($T_{\rm eff} = 3 \times 10^7$ K) the most significant changes occur in a wide region around the peak in the flux distribution, with a $\log g$ dependence that is of opposite sign compared to the $T_{\rm eff} = 8 \times 10^6$ K models. The models in Paper I are quite accurate numerically, therefore it is worthwhile to seek for a simplified analytical representation of both gravity effects, for the subsequent fitting of observed spectra of X-ray burst sources.

3. Fitting of theoretical X-ray burst spectra

The theoretical X-ray spectra for hot neutron star atmospheres (see table 1A – 1D in Paper I) can be quite accurately fitted by the expression

$$F_\nu = C_1 \, \nu^{\, C_2} \left[\exp\left(\frac{h\nu}{kT_{\rm c}} - \mu \right) - 1 \right]^{-1} , \qquad (1)$$

which is a modification of the Planck function with non-zero chemical potential μ and constant C_2 generally not equal to 3. Here F_ν denotes the flux measured in energy units (erg/cm^{2}sec Hz), and ν denotes frequency in Hz. Eq. (1) depends on 4 free parameters which have to be determined, C_1, C_2, $T_{\rm c}$, and μ. One of them, $T_{\rm c}$, is the color temperature of an X-ray burst. One should be aware, that $T_{\rm c}$ determined from Eq. (1) can differ slightly from color temperatures obtained from the standard blackbody fits, as made in many earlier studies.

Eq. (1) can be expressed in logarithmic form,

$$\log F_\nu = C_1 + C_2 \log \nu - \log \left[\exp\left(\frac{h\nu}{kT_{\rm c}} - \mu \right) - 1 \right] \qquad (2)$$

which is much more useful in the further fitting (decimal logarithms). Eq. (2) depends non-linearly on two parameters, $T_{\rm c}$

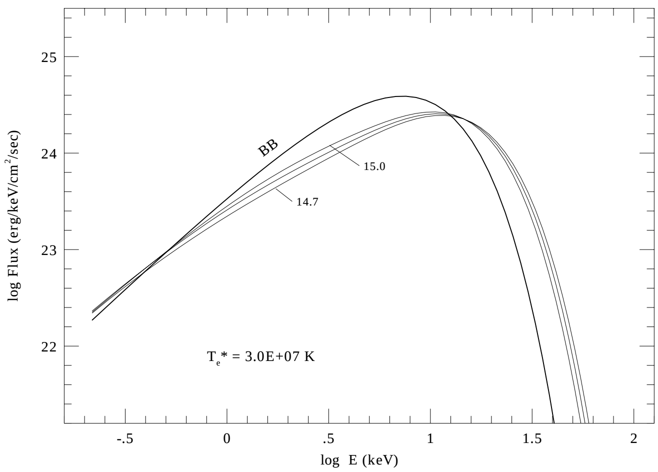

Fig. 2. Same for models with $T_{\rm eff} = 3 \times 10^7$ K = 2.585 keV. Here the evolution of spectra with $\log g$ is qualitatively different than in Fig. 1

and μ. Non-linear fitting procedures did not yield convergence in case of that formula. Therefore, for each X-ray spectrum given in tabular form (Paper I) a dense discrete mesh of $T_{\rm c}$ and μ has been defined within reasonable limits. Then, for each fixed pair of these parameters, the remaining C_1 and C_2 were determined with the usual linear least squares techniques, minimizing values of χ^2 in which all (unknown) errors of theoretical fluxes were arbitrarily set to unity, $\sigma = 1$.

Numerical FORTRAN procedures for linear fitting were directly taken from Press et al. (1986).

4. Recommended fitting coefficients

The resulting 20 sets of all 4 fit parameters were grouped in order to establish useful relations involving two parameters of a bursting star: $T_{\rm eff}$, and $\log g$, vs. observables. Perhaps the best method is defined by the following steps.

1. For each $T_{\rm eff}$, values of $T_{\rm c}$ corresponding to all listed $\log g$ were averaged to establish a unique relation $T_{\rm c} - T_{\rm eff}$, independent of gravity.
2. Parameters C_2 were averaged in all analysed spectra, which yielded the average value of $\overline{C_2} = 2.710 \pm 0.053$ (standard deviation).

Unfortunately, normalization factors C_1 are much less useful for averaging purposes. They exhibited significant scatter due to numerical roundoff errors in the fitting procedure, and will not be used in the following sections.

Table 2 gives four averaged values of $T_{\rm c}$, corresponding to each considered $T_{\rm eff}$. The averaged spectral hardening ratio, $\bar{t} = \overline{T_{\rm c}/T_{\rm eff}} = 1.547 \pm 0.023$. Such a value is fully consistent with earlier estimates, $t \approx 1.5$ (cf. Lewin et al. 1993). Note, however, that $T_{\rm c}$'s discussed in this paper were determined form fits, which are not strictly blackbodies.

Now the whole fitting procedure has been repeated for each model atmosphere, but with fixed $C_2 = 2.710$ and values of $T_{\rm c}$ given in Table 2. This has given the values of chemical potential μ, which are also listed in Table 2.

It is interesting to note, that the parameter μ is a function of surface gravity $\log g$ in models of each considered $T_{\rm eff}$. Only in some range of model parameters, determination of chemical potential μ does not provide unique determination of $\log g$ (e.g. at $T_{\rm eff} = 2 \times 10^7$ K).

5. Fitting of observed spectra

Let us temporarily ignore the existence of gravitational reddening. In such a case one can directly compare theoretical spectra discussed above to the observed spectra of X-ray burst sources.

Fitting coefficients C_1, computed here, are useless for the estimate of R/d, the radius to distance ratio. This important parameter can be estimated according to the usual blackbody fitting procedure (Lewin et al. 1993). Consequently, values of fitted C_1 are not essential in the following procedure, which does not include R/d determination.

An observed X-ray spectrum (expressed in erg/cm^{2}sec Hz) should be fitted by the Eq. (2) with the parameter C_2 set to 2.710. Then, the observed $T_{\rm c}$ can be either interpolated between values from Table 2 to find some estimate of $T_{\rm eff}$, or simply taken as $T_{\rm eff} = T_{\rm c}/1.547$. Both best fitted $T_{\rm c}$ and μ can be used for the surface gravity determination, by two dimensional interpolation between values given in Table 2.

The value of gravity, $\log g$, is measured at the photosphere of the X-ray burster, which does not necessarily coincide with the surface of the neutron star itself. In principle the above fitting offers a chance for tracing $\log g$ variations during a burst, and tracing of radius expansion or contraction.

Such a procedure can fail if matter emitting X-rays contains some amounts of heavy elements, like iron. In such case theoretical spectra of X-ray bursts must exhibit some discrete features of highly ionized ions, like Fe^{+24} and Fe^{+25}. Fitting parameters presented in this paper get irrelevant in such a situation. Also in case, when $\log g$ just approaches $\log g_{cr}$, values of $T_{\rm c}$ (or rather factors t) in Table 2 are too low, since t approaches 2 when $\log g \to \log g_{cr}$ (Babul & Paczyński 1987).

6. Nonzero gravitational redshift

Photons emitted from the surface of a neutron star lose part of their energy and change the rate at which they appear, while escaping to infinity. This is a general relativistic effect which is important in the extremely strong gravitational potential near neutron star. In case of nonrotating neutron star, all the corrections to observables are given by simple scalar factors (Thorne 1977).

The emerging spectrum of a star is squezeed along frequency axis, when measured at infinity (near the Earth). All photons decrease their initial frequency ν^* to ν, and

$$\nu = \nu^* \, (1+z_*)^{-1} = \nu^* \, \left(1 - \frac{2GM}{c^2 R}\right)^{+1/2} \, , \qquad (3)$$

Table 2. Recommended fitting parameters. The average values $\overline{C_2} = 2.710 \pm 0.053$, and $\overline{T_c^*/T_{\rm eff}^*} = 1.547 \pm 0.023$ (standard deviation).

$T_{\rm eff}$	8×10^6		1.257×10^7		2×10^7		3×10^7	
T_c^*	1.263×10^7		1.944×10^7		3.069×10^7		4.583×10^7	
$T_c^*/T_{\rm eff}^*$	1.579		1.547		1.535		1.528	
$\log g^*$	15.0	−0.205	15.0	−0.068	15.0	0.006	15.0	0.027
μ	14.5	−0.116	14.5	−0.030	14.5	0.021	14.8	0.023
	14.0	−0.068	14.0	−0.003	14.2	0.024	14.7	0.012
	13.5	−0.035	13.5	0.017	14.1	0.023		
	13.0	−0.007	13.3	0.019	14.0	0.011		
	12.5	0.018	13.2	0.013				
$\log g_{cr}^*$	12.396		13.177		13.964		14.643	

Asterisks indicate, that the parameters are measured at the photosphere of a neutron star

where ν is measured by a distant observer, and R is the actual radius of the photosphere measured locally.

Consider an arbitrary phase of a burst. If the redshift factor is guessed and the observed energy spectrum has been corrected for that factor, then the fitting procedure yields directly estimates of $T_{\rm eff}^*$ and $\log g^*$ in the photosphere at that phase. Both $T_{\rm eff}^*$ and g^* yield mass to luminosity ratio, M/L^*. In case if L is known (e.g. distance known) one may try to iterate and get momentary values of g^* and $(1 + z_*)$ consistent.

Lewin et al. (1993) review also independent methods, which can be used for the estimates of $(1 + z_*)$.

7. Conclusions

Hydrogen/helium model atmospheres of X-ray burst sources predict either the rise or decrease (at $T_{\rm eff} = 3 \times 10^7$ K) of a low-energy branch of the spectrum, in case when surface gravity $\log g^*$ decreases down to the critical gravity $\log g_{cr}^*$ (cf. Paper I). Simultaneously, there appears a slight flattening of the spectrum around the peak. Therefore such deviations of X-ray spectrum from a blackbody shape are signatures of gravity, $\log g^*$, in the photosphere.

Tabulated X-ray spectra are approximated by a new formula, Eq. (2), which involves two parameters (T_c and μ), which depend on the effective temperature $T_{\rm eff}^*$, and $\log g^*$. Both can be used for the fitting of the observed counts, and the temperature and gravity estimation. Numerical results are valid in case, when the atmosphere of a bursting neutron star consists of hydrogen and helium, and no heavy metals are present.

The referee pointed out, that the largest deviations of theoretical spectra from a blackbody shape occur at low energies, where the effects of interstellar absorption are most important. This is a very troublesome fact at the lowest temperatures of bursts (see Fig. 1, where all deviations from blackbody occur below 1 keV). However, with rising $T_{\rm eff}$ of a burst, predicted differences quickly shift above 1 keV, where interstellar extinction decreases (see Fig. 2, case of the extreme $T_{\rm eff}$). Therefore at higher $T_{\rm eff}$ determination of $\log g^*$ gets more reliable.

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