Astron. Astrophys. 320, 319–324 (1997)

ASTRONOMY
AND
ASTROPHYSICS

Simulation of collisions in planetary rings

Jan-Martin Hertzsch[1], **Hans Scholl**[2], **Frank Spahn**[1], **and Ines Katzorke**[1]

[1] Arbeitsgruppe „Nichtlineare Dynamik", Universität Potsdam, Am Neuen Palais, PF 60 15 53, D-14415 Potsdam, Germany
[2] Observatoire de la Côte d'Azur, Dept. C. G. Cassini, B.P. 229, F-06304 Nice, France

Received 20 June 1996 / Accepted 25 September 1996

Abstract. We investigate the evolution of structures in planetary rings in the neighbourhood of the orbit of an embedded moonlet (small satellite). The effects of collisions have been taken into account by introducing the velocity dependent restitution coefficient according to experimental and theoretical results. Here we present results of recent many-particle simulations which show a significant influence of the interparticle collisions on the formation and persistence of wake structures in a model ring. In contrary to the collision-free model, a stationary, non-mirrorsymmetric wake pattern with decreasing intensity in azimuthal direction is observed. The azimuthal damping rate of the wake intensity is estimated from the data obtained in the simulation.

Key words: planets and satellites: general; Saturn

1. Introduction

Collisions between particles in granular systems play an important role for the formation of dynamical structures and their persistence in time. A famous example are planetary rings, notably those of Saturn, where the Voyager spaceprobe observations revealed a wealth of interesting features. We took particular interest in the structures which showed in the Encke division where the satellite Pan (1981 S 13) is orbiting (Cuzzi & Scargle 1985, Showalter 1991). The gravitative influence of this moon causes three main features: a faint ringlet following the orbit of the satellite and spreading only over the immediate vicinity of this orbit (Spahn et al. 1992, Spahn et al. 1994), broad gaps flanking the ringlet, and wave-like structures, called wakes (Showalter et al. 1986), beyond the edges of the gaps.

Extending former work on the kinematics in planetary rings (Spahn et al. 1994) we have considered inelastic collisions in our numerical experiments in order to investigate their influence on the moon-induced structures mentioned above.

The model for the particle motion is introduced in Sect. 2. There we describe also the treatment of collisions between the particles. The results of the simulations are presented in Sect. 3. In Sect. 4 the damping rate of the wake intensity is estimated from the numerically obtained data, and a simple analytical explanation is proposed. Finally, we suggest a possibility to derive the damping rate from observational material.

2. Numerical modelling of the particle motion and treatment of collisions

We have simulated the evolution of a model ensemble of 2^{19} = 524288 particles of equal size on the Connection Machine CM-200 in Sophia-Antipolis. The computations are based on the solution of the elliptic three-body problem (Spahn et al. 1994). This means that we consider only the gravitational influence of the planet and the satellite (primaries) on each other and on the ring particles, but neither the influence of the latter on the primaries nor the self-gravity of the ring.

The mass ratio moonlet/planet was chosen to $M = M_m/M_p = 10^{-6}$ in order to provide a sufficiently rapid evolution of the system. This is a mere scale procedure so that the results can be mapped onto realistic circumstances (Hill 1878, Szebehely 1967, Petit & Hénon 1986). Initially, the particles were distributed homogeneously on a ring extending radially between $-0.1h^*a$ and $+0.1h^*a$ from the circular orbit of the moon. The orbital radius of the satellite is a, and $h^* \approx \left(\frac{M}{3}\right)^{1/3}$ is the radius of the Hill sphere of the moon, i.e. the region where its gravitational influence dominates.

The number of particles in our simulations is much higher than in other numerical investigations. This was required by the aim to resolve sufficiently well the structures of interest which extend over the whole range of azimuth. On the other hand, since we were mainly interested in the development of radial and azimuthal structures, we have assumed a monolayer of equally-sized particles. Other simulations did include a size distribution and inclined orbits (Salo 1992), or took into account the gravitational action of the smaller particles on the larger body (Ida & Makino 1992), but were performed with much lower particle numbers.

We have followed the evolution of the structures of a collision-dominated particle ensemble for 60 orbital periods of

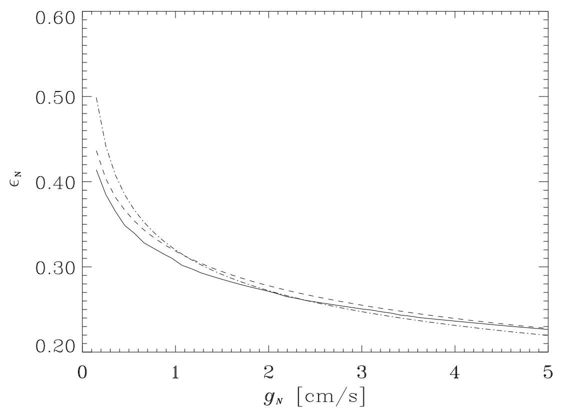

Fig. 1. Dependence of the normal restitution coefficient on the normal component of the impact velocity. Dash-dotted: experimental results (Bridges et al. 1984). Solid: theoretical results (Hertzsch et al. 1995). Dashed : theoretical results using a simplified model (Spahn et al. 1995).

the moon and compare it with the behaviour of a system without interactions between the ring particles.

For the treatment of the collisions we relate the relative velocities of two particles in normal and tangential direction after the collision to those before ($\boldsymbol{g}_N$ and $\boldsymbol{g}_T$, respectively) with the help of the restitution coefficients ϵ_N and ϵ_T :

$$\begin{aligned} \boldsymbol{g}'_N &= -\epsilon_N \cdot \boldsymbol{g}_N \qquad (0 \leq \epsilon_N \leq 1) \\ \boldsymbol{g}'_T &= \epsilon_T \cdot \boldsymbol{g}_T \qquad (-1 \leq \epsilon_T \leq 1) \end{aligned} \tag{1}$$

For icy particles, ϵ_N has been measured in the laboratory (Bridges et al. 1984). In order to find a theoretical explanation for the measured dependency $\epsilon_N(g_N)$, Hertz's contact theory (Hertz 1881) has been extended (Hertzsch et al. 1995, Brilliantov et al. 1996) towards dissipation. The experimentally obtained dependency $\epsilon_N = 0.32 \cdot g_N^{-0.234}$ (see Fig. 1) has been used in our simulations while we have kept $\epsilon_T = 1$ as first simple step since there is up to now no sufficient experimental evidence for the dependence $\epsilon_T(g_N, g_T)$.

The number of particles in our simulations is still too low to provide a sufficient collision frequency for realistic particle sizes in the order of centimeters. On the other hand, a higher number of particles would have required an impractical long computation time. Therefore, we increased the collision probability by enlarging the active cross section of the particles so that in the average each particle will undergo one collision per orbital period. This leads to an effective optical depth of $\tau_{eff} \approx 0.1$ (Shu et al. 1985). The rapidity of the evolution of a system of colliding particles in a gravitational field is proportional to the number of particles time the squared ratio of enlarged vs. original radii (Trulsen 1971). In the case of a monolayer, this means that the speed of evolution is proportional to the optical depth of the system. Since $\tau \approx 0.4$ at the borders of the Encke division, the evolution of our model system is slowed down again by a factor of $1/4$.

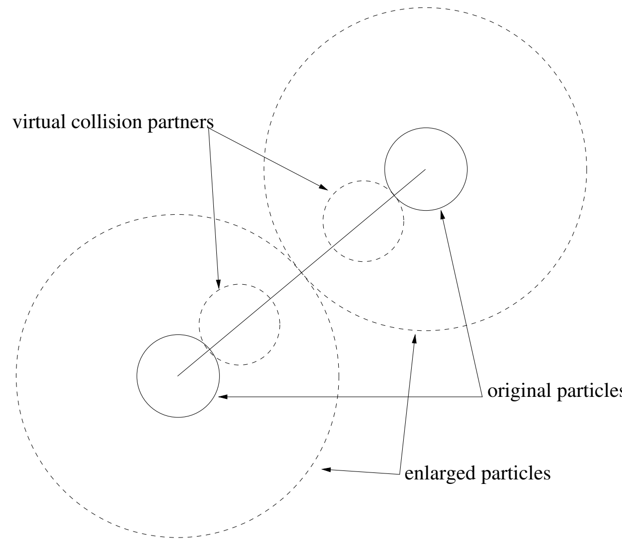

Fig. 2. The concept of "virtual particles": the radii of the particles are artificially enlarged in order to enhance the collision probability. After the detection of a collision, the simulation continues as if each particle had collided with another of the original size, but also at a distance which corresponds to the sum of the radii of the original and the "virtual" particles.

For the further treatment of the collisions, we follow the concept of "virtual particles" (see Fig. 2). Once a collision is detected, e.g. between particles A and B, then we assume that A collides with a a virtual particle B', and B with a virtual particle A'. Those virtual particles have the original (realistic) radii. The formulae for the restitution coefficient are applied in the following way: The virtual particle B' has the velocity of particle B, its position in the moment of the collision is on the line connecting the centres of particles A and B. The distance between A and B' is equal to the sum of the radii of the (original) particles A and B. This allows the application of the dependence of ϵ_N on the size of the particles in further numerical experiments.

However, due to the enlarged active cross sections the relative velocity of the particles is significantly enhanced due to the larger inter-particle radial distance. According to the relation $\epsilon_N(g_N)$ this leads to a higher energy dissipation rate and consequently to a more effective damping of the deviations of the particles from their circular orbits, which are induced by the passing moon and are responsible for the formation of the wake structures (Showalter et al. 1986, Spahn et al. 1994). In addition, due to the enlarged particles the nonlocal component of the transport processes is increased (Araki & Tremaine 1986). This causes problems in a scaled comparison with realistic systems.

3. Influence of the collisions

A first inspection of the density plots (Figs. 3 and 4) shows that all structures – ringlet, gaps, and wakes – which have been found in the collision–free case (Spahn et al. 1994) are also formed in

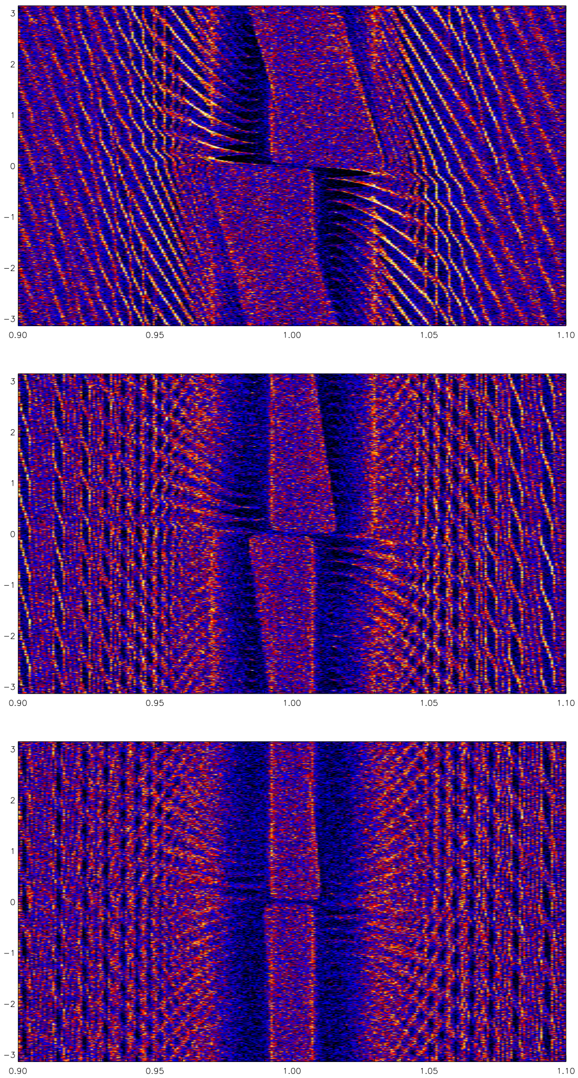

Fig. 3. Snapshots of the evolution of a gap forming under the influence of an embedded moon in an initially homogeneous planetary ring after 20 (top), 40 (middle), and 60 (bottom) orbital periods of the moon. Collisions are neglected. The moon is at (0,0). Dark: low density. Light: high density. Abscissa: impact parameter. Ordinate: azimuth.

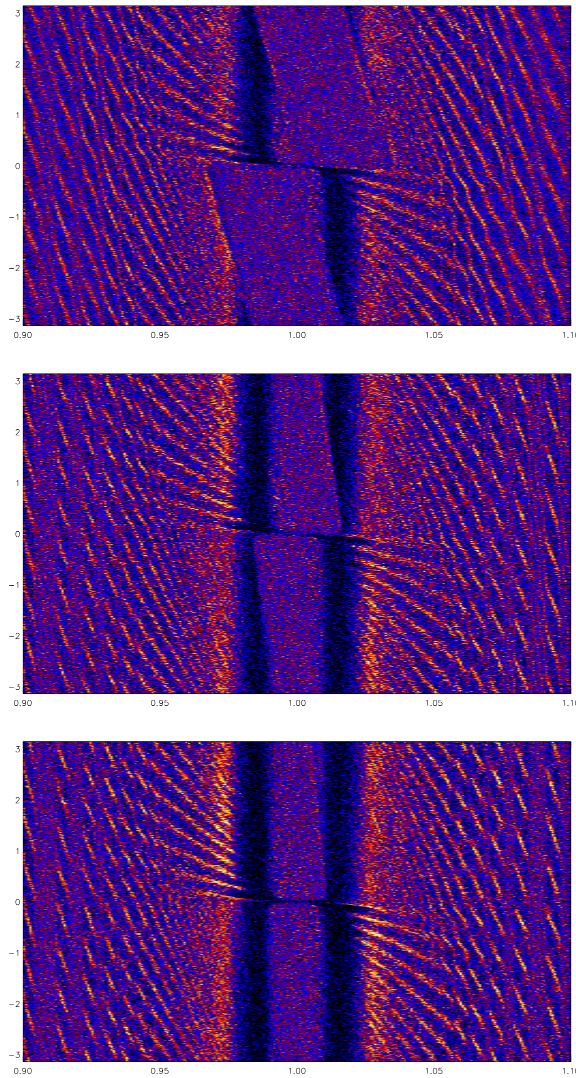

Fig. 4. The same as in Fig. 3, but inelastic collisions between the particles are taken into account: $\epsilon_N(g_N)$ according to Bridges et al. 1984, $\epsilon_T = 1$.

the case of collisions. However, there are several significant differences.

The most prominent one is that the formation of the forward-backward symmetry in the wake structures (superposition of successive wake generations in the collision-free case) is hindered by damping effects due to the inelasticity of the collisions. This difference is only the more important if one keeps in mind that for very long times of integration "phase mixing" comes into play which causes the structures to disappear altogether (Spahn et al. 1994). In this case, only resonant features will remain visible. Dissipation, on the other hand, leads to a damping of the wakes. If the damping is strong enough, the first wake generation at a certain point will be suppressed so much that its contribution to the superposition with the following generations is negligible. More rigorously, only the most recent wake generation will contribute considerably to the visible pattern which is, of course, an unsymmetric one (with respect to time). This stationary asymmetry accounts for a permanent torque between ring and moon and corresponds to the observed structures (Greenberg 1983). Therefore one can conclude that it is the energy dissipation in the system which causes the visibility of the wake structures.

The density in the gaps is considerably lower than in the collisionless (kinematic) case. In particular, the edges of the wake structures which continue into the gaps in the kinematic case have disappeared. Apparently, the inelastic character of the collisions enforces the depletion of these regions. Nevertheless, the gaps appear to be even narrower and the ringlet even larger than in the collisionless case. In reality, the gaps are wider, and the ringlet should be very faint. This discrepancy must be

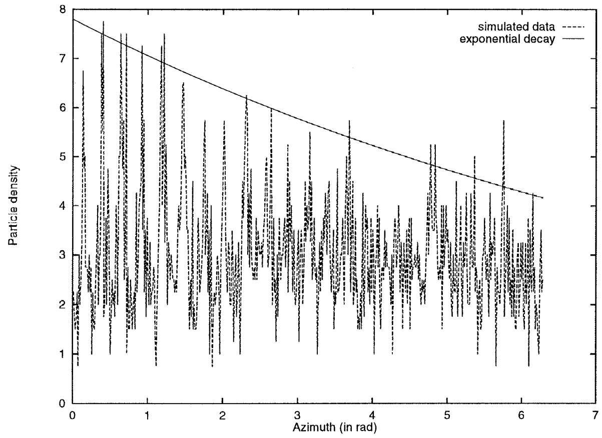

Fig. 5. Azimuthal section of the simulated ring in Fig. 4 near the edges of the gap showing the decay of the wake intensity after the passage of the moon. The strong fluctuations are due to the comparatively low number density of the particles. In order to reduce them, density values of points which are situated axisymmetric with respect to the moon have been summed. The solid line corresponds to a exponential decay of the density maxima according to $I = 7.8 \exp(0.1 \cdot \phi)$.

attributed to the neglect of some of the gravitational interactions in the system ring – satellite (Petit & Henon 1988).

A count of the particles after the end of the corresponding computer runs shows a decrease of the number of the particles in the latter case which must be attributed to the energy loss of particles during the inelastic collisions which causes a net inward migration of the particles. In addition, also in the collisionless case some particles are "lost" which fall onto the surface of the moonlet. Therefore, for the stability of the structures over very long time scales – even for a low collision probability – a mechanism is required which compensates the particle loss.

4. Estimation of the damping rate of the wakes

For an estimation of the damping rate of the particle density from our simulations we have examined sections of the computed density field in azimuthal direction. In Fig. 5 an azimuthal section at a distance of about $5 \cdot h^*$ from the moonlet's orbit is plotted. It shows the decay of the wake intensity with rising azimuthal distance from the moon. This plot illustrates also the main problem we have to deal with in our simulations: the comparatively low particle number causes considerable fluctuations of the density. Therefore, the estimation of the wake damping rate must be considered as an approximate one. In order to reduce the influence of the fluctuations on our results, we have summed the density values at locations point-symmetric with respect to each other (in an azimuth–impact-parameter coordinate system centered at the moon).

The decay of the intensity of the wake maxima follows approximately an exponential law $I(\phi) = I_0 \exp\left(D_{wake} \cdot \phi\right)$ where we denote the intensity with I and the azimuth with ϕ. The constants I_0 and D_{wake} are the (hypothetical) initial intensity which would be caused by the moonlet's action in the impulse approximation and the wake damping coefficient, respectively. We can derive from the ratio of the density values of subsequent wake maxima at the corresponding distance the values $I_0 \approx 7.8$ and $D_{wake} \approx 0.1\ \mathrm{rad}^{-1}$.

This exponential damping can be roughly explained by the action of a collective dissipative force on the ring particles caused by the inelastic collisions. As a first approximation, we assume this force F_{dis} to be proportional to the relative velocity c of the particles: $F_{dis} = \gamma \cdot c$. Here γ denotes the (hypothetical) internal friction coefficient of the "granular gas" which the ring consists of. The relative velocity of particles (corresponding to interacting streamlines) is roughly in the order of $c \approx \frac{2ae}{T}$. e is the eccentricity induced by the moon after the point of streamline crossing (Showalter et al. 1986, Borderies et al. 1982, Borderies et al. 1989). The decay of this induced eccentricity e with time (caused by F_{dis}) can be expressed by the Gauß equation (Burns 1977):

$$\frac{\mathrm{d}e}{\mathrm{d}t} = \sqrt{\frac{a(1-e^{2)}}{\mu}} \cdot \gamma \cdot \frac{2ae}{T} \tag{2}$$

In all cases of interest (embedded moons in planetary rings) holds $e \ll 1$. Omitting terms of higher than first order in e, the solution of this equation is found to

$$e(t) = e(0) \exp\left(-\sqrt{\frac{a}{\mu}} \cdot \gamma \cdot \frac{2a}{T} \cdot t\right) . \tag{3}$$

Since the azimuth of a particle with respect to the moonlet is proportional to time:

$$\phi = \frac{\Omega_s}{m}\, t \tag{4}$$

where Ω_s is the orbital period of the moon and m the inverse relative angular velocity of the particle with respect to the moon (see e. g. Spahn et al. 1994), we can also formulate an exponential dependence of e on ϕ:

$$e(\phi) = e(0) \exp\left(-D_{ecc} \cdot \phi\right) \tag{5}$$

with $D_{ecc} = \frac{m}{\Omega_s} \sqrt{\frac{a}{\mu}} \cdot \gamma \cdot \frac{2a}{T}$ being the eccentricity damping constant.

However, it must be noted that this is a rather simplified picture of the processes connected with collisional damping. In particular, this concerns the assumption of a velocity-proportional friction force which has to be proven for granular material like planetary rings. We have used it nevertheless since it explains the exponential decay fairly well. Also, the relationship between the damping constant D_{ecc} resp. the friction coefficient γ and the collisional properties (restitution coefficients) of the particles must be subject of further work.

We introduce the above dependency $e(\phi)$ in the streamline model which has already proven to be a useful tool for the description of wake phenomena in planetary rings (Showalter et al. 1986, Spahn et al. 1994). The gravitational action of the moonlet on a ring particle is expressed by the change of the eccentricity $e = kh^*$ of the particle's orbit which is determined by the Gauß equation (Showalter et al. 1986, Petit & Hénon 1986). Damping of the induced eccentricity of the ring particles leads to a damping of the induced density changes of the streamlines. The density of the streamlines can be expressed (Borderies et al. 1982) by

$$\sigma(r,\phi) = \frac{\sigma_0(r,\phi)}{|J(r,\phi)|}\,. \tag{6}$$

After n encounters of the streamline with the moon the factor $J(r,\phi)$ takes the form (Spahn et al. 1994):

$$J(r,\phi) = 1 - \mathrm{sgn}(b)\sum_{j=0}^{n} q_j \sin\left\{m\left(\phi - 2\mathrm{sgn}(b)j\pi\right) + \gamma_j\right\} \tag{7}$$

where the phase angle γ_j and the nonlinearity parameter q_j are given by $\tan\gamma_j = \frac{m(\phi-2\mathrm{sgn}(b)j\pi)}{2}$ and $q_j^2 = (3kmh^*)^2\left\{\left[\frac{m(\phi-2\mathrm{sgn}(b)j\pi)}{2}\right]^2 + 1\right\}$ with $m = \frac{2}{3bh^*}$ and $k = \frac{e}{h^*}$. This leads to the following relation between streamline density and induced eccentricity:

$$\sigma(r,\phi) = \frac{\sigma_0(r,\phi)}{1 - e(\phi)g(b,\phi)} \tag{8}$$

Using the above exponential dependency $e(\phi)$ we find:

$$\sigma(t) = \frac{\sigma_0}{1 - e(0)g(b,\phi)\exp(-D_{ecc}\phi)} \tag{9}$$

where $g(b,\phi)$ comprises all dependencies on the coordinates b and ϕ which are given in detail in an earlier article (Spahn et al. 1994). For small $e(0)$ the time dependence can be approximated as

$$\begin{aligned}\sigma(t) &\approx \sigma_0\left[1 + e(\phi)g(b,\phi)\right] \\ &= \sigma_0\left[1 + e(0)g(b,\phi)\exp(-D_{ecc}\phi)\right]\,. \end{aligned} \tag{10}$$

The streamline density can be considered proportional to the wake intensity resp. the particle density. Therefore, we can practically identify the damping rates D_{ecc} and D_{wake} for the eccentricity and the wake density, respectively.

Although the results of the simulations show a strong similarity with the observed structures, one has to be aware of the simplifications in the model which require a certain caution in a comparison with the real system:

The cross sections of the model particles have been enlarged artificially in order to provide a more rapid evolution (Hänninen & Salo 1992). This leads to relative velocities which are too high and consequently to lower values of the restitution coefficient than to expect in reality. In addition, the nonlocal transport component is enhanced, and too much of the collective hydrodynamic motion of the particles is transformed into thermal one. The effects of these deviations and the appropriate scalings have to be addressed in the future.

The assumption of a monolayer of particles may not be compelling. Although planetary rings have a very small thickness, it is unlikely that they consist of a single layer of small particles so that the observed structures could be attributed to a superposition of several sub-structures with different (but small) inclinations.

Furthermore, the values of the damping rate which are obtained from simulations and from observations can not be compared directly because of the scalings made in our simulations in order to obtain an evolution rapid enough to follow with the computer. In our example, we have to take into account the higher mass ratio of satellite and planet which affects the size of the Hill sphere. Since $D_{ecc} = \frac{3\Omega_0 bh^*}{2\phi}\ln\left(\frac{e(0)}{e(\phi)}\right)$ with $h^* = \frac{1}{a}\left(\frac{M_m}{3(M_p+M_m))}\right)^{1/3}$ and $\frac{M_m}{M_p} = 10^{-6}$ in our example compared with $\frac{M_m}{M_p} = 10^{-12}$ for the Encke gap moon, the mapping of our model system to the real one results in a value of $D_{ecc} \approx 10^{-3}\,\mathrm{rad}^{-1}$.

Unfortunately, a determination of the damping rate from observational material is difficult because of the following reasons:

The data from high-resolution measurements for the inner and outer wake of Pan for several azimuthal locations show even that the wakes survive for more than one synodic period (Horn et al. 1996), but were obtained by different methods (PPS and RSS scans) where different particle sizes contribute to the observed optical depth. If the distribution of particle sizes were known, one could derive from these data an "overall" optical depth whose changes will allow an estimation of the damping constant D_{ecc} of the induced eccentricity.

An approximate estimation of the damping coefficient using information on the Encke division from the Voyager image data (Cuzzi & Scargle 1985) is only possible – if at all – with a considerable uncertainty because the estimated amplitudes of the "waves" at the edges of the gap had to be normalized by the respective background brightness. This may cause deviations from the original amplitude ratios.

It is therefore necessary to re-examine the observation data in order to provide a solid base for a comparison with results of simulations and theoretical investigations.

5. Conclusions

Inelastic collisions between the particles in a planetary ring have considerable consequences for the evolution and stability of structures. Our model is able to take into account some of the effects in a realistic way. In particular, we found that the inelasticity of the collisions is responsible for the very existence of visible wake structures.

An exponential law for the decay of the wake intensity has been applied to estimate the damping rate from numerical data. The exponential dependence can be motivated by the assumption of a velocity-proportional friction force which is made responsible for the wake damping. For the purpose of this paper, this rather crude picture yields fairly sufficient results for the stimation of the damping coefficient. However, the influence of the dissipative collisions on the decay of the magnitude of collective structures should be investigated in greater detail in future.

Further topics of future interest are the investigation of the influence of tangential friction (rotational degrees of freedom) and of the self gravity of the ring matter on the formation of the observed structures, the derivation of the wake damping rate from observational material, and the more complete estimation of the dependence of the restitution coefficients on the particles' properties. This will reveal a deeper insight in the processes not only in planetary rings, but also (in particular the last topic) in granular materials.

Acknowledgements. We are grateful for the provided computing time on the parallel computer CM-200 at INRIA, Sophia-Antipolis, France. The authors wouls also like to thank Dr. R. Greenberg for his helpful comments.

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