

X-ray galaxy clusters: constraints on models of galaxy formation

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Abstract. We present a self-consistent approach to the modeling of X-ray clusters of galaxies in a flat universe. Employing the Press & Schechter (1974) formalism to derive the mass function and relating the observable properties of clusters to their virial mass allows us to study the cluster X-ray distribution functions and their evolution with redshift. This approach differs from some results based on the X-ray luminosity function, which we argue is subject to modeling uncertainties. We obtain stringent constraints on the power spectrum of the initial density perturbations assumed to seed galaxy formation by comparing the theoretical temperature function with available data: the amplitude σ_8 is found to be 0.57 ± 0.05 , while the local spectral index n is falling in the range $-2.4 \leq n \leq -1.5$ on scales between 5 and $15 \text{ h}^{-1} \text{ Mpc}$: the standard CDM model is clearly ruled out, independently of the normalization. We further examine the evolutionary properties of X-ray clusters. Our approach greatly clarifies the situation: contrary to some previous claims, we find that flat models are in reasonable agreement with the observed number of clusters at high redshifts. We also examine the contribution of clusters to the X-ray counts and to the soft X-ray background and compare the expected values in the case of the $\Omega_0 = 1$ and $\Omega_0 = 0.2$ universes. The number counts are in agreement with the observations, further confirming the relevance of our modeling. We conclude that although clusters are not the primary source of the soft X-ray background, their contribution is nevertheless non-negligible. This is particularly important for they could escape the constraints imposed on possible point sources contributing to the background. Finally, we briefly examine the case of non-Gaussian fluctuations and point out the degeneracy between the value of the power spectrum index and the nature of the fluctuations, that these models imply.

Key words: cosmology: theory – large scale structure of Universe – galaxy clusters – X-rays: galaxies

1. Introduction

The “standard” picture for the origin of the large-scale distribution of matter in the universe is based on the gravitational growth of initially small density perturbations assumed to be present

from the very earliest moments of cosmic time (Lemaître 1933). This idea has received considerable attention from theorists and recently some spectacular observational support: the detection of temperature fluctuations in the Cosmic Background Radiation (CBR) by the COBE instrument DMR (Smoot et al. 1992). To this discovery one may today add a host of other claimed detections (see, for example, Scott et al. 1995). In the gravitational instability scenario, the density field can be quantitatively described by the power spectrum of the perturbations and by their higher order moments. In this paper, although the non-gaussian case will be briefly discussed, we will be primarily concerned with Gaussian perturbations for which the power spectrum provides a sufficient description. The uncertain origin of the density perturbations translates into an inability to calculate from first principles the form of the power spectrum, and the lack of a specific value for the density parameter exacerbates the problem. The theory of inflation alleviates this theoretical uncertainty by predicting that the primordial spectrum follows a power-law: $P(k) \propto k^{n_p}$ with $n_p \leq 1$, the exact value of n_p depending upon the possible existence of gravitational waves (Lucchin et al. 1992). In addition, inflation sets the cosmic density to the critical value ($\rho_c = 3H_0^2/8\pi G$, where $H_0 = 100 \text{ km/s/Mpc}$ is the current value of the Hubble constant). Once the nature of the (necessarily non-baryonic) dark matter is specified, one may calculate the final, evolved power spectrum as a function of the amplitude and index n_p of the initial spectrum. This is the spectrum relevant to galaxy formation.

Among the possible scenarios based on inflation, the cold dark matter (CDM) model has proven very successful in explaining many observed properties of the universe on scales ranging from galaxies to galaxy clusters, i.e. between a few tens of kpc and a few Mpc (for a review on the subject, see Frenk 1991). The amplitude of the power spectrum is the only free parameter of the “standard” model: one adopts $n_p = 1$.

The galaxy cluster population provides some of the most stringent constraints on models of galaxy formation, essentially because clusters are rare objects and, hence, their properties are sensitive to the underlying density fluctuations. The obviously important role of baryons in the determination of the observed properties of clusters would seem to necessitate hydrodynamical simulations in order to derive any such constraints. Evrard

(1989) has pioneered this interesting and important approach. Nevertheless, the ensemble properties of clusters, like their optical and X-ray luminosity functions, or their velocity and temperature distribution functions, are difficult to address directly by numerical simulations because the size of the numerical box must be very large in order to contain a sufficient number of clusters; an analytical approach remains an effective alternative.

Kaiser (1986) made an important theoretical step in this direction by proposing and then using simple scaling laws for cluster properties to derive the evolution of the ensemble properties. The corroboration of the Press & Schechter (1974) (PS) mass function by some recent numerical simulations (Efstathiou et al. 1988, Carlberg & Couchman 1989, Gelb & Bertschinger 1994, Eke et al. 1996; see Brainerd & Villumsen 1992 for an alternate point of view) provides us with an even more powerful tool for constraining galaxy formation theories: the simple PS formula gives us the mass function of structures at any redshift for any theory of initially Gaussian fluctuations. Provided that the relation between some observed quantity (for instance, the luminosity) and the mass is known, perhaps provided by hydrodynamical simulations of a small number of clusters, both the cluster ensemble properties (e.g. the luminosity function) and their evolution can be predicted and compared to observations. For example, Schaeffer and Silk (1988) showed that the CDM scenario reproduces well the optical luminosity function of galaxy clusters. Evrard (1989), using the number density of galaxy clusters with velocity dispersions greater than 1350 km s^{-1} at redshifts lower than 0.1, concluded that the bias parameter b - which is defined to be the inverse of the rms value of mass fluctuations within spheres of radius $8h^{-1} \text{ Mpc}$ - must be of the order of 1.5, inconsistent with the higher values of b required to explain galactic properties. He pointed out that an even smaller value of the bias is necessary to explain the three high velocity dispersion clusters listed by Gunn (1989) at redshifts greater than 0.1. Peebles et al. (1989) reached a similar conclusion by using a variety of present day cluster properties. However, Frenk et al. (1990) argued, by constructing artificial cluster catalogues from numerical simulations, that the cluster velocity dispersion is not a property from which reliable constraints can be derived on models because projection effects along the line of sight can contaminate the galaxy samples and significantly increase the estimated velocity dispersion. Since then, the standard CDM model has met with some further serious problems, for example an inability to explain the angular correlations of galaxies detected with the Automatic Plate Measuring Machine (APM) (Maddox et al. 1990). This has shed doubt on the validity of “standard” CDM. In addition, the amplitude indicated by the COBE temperature fluctuations, corresponding to $b \sim 1$ rather than the advocated $b \sim 2$, is generally considered too high for the model to be viable (see, however, Bartlett & Blanchard 1994, 1996), although some other authors have suggested that this high normalization can explain the observations once non-linear effects have been properly accounted for (Couchman & Carlberg 1992).

To solve the various problems faced by the standard version of CDM, changes to the power spectrum have been proposed, such as suppressing the small-scale power by mixing in a small amount of hot dark matter (Bond et al. 1980, Bond & Szalay 1983, Dekel 1984, Schaefer & Shafi 1992, Davis et al. 1992) or by altering the primordial value of n_p (Cen et al. 1992, Cen & Ostriker 1993) (so-called “tilted” CDM models). A list of further other possibilities is given by McNally & Peacock (1995). All of this leads us to reconsider the form of the power spectrum and adopt the point of view that it is an unknown which we wish to constrain. This analysis constitutes an alternative to a direct analysis of galaxy surveys (Peacock & Dodds 1994) for which the amplitude of the bias is unknown and does not permit direct access to the mass distribution. In particular, we will use the cluster population for this purpose: we will assume that over cluster scales even the evolved power spectrum can be approximated by a power-law, and then we will use the ensemble cluster properties to place limits on the amplitude and spectral index n . The adoption of a power-law is not really restrictive as most currently considered models lend themselves to this approximation (this may not necessarily be the case in purely baryonic models, in which the Jeans mass may strongly influence the perturbations on cluster scales and in a manner dependent upon the ionization history).

In principle, both optical and X-ray data can be used to constrain models, but, as emphasized by Frenk et al. (1990), the optical properties are subject to projection effects. If indeed important, such effects can alter both the optical luminosity and the velocity distribution functions. Moreover, the relationship between the overall mass of a cluster and its constituent galaxies could very well be complicated by the non-linear physics of galaxy formation (Evrard et al. 1994). The X-ray properties of a cluster offer an interesting alternative as they should not suffer the same severe projection effects. However, the observed X-ray luminosity of clusters is dominated by their core radius, and the physical origin of this core is unknown. This makes it difficult to relate the X-ray luminosity to the cluster mass, a point we will discuss in greater detail below and which will lead us to focus on the temperature distribution function.

In recent years many authors have calculated the ensemble properties of X-ray clusters expected in various scenarios (Henry & Arnaud 1991, Blanchard & Silk 1991, Kaiser 1991, Pierre 1991, Lilje 1992, Oukbir & Blanchard 1992, Bahcall & Cen 1993, Bartlett & Silk 1993, Blanchard et al. 1994, Colafrancesco & Vittorio 1994, Balland & Blanchard 1995, Liddle et al. 1995, Eke et al. 1996, Pen 1996) and compared the results with observations (Edge et al. 1990, Henry & Arnaud 1991) in order to derive constraints on the power spectrum. Henry & Arnaud (1991) found that the spectral index n and the bias b of the density perturbations must be -2.1 and 1.7 , respectively, to reproduce their observed temperature distribution function. Blanchard & Silk (1991) claimed that the CDM model is marginally consistent with the Edge et al. (1990) data if the bias parameter is close to 1.5, but that $n = -2$ with $b \approx 1.7$ is favored over the CDM value of $n \approx -1$ on cluster scales. However, Kaiser (1991) argues that the observed evolution of the

luminosity function needs an index closer to -1 . Lilje (1992) has shown that $\Omega_0 = 0.2$, flat CDM models need to be antibiased in order to reproduce the temperature distribution function. He also noticed that at high redshifts the temperature distribution evolves differently depending on the value of λ_0 . Both of these conclusions are consistent with the results of Bartlett & Silk (1993). Oukbir & Blanchard (1992, 1996) have shown that an unbiased open universe with $\Omega_0 = 0.2$ is compatible with the observed temperature distribution and that the redshift distribution of X-ray clusters is a powerful test of the mean density of the universe. Colafrancesco & Vittorio (1994) investigated the constraints imposed by the cluster luminosity function on a variety of models normalized to COBE, extending the analysis of Bartlett & Silk (1993), although reaching quite different conclusions.

Given this large list of different analyses, it would seem difficult to derive a consistent set of constraints from observations of the cluster population. In this paper we re-examine the modeling of X-ray clusters to clarify the situation. We construct a self-consistent set of relations between observable cluster properties and the theoretically relevant virial mass. We then use these relations to obtain robust constraints on the power spectrum (i.e. on the amplitude and spectral index). These constraints are applicable on scales from 5 to 15 h^{-1} Mpc. In this work we consider only a flat, hierarchical, dark matter dominated universe with $\Omega_0 = 1$. We start in the next section with a presentation of the arguments supporting the PS formula. In the following section, we discuss the observed properties of individual clusters and then relate them to the mass appearing in the theoretical mass function (Sect. 3). In the fourth section we derive the power spectrum parameters which best reproduce the observed temperature distribution function. In the fifth section we discuss the expected evolution of the luminosity function and compare the model to the high redshift observations. The sixth section presents predictions for the cluster number counts as a function of X-ray flux and an estimate of the cluster contribution to the soft X-ray background. Finally, the seventh section contains a brief discussion of non-Gaussian fluctuations. In the last section we summarize our results.

2. Theoretical mass function

In the gravitational instability scenario, virialized objects like galaxy clusters form from initially small density fluctuations that grow under the influence of gravity. The density field is specified by its power spectrum and the statistical nature of the fluctuations. It is generally assumed that the field is Gaussian, although some consequences of non-Gaussianity have been examined (Weinberg and Cole 1992), a question to which we return in Sect. 7. For the power spectrum, we adopt a simple power law,

$$P(k) \propto k^n,$$

over the mass range corresponding to galaxy clusters. In hierarchical models, such as CDM, the variance of the density field diverges on small scales, and so one must work with a smoothed version (see, for instance, Bardeen et al. 1986). The variance of

the density field smoothed with some window function W on a scale corresponding to mass M is

$$\sigma^2(M) = \frac{2}{(2\pi)^2} \int_0^\infty dk k^2 P(k) \tilde{W}^2(kR),$$

where \tilde{W} is the Fourier transform of the window function. Davis & Peebles (1983) found that the variance of galaxy counts within spheres of radius $8h^{-1}$ Mpc, quoted as $\sigma_{\text{gal}}(8h^{-1}\text{Mpc})$, is close to one. If the galaxy distribution follows the mass distribution, then the variance of the density perturbations on the same scale would also be equal to unity. However, if the galaxy distribution is biased relative to the mass distribution, then the variance of mass fluctuations in a sphere of radius R containing a mass $M = 4/3\pi\rho R^3$ can be written in the following way:

$$\sigma(M) = \frac{1}{b} \left(\frac{M_8}{M} \right)^{(n+3)/6}, \quad (1)$$

Accordingly, in the following, the value of b will correspond to $1/\sigma_8$ where b is the bias parameter. The value of b advocated to explain the observed abundance of galaxies, their correlations and their velocity dispersion was in the range 2 – 2.5. This large value of b met with difficulty in other quarters (Valls-Gabaud et al. 1989).

Since in one dimension the early stage of the non-linear collapse is entirely determined by the amplitude of the local mean density, the exact solution for the collapse of an overdensity is calculable until the first orbit crossing occurs. In three dimensions, the solution is also known for the case of a spherical matter distribution (Lemaître 1933, Gunn & Gott 1972, Peebles 1980). This so-called spherical model can be used to model the non-linear collapse of a cluster, which one then finds is driven by the value of the density field smoothed with a top-hat window of size comparable to that of the cluster. It is well known that when the linear density field reaches the value 1.68, the density becomes singular for a purely spherical collapse. In reality, the spherical symmetry is broken by the development of substructures which instead leads to the formation of a stationary state, the “virial” equilibrium. The final radius of the collapsed object is expected to be half of its maximum expansion radius, corresponding to a density contrast of the order of 200. In the absence of significant fragmentation during the collapse, initial density fluctuations of the field smoothed on the scale R are expected to collapse to structures with a typical mass $M = 4/3\pi\rho R^3$, where ρ is the mean cosmological background density. Within this framework, it is in principle possible to relate the ensemble characteristics of non-linear objects to the statistical properties of the initial density field.

In practice one deals with the initial density field linearly extrapolated to the present epoch $z = 0$. For instance, the rms fluctuation on some scale M is:

$$\sigma_0(M) = \frac{D(z=0)}{D(z_i)} \sigma_i(M).$$

In this relation D is the growing mode solution of the linearized growth equation and z_i is the redshift corresponding to some

early time at which the fluctuations in the universe were still linear. For the case of $\Omega_0 = 1$ and a vanishing cosmological constant, $\lambda_0 = 0$, $D \propto a$, where a is the expansion factor.

Despite the fact that the spherical top-hat model permits a considerable simplification of the actual development of non-linearities, the precise calculation of the number density of collapsed objects of mass M remains an extremely complicated problem. One major difficulty comes from the fact that a given region of space identified as non-linear on some scale might in fact form part of a still larger non-linear structure. This is the so-called ‘‘cloud-in-cloud’’ problem. However, one may assume that, being rare, massive objects, such as galaxy clusters, originate from nearly isolated density fluctuations, for which the cloud-in-cloud effect should be less important. In this case, the spherical model is likely to be a good description of the nonlinear evolution. Indeed, Bernardeau (1994) has shown that the rare density fluctuations of a Gaussian random field follow exactly the dynamics of the spherical model.

Using the spherical model, Press & Schechter (1974) proposed a derivation of the mass function of virialized objects. They argued that the fraction of matter in the form of non-linear objects with mass greater than M may be evaluated by:

$$\int_{\nu_t}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(-\nu^2/2) d\nu,$$

where ν_t is the threshold for non-linear collapse, which in the spherical model is given by

$$\nu_t = \frac{\delta_{c,0}(z)}{\sigma_0(M)}$$

For an Einstein-de Sitter universe, $\delta_{c,0} = 1.68(1+z)$. The mass distribution function is then easily derived:

$$N(M, z) dM = -\sqrt{\frac{1}{2\pi}} \frac{\rho_b}{M} \frac{\delta_{c,0}(z)}{\sigma_0^2} \frac{d\sigma_0}{dM} \exp\left(-\frac{\delta_{c,0}^2}{2\sigma_0^2}\right) dM.$$

Unfortunately, this result predicts that only half of the mass of the universe ends up in virialized objects, whereas one expects that at each epoch in a hierarchical scenario all of the mass should be bound in virialized objects. Press & Schechter (1974) side-stepped this problem by arbitrarily multiplying this mass function by a factor of 2. Several authors have recently re-examined the issue and proposed solutions (Peacock & Heavens 1990, Cole 1991, Blanchard et al. 1992a).

The problem has also been considered by Bond et al. (1991). Instead of smoothing the density contrast $\delta(x)$ with a filter $W(x, R)$, they adopted a low-pass, top-hat filter in Fourier space. At a given point x in real space, the value δ_s of the smoothed density contrast executes a Gaussian random walk as the size of the filter is increased from k_c to $k_c + \Delta k_c$ (corresponding to a decrease in the size of the window in real space). The step size of the random walk is a Gaussian random variable whose variance depends only on the power spectrum $P(k)$ and on the value of k_c (all of this applies only for Gaussian density fields). By identifying points which first pass the critical density δ_c at a particular value $k_c = K_c$ as objects of mass $M = 6\pi^2 \rho_b a^3 K_c^{-3}$,

the authors were able to recover the PS formula, including the troublesome factor of 2.

On the other hand, Blanchard et al. (1992a) point out that with the usually adopted simplifications (i.e. the collapse being driven essentially by the mean local density and the neglect of fragmentation), it is always possible to write the mass function in an exact way as:

$$N(M, z) dM = -\frac{\rho_b}{M} \frac{\delta_{c,0}(z)}{\sigma_0^2} \frac{d\sigma_0}{dM} F\left(\frac{\delta_{c,0}^2}{2\sigma_0^2}\right) dM, \quad (2)$$

where $F(\nu)$ represents the fraction of volume covered by non-linear spheres of radius greater than R (the smoothing scale corresponding to mass M). Although the function F is well defined, its calculation represents an extremely complicated statistical problem for Gaussian random fields. It would appear, then, that the normalization problem of the PS formula is due to the assumption that only points which are at the center of non-linear spheres of radius R are counted, rather than all points (not just the centers) residing in spheres of radius R or greater.

We justify our use of the PS formalism by its surprisingly good fit to the mass functions found in numerical simulations. This was first emphasized by Efstathiou et al. (1988), who examined the cluster multiplicity function for different values of n . Let us consider the more recent simulations of White et al. (1993). With an error of less than 50% in the mass, the simulation distributions agree with the PS formula for masses above $5 \times 10^{13} M_\odot$, where the abundance is $10^{-7} h^{-1} \text{Mpc}^{-3}$. We may take this to mean that the PS mass function is reliable into the regime where it accounts for $\approx 10^{-4}$ of the total mass. Roughly speaking, real clusters with a mass of a couple of $10^{15} M_\odot$ have an abundance of a few $10^{-8} (h^{-1} \text{Mpc})^{-3}$ and thus represent about 10^{-4} of the total mass. For clusters with $T \approx 14 \text{ keV}$ this means an abundance of $10^{-9} (h^{-1} \text{Mpc})^{-3}$. We conclude that the PS formula can be applied to confidently predict the abundance of X-ray clusters over the full range of observed temperatures.

3. The $T - M$ and $L_X - M$ relations

3.1. The $T - M$ relation

We cannot directly relate the mass function to observations because we have very little information on the actual virial mass of clusters. Note that in the spherical top-hat model, the virial radius is

$$R_V = 1.69 h^{1/3} M_{15}^{1/3} (1+z)^{-1} h^{-1} \text{Mpc},$$

which extends out beyond the region of currently available mass determinations. In order to obtain a fruitful comparison of the theoretical mass function with the observations, it is therefore necessary to construct trustworthy relations between X-ray properties and cluster virial masses. This ‘‘virial’’ mass should be understood in the sense in which it is employed in the mass function: it is the mass contained within the region of mean contrast density ~ 200 . Here we consider the $T - M$ and $L_X - M$ relations. We shall see that the former is more reliable.

Spectroscopic studies have demonstrated that the X-ray emission is produced by thermal bremsstrahlung in an hot, optically thin intracluster plasma with a temperature of approximately 10^8 K. The detailed history of this gas is not well known. The presence of the 7 keV iron emission line indicates that the intracluster medium (ICM) has been partially processed through the stars of the cluster galaxies. However, the large mass of the ICM, typically several times greater than the cluster stellar mass, leads one to believe that the majority of the gas is primordial in origin, since it seems difficult that the galaxies lose through winds or ram pressure stripping more than 50% of their initial mass. In addition, the measured metallicities close to third Z_{\odot} can be accounted for by a bimodal star formation model (Arnaud et al. 1992a).

We will work under the hypothesis that the cluster gas is in hydrostatic equilibrium with an isothermal temperature profile. Despite the lack of rigorous evidence, the latter point is at least consistent with the majority of data. Under these conditions we may write

$$\frac{kT}{\mu m_p} \frac{d \ln \rho_{\text{gas}}(r)}{d \ln r} = - \frac{GM(r)}{r},$$

where kT and ρ_{gas} are, respectively, the temperature and the density of the gas, μ is the mean molecular weight, m_p is the proton mass and $M(r)$ is the binding mass of the cluster. The observed X-ray surface brightness profile can be directly converted to a three dimensional density profile:

$$\rho_{\text{gas}}(r) = \rho_{0,\text{gas}} \left(1 + (r/r_{c,\text{gas}})^2\right)^{-3\beta_{\text{in}}/2}.$$

This is just the deprojection of the isothermal β form known to fit the surface brightness profiles of clusters. Best fit values for β_{in} are typically around 0.6 (Jones & Forman 1984). Using this value, the relation between the temperature and the virial mass can be evaluated as

$$T = 2 \frac{\mu m_p}{k} \frac{GM}{R}$$

This result is in reasonable agreement with the hydrodynamic simulations of Evrard (1990a, 1990b), although he obtained a constant of proportionality which is about 20 % lower. Taking this into account, the relation between mass and temperature becomes

$$kT = 6.35 M_{15}^{2/3} (1+z) h^{2/3} \text{ keV}$$

In this expression M_{15} is the cluster virial mass in units of $10^{15} M_{\odot}$. More recently, Evrard et al. (1996) showed that in the CDM case, this relation between mass and temperature holds with a very good accuracy. This equation will allow us to transform the mass function into a temperature function which we can compare to observations.

The temperature can also be related to the initial, comoving radius containing the mass:

$$kT = 4.5 \text{ keV} \left(\frac{R}{8 h^{-1} \text{ Mpc}} \right)^2 (1+z).$$

This last relation illustrates an important point: as the temperature is independent of the Hubble constant when scales are

expressed in $h^{-1} \text{ Mpc}$, a model corresponding to a given power spectrum and normalized to a scale also measured in $h^{-1} \text{ Mpc}$ (such as $\sigma(M_8)$) will produce the same cluster abundance per $(h^{-1} \text{ Mpc})^{-3}$. Therefore, the constraints on n and b inferred from the observed abundances of clusters are independent of the value of the Hubble constant.

3.2. The $L_X - M$ relation

The bolometric X-ray luminosity of a galaxy cluster due to thermal bremsstrahlung depends strongly on the gas density profile:

$$L_X \propto \int_0^{\infty} \rho_{\text{gas}}^2(r) T^{1/2} 4\pi r^2 dr.$$

Usually, one assumes that the ICM represents a constant fraction f_g of the cluster virial mass. Adopting this hypothesis and assuming an identical radial distribution for both the gas and the dynamical mass, Kaiser (1986) derived a scaling law for the X-ray luminosity: $L_X \propto M^{4/3} (1+z)^{3.5}$. However, there are reasons to suspect this scaling law. As emphasized by Blanchard et al. (1992b), the total luminosity of a cluster is dominated by the mass of the gas core and the self-similar scaling applies only if the mass of the gas core scales as the virial mass. However, the formation of a core in the gas distribution is not well understood and may result from any of several physical processes, including cooling in the center of the cluster or in the smaller structures from which the cluster was built, preheating by a first generation of collapsed objects, or gas ejection from galaxies. Thus, theoretical modeling of the X-ray luminosity is dangerously uncertain. Indeed, if one combines the self-similar $L_X - M$ relation with the highly reliable theoretical $T - M$ relation, then one obtains a $L_X - T$ correlation whose shape is in severe conflict with local data. In addition, Blanchard & Silk (1991) and Evrard & Henry (1991) have shown that self-similar scaling produces a luminosity function which disagrees with observations for both the CDM model and for a model with a power-law power spectrum with index $n = -2$. An alternative to the self-similar scheme is to parametrize the luminosity-mass relation as a power law of the cluster mass and redshift, $L_X \propto L_0 M^p (1+z)^s$, and to consider values of the free parameters that fit the observed luminosity function. But as these authors have noted, there is an observational degeneracy between the shape of the assumed initial power spectrum and the chosen $L_X - M$ relation. Perhaps the best way to deduce the true $L_X - M$ relation is through the observed $L_X - T$ relation. This seems a more trustworthy approach if one believes that the temperature reflects the virial energy of the cluster. In the remainder of this paper, we follow this procedure and use the local $L_X - T$ relation. Edge & Stewart (1991) have found that $L_X = (10^{43.05} \text{ erg s}^{-1}) T_{\text{keV}}^{2.62}$ using EXOSAT data. This relation is very close to that found by Henry & Arnaud (1991) using a compilation of data from EXOSAT, HEAO/OSO and the Einstein satellite. Additionally, Edge & Stewart (1991) have also given the correlation $T = (10^{-12.73} \text{ keV}) L_{X,\text{erg s}^{-1}}^{0.30}$ resulting from a minimization of the residuals in $\log T$. Using the result of a double regression fit in which the slope is defined as

the square-root of the product of the two individual regression slopes, we find $L_X \propto T^3$. We normalize this relation at 7 keV, the temperature at which the two regressions cross each other, to finally obtain

$$L_X = (10^{42.7} \text{ erg s}^{-1}) T_{\text{keV}}^3. \quad (3)$$

This expression represents the bolometric X-ray luminosity and we must correct for the fraction

$$f_{\text{band}}(z) = \int_{E_1(1+z)}^{E_2(1+z)} dE e^{-E/kT} / kT$$

actually collected in the relevant energy band $[E_1 - E_2]$.

4. Constraints on n and b

As we have just emphasized, the relation between the ICM temperature and the cluster virial mass is, in contrast to the luminosity-mass relation, relatively well understood. For this reason, we prefer in the following to use the temperature distribution function rather than the luminosity function to draw our conclusions on the power spectrum of density fluctuations.

The two existing cluster temperature distribution functions were derived from X-ray all-sky surveys. The Edge et al. (1990) temperature function was derived from 55 clusters with fluxes greater than $1.7 \times 10^{-11} \text{ erg cm}^{-2} \text{ s}^{-1}$ in the 2-10 keV band. It was constructed by correcting the Piccinotti et al. (1982) survey (which is supposed to be complete down to $3.1 \times 10^{-11} \text{ erg cm}^{-2} \text{ s}^{-1}$), for both mis-identification and confusion, and then by including clusters with fluxes greater than $1.7 \times 10^{-11} \text{ erg cm}^{-2} \text{ s}^{-1}$. The supposed clusters with insufficient information were excluded from the analysis, but the authors estimated from the $\log N - \log S$ and V/V_{max} distributions that their sample is 100% complete down to $3.1 \times 10^{-11} \text{ erg cm}^{-2} \text{ s}^{-1}$ and 70 - 90% complete down to $1.7 \times 10^{-11} \text{ erg cm}^{-2} \text{ s}^{-1}$.

Henry & Arnaud (1991) also used the Piccinotti et al. (1982) all sky sample with corrections for source confusion to derive the temperature distribution function. They find that a power-law with index similar to that derived by Edge et al. (1990) fits the data, but with a normalization twice as large.

Before pursuing our analysis, it is interesting to examine the luminosity function by translating it to a temperature function via the observed $L_X - T$ relation. In Fig. 1 we show the results of converting various published luminosity functions by our determination of the local $L_X - T$ relation (Eq. 3). The luminosity functions considered are from Edge et al. (1990), Henry & Arnaud (1991) and from the EMSS sample (Gioia et al. 1990, Henry et al. 1992). The temperature data, represented by the points in the figure, come from the two former sets of authors (Edge et al. 1990 and Henry & Arnaud 1991). Thus, only the EMSS data is entirely independent of the temperature observations. The converted temperature distribution functions from both Henry & Arnaud (1991) and the $z = 0.17$ EMSS give slightly flatter functions than the direct temperature observations of the temperature distribution. One can see that all of the luminosity functions are consistent with the direct determinations of the temperature distribution. Therefore, one would get

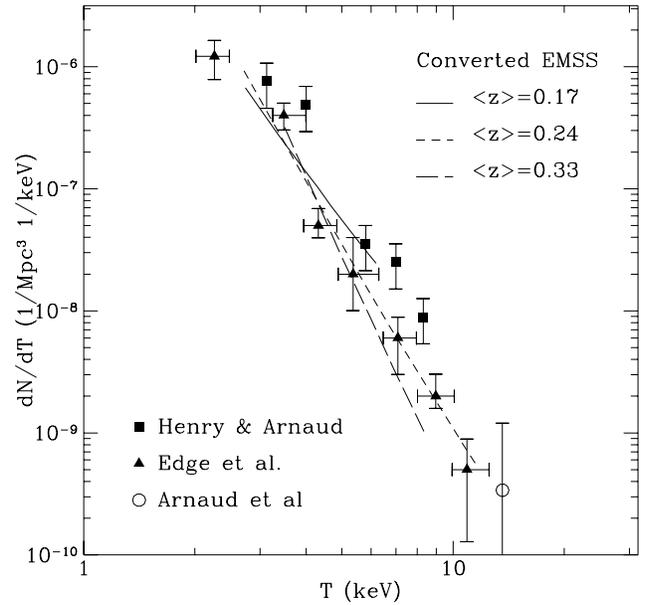


Fig. 1. The two local determinations of the temperature distribution function, one by Henry and Arnaud (1991), shown as the squares, and the other by Edge et al. (1990), shown as the triangles, are compared to the temperature functions deduced from the EMSS (Gioia et al. 1990, Henry et al. 1992) by application of our $L_X - T$ relation (see text). In this comparison, the $L_X - T$ relation is assumed NOT to evolve with redshift. The different line-types display the results for different redshifts, as indicated.

similar constraints by using luminosity functions instead of the temperature distribution functions.

In the following, we will compare the theoretical models to the observed temperature distribution functions and derive constraints on the parameters of the models. The free parameters are the index n of the power spectrum and the bias parameter b , both of which appear in the expression for the rms mass fluctuations (Eq. 1). The parameters are derived by chi-square fitting. We fit the model to each of the two temperature distribution functions, which we denote by ESFA and HA for the results of Edge et al. (1990) and Henry & Arnaud (1991), respectively. The differences will be considered as indicative of the uncertainties. We also add an additional point to each data set to represent A2163 which is the hottest known cluster. The temperature of this cluster was first determined by Arnaud et al. (1992b) who found that it is of the order of 14 keV with an uncertainty of about 1 keV. The importance of this measurement resides in the precision of the temperature determination. Indeed, the uncertainties of the cluster temperature measurements within the samples of Edge et al. (1990) and Henry & Arnaud (1991) were quite large. This has left open the possibility that the existence of such high temperature clusters were not real and that they were due to the tail of the temperature error distribution. On the contrary, although the statistical weight of A2163 is small, the precision in the temperature measurement does give us confidence in the overall shape of the distribution functions we used.

Table 1. Best-fit parameters

	n	b
ESFA ₁	-2.02	1.84
ESFA ₂	-2.03	1.85
HA ₁	-1.85	1.65
HA ₂	-1.85	1.67

We have calculated the number density by assuming that A2163 is the only such cluster in the Abell survey volume extended to a depth of $z \approx 0.3$. The error bars represented on the data points are the 90% confidence limits. Poisson statistics were used in the case of HA and A2163. In Table 1 we give the best fit values of n and b for each data set along. The subscripts 1 and 2 refer to data sets with and without the A2163 point, respectively.

It is difficult to assess the real nature of the detection statistics for clusters and Poisson statistics may not be relevant. Accordingly, the error bars should be treated with some caution. In the absence of further knowledge we assume that the errors are normally distributed and that the given error bars correspond to the rms deviation. We then draw the confidence contours which would contain 68.3%, 90% and 95.4% of the normally distributed data in the $b - n$ plane (Fig. 2). Since the error distribution function is unlikely to be gaussian, the actual probability associated with these contours cannot be evaluated. However, we estimated the reliability of the models by checking by eye the goodness of fit for different values of (b, n) on the 1σ contour.

From the ESFA data set we find:

$$1.8 \leq b \leq 1.9$$

$$-2.25 \leq n \leq -1.8$$

while with the HA data we find:

$$1.6 \leq b \leq 1.8$$

$$-2.4 \leq n \leq -1.5$$

These constraints are similar to those drawn by Blanchard & Silk (1991), Henry & Arnaud (1991) or more recently by Bartlett et al. (1995). Because Henry & Arnaud (1991) use a smaller number of clusters spread over a shorter range of temperatures, their data set provides less stringent constraints on the parameters. We finally consider as robust the following intervals:

$$1.6 \leq b \leq 1.9$$

$$-2.4 \leq n \leq -1.5$$

The temperature distribution does a good job in constraining both the shape and the amplitude of the power spectrum. This constraints were derived using clusters with temperatures between 2 and 14 keV, so the shape of the fluctuation spectrum is actually constrained over the range 5 and $10 h^{-1} \text{Mpc}$. One important implication is that the CDM model cannot explain the shape of the temperature distribution function: its power spectrum is too steep on galaxy cluster scales. Instead of the

CDM value of $n \approx -1$, the data suggest that n is closer to -2 over these scales. This is in agreement with analyses based on other methods, for example the power spectrum determination of Hamilton et al. (1991) and Peacock and Dodds (1994). This conclusion applies independently of the normalization of the spectrum. In considering the cosmic microwave background temperature fluctuations, we may make the additional statement that the normalization required by the clusters does not conform to the normalization demanded of CDM by the COBE measurements: given the CDM spectrum, the latter favors bias factors of order unity or less.

Our conclusions mainly rely on the validity of the temperature-mass relation. It should be emphasized that any error in this relation enters the exponential of the Gaussian in the mass function. As discussed above, the $T - M$ relation may be derived from the assumptions of isothermality and hydrostatic equilibrium and has been checked further by numerical simulations. However, standard mass estimates from hydrostatic equilibrium may underestimate the actual masses of clusters (Balland & Blanchard 1996). One may make several remarks here concerning future studies on the relation between the state of the ICM and the underlying dark matter. One will eventually be able to measure the temperature profile of the gas using the spatially resolved spectroscopic data of XMM and AXAF. For the present, one may attempt to constrain this profile by combining X-ray images and radio maps of the Sunyaev-Zel'dovich effect (assuming sphericity). Once given a temperature profile, the temperature-mass relation may in principle be deduced only from the assumption of hydrostatic equilibrium. From our point of view, the most exciting prospect employs the weak distortion of gravitationally lensed background galaxies to probe the cluster binding mass. By examining a sample of clusters with lensing data, X-ray images and even maps of the Sunyaev-Zel'dovich effect, one can directly constrain the temperature-mass relation.

5. Evolution with redshift

In the above section, we have shown that the local X-ray data allow one to fully determine the power spectrum of the density fluctuations. Oukbir & Blanchard (1996) used a similar analysis to constrain the same quantity in the case of an open universe (see also Oukbir & Blanchard 1992, Viana & Liddle 1995, Eke et al. 1996). Within our framework, the models are then completely specified and we can now predict the temperature distribution function of galaxy clusters at any redshift. In principle this is a powerful test of the mean density of the universe, since Oukbir & Blanchard (1996) demonstrated that the evolution of the comoving number density of X-ray temperature selected galaxy clusters depends solely on Ω_0 . Nevertheless, such information is not yet available and we can only investigate the evolution of the luminosity function. The most straightforward way to achieve this is to use the observed $L_X - T$ relation. However, this correlation is determined only at low redshift. On the other hand, the standard scaling relation, $L_X \propto M^{4/3}(1+z)^{3.5} \propto T^2(1+z)^{1.5}$, predicts the evolution with the redshift of the $L_X - T$ relation; but as we have discussed

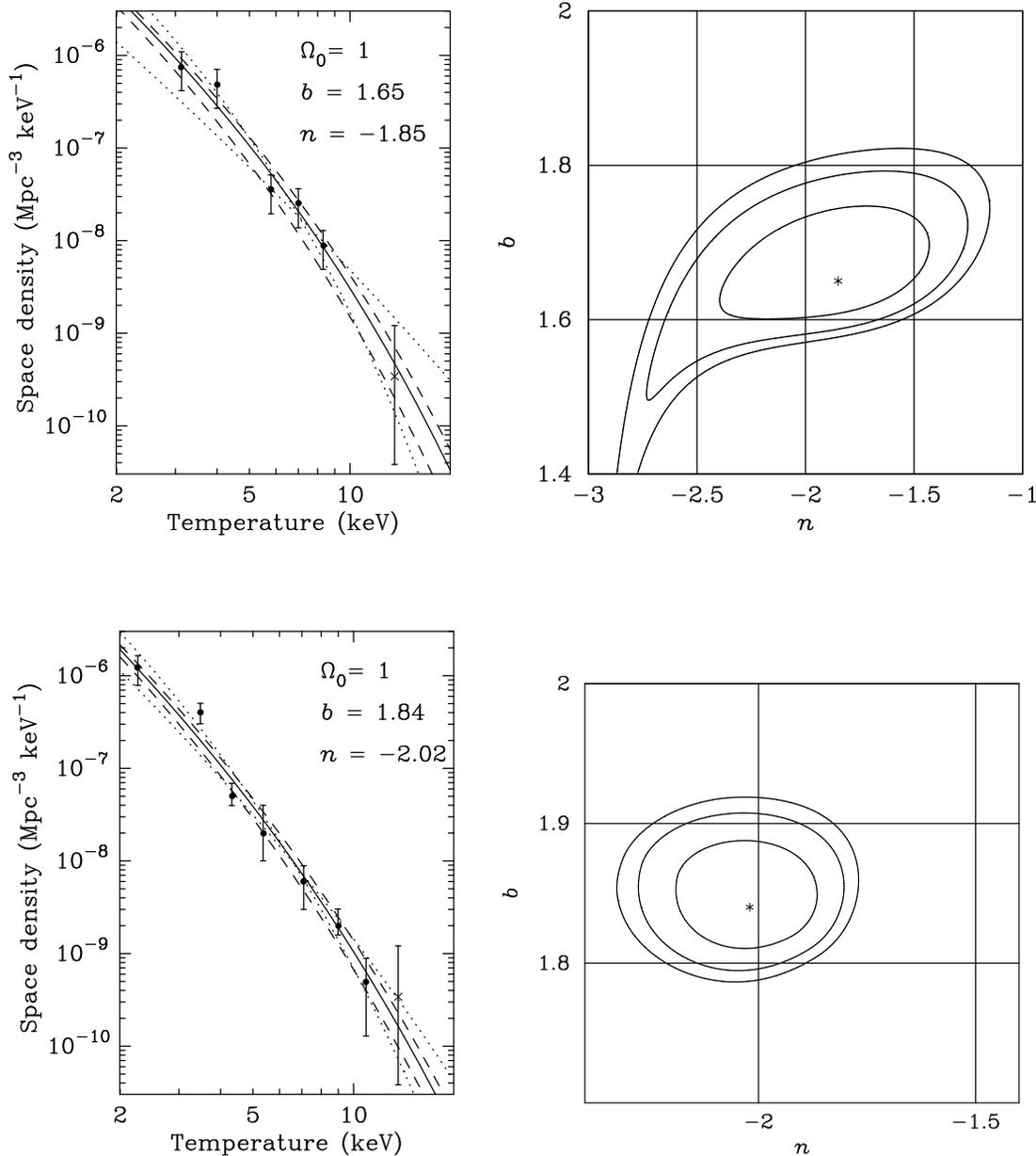


Fig. 2a and b. The left hand side column shows observed temperature distribution functions (points) with the corresponding best fitting theoretical functions (solid lines). From top to bottom: **a** Henry & Arnaud (1991) data. **b** Edge et al. (1990) data. Temperature distribution functions fitting the data at 1σ level are also represented: **a** The dashed lines represent $b = 1.6$ and 1.75 in the case $n = -1.85$ and the dotted lines represent $n = -2.4$ and $n = -1.5$ in the case $b = 1.65$. **b** The dashed lines represent $b = 1.8$ and 1.9 in the case $n = -2.02$ and the dotted lines represent $n = -2.25$ and $n = -1.8$ in the case $b = 1.84$. The right hand side column shows confidence region ellipses corresponding to $\Delta\chi^2 = 2.30, 4.61, 6.17$. These contours correspond to 68.3%, 90% and 95.4% respectively, for normally distributed data.

in Sect. 3.2, this relation is not in agreement with local data. Actually, there is only little information concerning the temperature of high redshift galaxy clusters and the existing data seem to indicate that the $L_X - T$ correlation is independent of redshift (Henry et al. 1994). However, due to the small number of clusters and to the large error bars on the temperatures, the uncertainties are quite high and do not lead to robust constraints. Investigating the possible evolution of the $L_X - T$ relation, Oukbir & Blanchard (1996) determined the parameters which

best fit the observed redshift distribution of the EMSS clusters (Gioia & Luppino 1994). In the case of the $\Omega_0 = 1$ universe, they found that a non-evolving $L_X - T$ relation is in acceptable agreement with the observations, although a slight positive evolution $L_X = L_0(1+z)$ better fits the data (here, L_0 is the luminosity that a cluster of given temperature would have at $z = 0$ according to the local $L_X - T$ correlation). This latter relation is the one we will use in the following.

The observed evolution of the X-ray cluster population has been investigated and discussed in detail (Gioia et al. 1990, Edge et al. 1990, Henry et al. 1992, Luppino & Gioia 1995). The situation, however, is not very clear: the first results suggested a strong negative evolution, in the sense that for a given luminosity, fewer clusters were observed at high redshifts. On the other hand, Ebeling et al. (1995) claim that previous investigations were undermined by non-uniform selection procedures and they found no convincing evidence for any evolution within a sample of X-ray selected intermediate redshift ROSAT clusters, up to $z \sim 0.3$. In fact, due to the extended nature of these objects, the interpretation of an X-ray selected cluster sample is not straightforward: apparent fluxes have to be corrected by a factor which depends on the assumed geometry of the source. This procedure has been used both by Gioia et al. (1990) and Henry et al. (1992), as well as by Luppino & Gioia (1995). The correction is very large for low redshift clusters and becomes moderate at higher redshifts. For instance, the mean correction factor used by Gioia & Luppino (1994) is 7; it could be as high as 15 for clusters with redshifts smaller than 0.15, but is less than 1.5 in the highest redshift bins. It seems therefore possible that a moderate systematic error in this correction could alter the inferred luminosity function.

It is interesting to compare our best fitting model with the observed luminosity function at high redshifts. This is presented in Fig. 3, where the luminosity function has been computed assuming the above mentioned evolutionary law for the $L_X - T$ relation. The most impressive aspect of the observations is the fast apparent evolution of the slope of the observed luminosity function over the moderate redshift range from $\bar{z} = 0.17$ to $\bar{z} = 0.33$, which is not reproduced by the models. This is manifest by the fact that the model curve at $\bar{z} = 0.17$ is already steeper than the data at this redshift; we have already noted this earlier in our discussion of Fig. 1. However, the models remains consistent with the data, when the uncertainty are taken into account. It is also interesting to note that the data from higher redshifts, shown in the inset, demonstrate similar or less evolution than the models; as pointed out by Gioia & Luppino (1994), the difference between $\bar{z} = 0.33$ and $\bar{z} = 0.66 - 0.8$ is consistent with no-evolution. From all of this, it seems reasonable to us to conclude that the observations are globally consistent with a moderate negative evolution, and that this evolution is weaker than previously estimated.

6. The X-ray background and the X-ray counts

Two interesting probes of cluster evolution are the X-ray source counts and the contribution of clusters to the X-ray background.

In order to compute the contribution of clusters to the X-ray counts, we must assign a luminosity to each mass in the PS mass function. According to Oukbir & Blanchard (1996), we use the locally observed $L_X - T$ correlation and we assume that it evolves such as to best fit the EMSS cluster redshift distribution (Gioia & Luppino 1994). As we mentioned in the previous section, in the case of the $\Omega_0 = 1$ universe, a non-evolving $L_X - T$ relation is in acceptable agreement with the observations, al-

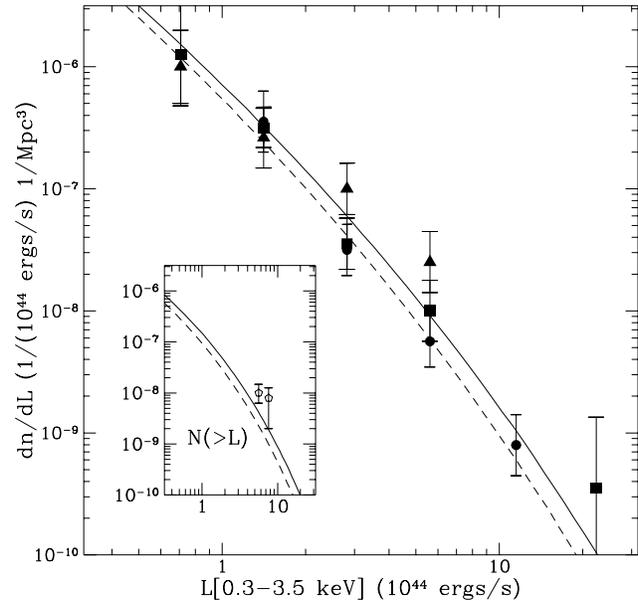


Fig. 3. The X-ray temperature-luminosity function at different redshifts. The triangles show the EMSS data at $z = 0.17$ and the squares correspond to $z = 0.33$; the solid curve shows the model redshift-zero luminosity function, while the dashed line shows the result for a redshift of 0.33. The model has been normalized to the local temperature function, and the luminosity functions have been constructed by application of $L \sim T^3(1+z)$ to this local temperature function. The small inset shows the integrated number of clusters as a function of in-band luminosity. The slightly higher data point corresponds to a redshift of 0.66 and the lower point to a redshift of 0.8, both given by Luppino & Gioia (1995). The solid and dashed lines show the corresponding model predictions for $z = 0.66$ and $z = 0.8$, respectively.

though a slight positive evolution $L_X = L_0(1+z)$ better fits the data. In the case of an open universe, a strong negative evolution is needed to reproduce the same data, and $L_X = L_0(1+z)^{-2.3}$. These latter relations are the one we will use in the following.

As the models are forced to match the Einstein redshift distribution as well as the local data, we do not expect a significant difference in the predicted quantities between the two models.

In Fig. 4, we show the expected $\log N(>S) - \log S$ in the energy band 0.5-2 keV. The triangle at $2 \times 10^{-12} \text{erg s}^{-1} \text{cm}^{-2}$ comes from the ROSAT cluster number counts in the northern sky (Burg et al. 1994), and the arrow at $10^{-14} \text{erg s}^{-1} \text{cm}^{-2}$ is a lower limit inferred by Rosati et al. (1995) from deep ROSAT PSPC observations. The solid and the dashed lines are the predicted cluster counts from our models in the case of the $\Omega_0 = 1$ and $\Omega_0 = 0.2$ models respectively. The thick lines are computed assuming the evolution of the $L_X - T$ correlation which best fits the EMSS redshift distribution, whereas the thin lines correspond to a non-evolving $L_X - T$ relation. Although the models are slightly above the observed number counts at low fluxes, our self-consistent modeling leads to predicted number counts which are in agreement with the data, and as expected, the flat and the open case are then almost identical. The strong negative evolution that is necessary in open models is again

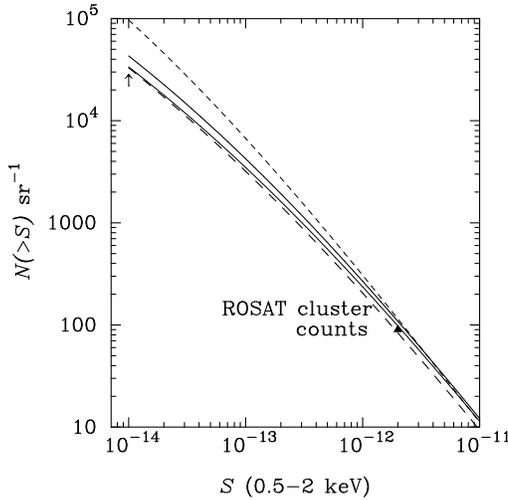


Fig. 4. The galaxy cluster number counts in the 0.5 -2 keV energy band. The triangle comes from the ROSAT cluster number counts in the northern sky (Burg et al. 1994), and the arrow is a lower limit inferred by Rosati et al. (1995) from deep ROSAT PSPC observations. The solid and the dashed lines are the predicted cluster counts from our models in the case of the $\Omega_0 = 1$ and $\Omega_0 = 0.2$ models respectively. The thick lines are computed assuming that $L_X = L_0(1+z)$ in the case $\Omega_0 = 1$ and $L_X = L_0(1+z)^{-2.3}$ in the case $\Omega_0 = 0.2$. The thin lines correspond to a non-evolving $L_X - T$ relation.

emphasized: if one assumes a non-evolving $L_X - T$ correlation at high redshifts in the $\Omega_0 = 0.2$ universe, then one overproduces the number of expected clusters at low fluxes by a factor close to five. Although the counts at $10^{-14} \text{ erg s}^{-1} \text{ cm}^{-2}$ could constitute a lower limit, it is unlikely that they were underestimated by such a large factor.

With our approach we can also estimate the contribution of clusters to the X-ray background.

The contribution of X-ray clusters to the XRB has already been discussed at length in the literature. From the X-ray luminosity function of Abell clusters, McKee et al. (1980) have put an upper limit of 5% to this contribution in the 2-10 keV band. Piccinotti et al. (1982) have used a complete X-ray survey of the HEAO experiment to derive a similar result for the same energy band. At energies less than 1 keV, Schaeffer & Silk (1988) derived a contribution as high as 50% coming from small objects with large redshifts. However, this was based on the self-similar model. Blanchard et al. (1992b) investigated the contribution of X-ray clusters to the XRB within the framework of different cosmological models. Using parameters in the luminosity-mass relation which reproduce the local luminosity function, they found a contribution of approximately 10% in the 2-10 keV band. Burg et al. (1993) have also estimated this contribution, but do not attempt to reproduce the locally observed quantities. Fig. 5 shows our calculation of the cluster contribution to the X-ray background in the energy range 0.07-10 keV. The two crossing, thin, solid lines correspond to the region where the ROSAT background lies (Hasinger 1992). The two parallel, solid lines are power laws with energy index of -0.4 and two different

normalizations at 1 keV: $8 \text{ keV cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \text{ keV}^{-1}$, as determined from the HEAO spectrum by Marshall et al. (1980), and $11 \text{ keV cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \text{ keV}^{-1}$ from the Wisconsin results of McCammon et al. (1983). The triangles are the values inferred by Wu et al. (1991). The solid and the dashed lines are the predicted cluster counts from our models in the case of the $\Omega_0 = 1$ and $\Omega_0 = 0.2$ models, respectively. As in Fig. 4, the thick lines are computed assuming the evolution of the $L_X - T$ correlation which best fits the EMSS redshift distribution, whereas the thin lines correspond to the case of the non-evolving $L_X - T$ correlation. As for the contribution of clusters to the X-ray counts, if the self-consistent modeling is used, then the contribution of clusters to the X-ray background is very similar in the case of the critical and open models. In the 1 - 2 keV band, this contribution is about 10%. If one considers the hypothesis of a non-evolving $L_X - T$ correlation in the case of the $\Omega_0 = 0.2$ model (recall that this model does not fit the redshift distribution of the EMSS data), the contribution in the same band reaches 30%, which is still lower than the boundary which is allowed by considering that approximately 50% of the background in the 1 - 2 keV band is already resolved by point sources (Hasinger 1992). Nevertheless, Barcons et al. (1994) showed from fluctuation analysis that the extrapolated counts of the present known point sources could explain 90% of the background, but there still remains an unidentified component. In this context, a contribution of 10% is far from negligible. This is especially true since clusters are extended objects and they probably escape detection by the point source detection algorithms routinely employed (Blanchard et al. 1992b).

We conclude that the self-consistent modeling of galaxy clusters is in agreement with the observed X-ray cluster counts, and, contrary to other claims (Evrard & Henry 1991, Burg et al. 1993), that the X-ray background does not provide us with stringent constraints on the power spectrum or the evolution of X-ray properties of galaxy clusters.

7. Non-Gaussian fluctuations

In all of the above analysis, we have assumed that the fluctuations were Gaussian. This was implicit when we adopted for the function F in Eq. 2 the PS formula, which is known to reproduce the results of numerical simulations in the case of Gaussian fluctuations. Notice, however, that the appearance of the exponential in this formula is not trivial and should be considered as fortuitous since the function F is an extremely complex quantity to evaluate, and that this has not yet been achieved even in the Gaussian case. It is not our goal to investigate any specific case of non-Gaussian fluctuations, since there is little physical motivation for any specific model. We would rather like to point out some differences that may result in such a case. For non-Gaussian fluctuations it is still possible to write the mass function as :

$$\int_M^\infty m \Phi(m) dm = \rho \int_{\nu_{\text{NG}}}^\infty F_{\text{NG}}(\nu) d\nu,$$

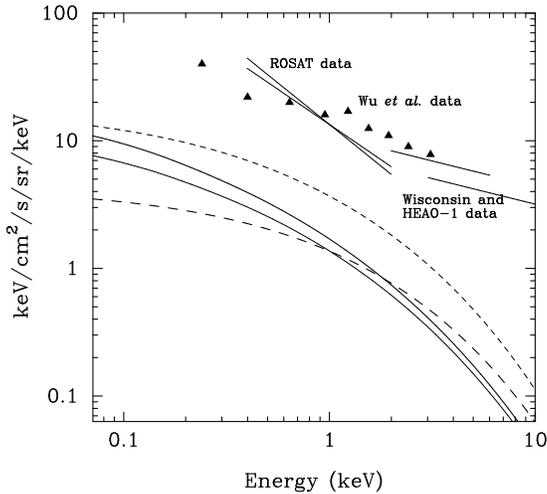


Fig. 5. Contribution of galaxy clusters to the XRB in the case of the critical model (solid lines) and in the case of the open model (dashed lines). As in Fig. 4, the thick lines are computed assuming that $L_X = L_0(1+z)$ in the case $\Omega_0 = 1$ and $L_X = L_0(1+z)^{-2.3}$ in the case $\Omega_0 = 0.2$. The thin solid lines in the range 0.5 - 2 keV represent the ROSAT background (Hasinger 1992) and the thin solid lines in the range 2 to 10 keV are the HEAO 1 background (Marshall et al. 1980) and the Wisconsin data (McCammon et al. 1983) with same power law index and different normalisations (see text). The triangles are the values inferred by Wu et al. (1991).

with

$$\nu_{\text{NG}} = \frac{\delta_c}{\sigma_{\text{NG}}(M)}.$$

However, the function F_{NG} is now arbitrary. It is then rather simple to show that the mass function will mimic a Gaussian fluctuation spectrum σ_G which is related to the non-Gaussian perturbation spectrum by the following relation:

$$\int_{\nu_{\text{NG}}}^{\infty} F_{\text{NG}}(\nu) d\nu = \int_{\delta_c/\sigma_G(M)}^{\infty} F_G(\nu) d\nu$$

Because of the arbitrariness of the function F_{NG} , it is possible to fit the local properties of clusters, whatever the spectrum is, by using an appropriate distribution function. As an illustration, we have computed the distribution function for which an $n = -1$ spectrum would mimic an $n = -2$ spectrum. This is represented on Fig. 6. As is naively expected, the distribution function presents a tail towards high ν which favors the formation of massive clusters.

8. Conclusions

The ensemble properties of clusters (and groups) are potentially useful for the determination of the various ingredients of cosmological models. However, in practice there are several problems which must be addressed in order to fruitfully use this method. As clusters are the result of the non-linear collapse of the largest fluctuations, one might think that their physical properties would be difficult to understand, and that any modeling would suffer

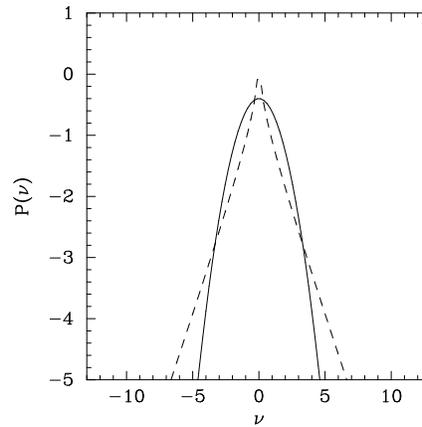


Fig. 6. Probability distribution functions: the Gaussian case (solid line) and the distribution function for which an $n = -1$ spectrum would mimic the mass function obtained with an $n = -2$ spectrum (dashed line)

from this limitation. This is even more important when one is trying to account for the evolution of the ensemble properties. The optical properties are certainly difficult to model: as pointed out by Frenk et al. (1990), projection effects can alter both the optical richness and inferred velocity dispersions. Other problems that make the modeling of cluster evolution difficult are 1) evolution of the member galaxies; 2) merging; 3) environmental effects likely to have played a major - but yet unclear - role in the galaxy formation history. Therefore any conclusions inferred from optical observations should be regarded as only tentative.

On the other hand, X-ray observations appear to provide a more reliable test of cosmological models because they are much less subject to these optical biases. Nevertheless, a substantial uncertainty remains in the modeling of cluster X-ray luminosities because the luminosity depends mainly on the cluster core properties. As we have emphasized, the gas temperature is better understood from a theoretical point of view, and present day data are of a good enough quality to allow reliable modeling. We find that the temperature distribution function indicates a power spectrum index of the order of -2 , and a bias parameter of about 1.7, in agreement with other power spectrum determinations. Notably, this conflicts with the standard CDM prediction of $n \approx -1$ on cluster scales. We have also investigated the case of non-gaussian fluctuations: we show that the local data implied by a given spectrum can be reproduced by any other spectrum, provided that the distribution function of the fluctuations is adequately chosen. Therefore, only specific models can be further investigated. Calculations of the luminosity evolution of cluster remains more uncertain, mainly because this needs to model the cluster core. One way to avoid this problem would be to obtain a temperature limited rather than flux limited sample of clusters, but this seems rather difficult to achieve. Nevertheless, information from the redshift distribution of flux limited clusters allows one to remove this difficulty. We find that the redshift distribution of the EMSS clusters can be fitted

with a moderate positive evolution of the $L_X - T$ (clusters of a given temperature were brighter in the past) whereas a strong negative evolution is needed in the case $\Omega_0 = 0.2$ (Oukbir & Blanchard, 1996).

Using the best-fit model to the cluster X-ray data, we have estimated the predicted cluster number counts as well as the contribution of clusters to the X-ray background in the case $\Omega_0 = 1$ and $\Omega_0 = 0.2$. We confirm previous results: clusters could represent a significant fraction of the faint sources, and are expected to contribute about 10% of the X-ray background at energies of the order of a few keV. Since our modeling matches the local data as well as the high redshift observations, there is not a noticeable difference among the two models in the X-ray counts and the contribution of clusters to the X-ray background: within self-consistent modeling, these two quantities cannot provide stringent constraints on the various models. Further observations will help to remove the substantial uncertainty in the temperature distribution function and therefore will allow a better evaluation of the characteristics of the spectrum, while redshift information will lead to unambiguous information on the mean density of the universe (Oukbir & Blanchard, 1996).

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Note added in proof: Recently, as a part of the Wide Angle ROSAT Pointed Survey Scharf et al. 1996 have given the sky density of extended objects at fluxes greater than $3.5 \times 10^{-14} \text{ erg cm}^2 \text{ s}^{-1}$ in the 0.5–2 keV band. They find a value in the range 2.8 to 4.0 (± 0.4) deg^{-2} , in excellent agreement with our predictions.