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The energy balance in contact binaries

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Abstract. We derive the global energy balance in contact binaries evolving on a thermal time-scale, including the semi-detached phase. Effects of accretion and rotation are taken into account. Within the framework of standard assumptions (uniform rotation, conservation of mass and angular momentum, spherically averaged components, each of them in hydrostatic equilibrium) the discussion is exact.

Key words: stars – binaries – close

1. Introduction

As shown in an accompanying paper (Kähler 1996), most (possibly all) contact binaries evolve on a thermal time scale and thus probably in thermal cycles, alternating between a contact phase and a semi-detached phase as first proposed by Lucy (1976) and Flannery (1976). In evolutionary calculations it is usually assumed that the total luminosity L can be written in the form

$$L = \sum_{i=1}^{2} \int_0^{M_i} (\varepsilon_i - T_i \dot{s}_i) \, \mathrm{d}m_i, \tag{1}$$

where the index i denotes the component, m_i is the mass variable, s_i is the specific entropy and the notation is standard otherwise. This amounts to assuming that

$$\dot{E} = \sum_{i=1}^{2} \int_0^{M_i} T_i \dot{s}_i \, \mathrm{d}m_i, \tag{2}$$

where E is the total energy content except for nuclear energy. In these equations effects of accretion are neglected which can be important at least in the semi-detached phase. Accretion is expected to give a contribution

$$\dot{M}_2(\Psi_1 - \Psi_2) = \dot{M}_1(\Psi_2 - \Psi_1) \tag{3}$$

to the luminosity, where Ψ_i denotes the (averaged) Roche potential at the surface of the component i. Thermodynamic properties (e.g. entropy or heat content) of the accreted matter should also have some influence. Finally, changes in the rotational energy should be properly taken into account.

The purpose of this paper is to derive the correct global energy balance within the framework of some standard assumptions. For this purpose we apply a self-consistent treatment of the mechanical structure of contact binaries (Kähler 1986, hereafter K86).

2. Assumptions and basic equations

2.1. Assumptions

The system may be either in contact or semi-detached. We assume uniform rotation and adopt a spherically averaged treatment of the components as described in K86. Each component is assumed to be in hydrostatic equilibrium, but the Roche equipotential condition $\Psi_1 = \Psi_2$ is not assumed to be satisfied. With these simplifying assumptions the mechanical structure of a system is well-defined for given mass, mass ratio, angular velocity, composition and entropy distribution. Note that these assumptions are almost unavoidable in evolutionary calculations.

The real problem is, of course, more complex. The exchange of mass and energy between the components requires internal mass motions and thus departures from strictly uniform rotation. In the semi-detached phase there are also departures from hydrostatic e quilibrium in the secondary's outermost layers. For a discussion of these points cf. Sect. 4.

Concerning the evolution we shall assume conservation of total mass and total angular momentum (the sum of orbital angular momentum and spin angular momentum).

2.2. The Roche potential

Total mass $M = M_1 + M_2$, angular velocity ω and the separation A of the components are connected by Kepler's law

$$\omega^2 = \mathrm{G}M/A^3. \tag{4}$$

As shown in K86, the (spherically averaged) Roche potential at the surface of the component i can be written in the form

$$\Psi_i = -\frac{\mathrm{G}M}{A}\left(\frac{\lambda_i^2}{3} + \frac{\mu_i}{\lambda_i} + \frac{(1-\mu_i)(3-\mu_i)}{2}\right), \tag{5}$$

where μ_i and λ_i are, respectively, normalised mass and radius of the component i defined by

$$\mu_i = M_i/M, \quad \lambda_i = R_i/A. \tag{6}$$

In the contact phase the equipotential condition is approximately satisfied ($\Psi_1 \approx \Psi_2$). Assuming $M_1 > M_2$, in the semi-detached phase we have

$$\Psi_1 > \Psi_2. \tag{7}$$

2.3. Structure equations

Two of the structure equations are well-defined. The continuity equation in the component i is

$$\frac{\partial r_i}{\partial m_i} = \frac{v_i}{4\pi r_i^2}, \tag{8}$$

where v_i denotes the specific volume, and the momentum balance (equation of hydrostatic equilibrium) is

$$\frac{\partial P_i}{\partial m_i} = -\frac{1}{4\pi r_i^2}\left(\frac{\mathrm{G}m_i}{r_i^2} - \frac{2}{3}\omega^2 r_i\right). \tag{9}$$

The balance of energy is more complex. Let $l_i(m_i)$ denote the luminosity in the component i. The effects of internal mass motions in the system are sources or sinks of energy in both components. Let $\sigma_i(m_i)$ denote the source (when positive) or sink (when negative) of energy per unit of mass, and

$$\Lambda_i = -\int_0^{M_i} \sigma_i \, \mathrm{d}m_i. \tag{10}$$

The local balance of energy is then

$$\frac{\partial l_i}{\partial m_i} = \varepsilon_i - T_i \dot{s}_i + \sigma_i, \tag{11}$$

and the global balance is

$$L = \sum_{i=1}^{2} \int_0^{M_i} (\varepsilon_i - T_i \dot{s}_i) \, \mathrm{d}m_i - \Lambda_1 - \Lambda_2. \tag{12}$$

In the special case of

$$\Lambda_1 + \Lambda_2 = 0, \tag{13}$$

we recover Eq. (1), and Λ_1 can be interpreted as the luminosity transferred from the primary to the secondary. In the general case, however, Eq. (13) is not satisfied as we shall see.

The functions σ_i are unknown apart from some restrictions. Below the turbulent envelope σ_i is zero (or at least small) since circulation currents in radiative regions are ineffective (Lucy 1968). In the adiabatic part of a turbulent envelope the run of σ_i is unimportant. In fact, the essential problem is to determine the integrals Λ_1 and Λ_2.

2.4. The energy content

Let $\mu = M_1 M_2 / M$ be the reduced mass of the system. The index i denotes the component, and the index 0 refers to the orbital motion. Within the framework of our assumptions the energy E takes the form

$$E = U + \Omega + K, \tag{14}$$

where

$$U = U_1 + U_2, \quad U_i = \int_0^{M_i} u_i \, \mathrm{d}m_i \tag{15}$$

is the internal energy,

$$\Omega = \Omega_0 + \Omega_1 + \Omega_2 \tag{16}$$

with

$$\Omega_0 = -\frac{\mathrm{G}M\mu}{A}, \quad \Omega_i = -\int_0^{M_i} \frac{\mathrm{G}m_i}{r_i} \, \mathrm{d}m_i \tag{17}$$

is the gravitational energy, and

$$K = \frac{\omega^2}{2}\Theta \tag{18}$$

is the rotational energy, where

$$\Theta = \Theta_0 + \Theta_1 + \Theta_2 \tag{19}$$

with

$$\Theta_0 = \mu A^2, \quad \Theta_i = \frac{2}{3}\int_0^{M_i} r_i^2 \, \mathrm{d}m_i \tag{20}$$

denotes the moment of inertia.

2.5. Virial theorems and conservation laws

Multiplying Eq. (9) by $4\pi r_i^2$ and integrating over m_i we obtain the virial theorem for the component i

$$\Omega_i = -3\int_0^{M_i} P_i v_i \, \mathrm{d}m_i - 2K_i, \quad K_i = \frac{\omega^2}{2}\Theta_i, \tag{21}$$

where K_i is the rotational energy of the component. The virial theorem for the orbital motion

$$\Omega_0 = -\omega^2 \Theta_0 \tag{22}$$

is equivalent to Kepler's law (4). Since total mass M and angular momentum $J = \Theta\,\omega$ are conserved, we have

$$\dot{M}_1 + \dot{M}_2 = 0 \tag{23}$$

and

$$\dot{\Theta}\,\omega + \Theta\,\dot{\omega} = 0. \tag{24}$$

3. Energy changes

We are now prepared to derive expressions for the energy changes. To begin with the internal energy, from Eq. (15) we find

$$\dot{U}_i = \int_0^{M_i} \dot{u}_i \, \mathrm{d}m_i + u_{is}\dot{M}_i, \tag{25}$$

where the index s refers to the surface ($m_i = M_i$), and finally, making use of a thermodynamic relation,

$$\dot{U}_i = \int_0^{M_i} (T_i \dot{s}_i - P_i \dot{v}_i) \, \mathrm{d}m_i + u_{is}\dot{M}_i. \tag{26}$$

Concerning the gravitational energy Ω_i, from the virial theorem (21) we obtain

$$\dot{\Omega}_i = -3 \int_0^{M_i} (\dot{P}_i v_i + P_i \dot{v}_i) \, \mathrm{d}m_i - 3(P_i v_i)_s \dot{M}_i - 2\dot{K}_i. \tag{27}$$

To replace the term involving $\dot{P}_i v_i$ we differentiate Eq. (9) with respect to time, multiply by $4\pi r_i^3$ and integrate over m_i. Performing an integration by parts and making use of Eq. (8) we obtain

$$-3 \int_0^{M_i} \dot{P}_i v_i \, \mathrm{d}m_i = -4\pi R_i^3 \dot{P}_{is} + \int_0^{M_i} \left(4 \frac{\mathrm{G}m_i}{r_i^2} \dot{r}_i + \frac{4}{3} \omega \dot{\omega} r_i^2 - \frac{2}{3} \omega^2 r_i \dot{r}_i \right) \mathrm{d}m_i. \tag{28}$$

To simplify the RHS we note that Eq. (9) implies

$$\dot{P}_{is} = \frac{1}{4\pi R_i^2} \left(\frac{\mathrm{G}M_i}{R_i^2} - \frac{2}{3} \omega^2 R_i \right) \dot{M}_i \tag{29}$$

and that Eqs. (17) imply

$$\dot{\Omega}_i = \int_0^{M_i} \frac{\mathrm{G}m_i}{r_i^2} \dot{r}_i \, \mathrm{d}m_i - \frac{\mathrm{G}M_i}{R_i} \dot{M}_i. \tag{30}$$

Furthermore we have

$$\dot{K}_i = \omega \dot{\omega} \Theta_i + \frac{\omega^2}{2} \dot{\Theta}_i \tag{31}$$

with

$$\dot{\Theta}_i = \frac{4}{3} \int_0^{M_i} r_i \dot{r}_i \, \mathrm{d}m_i + \frac{2}{3} R_i^2 \dot{M}_i. \tag{32}$$

With these equations the RHS of Eq. (28) becomes

$$\left(3 \frac{\mathrm{G}M_i}{R_i} + \omega^2 R_i^2 \right) \dot{M}_i + 4\dot{\Omega}_i + 2\dot{K}_i - \frac{3}{2} \omega^2 \dot{\Theta}_i, \tag{33}$$

and as the final result for the change in the gravitational energy of the component i we obtain

$$\dot{\Omega}_i = \int_0^{M_i} P_i \dot{v}_i \, \mathrm{d}m_i + \left((P_i v_i)_s - \frac{\mathrm{G}M_i}{R_i} - \frac{\omega^2}{3} R^2 \right) \dot{M}_i + \frac{\omega^2}{2} \dot{\Theta}_i. \tag{34}$$

Eqs. (26) and (34) give

$$\dot{U}_i + \dot{\Omega}_i = \int_0^{M_i} T_i \dot{s}_i \, \mathrm{d}m_i + \left(h_{is} - \frac{\mathrm{G}M_i}{R_i} - \frac{\omega^2}{3} R_i^2 \right) \dot{M}_i + \frac{\omega^2}{2} \dot{\Theta}_i, \tag{35}$$

where h_i denotes the specific enthalpy. It remains to determine the changes $\dot{K}$ and $\dot{\Omega}_0$. Allowing for angular momentum conservation we find

$$\dot{K} = -\frac{\omega^2}{2} (\dot{\Theta}_1 + \dot{\Theta}_2 + \dot{\Theta}_0). \tag{36}$$

From Eqs. (22) and (23) and from the definitions of Ω_0 and Θ_0 we obtain

$$\dot{\Omega}_0 = \frac{\omega^2}{2} \dot{\Theta}_0 - \frac{3}{2} \frac{\mathrm{G}M}{A} \dot{\mu}, \quad \dot{\mu} = (\mu_1 - \mu_2) \dot{M}_2. \tag{37}$$

Collecting all contributions to $\dot{E}$ it can be verified that

$$\dot{E} = \sum_{i=1}^{2} \int_0^{M_i} T_i \dot{s}_i \, \mathrm{d}m_i + (h_{2s} - h_{1s} + \Psi_2 - \Psi_1) \dot{M}_2. \tag{38}$$

Making use of the equation

$$L = \sum_{i=1}^{2} \int_0^{M_i} \varepsilon_i \, \mathrm{d}m_i - \dot{E} \tag{39}$$

we find

$$L = \sum_{i=1}^{2} \int_0^{M_i} (\varepsilon_i - T_i \dot{s}_i) \, \mathrm{d}m_i + L_{\mathrm{acc}} \tag{40}$$

with

$$L_{\mathrm{acc}} = (\Psi_1 - \Psi_2 + h_{1s} - h_{2s}) \dot{M}_2. \tag{41}$$

A comparison with Eq. (12) shows that

$$\Lambda_1 + \Lambda_2 = -L_{\mathrm{acc}}. \tag{42}$$

4. Discussion

Assuming synchronised rotation and neglecting departures from hydrostatic equilibrium in the components we have derived an approximation for the total luminosity of a contact binary. The exchange of mass between the components provides two contributions to the luminosity. The first contribution $(\Psi_1 - \Psi_2)\dot{M}_2$ is the accretion luminosity as expected in Sect. 1. This contribution is obviously non-negative. The second contribution $(h_{1s} - h_{2s})\dot{M}_2$ is a correction which depends on thermodynamic properties of the accreted matter. This contribution may have any sign. Note that the specific enthalpy (in contrast to the specific entropy) i s finite and well-defined in the surface layers of the components.

If a system as a whole is in strict hydrostatic equilibrium, both contributions vanish since the equipotential condition $\Psi_i = \Psi_2$ is satisfied and since $h_{1s} = h_{2s}$ on account of Poincaré's theorem. In reality the exchange of mass and energy requires departures from strict hydrostatic equilibrium.

The physical meaning of the second contribution becomes clear when considering spherically symmetric accretion onto a star of mass M and radius R. If all kinetic energy of infalling matter is converted to radiation at the stellar surface, the accretion luminosity is

$$L_{\rm acc} = \frac{\mathrm{G}M}{R}\dot{M} = -\Psi\dot{M}, \qquad (43)$$

where Ψ is the gravitational potential at the surface. In reality not all kinetic energy is transformed to radiation. Some fraction is used to adjust the heat content of the accreted matter. (Note that the enthalpy is a continuous function of the mas s variable.) Indeed, proceeding as in Sect. 3 or making use of Eq. (35) we find

$$L_{\rm acc} = \left(\frac{\mathrm{G}M}{R} - h_s\right)\dot{M} = -(\Psi + h_s)\dot{M}. \qquad (44)$$

Thus, the second contribution (which is usually small) allows for the thermal adjustment of the accreted matter.

In our idealised treatment the kinetic energy of internal mass motions is neglected. The exchange of mass is assumed to proceed as follows. In a small time interval a spherical infinitesimal mass shell is removed from the surface of the component with the higher value of the (spherically averaged) Roche potential and instantane ously deposited on the surface of the other component (the 'gainer'). It is assumed that the adjustment of the components to hydrostatic equilibrium and synchronised rotation occurs instantaneously. In reality gravitational energy is first converted to kinetic energy, then dissipated and finally converted to radiation. Moreover, the adjustment of the system (particularly the tidal adjustment) occurs on a finite time-scale. Apart from some time delay, the net result is essentially the same as in in our simplified discussion.

We have derived an approximation for the accretion luminosity $L_{\rm acc}$ and thus for the sum $\Lambda_1 + \Lambda_2$. It is manifest that $L_{\rm acc}$ must be represented by an energy source in the gainer's surface layers. In evolutionary calculations we need Λ_1 and Λ_2 separately, i.e. we need a second relation between these quantities.

In systems in good thermal contact an approximate relation follows from the condition that the surface temperatures of the components be similar. In such systems $L_{\rm acc}$ is small since the Roche equipotential condition is approximately satisfied and since the difference in the enthalpy is also small. Eq. (13) is therefore a reasonable approximation. If the energy transfer between the components can be assumed to occur in the adiabatic parts of the convective envelopes, the run of the functions $\sigma_i(m_i)$ in the outer layers is also known.

In systems in shallow contact the situation is much more complex. Since the difference in enthalpy is larger, accretion effects are more important. Additional considerations are necessary to derive a second relation between Λ_1 and Λ_2. A further problem is the run of the sources and sinks $\sigma_i(m_i)$. In some evolutionary phases this problem is serious, particularly in the secondary's outer layers after contact has been reestablished during a thermal cyc le.

In semi-detached systems, finally, the secondary is the gainer and the energy transfer between the components occurs solely by accretion. Thus we have simply

$$\Lambda_1 = 0, \quad \Lambda_2 = -L_{\rm acc}, \qquad (45)$$

and in more detail

$$\sigma_1(m_1) \equiv 0, \quad \sigma_2(m_2) = L_{\rm acc}\,\delta(m_2 - m_*), \qquad (46)$$

where δ is the Dirac function and m_* denotes a point in the secondary's outermost layers. The difference in the Roche potential $\Psi_1 - \Psi_2$ and the rate of mass exchange $\dot{M}_2$ can be large. Under these circumstances accretion can be important in decreasing the temperature difference between the components.

Summarising, additional equations are needed to determine the rate of mass exchange $\dot{M}_2$. Within the framework of our approximations and for given $\dot{M}_2$, the balance of energy is known in the semi-detached phase and approximately known in systems in good thermal contact, but largely unknown in systems in shallow contact.

5. Concluding remarks

Contact binaries evolving in thermal cycles oscillate about a state of marginal contact, changing between a contact phase and a semi-detached phase. This raises the difficulty that the semi-detached phase is – at least apparently – very rare among observed shortest-period systems ($P \lesssim 0.35$ days). Two ways out of this difficulty are possible. The fraction of time spent in the semi-detached phase may be very small as proposed by Robertson & Eggleton (1977), and/or the paucity of shortest-period systems with EB light curves may be spurious as proposed by Eggleton (1996). The present results support the second way out. On account of accretion the temperature difference in the semi-detached phase is possibly much smaller that hitherto expected. According to Flannery (1976), accretion during t he semi-detached phase increases the secondary's luminosity by about 10 percent only. However, there are still many uncertainties in evolutionary calculations. Preliminary results suggest that the effects of accretion can be much larger. Accordingly, some observed systems classified as EWs are possibly semi-detached.

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