Astron. Astrophys. 320, 799–810 (1997)

ASTRONOMY AND ASTROPHYSICS

An improved calibration of Cepheid visual and infrared surface brightness relations from accurate angular diameter measurements of cool giants and supergiants

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Received 2 August 1996 / Accepted 7 October 1996

Abstract. We have calibrated optical and near-infrared surface brightness – colour relations for cool giant and supergiant stars using high-precision angular diameters of these stars determined from Michelson interferometry. We find that the giant and supergiant relations are undistinguishable over a wide range of intrinsic colours. We independently determine the slopes of these relations obeyed by Cepheid variables and find that in all the diagrams considered, these agree very well with the slopes derived from the stable giants and supergiants. Forcing the slopes to the values derived from the Cepheids, we determine a very precise value of the zero point of the surface brightness – colour relations valid for Cepheid variables, which is 3.947 ± 0.003. This value is in agreement with the one derived from the Cepheid effective temperature scale of Pel (1978), and from the lunar occultation angular diameter of the Cepheid ζ Gem (Ridgway et al.1982).

We apply our newly calibrated surface brightness – colour relations to the cluster Cepheid U Sgr to find its radius and distance from the optical V, $V - R_J$ and the infrared V, $V - K$ and K, $J - K$ Barnes-Evans technique. While the numerical values derived from the three different versions of the technique do agree within 1σ, the near-infrared distance and radius values are $5 - 10$ times more accurate than the optical one; in particular, the distance and radius of the star derived from the V, $V - K$ solution are 592 ± 4 pc and 48.7 ± 0.3 solar radii, respectively, with errors less than 1 percent. We briefly discuss the potential of the near-infrared versions of the Barnes-Evans technique to set a very accurate (0.02 mag) zero point to Cepheid period-luminosity relations and thus make a very important contribution to the absolute calibration of the extragalactic distance scale.

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Key words: cepheids – stars: distances – stars: fundamental parameters – stars: individual: U Sgr – infrared: stars

1. Introduction

The relationship between the surface brightness of a Cepheid variable, in an optical or near-infrared passband, and a suitably chosen colour index, is of considerable astrophysical interest because it permits to establish important astrophysical parameters like the effective temperature, the mean radius and the distance, and hence the luminosity, of the variable. In particular, if there exists a well-defined relation between surface brightness and colour index, this relationship may be used to infer the angular diameter variation of Cepheids during their pulsation cycles from their observed colour variations, and combining this information with the linear displacements of the stellar surface, derived from an integration of the observed radial velocity curves, the distances and mean diameters of Cepheid variables can be found from least-squares solutions. This technique is called the surface brightness or Barnes-Evans method and its foundations have been introduced originally by Barnes & Evans (1976). The optical version of this technique which uses the $V - R$ colour index on the Johnson system as the most suitable surface brightness indicator has been extensively employed by Gieren, Barnes & Moffett (1993; hereafter GBM) to derive the distances to a large number (100) of galactic Cepheid variables, from which a period-luminosity relation in the V band was calibrated. Earlier, Gieren, Barnes & Moffett (1989) had used the same approach to calibrate the period-radius relation followed by galactic Cepheid variables. A first near-infrared version of this technique was calibrated by Welch (1994) who used the $V - K$ colour index as a surface brightness indicator. Discussion of the systematic effects involved in the determination of Cepheid radii and distances in the optical and near-infrared wavelength ranges, as given by Welch (1994), Laney & Stobie (1995) and Madore & Freedman (1991) lead to the expectation that Cepheid radii and distances can be determined with higher accuracy, both intrinsic and systematic, when near-infrared colours are used. This seems to be borne out by the Barnes-Evans near-infrared analysis of the cluster Cepheid U Sgr carried out by Welch (1994) which produces a distance and radius which is consistent with the optical determination of GBM, but of higher accuracy.

In view of the vital importance of Cepheid variables to establish the extragalactic distance scale, and of the 0.2 mag discrepancy detected by Gieren & Fouqué (1993) between the distance moduli of Cepheids derived from the optical Barnes-Evans method and from ZAMS-fitting to the colour-magnitude diagrams of galactic open clusters and associations containing Cepheid variables, it is very important to calibrate Cepheid surface brightness relations, both in the optical and in the near-infrared, with the highest possible accuracy. While GBM had to rely on model atmosphere calculations and effective temperature scales for Cepheids to find the zero point of the $F_{\rm V}$ vs. $(V-R_{\rm J})_\circ$ relation, Welch (1994) was the first to provide a calibration which was based solely on interferometrically measured angular diameters of cool giants and supergiants of high precision (typically 5% or better). Since then, more precise angular diameter measurements from interferometric methods have become available (e.g. Dyck et al.1996) which can be used to improve the calibration of surface brightness – colour relations for giants and supergiants, and for Cepheid variables. This is the principal aim of this paper. Besides of re-calibrating the $F_{\rm V}$ vs. $V-R$ and $F_{\rm V}$ vs. $V-K$ relationships, we also provide a calibration of the $F_{\rm K}$ vs. $J-K$ relationship which can be very useful in applications where only near-infrared JK photometry and radial velocity curves are available.

As a test case, we apply the different surface brightness – colour relationships to the Cepheid U Sgr which is a member of the open cluster M 25 and has a well-determined independent (ZAMS-fitting) distance. From a comparison of the results, conclusions can be drawn as to the relative usefulness and accuracy of the different versions of the Barnes-Evans technique we employ.

2. Surface brightness relations for cool giants and supergiants

2.1. Selection criteria

We have compiled a list of stars with accurate measures of angular diameters, mostly obtained by Michelson interferometry. Original sources are given by di Benedetto (1993), and Dyck et al.(1996). Diameters are corrected for limb-darkening. The relative precision of these measurements is generally better than 5%. When several measurements exist, a weighted mean has been adopted.

For sake of homogeneity in the photometry, only stars measured by Johnson et al.(1966) in V, $R_{\rm J}$, J and K bands have been retained. These authors attribute the following mean accuracy to their measurements: 0.015 in V, 0.030 in K, 0.018 in $V-R_{\rm J}$, and 0.025 in $J-K$. Three double stars, namely HR 603, HR 1708 and HR 6406 have been rejected, because Johnson's V measurements refer to the whole system. Too few of these stars also have photometry in the VRI Cousins system (Cousins 1980) to allow an accurate calibration of the $V-R_{\rm C}$ and $V-I_{\rm C}$ surface brightness relations.

Forty stars are a priori available for calibrating the visual and infrared surface brightness variations with colour. They range from supergiants to dwarfs, and from F0 to M6 spectral types. We first reject variable stars. For this purpose, we use two criteria: the Bright Star Catalogue (Hoffleit & Jaschek 1982) gives the variable classification and its amplitude: we keep stars with less than 0.2 mag V amplitude; Cousins (1980) gives a raw estimate of the photometric variability by the number of decimal places: we reject stars whose V magnitude is given with only one significant decimal place. Both criteria generally agree, except for HR 8308, which is a flare star, and therefore may be considered as not variable for photometric purposes at a given epoch. HR 4902 and HR 8698 are rejected according to Cousins, although the Bright Star Catalogue classifies them as low amplitude variables (Lb 0.1 V for both). Applying this rejection criterion, 27 stars remain in our catalogue. Among these, 19 have high precision diameters (accuracy better than 6%). Remember that a 10% accuracy in angular diameter corresponds to 0.22 mag uncertainty in magnitude. Table 1 lists these 27 stars, among which 13 are giants (luminosity class III), and 13 are supergiants (luminosity classes I, I-II and II). The Sun has been added as a reference point, with photometry from Johnson (1965). HR number, star common name, spectral type and variability class and amplitude come from the Bright Star Catalogue (1982); the limb-darkened mean diameter (in milliarcsec) and its accuracy (in %) are from di Benedetto (1993, source 1) or Dyck et al.(1996, source 2): for details of the angular diameter measurements (technique, wavelength, limb-darkening correction), we refer the reader to these references; all the photometry comes from Johnson et al.(1966), followed by a cross if the star has also been measured by Cousins (1980); the source of the visual absorption value is given as a code in the last column, which is described in the Notes to the Table.

The next criterion deals with the amount of absorption (both interstellar and circumstellar) suffered by these stars. Various sources of $A_{\rm V}$ are used, as detailed in the Notes to Table 1. According to the published values, 9 stars suffer more than 0.2 mag V of absorption, while 2 do not have an absorption determination. All but one are supergiants. Rejecting those stars drastically reduces our sample and, moreover, nearly eliminates all the supergiants. This is annoying, as the coefficients of the surface brightness relations may differ for giants and supergiants. We therefore adopt the following philosophy: for supergiants (luminosity classes I to II), we accept all stars and correct their magnitudes and colours for absorption, after discussing the choice of the reddening law. For giants (luminosity class III), we restrict our sample to those stars with **a)** a low absorption value ($A_{\rm V} < 0.2$ mag), to avoid the dependence on an adopted reddening law (this eliminates only one star, namely HR 5340), and **b)** an accurate measure of diameter ($< 6\%$, this eliminates only one additional star, namely HR 7951).

2.2. The giants surface brightness relations

The surface brightness $F_{\rm V}$ (resp. $F_{\rm K}$) is defined from the measured magnitude and angular diameter (in milliarcsec) as:

$$F_{\rm V} = 4.2207 - 0.1\,V - 0.5 \log \oslash_{\rm LD}, \quad (1)$$

$$F_{\rm K} = 4.2207 - 0.1\,K - 0.5 \log \oslash_{\rm LD}, \quad (2)$$

where the coefficient 4.2207 only depends on the bolometric absolute magnitude $M_{\rm bol\odot}$ and the total integrated flux $f_\odot$ (solar constant) of the Sun, and on the Stefan-Boltzmann constant σ. Therefore, it does not depend on the chosen photometric band. It is given by:

$$4.2207 = 0.1\,M_{\rm bol\odot} + 1 + 0.25\,\log \frac{4 f_\odot}{\sigma}. \quad (3)$$

As di Benedetto (1993) concludes that M giants follow a different relation than G to K giants, we a priori exclude the reddest giant, namely HR 5299 (M4 III). In spite of the low absorption values of the selected giants which make absorption corrections unimportant, we apply these corrections (as discussed in the following Sect. 2.3), for the sake of homogeneity. We then find the following dereddened relations for our giants sample, by linear least-squares fits, assuming all the errors in $F_{\rm V\circ}$ (resp. $F_{\rm K\circ}$):

$$F_{\rm V\circ} = 3.925_{\pm 0.017} - 0.379_{\pm 0.016}\,(V - R_{\rm J})_\circ, \quad (4)$$

with N = 10, rms = 0.012, and colour range = 0.72 – 1.32.

$$F_{\rm V\circ} = 3.930_{\pm 0.012} - 0.124_{\pm 0.004}\,(V - K)_\circ, \quad (5)$$

with N = 10, rms = 0.008, and colour range = 2.22 – 4.11. For comparison, di Benedetto (1993) gives $3.927 - 0.122\,(V - K)$ for G0 to K5 giants.

$$F_{\rm K\circ} = 3.940_{\pm 0.013} - 0.100_{\pm 0.016}\,(J - K)_\circ, \quad (6)$$

with N = 10, rms = 0.008, and colour range = 0.60 – 1.06.

2.3. Adopted reddening law

For the supergiants sample, a reddening law has to be adopted to convert $A_{\rm V}$ values to $E(V - R_{\rm J})$, $E(V - K)$ and $E(J - K)$. The agreement of published values for some of these ratios is not satisfying. For instance, in the case of M supergiants, the ratio $\frac{E(V-R_{\rm J})}{E(B-V)}$ is 1.28 ± 0.10 according to Lee (1970), but 0.84 according to Cardelli et al.(1989), using $R_{\rm V} = 3.6 \pm 0.3$ from Lee. We adopt an intermediate value of 0.97 from Hindsley & Bell (1989). For the mean $R_{\rm V}$ value of Cepheids, namely 3.26 from Gieren & Fouqué (1993), this gives $\frac{E(V-R_{\rm J})}{A_{\rm V}} = 0.30$.

The agreement is better for the $\frac{E(V-K)}{E(B-V)}$ ratio: Lee gives 3.22 ± 0.12 for M supergiants, Cardelli et al.3.17 for $R_{\rm V} = 3.6$, and Laney & Stobie (1993) argue that $\frac{A_{\rm V}}{E(V-K)} = 1.1$ for a large variety of grains, which leads to $\frac{E(V-K)}{E(B-V)} = 3.27$ for $R_{\rm V} = 3.6$. Adopting the Cardelli et al.law and $R_{\rm V} = 3.26$ for Cepheids gives $\frac{E(V-K)}{A_{\rm V}} = 0.88$.

For $\frac{E(J-K)}{E(B-V)}$, Lee gives 0.59 for M supergiants, Cardelli et al.0.64 for $R_{\rm V} = 3.6$, while Laney & Stobie derive 0.485 for a zero-colour, zero-extinction star, which leads to 0.57 for $R_{\rm V} = 3.6$. Again adopting the Cardelli et al.law with $R_{\rm V} = 3.26$ for Cepheids gives $\frac{E(J-K)}{A_{\rm V}} = 0.17$. In summary, we use for Cepheids:

$$E(V - R_{\rm J}) = 0.30 A_{\rm V} \quad (8)$$

$$E(V - K) = 0.88 A_{\rm V} \quad (9)$$

$$E(J - K) = 0.17 A_{\rm V}, \quad (10)$$

or equivalently:

$$E(V - R_{\rm J}) = 0.97 E(B - V) \quad (11)$$

$$E(V - K) = 2.88 E(B - V) \quad (12)$$

$$E(J - K) = 0.56 E(B - V). \quad (13)$$

2.4. The supergiants surface brightness relations

The following relations for supergiants, corrected for absorption as described above, are derived by linear least-squares fits, assuming all the errors in $F_{\rm V\circ}$ (resp. $F_{\rm K\circ}$):

$$F_{\rm V\circ} = 3.943_{\pm 0.026} - 0.392_{\pm 0.025}\,(V - R_{\rm J})_\circ, \quad (14)$$

with N = 12, rms = 0.033, and colour range = 0.23 – 1.78. This equation is established on only 12 points, because of the rejection of discrepant data for HR 7735.

$$F_{\rm V\circ} = 3.914_{\pm 0.023} - 0.119_{\pm 0.007}\,(V - K)_\circ \quad (15)$$

with N = 13, rms = 0.032, and colour range = 0.52 – 5.53.

$$F_{\rm K\circ} = 3.938_{\pm 0.023} - 0.101_{\pm 0.027}\,(J - K)_\circ \quad (16)$$

with N = 13, rms = 0.027, and colour range = 0.17 – 1.21.

First, note that the supergiants relations are far less precise than the corresponding giants relations. Second, there is no evidence of different slopes for supergiants and giants, contrary to di Benedetto's claim (1993), who gives: $F_{\rm V\circ} = 3.958 - 0.139\,(V - K)_\circ$.

In order to settle more definitely this question, let us compare these slopes from the giants and supergiants samples, with those directly determined from Cepheids. Indeed, short-period Cepheids are like type II supergiants, on the basis of their mean effective surface gravities, while long-period Cepheids are classified spectroscopically as type Ib, Iab, or – for the very longest periods – Ia.

Table 1. Input parameters for the 28 stars with measured angular diameters

HR	Name	Spectral Type	Variability	$\oslash_{\rm LD}$	acc	ref	V	$V-K$	K	$J-K$	$V-R_{\rm J}$	$A_{\rm V}$	ref
165	δ And	K3 III		4.12	1.0	1	3.28	2.80	0.48	0.76	0.92	0.05	c
168	α Cas	K0 IIIa		5.64	0.9	1	2.23	2.48	-0.25	0.67	0.78	0.08	c
337	β And	M0 IIIa		13.98	0.8	1	2.05	3.88	-1.83	1.01	1.24	0.03	b
617	α Ari	K2 IIIab Ca-1		6.85	0.9	1	2.00	2.64	-0.64	0.74	0.84	0.03	c
834	η Per	M3- Ib-IIa		4.6	11	1	3.79	3.70	0.09	0.96	1.23	1.0	e
911	α Cet	M1.5 IIIa	0.06 V	13.23	1.2	1	2.53	4.21	-1.68	1.08	1.35 x	0.11	c
1612	ζ Aur	K4 II + B8 V	EA 0.15 V	5.51	6.2	1	3.75	3.60	0.15	0.95	1.13	0.36	a
2326	α Car	F0 II		6.6	12	1	-0.75	0.56	-1.31	0.18	0.24 x	0.05	c
2473	ε Gem	G8 Ib		4.70	2.1	1	2.98	2.76	0.22	0.77	0.96	0.08	c
2693	δ CMa	F8 Ia		3.60	14	1	1.84	1.42	0.42	0.38	0.51 x	0.10	f
2943	α CMi	F5 IV-V		5.51	0.9	1	0.37	1.01	-0.64	0.24	0.42 x	0.00	c
2990	β Gem	K0 IIIb		8.03	1.0	1	1.14	2.23	-1.09	0.60	0.75	0.01	b
3705	α Lyn	K7 IIIab		7.98	3.9	1	3.13	3.74	-0.61	1.00	1.23	0.06	b
5299	BY Boo	M4 III	Lb? 0.18 V	6.9	4.3	2	5.28	5.65	-0.37	1.17	1.85	0.05	c
5340	α Boo	K1 IIIb CN-1		20.89	0.9	1	-0.05	2.95	-3.00	0.82	0.97	0.34	d
6056	δ Oph	M0.5 III		9.5	5.3	2	2.75	3.97	-1.22	1.02	1.29 x	0.04	c
6705	γ Dra	K5 III	0.08 V	10.00	1.9	1	2.22	3.56	-1.34	0.95	1.14	0.05	b
7139	δ^2 Lyr	M4 II	SRc? 0.11 V	9.8	4.1	2	4.30	5.53	-1.23	1.21	1.78		
7525	γ Aql	K3 II		7.8	3.8	2	2.72	3.31	-0.59	0.89	1.07 x	0.37	d
7536	δ Sge	M2 II + A0 V	0.09 p	8.0	3.8	2	3.83	4.62	-0.79	1.16	1.44		
7735	31 Cyg	K2 II + B3 V	EA 0.11 V	5.85	7.4	1	3.80	3.31	0.49	0.92	0.97	0.34	a
7751	32 Cyg	K3 Ib + B3 V	EA 0.03 V	5.83	6.6	1	3.98	3.82	0.16	1.03	1.20	0.41	a
7949	ε Cyg	K0 III		4.62	0.9	1	2.46	2.35	0.11	0.66	0.73	0.02	c
7951	EN Aqr	M3 III	Lb 0.04 V	5.6	12	2	4.44	4.62	-0.18	1.17	1.47 x	0.13	c
8079	ξ Cyg	K4-5 Ib-II		6.59	5.0	1	3.70	3.75	-0.05	1.05	1.20	0.34	d
8308	ε Peg	K2 Ib	flare 2.8 V	7.5	5.3	2	2.39	3.20	-0.81	0.84	1.05 x	0.58	d
8465	ζ Cep	K1.5 Ib		5.5	18	1	3.35	3.24	0.11	0.86	1.08	1.0	e
	Sun	G2 V		1919260			-26.74	1.41	-28.15	0.35	0.52	0	

Sources of visual absorption values:

Ref. a: corresponds to Table 12 of di Benedetto (1993), original ref. e (di Benedetto & Ferluga 1990): absorption is computed from adopted distance and galactic latitude; we keep the original value, but without rounding it.

Ref. b: corresponds to di Benedetto & Rabbia (1987), where absorption is computed from adopted distance (in kpc) and galactic latitude, according to:

$$A_{\rm V} = 0.14 \times \frac{1-\exp(-10 \times d \times |sinb|)}{|sinb|} \tag{7}$$

For $b \rightarrow 0$, this gives 1.4 mag per kpc. Other authors (Blackwell et al.1990) adopt a smaller coefficient (0.8 or even 0.6 mag per kpc). For α Boo, ref. d value is preferred.

Ref. c: corresponds to the above method (ref. b), but computed by us (d and b are taken from the Bright Star Catalogue). For ε Gem, Table 12 of di Benedetto (1993) gives 0.1 under ref. d (Blackwell et al.1990), but the original value is 0.00, because Blackwell et al.adopt 0.00 for $d < 75$ pc; we therefore recompute the value from adopted distance and galactic latitude, which gives 0.08.

Ref. d: corresponds to Dyck et al.(1996).

Ref. e: corresponds to Table 12 of di Benedetto (1993), but original ref. is not given; adopted as such, except for ξ Cyg, where ref. d value is preferred.

Ref. f: from $E(B-V)$ = 0.03 in Arellano Ferro & Parrao (1990), which gives $A_{\rm V}$ = 0.10.

3. Surface brightness relations for Cepheid variables

Rather than assuming that Cepheid variables exhibit the same slopes on the various surface brightness – colour diagrams as the stable giants and supergiants, it is a better approach to determine the slopes from the Cepheid data themselves. This allows an independent check on whether or not Cepheids and stable giants and supergiants of similar effective temperatures do behave in an identical way on these diagrams, and it ensures that the surface brightness solutions (see Sect. 4) do not depend on a possible curvature of the surface brightness – colour relation at the very red end (M stars).

The surface brightness $S_{\rm V}$ of a Cepheid in the V band, at a given phase, can be determined from its observed V magnitude and the displacement r from the mean radius $R_\circ$ according to the formula given by Thompson (1975):

$$S_{\rm V} = V + 5\log(1 + \frac{r}{R_\circ}). \tag{17}$$

The displacements can be calculated from the integrated radial velocity curve of the variable, and the mean value of the radius has to be known. Similarly, the surface brightness $S_{\rm K}$ in the K band can be calculated from

$$S_{\rm K} = K + 5\log(1 + \frac{r}{R_\circ}). \tag{18}$$

Using photometric and radial velocity data and assuming a value for the mean radius, the variation of $S_{\rm V}$ or $S_{\rm K}$ with a colour index can thus be established for the variable, and the slope on this diagram be determined by least-squares fits. From this, the slopes on the $F_{\rm V}$ or $F_{\rm K}$ vs. colour diagrams are obtained by multiplication by -0.1, according to the definition of these surface brightness parameters:

$$F_{\rm V} = 4.2207 - 0.1\,S_{\rm V}, \tag{19}$$
$$F_{\rm K} = 4.2207 - 0.1\,S_{\rm K}, \tag{20}$$

3.1. The slope on the $F_{\rm V}$ vs. $V - R_{\rm J}$ diagram

The values of the slopes on this diagram have been established for 52 galactic classical Cepheids by Gieren (1988), using Thompson's method and the extensive and accurate photometric and radial velocity data of Gieren (1981a, b) and Coulson & Caldwell (1985). From these results, Gieren found that the slope m could be represented by:

$$m = -0.359 - 0.020\log P, \tag{21}$$

where P is the pulsation period in days. However, the period dependence turned out to be only marginally significant. Moffett & Barnes (1987) conducted a similar study on a sample of northern galactic Cepheids and found somewhat more positive values for the slopes, but with no significant dependence on period either. While in the GBM study on the distances of galactic Cepheid variables a mean value from both determinations was adopted, we prefer here adopting the determination of Gieren (1988), because of the clearly higher accuracy of the radial velocity data he used in his work, which permitted to establish the radial displacements, and thus the surface brightnesses, with a better accuracy than in the work of Moffett & Barnes (1987). Neglecting the marginal period dependence, we adopt a mean value of -0.380 ± 0.003 for the slope shown by Cepheid variables on the $F_{\rm V}$ vs. $V - R_{\rm J}$ diagram, which is in excellent agreement with the value determined from the stable giants and supergiants in the preceding Section.

3.2. The slope on the $F_{\rm V}$ vs. $V - K$ diagram

To determine the slope shown by Cepheid variables on this diagram, we chose 10 Cepheids with very accurate K band observations by Laney & Stobie (1992), spanning a period range between 4 and 39 days, which have also very accurate V band light curves and radial velocity curves by Coulson & Caldwell (1985), Gieren (1981a, b), and by Gieren et al.(1996). These stars are listed in Table 2.

Here, a potential problem may arise, due to the use of different infrared systems, namely Carter's one, on which Laney & Stobie measurements are defined, and Johnson's one, on which stars with measured angular diameters are measured. As Bessell & Brett (1988) do not give definitive conversion relations between these two systems (they compared the Johnson system to an older SAAO system, defined by Glass), we have conducted a comparison on 87 common standards. Results are the following:

$$K_{\rm J} = K_{\rm C} - 0.004_{\pm 0.003}\,(V_{\rm J} - K_{\rm C}) + 0.014_{\pm 0.006}, \tag{22}$$

with N = 87 and rms = 0.032.

$$(J-K)_{\rm J} = 0.962_{\pm 0.009}\,(J-K)_{\rm C} + 0.007_{\pm 0.006}, \tag{23}$$

with N = 86 and rms = 0.028.

As the slope of the first relation and the intercept of the second one are not significant, we finally adopt:

$$K_{\rm J} = K_{\rm C} + 0.006_{\pm 0.003}, \tag{24}$$

with N = 87 and rms = 0.032.

$$(J-K)_{\rm J} = 0.973_{\pm 0.008}\,(J-K)_{\rm C}, \tag{25}$$

with N = 82 and rms = 0.023.

We conclude that the K filters (including atmosphere and detectors) are almost identical (the same conclusion was reached by Bessell & Brett), while the J filters slightly differ. We will test in Sect. 3 whether this has a measurable effect on the adopted surface brightness relations.

Since the V and K observations were not obtained simultaneously, we adopted the procedure to fit Fourier series of appropriate orders to the K light curves and from this calculate the K values at the phases of the observed V magnitudes. The precision of such a K magnitude is generally better than 0.01 mag. Also, there was little problem with the adopted period values because these are well established and the epoch difference between the Laney & Stobie K observations and the Gieren and

Table 2. Slopes of near-infrared surface brightness relations from Cepheids

Cepheid	Period	$V-K$		$J-K$	
	(days)	slope	correlation	slope	correlation
BF Oph	4.07	-0.133	0.996	-0.094	0.879
T Vel	4.64			-0.130	0.942
CV Mon	5.38	-0.121	0.999	-0.077	0.903
V Cen	5.49	-0.125	0.999	-0.091	0.924
U Sgr	6.74	-0.132	0.998	-0.104	0.949
U Nor	12.64	-0.127	0.999	-0.111	0.960
RZ Vel	20.40	-0.132	0.998	-0.133	0.966
WZ Sgr	21.85	-0.131	0.998	-0.118	0.963
SW Vel	23.44	-0.132	0.997	-0.112	0.973
RY Vel	28.14	-0.135	0.998	-0.108	0.929
U Car	38.81	-0.137	0.998	-0.134	0.982

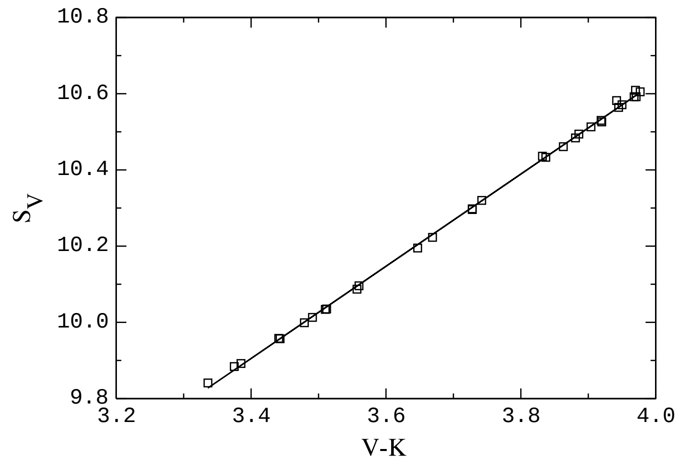

Fig. 1. The variation of the visual surface brightness of the cluster Cepheid CV Mon with its $V-K$ colour index during its pulsation cycle. Overplotted is a least-squares fit to the data

Coulson & Caldwell V and radial velocity observations is quite small. From this, we established $S_{\rm V}$ and $V-K$ pairs at different phases and calculated the slope of the relation followed by each variable. The resulting values are given in Table 2. For all the variables, we found excellent linear relationships with typical correlation coefficients of 0.998. A typical example is given in Fig. 1. These excellent linear relationships indicate that the adopted mean radii, which were taken from Laney & Stobie (1995), are very nearly correct, although the adopted value of the mean radius is not critical in the calculations of the surface brightness: for the longest period Cepheids, where the relative radius variation is largest, a change of the adopted radius of 15% produces a change in the slope of 0.003. A plot of the resulting slope values against pulsation period shows that there is no significant dependence of the slope on period, and we adopt a constant mean value of -0.131 ± 0.002 for the slope shown by Cepheids on the $F_{\rm V}$ vs. $V-K$ diagram, which is valid at least for the period range from 4 to 40 days. Within 2σ, this value is compatible with the one found to hold for giants and supergiants in the preceding Section.

3.3. The slope on the $F_{\rm K}$ vs. $J-K$ diagram

To determine the slope shown by the Cepheids on this diagram, we used the same variables given in Table 2. We took the $J-K$ values from the tabulation of Laney & Stobie (1992) and calculated the surface brightnesses in the K band from the K magnitudes and the displacements calculated from the radial velocity curves at the phases of the J and K observations. For all the variables, a linear relationship between $S_{\rm K}$ and $J-K$ is followed during the pulsation cycles, but there is more scatter than in the near-dispersionless relationships between $S_{\rm V}$ and $V-K$, which reflects itself in a larger scatter among the slope values determined for the different variables, which are given in Table 2. However, as in the case of the other diagrams discussed above, there is no significant dependence of the slope on pulsation period. From our determinations, we adopt as the appropriate value for Cepheid variables -0.110 ± 0.006 for the slope on the $F_{\rm K}$ vs. $J-K$ diagram, which again is in excellent agreement with the value determined in the preceding Section from the stable giants and supergiants.

3.4. Adopted surface brightness relations for Cepheid variables

We now adopt the slopes derived from the Cepheid variables themselves in the preceding paragraphs and force these slope values separately to the same samples of giants and supergiants which were used to establish the surface brightness relations of Sect. 2. We do not apply the conversion relations between Johnson and Carter infrared photometric systems, but test the influence of such conversions. All this yields the zero points and standard deviations given in Table 3. Note that the supergiants zero point of the $F_{\rm V\circ}$ vs. $(V-R_{\rm J})_\circ$ relation is obtained after rejection of discrepant data for HR 7735 (without this rejection, the zero point is 3.924 and the rms 0.043), while the giants zero point of the $F_{\rm V}$ vs. $V-K$ relation is obtained after rejection of HR 6056 and HR 6705 (without these rejections, the zero point is 3.953 and the rms 0.010).

Clearly, there is no significant zero point difference between the giants and supergiants in any of these relations. As our final value , we adopt a weighted mean of the six zero point determinations, which yields 3.947 $\pm$ 0.003. Should we correct Johnson's $J-K$ values to Carter system, the zero points in the last line of Table 3 would become 3.951 and 3.947, leading to an unchanged weighted mean of the six zero points.

This new value 3.947 is slightly smaller than the GBM adopted value of 3.964 ± 0.003, which was an average of values coming from model atmosphere calculations (3.977 $\pm$ 0.008), and from an effective temperature scale established from the work of Pel (1978) on short-period Cepheids (3.953 $\pm$ 0.003). Using the Pel $T_{\rm eff}$ data on 17 Cepheids with well determined $V-R_{\rm J}$ mean colours, together with the mean slope of -0.380 and the reddening law adopted in this paper, the modified

Table 3. Mean zero points of the 3 surface brightness relations

Relation	Slope	Giants			Supergiants		
		zero point	rms	N	zero point	rms	N
$F_{V\circ}$ vs. $(V-R_J)_\circ$	-0.380	3.927	0.011	10	3.932	0.031	12
$F_{V\circ}$ vs. $(V-K)_\circ$	-0.131	3.949	0.004	8	3.949	0.034	13
$F_{K\circ}$ vs. $(J-K)_\circ$	-0.110	3.948	0.008	10	3.945	0.026	13

zero point of the $F_{V\circ}$ vs. $(V-R_J)_\circ$ relation turns out to be 3.949 ± 0.003, in excellent agreement with our adopted surface brightness zero point from the measured angular diameters of giants and supergiants. We therefore conclude that the value from the model atmosphere calculations is too high.

Another possible check on our surface brightness zero point comes from the angular diameter of the Cepheid ζ Gem which was measured with the lunar occultation technique by Ridgway et al.(1982). This is so far the only Cepheid for which a direct angular diameter measurement has been obtained. From the measured value of 1.81 ± 0.31 milliarcsec and the $VRJK$ magnitudes of the variable corresponding to the pulsation phase at the time of the angular diameter measurement as listed by Ridgway et al., together with a colour excess of $E(B-V) = 0.018$ (Fernie 1990), we find from Eqs. 1 and 2 and our adopted slopes a zero point of 3.935 ± 0.038 from the $F_{V\circ}$ vs. $(V-R_J)_\circ$ relation, a value of 3.931 ± 0.038 from the $F_{V\circ}$ vs. $(V-K)_\circ$ relation, and a value of 3.928 ± 0.038 from the $F_{K\circ}$ vs. $(J-K)_\circ$ relation, respectively. The quoted uncertainties correspond to the uncertainty of the angular diameter measurement (which is, unfortunately, rather large). Clearly, within these uncertainties the zero point values agree with the zero point derived in our present study and lend some further support to our hypothesis that Cepheids and nonvariable giants and supergiants of similar colours do follow the same surface brightness – colour relations.

The final surface brightness relations we adopt for Cepheid variables are then the following:

$$F_{V\circ} = 3.947 - 0.380\,(V-R_J)_\circ, \quad (26)$$
$$F_{V\circ} = 3.947 - 0.131\,(V-K)_\circ, \quad (27)$$
$$F_{K\circ} = 3.947 - 0.110\,(J-K)_\circ. \quad (28)$$

These relations are shown in the three following figures, one for each relation, namely Figs. 2 – 4. Superimposed are the data points for the 28 stars with measured angular diameters with different symbols described in each figure caption.

3.5. Validity of the adopted reddening law

From the slope ratios, we can obtain a check on the adopted reddening ratios. Indeed, $F_{V\circ}$ (resp. $F_{K\circ}$) should be the same using any of the relations. We get:

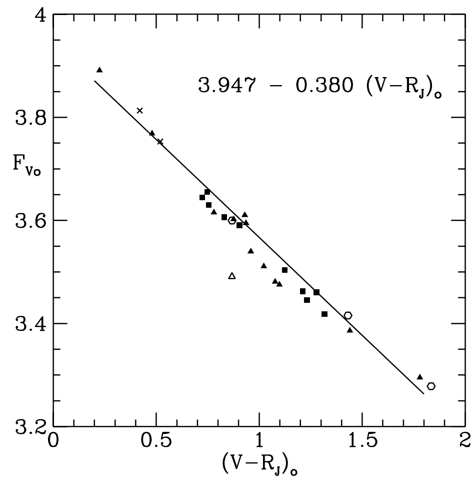

Fig. 2. The $F_{V\circ}$ vs. $(V-R_J)_\circ$ surface brightness relation, with 28 stars with measured angular diameters superimposed. The 10 giants which were used to establish the giants relation and the mean zero point are shown with filled square symbols; the 12 supergiants which were used to establish the supergiants relation and the mean zero point are shown with filled triangle symbols; the rejected supergiant (HR 7735) is shown with an open triangle symbol; the three a priori excluded giants (HR 5299, HR 5340, HR 7951) are shown with open hexagon symbols; and the two dwarfs, namely HR 2943 and the Sun, are shown with crosses. Cepheids are found in the colour range 0.4 – 0.9

$$\frac{E(V-R_J)}{E(V-K)} = \frac{0.131}{0.380} = 0.34, \quad (29)$$
$$\frac{E(J-K)}{E(V-K)} = \frac{0.131-0.1}{0.110} = 0.28. \quad (30)$$

Using our adopted value of $\frac{E(V-K)}{A_V} = 0.88 \pm 0.02$, this gives:

$$\frac{E(V-R_J)}{A_V} = 0.30 \pm 0.02, \quad (31)$$
$$\frac{E(J-K)}{A_V} = 0.25 \pm 0.06. \quad (32)$$

The first of these results justifies our choice of $\frac{E(V-R_J)}{E(B-V)} = 0.97$ from Hindsley & Bell (1989), among various discrepant values discussed in Sect. 2.3 of this paper. The second result is

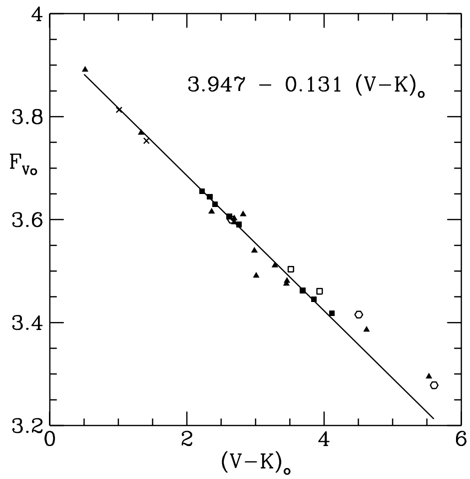

Fig. 3. The $F_{\mathrm{V}\circ}$ vs. $(V-K)_\circ$ surface brightness relation, with 28 stars with measured angular diameters superimposed. The 10 giants which were used to establish the giants relation are shown with square symbols: the 2 giants rejected in the zero point determination (HR 6056, HR 6705) are shown with open square symbols, and the remaining 8 giants with filled square symbols; the 13 supergiants which were used to establish the supergiants relation and the mean zero point are shown with filled triangle symbols; the 3 a priori excluded giants (HR 5299, HR 5340, HR 7951) are shown with open hexagon symbols; and the two dwarfs, namely HR 2943 and the Sun, are shown with crosses. Cepheids are found in the colour range 0.8 – 2.4

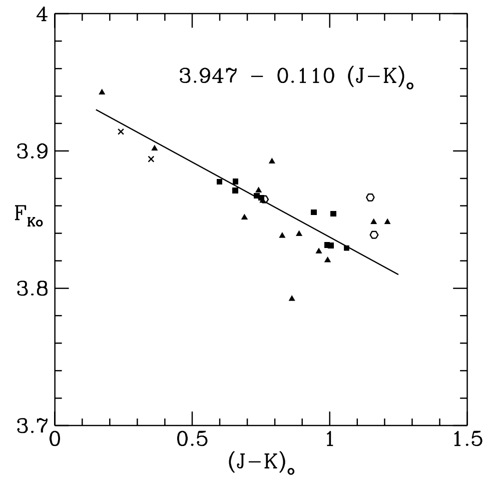

Fig. 4. The $F_{\mathrm{K}\circ}$ vs. $(J-K)_\circ$ surface brightness relation, with 28 stars with measured angular diameters superimposed. The 10 giants which were used to establish the giants relation and the mean zero point are shown with filled square symbols; the 13 supergiants which were used to establish the supergiants relation and the mean zero point are shown with filled triangle symbols; the three a priori excluded giants (HR 5299, HR 5340, HR 7951) are shown with open hexagon symbols; and the two dwarfs, namely HR 2943 and the Sun, are shown with crosses. Cepheids are found in the colour range 0.4 – 0.7. Note the different ordinate scale, as compared to Figs. 2 and 3

slightly larger than our adopted value 0.17, but other choices would have led to even smaller values of this ratio. Given the relatively large uncertainty of the value derived from the slope ratio in this case (a ratio of small numbers is involved), we do not regard this slight discrepancy as real.

4. The distance and radius of the cluster Cepheid U Sgr

In order to carry out a first test of the new surface brightness relations obtained in the preceding Section, we apply the Barnes-Evans optical and near-infrared technique to the cluster Cepheid U Sgr. This particular Cepheid has not only excellent observational data for this kind of analysis, but is furthermore a member in the open cluster M 25 and thus has an independent determination of its distance from cluster ZAMS-fitting which can be compared to the surface brightness results. U Sgr has also been used by Welch (1994) to test his calibration of the K, $V-K$ relation obtained from giant and supergiant angular diameters then available for calibration.

4.1. Observational data

Since the calibrating angular diameters are good to a few percent, we need observations of the highest possible quality to ensure that the resulting radius and distance are not degraded further. We use the V and $V-R_{\mathrm{J}}$ observations of Moffett & Barnes (1984) on the Johnson system and combine them with the observations of Gieren (1981a) to produce a high-quality V light curve and $V-R_{\mathrm{J}}$ colour curve of the Cepheid, consisting of 81 points providing excellent phase coverage. The Gieren R, I observations were obtained on the Cousins system and were transformed to the Johnson system using the relation (Cousins 1981):

$$V-R_{\mathrm{J}} = 0.587\,(V-R_{\mathrm{C}}) + 0.413\,(V-I_{\mathrm{C}}) + 0.055. \qquad (33)$$

In fact, the original Cousins relation has a mean offset of 0.03. But this offset varies e.g. with right ascension, and we found that for our particular star the appropriate value was 0.055, giving an excellent fit to the Moffett & Barnes $V-R_{\mathrm{J}}$ curve. No zero point shift was necessary in V. We further use the excellent J and K light curves of Laney & Stobie (1992; 30 points) which were measured on the Carter system. Finally, we adopt the excellent CORAVEL radial velocity curve of the variable of Mermilliod, Mayor & Burki (1987). While the radial velocities and the optical observations were obtained nearly simultaneously, the near-infrared photometry was obtained a few years later, but in view of the apparently very stable and precisely known period of U Sgr, confirmed by the adoption by Laney & Stobie of Pel's period (1976), this does not introduce the possibility of a significant phase mismatch between the infrared light curves and the optical light curve and radial velocity curve (anyway, a phase mismatch, if existent, can be detected in the surface brightness solutions, representing one of the strengths of the Barnes-Evans technique).

4.2. The method

Using our new calibrations of the surface brightness parameter in V and K with colour (Eqs. 26 – 28) and combining them with Eqs. 1 and 2, we get the following equations for the angular diameter of a Cepheid in terms of the different magnitudes and colours:

$$\phi = 10^{0.5474 - 0.2\, V_\circ + 0.760\, (V - R_J)_\circ}, \quad (34)$$

$$\phi = 10^{0.5474 - 0.2\, V_\circ + 0.262\, (V - K)_\circ}, \quad (35)$$

$$\phi = 10^{0.5474 - 0.2\, K_\circ + 0.220\, (J - K)_\circ}. \quad (36)$$

Note that while the adopted zero point 3.947 depends on the precise value of the adopted constant 4.2207 in Eq. 3, the resulting diameters are independent of it, therefore do not depend of a given choice of the bolometric absolute magnitude of the Sun.

The unreddened magnitudes and colours are calculated with the relations adopted in Sect. 2.3, using $E(B-V) = 0.438$ (Laney & Stobie 1992). We then get the angular diameter variation of U Sgr from each of these formulae from the pairs of a magnitude and colour index obtained at the same phases. While the V, $V - R_J$ and J, K pairs were obtained simultaneously, this is not the case of the V and K observations. In order to apply Eq. 35, we therefore fitted a Fourier series of order 3 to the Laney & Stobie K light curve, which gave an excellent representation of the measured data, and calculated from this the K magnitudes at the phases of the actually observed V datum points; from this, the $V - K$ values at the phases of the V observations were obtained. To illustrate the precision of this process, we show the resulting $V - K$ colour curve of U Sgr in Fig. 5. To obtain the linear displacements of the stellar surface at the phases of the photometric observations, we integrated the radial velocity curve adopting a projection factor of 1.365 which was calculated from the formula given by GBM. The adopted period value for U Sgr is 6.744925 days (Pel 1976). The mean radius and distance of the Cepheid variable is then obtained from a linear regression of the relation:

$$D_\circ + \Delta D = 10^{-3}\, d\, \phi, \quad (37)$$

where $D_\circ$ is the mean linear diameter in AU, ΔD the displacement from the mean in AU, d the distance of the star in pc, and ϕ the angular diameter in milliarcsec.

4.3. Results

The radius and distance values for U Sgr obtained from the three different surface brightness relations, and their errors, are summarized in Table 4. The corresponding plots for the three cases, showing **a)** the angular diameters together with the linear displacement curve and **b)** the linear displacements vs. the angular diameters (the slope of which gives the distance of the star) are shown in Figs. 6 – 11. There is no need to apply any phase shift between the linear displacement and the angular diameter curves, in none of the three solutions. It is impressing to see how

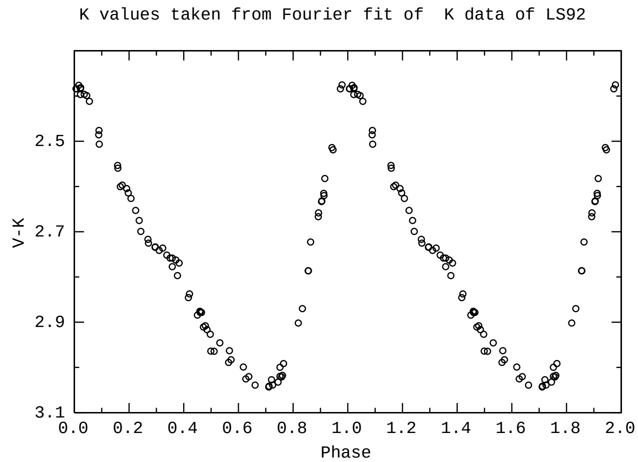

Fig. 5. The $V - K$ colour curve of U Sgr. A third-order Fourier series was fitted to the K light curve of Laney & Stobie (1992) and from this the K values at the phases of the V observations of Gieren (1981a) and Moffett & Barnes (1984) were obtained

the angular diameters obtained from the V, $V - K$ pairs adjust to the linear displacement curve obtained from the integrated radial velocity curve, in a diagram which is almost scatter-free. The error in the distance and radius is less than 1% in this case. This very small error is due to a combination of two effects: first, to the shallow slope of the $F_{V_\circ}$ vs. $(V-K)_\circ$ surface brightness relation, as compared to the $F_{V_\circ}$ vs. $(V - R_J)_\circ$ relation, which permits a more accurate correction for the temperature-induced variation of the surface brightness of a Cepheid, and second, to the much reduced sensitivity of the $V - K$ colour index to variations in gravity and microturbulence during the pulsation cycle, as shown by Laney & Stobie (1995). The situation is somewhat less favourable in the K, $J - K$ solution where the error is 3%, but this is still very good taking into account that only 30 J, K data points have been available for this analysis, and that the fit of the K light curve for the V, $V - K$ solution artificially reduces the observational scatter; with a comparable number of data as in the V, $V - K$ solution, the error would probably decrease to 1-2%. Also very importantly, there are no systematic effects in these two solutions employing a near-infrared colour index which could produce a significant difference in the radius and distance if one restricts the solution to a particular phase range, like descending or ascending branches of the light curve, which is generally not true for the V, $V - R_J$ solution. The results given in Table 4 are derived from bisector solutions to Eq. 37 which assumes equal errors in the angular diameters and linear displacements. In the case of the near-infrared solutions, this assumption is not critical, and simple least-squares fits produce the same radius and distance results (the difference is smaller than 0.5σ). In the case of the V, $V - R_J$ solution, however, the difference between the bisector and least-squares solution is 3σ in the case of U Sgr and thus clearly significant.

There is clearly much more scatter in the optical V, $V - R_J$ solution. However, and perhaps surprisingly, the distance and radius obtained from the optical version of the method agrees

Table 4. Surface brightness solutions for the distance and radius of U Sgr

Mag / colour	distance pc	acc %	$\mu_\circ$	err	radius $R_\odot$	acc %	phase shift	N
$V, V-R_J$	590	6.3	8.85	0.13	48.1	6.3	0	81
$V, V-K$	592	0.7	8.86	0.02	48.7	0.7	0	81
$K, J-K$	610	3.0	8.93	0.06	49.5	2.8	0	30

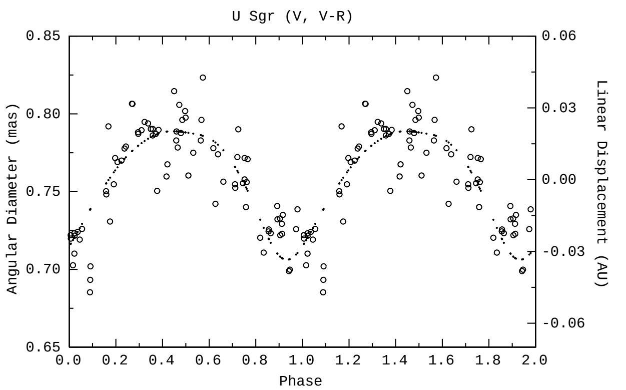

Fig. 6. The angular diameters (open circles) of the Cepheid U Sgr calculated from the V, $V-R_J$ photometry according to Eq. 34 (see text), plotted against phase. Overplotted (dots) is the linear displacement variation calculated from the integrated radial velocity curve of U Sgr

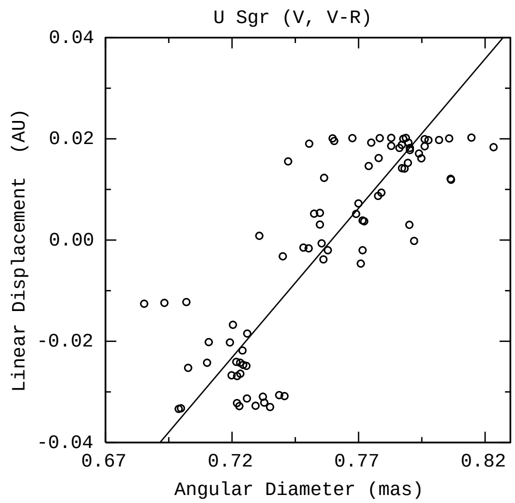

Fig. 7. The linear displacements vs. the angular diameters, calculated from V, $V-R_J$, for U Sgr. Overplotted is a bisector fit to the data which assumes equal errors in both axes. The slope of this line yields the distance of the Cepheid

very well with the results from the near-infrared solutions, albeit with an error about 5-10 times larger. Whether this is a coincidence or a general feature remains to be further investigated.

It is very satisfactory to see that the three different surface brightness solutions all agree within 1σ. A weighted mean for the distance and radius of U Sgr from the present results is d = 593 $\pm$ 4 pc (corresponding to a distance modulus of 8.87) and $R_\circ$ = 48.7$\pm$0.3 solar radii. The mean radius compares very well with the Laney & Stobie (1995) infrared value $R_\circ$ = 51.4 solar radii. The distance may be compared to the ZAMS-fitting distance modulus of M 25 which is 8.95 (Pel 1985). It may also be compared to the distance of 660 $\pm$ 24 pc obtained by Welch (1994) from his calibration of the K, $V-K$ surface brightness relation: we attribute the difference with our result to our improved calibration which was able to take advantage of an increased number of stars with accurate angular diameters, as well as to the inclusion of the Gieren (1981a) V data in the surface brightness analysis of this paper.

We will give a detailed analysis of possible systematic errors in the method in a follow-up paper (Gieren, Fouqué, & Gómez 1997), where a large sample of cluster Cepheids will be analysed in the same way, which will permit to identify possible trends and problems more safely than from the analysis of just one star. This discussion will include the problem of a possibly variable p-factor (Sabbey et al.1995; Sasselov & Karovska 1994) which is common to all Baade-Wesselink – type methods. We just note here that the close agreement in shape between the linear displacement curve of U Sgr (derived in our study under the assumption that p = const) and the angular diameter curve of the star obtained from infrared photometry (see Figs. 8 and 10) lends support to the idea that p = const is a good assumption in the case of U Sgr: neglect of a significant variation of p during the Cepheid's pulsation cycle should have distorted the linear displacement curve from its true shape, an effect for which there is no indication in Figs. 8 and 10, and it thus appears that a variable p-factor is an effect of second-order importance in the determination of the radius and distance of U Sgr from the Baade-Wesselink infrared technique.

5. Conclusions

Our work shows that the zero point of the surface brightness - colour relations valid for cool giants and supergiants can now be obtained with high accuracy from the many precise angular diameters now available for such stars from Michelson interferometry. Our results indicate that giants and supergiants follow the same relations. The slopes of the various surface brightness – colour diagrams as determined from Cepheid variables are

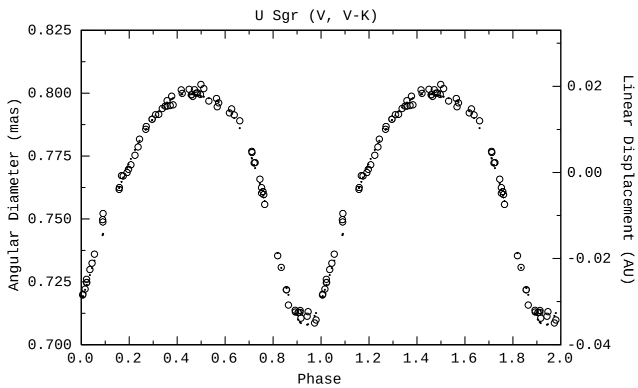

Fig. 8. The angular diameters (open circles) of the Cepheid U Sgr calculated from $V, V-K$ photometry according to Eq. 35 (see text), plotted against phase. Overplotted (dots) is the linear displacement variation of the Cepheid. Note how the angular diameters adjust in an almost dispersionless fashion to the linear displacement curve of the variable, leading to errors of less than 1% in the determination of the distance and radius of the Cepheid from Eq. 37 (see text)

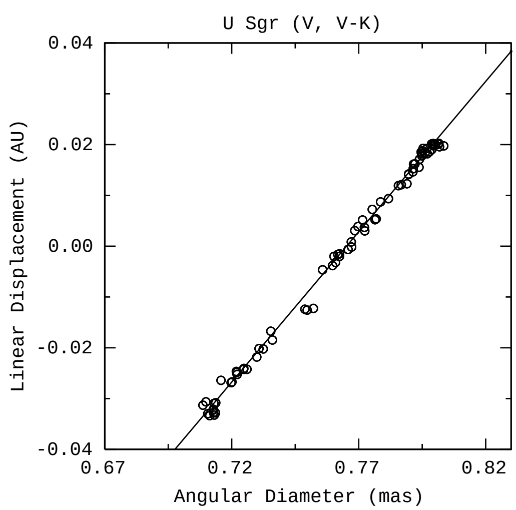

Fig. 9. The linear displacement vs. angular diameter plot, corresponding to the data in Fig. 8. Overplotted is the bisector fit to the data (identical to the least-squares fit in this case) which yields the distance of U Sgr

indistinguishable from the slopes defined by the stable giants and supergiants, and do not depend in a significant way on the pulsation period or luminosity of the Cepheids, in agreement with the fact that the slopes are also identical for giants and supergiants. The agreement of our surface brightness – colour zero point with the one derived from the measured angular diameter of the Cepheid ζ Gem further supports the evidence that Cepheids and stable giants and supergiants do follow identical surface brightness – colour relations.

The results of the optical and near-infrared Barnes-Evans solutions on the Cepheid U Sgr suggest that there is no significant systematic difference between the distance and radius results as obtained from the three versions of the technique we employ, but

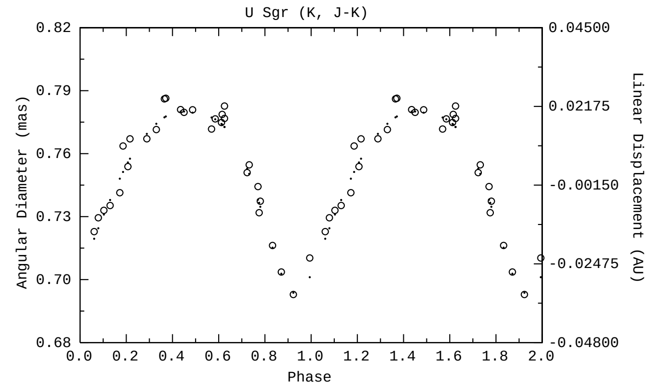

Fig. 10. The angular diameters (open circles) of the Cepheid U Sgr calculated from near-infrared $K, J-K$ photometry according to Eq. 36 (see text), plotted against phase. Overplotted (dots) is the linear displacement variation calculated from the integrated radial velocity curve of U Sgr. Note the marked increase in the accuracy of the angular diameters, as compared to Fig. 6

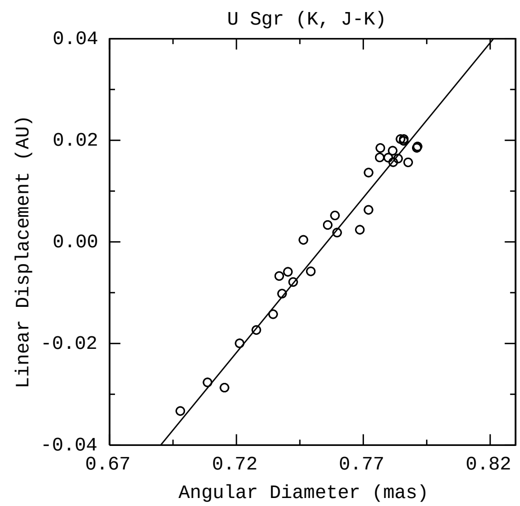

Fig. 11. The linear displacements vs. the angular diameters, calculated from $K, J-K$, for U Sgr. Overplotted is the bisector fit to the data

there is a huge gain in accuracy going to the infrared. Whether there is a systematic difference between the optical and near-infrared solutions will have to be further investigated, applying the different techniques to many more variables. However, even if it turns out that there is no systematic offset between optical and near-infrared Barnes-Evans distances and radii of Cepheid variables, it is clear that the infrared is the region to work because of the very large increase in the accuracy of the results.

We believe that the application of our near-infrared calibrations of the Barnes-Evans technique has the potential to settle the zero point of the Cepheid period-luminosity relation to about 0.02 mag. The most precise distances may be expected to the Cepheid-rich Magellanic Cloud clusters which are clearly within the reach of the technique, and where the individual distances to cluster Cepheids can be averaged to yield very accurate distances of these clusters. From this kind of work, it should be possible in the near future to eliminate the weakest link in the application of Cepheids as our most trusted extragalactic distance indicator, which is the relatively uncertain local calibration of Cepheid distances.

Acknowledgements. This work was completed while W.P. Gieren was a Senior Visitor at E.S.O. He gratefully acknowledges the support received during his stay.

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