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The distance scale to globular clusters through new horizontal branch models

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Abstract. As a completion of preceding work (Mazzitelli et al. 1995), we present new horizontal branch models with mass fractions of heavy element content Z= 2×10^{-4} and 6×10^{-4}; we discuss the complete set of models with Z from 1×10^{-4} to 4×10^{-3}, and their implications for the ages of galactic globular clusters.

Models at low Z are ~ 0.15 mag more luminous than equivalent models of the previous generation, implying a reduction of globular cluster ages of ~ 2 Gyr. Half of this effect depends on the core masses at the helium flash, which are larger by 0.007 $M_\odot$ at Z=1×10^{-4}; the remaining half depends on the new microphysical inputs (equation of state and opacities) in present models.

The distance scale resulting from these new models for $[Fe/H] \lesssim -1$ is consistent with Sandage (1993) and Walker (1992) scales, but not with Carney et al. (1992) scale. The relation M_v(ZAHB) *vs.* $[Fe/H]$ is not linear, so that the notion of slope turns out to be an elusive one, and in any case depending on the range of $[Fe/H]$ considered.

New computations of the core mass at the helium flash are presented and discussed. We finally show that the luminosity at the tip of the giant branch obtained in our computations is compatible with the maximum luminosity of giants observed in globular clusters by Frogel et al. (1983).

Key words: stars: evolution – stars: horizontal branch models – globular clusters – distance scale

1. Introduction

The problems of the distance scale and of the age of globular clusters are strictly intertwined, since age is determined through the absolute magnitude of the turn–off region of a cluster. A common procedure to derive the age is to adopt the distance scale based on the luminosity of RR Lyrae variables (e.g. VandenBerg et al. 1996). This empirical luminosity can be checked on the basis of the luminosity of the zero age horizontal branch (ZAHB) models at the position of the variable gap (taking into account also the generally small evolutionary corrections). The relevance of an accurate evaluation of the luminosity of horizontal branch (HB) models is therefore evident.

In Mazzitelli et al. (1995), we proceeded to an update of the physical inputs and computed a first set of HB evolutionary tracks ($Z = 1 \times 10^{-4}, 3 \times 10^{-4}, 1 \times 10^{-3}, 2 \times 10^{-3}$ and 4×10^{-3}). We give here additional models with $Z = 2 \times 10^{-4}$ and 6×10^{-4}, which complete the description of HB behavior for crucial values of the heavy element content in the comparison with globular cluster color magnitude diagrams, and discuss the results in the framework of present theoretical and observational constraints. Since small changes (mainly minor improvements in the numerics) had been introduced in the ATON stellar evolutionary code, we recomputed all the models: the ZAHB locations are unchanged with respect to Mazzitelli et al., only a minor reshuffling of some of the larger masses along the ZAHB locus was found. The full tables of models are available via Internet at the address: http://www.mporzio.astro.it.

2. The models

The evolutionary tracks computed are summarized in Table 1. The ATON code and the input physics are described in Mazzitelli et al. (1995): they include Kurucz (1993) and OPAL updated opacities (Rogers and Iglesias 1992a, 1992b and 1993) and the equation of state by Rogers et al. (1996); convection treatment is according to Canuto and Mazzitelli (1991). The theoretical values were transformed into M_v and $B-V$ *via* Kurucz (1993) relations.

Since thermodynamics and opacities for the metal abundances Z=2×10^{-4} and 6×10^{-4} considered in the present work are not among the original OPAL tables, we interpolated

Send offprint requests to: F. D'Antona linearly the logarithm of the Rosseland mean opacity (log κ) *vs.* log Z among the standard OPAL tables, and used the code provided by Rogers et al. (1996) to evaluate thermodynamic tables for each chemical abundance. On the thermodynamics, see later in the section about helium cores at the flash.

We find HB luminosities larger than in previous published computations; recent unpublished models by Straniero and Chieffi (1996) and Salaris et al. (1996), —in which analogous updates in the input physics have been included— are in close agreement with our results. We shall examine the reasons for such a difference. For the sake of argument, we chose the models by Castellani et al. (1991) (comparisons with results by Dorman (1992) would have been more tricky, due to the large oxygen enhancement in these latter HB models.)

For every Z, models by Mazzitelli et al. have a core mass M_c larger than in Castellani et al.; for $Z = 1 \times 10^{-4}$, we have M_c= 0.512 and $\log L/L_\odot$ (at $T_{\rm eff}$ = 3.83) = 1.779, while in Castellani et al. one has M_c= 0.505 and $\log L/L_\odot$ = 1.718. [1] So the difference, at a given $T_{\rm eff}$, is $\sim$ 0.15 mag. This difference alone implies ages about 2 Gyr younger than those derived from Castellani et al. if equal clocks in the two turn–off isochrone sets are assumed.

Less than half of this difference in HB luminosity is due to the core mass, larger in our case by 0.007 $M_\odot$. In fact, according to the well established relation – which we also confirm with present models – $\delta logL/L_\odot/\delta M_c \sim 3$ (e.g. Sweigart and Gross 1976), the increase in M_c accounts for an increase in the ZAHB level of $\sim$ 0.06 mag only. The remaining difference (0.09 mag) must be attributed to the difference in the micro-physics employed, and mainly to the inclusion of Debye's screening in Rogers et al. (1996) EoS.

3. The distance scale and the $M_v(ZAHB)$ vs. [Fe/H] relation

3.1. *The zero point of the absolute magnitudes and the role of bolometric corrections*

Fig. 1 illustrates the situation in the observational reference frame. The upper panel shows the observational results, summarized by the two (extreme) relations for RR Lyrae star luminosity: by Carney et al. (1992; dotted line):

$$M_v(RR) = 0.15[Fe/H] + 1.01; \quad (1)$$

and by Sandage (1993; dash–dotted line):

$$M_v(RR) = 0.30[Fe/H] + 0.94; \quad (2)$$

The relation by Walker (1992; dashed line) is also shown: it has the same slope as in Carney et al. but a zero point equal to 0.73 mag (instead of 1.01). Remember however that Walker *did not* derive a *slope* for his relation, but simply revised Carney's zero point for the absolute magnitude scale, on the basis of observations of RR Lyrae variables in the Large Magellanic Cloud (adopted distance modulus: 18.50 mag).

In Table 2 we give theoretical luminosities and masses for ZAHB models, at log$T_{\rm eff}$ 3.83 and 3.85. We also give the M_v values at the same $T_{\rm eff}$'s, obtained by means of Kurucz (1993) bolometric corrections. Notice that the BCs are normalized to $BC_\odot = -0.197$ (Kurucz 1991); we adopt $M_{v\odot}$=4.79, so that $M_{bol\odot}$=4.59. We refer to these values $M_v([Fe/H])$ as to our "standard" ZAHB relation. In Fig. 1 these values are shown as full squares.

Also shown (lower panel) are data derived from Castellani et al. (1991, CCP), by means of two different relations found in the literature: the dash–dotted line follows the interpolation formula given in Salaris et al. (1993); the dashed line follows the relation given by Castellani et al. (1993). Dorman (1992) models (not shown here) are less luminous at a given $[Fe/H]$, but this is mainly due to the high α enhancement (see Fig. 10 in Mazzitelli et al. 1995).

Alerted by the evidence that somewhat different M_v *vs.* $[Fe/H]$ relations may originate from the same theoretical relation, we looked into the possible uncertainty on M_v one may expect from the most common transformations from (log $L/L_\odot$, log$T_{\rm eff}$) values to ($B-V$, M_v). We had in mind also the remark by Walker (1994), on the caution with which one has to handle the values which depend on "the details of the bolometric corrections and temperature conversion".

Since we were aware of the problems still existing with atmospheric models by Kurucz, and so with colour–temperature transformations and bolometric corrections derived from them, we considered also the transformations by VandenBerg (1992). Assuming these latter BCs, we have $BC_\odot = -0.12$ and we must assume $M_{bol\odot}$ = 4.67 to get the same $M_{v\odot}$, which is the relevant observational value. (Of course, making an overall scaling does not warrant against systematic differences in the model atmospheres which can *vary* with $T_{\rm eff}$). By adopting VandenBerg relations we obtain, for our models, the dotted line presented in the upper panel of Fig. 1, $\sim$ 0.08 mag more luminous than our standard sequence.

At present, the most widely accepted value for $M_{v\odot}$ is 4.82$\pm$ 0.03 (Lang 1991; our estimate for the error). Then the value chosen in the present paper ($M_{v\odot}$ = 4.79) leads to the brightest possible values for M_v for a given set of BCs. If we instead assume the other extreme value ($M_{v\odot}$ = 4.85), there will be a systematic shift of 0.06 mag, which is shown by the dotted line below our standard relation. Of course, assuming VandenBerg (1992) BCs and $M_{v\odot}$ = 4.85, we would end up with values much closer to our standard relation. We cannot but conclude that there is still an intrinsic uncertainty of about $\pm$0.07 mag, due to the (log $L/L_\odot$, log$T_{\rm eff}$) transformations, on the zero point of the absolute visual magnitude location of ZAHB models.

A linear regression on the data of Table 2 provides a very good fit to our data (r=0.997). Allowing for the possible uncertainty on the zero point, our relation is:

$$M_v(ZAHB) = 0.26[Fe/H] + (0.905 \pm 0.07) \quad (3)$$

[1] However, Castellani et al. 1991 used weak screening for the 3α reactions, in place of the intermediate one, and this alone accounts for their smaller core mass (section 5).

Table 1. Computed HB models. The evolution for each given mass, until the central helium abundance is reduced to 10%, is available in Internet at the address http://www.mporzio.astro.it .

Z	Y	$M_c/M_\odot$	Label	Masses ($M_\odot$)
1×10^{-4}	0.24	0.5120	CDM14	0.52, 0.54, 0.56, 0.58, 0.62, 0.66, 0.70, 0.75, 0.80, 0.85, 0.90
2×10^{-4}	0.24	0.5078	CDM24	0.51, 0.52, 0.54, 0.56, 0.58, 0.62, 0.66, 0.70, 0.73, 0.75, 0.80, 0.85
3×10^{-4}	0.24	0.5053	CDM34	0.51, 0.52, 0.54, 0.56, 0.58, 0.60, 0.62, 0.64, 0.66, 0.68, 0.70, 0.72, 0.75, 0.80, 0.85
6×10^{-4}	0.24	0.5011	CDM64	0.51, 0.52, 0.54, 0.56, 0.58, 0.60, 0.62, 0.64, 0.66, 0.68, 0.70, 0.75, 0.80, 0.85
1×10^{-3}	0.245	0.4980	CDM13	0.51, 0.52, 0.54, 0.56, 0.58, 0.60, 0.62, 0.64, 0.66, 0.68, 0.70, 0.75, 0.80, 0.85
2×10^{-3}	0.245	0.4935	CDM23	0.50, 0.51, 0.52, 0.54, 0.56, 0.58, 0.60, 0.62, 0.63, 0.64, 0.66, 0.68, 0.70, 0.75, 0.80, 0.85
4×10^{-3}	0.245	0.4890	CDM43	0.50, 0.52, 0.54, 0.56, 0.57, 0.58, 0.59, 0.60, 0.62, 0.66, 0.68, 0.70, 0.75, 0.80, 0.85

Table 2. ZAHB luminosities at $\log T_{eff}$=3.83 and 3.85 (columns 3 and 4), the corresponding M_v (5 and 6) and masses (7 and 8) are given as function of Z(column 2) and $[Fe/H]$(column 1). We assume $Z_\odot$=0.018.

$[Fe/H]$	Z	$\log L/L_\odot$(3.85)	$\log L/L_\odot$(3.83)	M_v(3.83)	M_v(3.85)	$M/M_\odot$(3.83)	$M/M_\odot$(3.85)
−2.255	0.0001	1.764	1.779	.310	.324	0.836	0.816
−1.954	0.0002	1.731	1.741	.401	.401	0.760	0.748
−1.778	0.0003	1.711	1.720	.447	.447	0.727	0.717
−1.477	0.0006	1.674	1.682	.537	.541	0.677	0.671
−1.255	0.001	1.664	1.671	.563	.558	0.650	0.646
−0.954	0.002	1.621	1.629	.650	.660	0.617	0.614
−0.653	0.004	1.580	1.586	.740	.742	0.591	0.589

If we are interested only in the luminosity level at a given $[Fe/H]$, the flattening of the data close to Z=1×10^{-3} does not affect the relation too much, the maximum difference between the tabulated values and the computed values being -0.014 mag at Z=6×10^{-4}. However, see section 3.3.

In order to compare the ZAHB luminosity level with that of the RR Lyrae variables, one has to allow for the evolutionary increase in luminosity over the ZAHB, an effect probably varying with $[Fe/H]$. In the literature we can find a large variety of estimates for this effect. From inspection of our models, RR Lyrae variables average magnitude should be $\sim$ 0.06 mag lower than the ZAHB level, with a weak dependence on Z. Similar conclusions (~ -0.1 mag luminosity increase in the range Z=0.0001 to 0.0004) are reached, e.g., by Caputo and Degl'Innocenti (1995) by means of synthetic HB models.

Thus the M_v at the RR Lyrae gap from present ZAHB models is close to values given by the relations by Sandage and Walker quoted above. In particular, the zero point is certainly not consistent with Carney et al. (1992) relation, in spite of the uncertainty in BCs.

If we use our models to fix the distance to a cluster, we shall find ages $\sim$ 2 Gyr lower than had we used, for example, the models by Castellani et al. (1991), (*with the same assumptions about the BCs!*), due to the increased distance scale. Since we have shown (Mazzitelli et al. 1995, D'Antona et al. 1996) that also the turn–off age is lowered, for a given distance modulus, by the improvements in the input physics, we find that, on the whole, the age of galactic globular clusters can be decreased to values ($\sim$ 12 Gyr), no longer inconsistent with the age of the Universe from the determination of the Hubble constant H_0, as long as the latter turns out to be lower than $\sim$ 65 Km s^{-1} Mp^{-1} (open universe) or lower than 60 (Ω = 1).

3.2. Role of the core mass at the helium flash

The present results depend in part on the mass of the hydrogen exhausted core at the helium flash, M_c, being larger than previously assumed (see next section). This is a crucial input in HB computations, but we remark that present differences in the values of M_c justify only a fraction of the increase in the HB luminosity we have reported above, as we have shown in the comparison with Castellani et al. (1991). Let us consider even smaller M_c's, taking, e.g., the core mass values from Sweigart (1994): he would have obtained M_c=0.503 $M_\odot$ (for Z=1×10^{-4} and Y=0.23), so his models would have been $\lesssim$ 0.08 mag less luminous. Still lower is VandenBerg (1996) result: he gets M_c=0.498 $M_\odot$ (for Y=0.235 and Z=1×10^{-4}), so that his HB should be $\sim$ 0.1 mag less luminous than ours.

Summing up all the uncertainties in the same direction, namely: assuming $M_{v\odot}$=4.85, Kurucz (1993) BCs, and Vandenberg's core mass, the ZAHB location remains *at least* 0.15 mag more luminous than the RR Lyrae magnitudes given by Carney et al. relation, which then appears ruled out by all recent HB models (see Fig. 1). A similar conclusion is reached by VandenBerg et al.(1996).

3.3. The slope of the M_v(ZAHB) vs. $[Fe/H]$ relation

The *slope* of the M_v *vs.* $[Fe/H]$ relation is very important in connection to the problems related to the pulsational properties of RR Lyrae stars. We must warn against the use of an "average" slope from our stellar models: the global slope given in equation 3 (0.26) can vary significantly if we select a narrower range of metallicity. It grows up to 0.29 (very close to that given by Sandage 1993) if one considers values from Z=1×10^{-4} to 6×10^{-4}, or from Z=1×10^{-3} to 4×10^{-3}, while it drops to

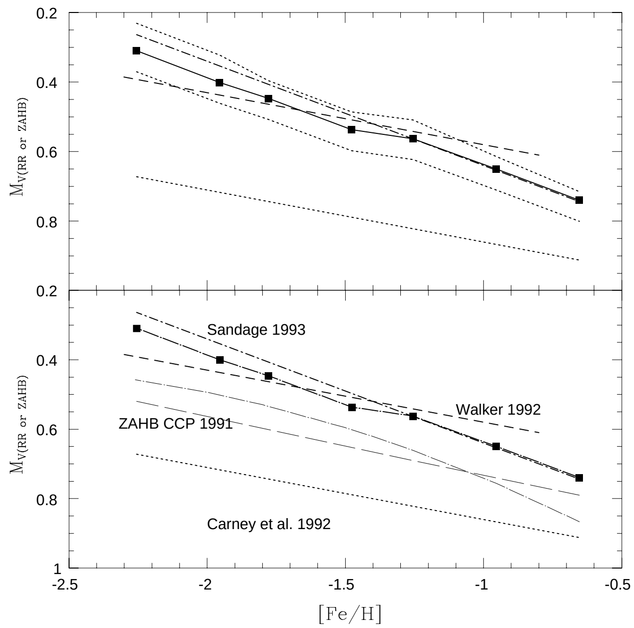

Fig. 1. M_v vs. $[Fe/H]$ relation for ZAHB models at log$T_{\rm eff}$=3.83 from present results (full squares). Upper panel: the variations induced by the adoption of a different conversion law or of a different M_v for the Sun (dotted lines above and below the standard models; see text). Also shown are the observational relations for RR Lyrae stars by Sandage (1993, dot-dashed), Walker (1992: dashed) and Carney et al. (1992: lowest dotted line). Lower panel: besides the observational relations as in the upper panel, the relations from Castellani et al.(1991, see text).

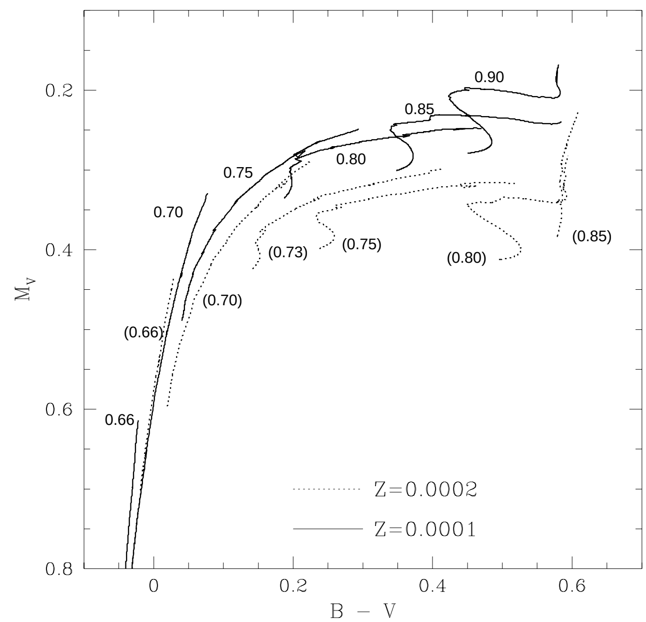

Fig. 2. Variations in luminosity and mass distribution among HB models with Z=1 $\times$ 10^{-4} and 2 $\times$ 10^{-4}.

0.22 (intermediate between Carney et al. 1992 and Sandage's value) in the range from Z=3$\times$$10^{-4}$ to 1$\times$$10^{-3}$. (Notice that this is the ZAHB slope, which should not be directly related to the RR Lyrae slope before allowing for the evolutionary effects.) So we do not consider decisive the very low slopes found by Ajhar et al.(1996) and by Fusi Pecci et al. (1996) from the HBs of globular clusters in M31, until the range in metal content is well established, and the observational errors will be substantially reduced.

In summary, these new models provide on average a larger slope M_v(ZAHB) vs. $[Fe/H]$ than previous computations do. Again we must check how much of this result depends on present core masses being *differentially* larger at low Z than at high Z. We again take advantage of Castellani et al. (1991) results, in the low metallicity range. At Z=1$\times$$10^{-4}$, a decrease of M_c to 0.505 $M_\odot$ would decrease the present ZAHB luminosity by $\simeq$ 0.053 mag. At Z=1$\times$$10^{-3}$ the difference between core masses is only 0.003 $M_\odot$, thus the luminosity of the ZAHB would decrease by $\sim$ 0.022 mag. The slope in equation 3 would decrease from 0.26 to 0.22. Notice however that the core sizes by Castellani et al. (1991) have to be considered superseded by the update in Salaris et al. (1993), who give values very close to those used here (0.511 for Z=1 $\times$ 10^{-4}, 0.500 for Z=1 $\times$ 10^{-3}; see later the section on helium core masses).

4. Description of model results

4.1. The mass distribution on the horizontal branch

The HB models with Z=2 $\times$ 10^{-4} were computed in order to provide a finer grid for the interpretation of very metal poor globular clusters, in which an observed $[Fe/H]$ value of –2.25 (corresponding to Z=1 $\times$ 10^{-4}) may have to be compared with theoretical tracks computed with somewhat larger *equivalent* Z due to the enhancement in the α–elements (Salaris et al. 1993).

An important, and well known, feature in ZAHB population is the dramatic change in ZAHB mass at a given $T_{\rm eff}$ when $[Fe/H]$ is changed. This effect is exemplified, for the low metallicity regime, in Fig. 2, where the mass distributions along the ZAHBs with Z = 1 $\times$ 10^{-4} and 2 $\times$ 10^{-4} are shown: at the RR Lyrae gap, the mass moves from $\sim$ 0.84 to $\sim$ 0.76, a fact of consequence in the interpretation of observed HBs, specially when considering the consistency of RR Lyrae masses with the turn–off ones.

In fact, the age of the most metal poor globular clusters (Z=1$\times$$10^{-4}$), assuming the distance scale suggested here for the HBs, results to be $\sim$ 12 Gyr (Mazzitelli et al. 1995, D'Antona et al. 1996). In this case, the turn off mass is $\sim$ 0.806 $M_\odot$ (for Z=1 $\times$ 10^{-4}) and thus the mass at the red giant tip is $\sim$ 0.84 $M_\odot$. This mass is not inconsistent with the ZAHB mass at the RR Lyrae gap –see Fig. 2 and Table 2. However, a slight increase in the adopted *equivalent* Z for these clusters, while not changing the red giant mass, decreases the HB mass for RR Lyrae variables, further easing the mass consistency problem.

This point is of relevance for very metal poor clusters like M15 and M68, in which also the red part of the HB is populated. While the few RR Lyrae variables in M92 may be evolved from

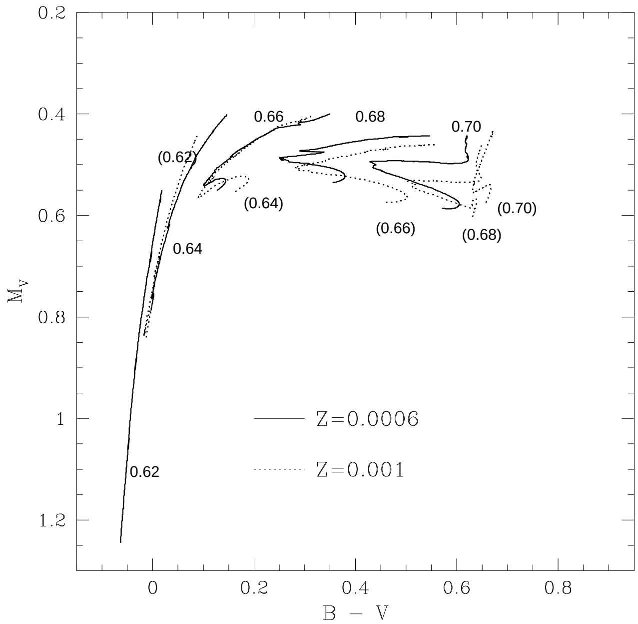

Fig. 3. Variations in luminosity and mass distribution among HB models with Z=6 × 10^{-4} and 1 × 10^{-3}.

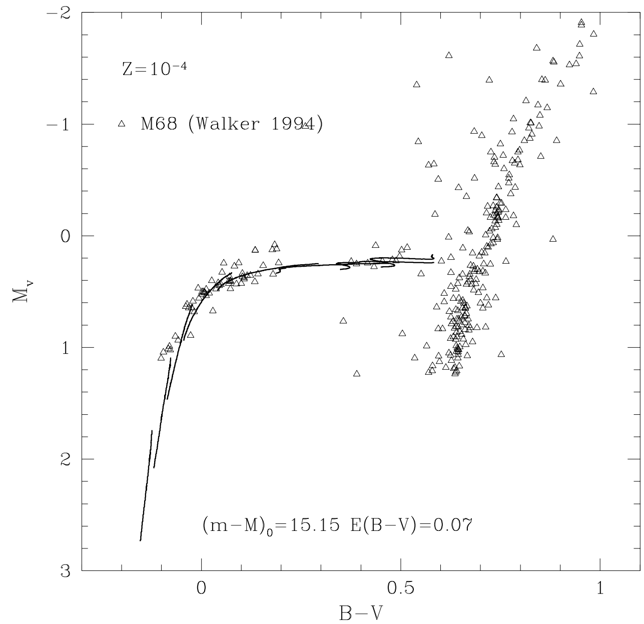

Fig. 4. Transformed theoretical HB models for Z=1 × 10^{-4} and the observed HB of M68.

blue HB stars, and so may have (quite) low mass, surely there are at least some ZAHB RR Lyrae stars in M68 and M15.

The shift in mass with Z along the ZAHB accounts for the similarity between ZAHB luminosities for $Z = 6 \times 10^{-4}$ and 1×10^{-3} . In fact, as shown in Fig. 3, it happens that the change in mass from one Z to the other makes up for the differences in core mass and shell efficiency, so to give two sequences almost superimposed. We warn that the wiggles that appear here and there in the HB tracks are due to a relatively poor zoning in the super–adiabatic region of these models, and have no effect on the luminosity or on the average value of $T_{\rm eff}$.

4.2. Comparison with observations and age estimate

In Fig. 4 the comparison is shown with the metal poor, RR Lyrae rich cluster M68, and in Fig. 5 with the intermediate metal poor cluster M5. For NGC 6397, with an HB all to the blue of the variable gap, we show the comparison with two independent estimates of the positions of HB members: in Fig. 6 with photographic photometry (Alcaino et al. 1987), and in Fig. 7 with $T_{\rm eff}$'s estimated from visual spectra (de Boer et al. 1995). All the fits have been performed with currently accepted values for E_{B-V}. Particularly satisfactory are the fits in Figs. 6 and 7, which adopt the same values for $(M-m)_0$ and E_{B-V}: the two distributions, obtained with completely independent methods, are both reproduced by our models.

Once the distance modulus has been determined by superimposing theoretical and observed HBs (see above), the age can be derived from the ΔV between the HB and the turnoff, and the relation between M_v of the turnoff and age. From the data in D'Antona et al. (1996), we find ages of $\sim 12 \pm 2$ Gyr for the clusters examined. Thus the present HB models help to provide a coherent framework in which the age of globular clusters is significantly reduced.

5. Helium cores at the tip of the red giant branch

We have mentioned that the difference in core masses between our models and other computations in the literature amount to a maximum of 0.014 $M_\odot$ for the Z=1 × 10^{-4} case, and thus may account only for HB luminosities larger by a maximum of 0.1 mag. Nevertheless, the size of the helium core is considered one of the crucial problems in the astrophysics of globular clusters, and it is necessary to present once more the results, the way they have been obtained, and infer the reasons for the discrepancies.

In Mazzitelli et al. (1995) we evolved models up to the red giant tip for various $[Fe/H]$ to evaluate consistently the helium cores appropriate for the HB structures, obtaining the hydrogen exhausted cores given in Table 1. These values turn out to be very close to those obtained by Salaris et al. (1993), but larger than the sizes otherwise found in the literature and used for synthetic HB models (f.e., Caputo et al. 1993 and references therein).

Table 3 shows our most recent numerical results. The models have been evolved from the main sequence to the helium flash with $\simeq$ 15000 *physical* time steps, without any shell–shift procedure, and without assuming CNO equilibrium. As described in Mazzitelli et al., each physical timestep is divided into ten *chemical* evolutionary time steps. In this way, even a very thin hydrogen shell is advanced so slowly that, after each *chemical* time step, the local hydrogen abundance is changed by less than 1%. According to Sweigart (1994), this corresponds to a possible underestimate of the fuel consumption not larger than 0.3%.

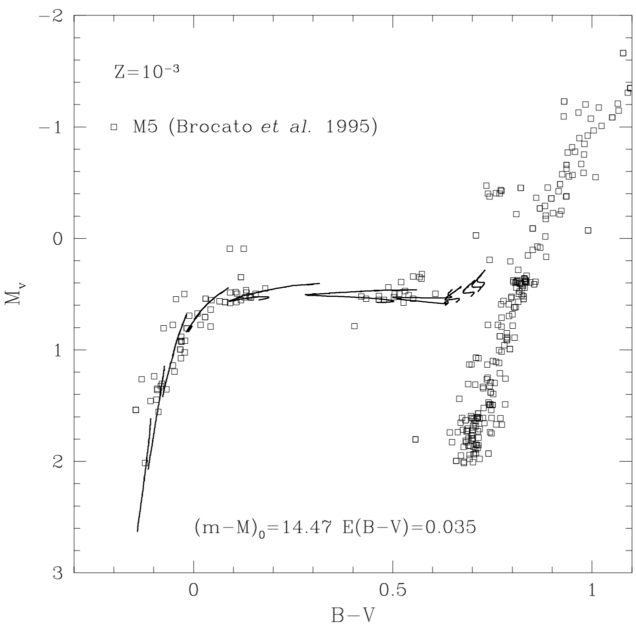

Fig. 5. Transformed theoretical HB models for Z=1 × 10^{-3} and the observed HB of M5.

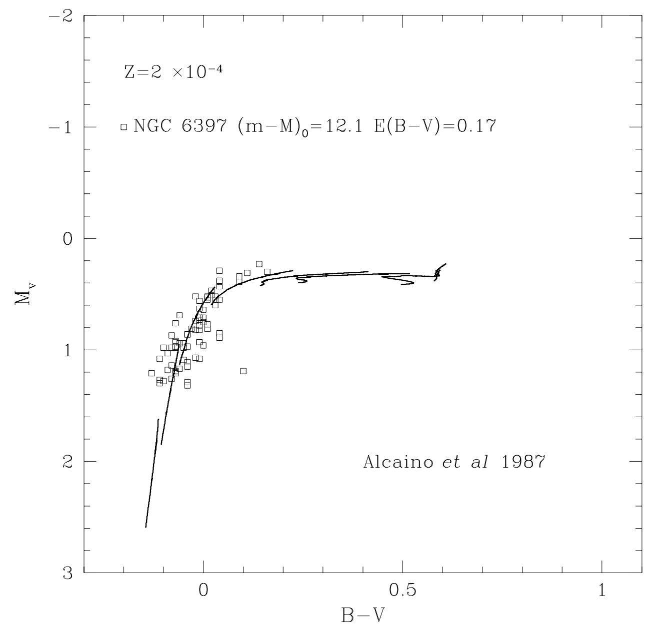

Fig. 6. Transformed theoretical HB models for Z=2 × 10^{-4} and the observed HB of NGC 6397.

The procedure implies that about 150000 chemical time steps have been used for each giant evolution.

Both direct comparison and interpolation on the data in Sweigart (1987) through the formulae in Sweigart (1994), give values $\sim$ 0.01 $M_{\odot}$ smaller than present values. Similar differences exist with the helium cores in Dorman (1992), when taking into account models with comparable Z (Dorman's models include a strong α–enhancement). As mentioned before, the core sizes by Castellani et al. (1991) have been updated by Salaris et al. (1993), who obtain values very close to those we find. The difference is to be attributed to the use of the (more appropriate) intermediate screening for 3–α reactions, instead of the weak one (Chieffi 1996). We independently tested the influence of the choice of the screening regime on the final size of helium cores, obtaining a difference of 0.0055 $M_{\odot}$, identical to that mentioned by Chieffi. In any case, recent computations by Straniero and Chieffi (1996), with updated micro–physics, give core masses in close agreement with ours.

Recently, VandenBerg (1996) has suggested that the core mass at the flash can turn out larger if relativistic corrections are not included in the EoS. We recall that we use the thermodynamics tables from Rogers et al. (1996) which do not include relativistic effects; however, these tables reach up only a density ρ = 10^5, where no relativistic effect is as yet present. For larger densities (the flash occurs at $\rho \geq 10^6$) we use a recent and improved recomputation of the thermodynamic tables by Magni and Mazzitelli (1979), which do include relativistic effects and Coulomb corrections.

Finally, it would be interesting to evaluate (but this is not within our grasp) if the core masses are sensitive to the coupled (e.g. Eggleton 1971) or uncoupled (as in this case) chemical evolution schemes. What we can say is that, with the more updated micro–physical inputs, and working with a code free at least from trivial mistakes, with a number of grid points in the range 1000 ÷ 1500, and with more than 10^5 chemical evolutionary time steps, we find the results collected in Table 3.

Table 3. Computed values of core mass at the helium flash. *: model with weak screaning in place of the intermediate one.

Z	Y	M_{RG}	$M_{core}/M_{\odot}$	$\log L_{tip}/L_{\odot}$
0.0001	0.230	0.800	0.5114	3.311
0.0001*	0.230	0.800	0.5059	3.290
0.0001	0.230	0.900	0.5087	3.302
0.0003	0.230	0.800	0.5048	3.346
0.001	0.235	0.800	0.4972	3.381
0.002	0.235	0.800	0.4940	3.404
0.004	0.240	0.800	0.4899	3.423

6. The luminosity at the tip of the Red Giant Branch

Present ZAHBs require an increase in RR Lyrae variable luminosity, that has to be checked for consistency (see, e.g., Castellani et al. 1993). Summarising, the theoretical dependence of the luminosity of the red giant branch tip on $[Fe/H]$, must agree with the luminosity obtained by Frogel et al. (1983) for the most luminous cluster giants, once reported at the distance estimated through comparison among observed HBs and theoretical ZAHBs.

Table 4 gives, for many of the globular clusters considered by Frogel et al. (1983), the value of M_{bol}(RG tip) obtained through this procedure, and Fig. 8 shows the result compared with the theoretical line M_{bol}(RG tip) *vs.* $[Fe/H]$ derived from

Table 4. M_{bol} at the tip of the red giant branch as function of $[Fe/H]$. Metal content from Djorgovski (1993), M_v(HB) from relation 3 (column 3); column 4 gives the difference with Frogel et al. distance modulus, columns 5 and 6 give estimates of M_{bol} by Frogel et al. and present paper, respectively. Column 7 reports our estimate of the error on M_{bol} on the basis of the quality of HB observations.

Name	$[Fe/H]$	M_v(HB)	$\delta(m-M)_v$	M_{bol}(F)	M_{bol}	err
104	−0.71	0.72	0.08	−3.60	−3.68	0.15
288	−1.40	0.54	0.06	−3.53	−3.59	0.15
362	−1.28	0.57	0.03	−3.52	−3.55	0.10
1261	−1.31	0.56	0.04	−3.42	−3.46	0.10
1851	−1.29	0.57	0.03	−3.71	−3.73	0.10
2808	−1.37	0.55	0.05	−3.55	−3.59	0.15
3201	−1.61	0.49	0.11	−3.38	−3.49	0.15
4590(M68)	−2.09	0.36	0.24	−3.16	−3.40	0.05
5272(M3)	−1.66	0.47	0.13	−3.34	−3.47	0.05
5904(M5)	−1.40	0.54	0.06	−3.33	−3.39	0.05
6121	−1.28	0.57	0.03	−3.45	−3.48	0.15
6171	−0.99	0.65	0.05	−3.42	−3.47	0.15
6205(M13)	−1.65	0.48	0.12	−3.39	−3.51	0.15
6341(M92)	−2.24	0.32	0.28	−3.22	−3.50	0.10
6397	−1.91	0.41	0.19	−3.09	−3.28	0.10
6752	−1.54	0.50	0.10	−3.52	−3.62	0.15
7078(M15)	−2.17	0.34	0.26	−3.29	−3.55	0.05

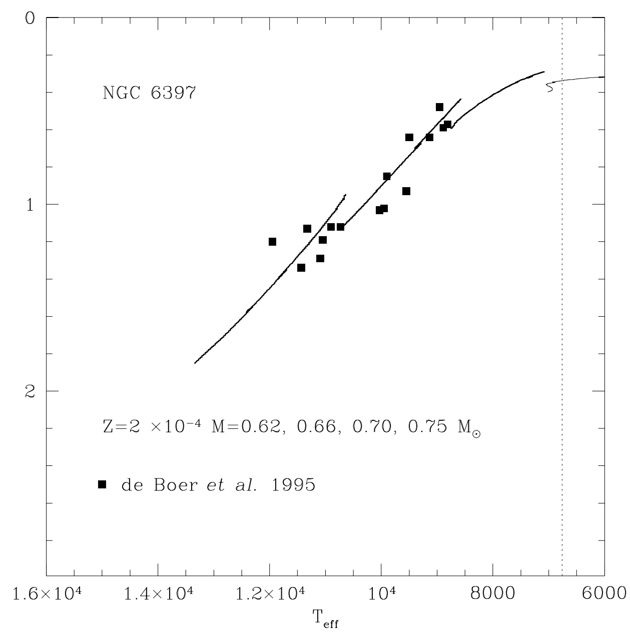

Fig. 7. Transformed theoretical HB models for Z=2 × 10^{-4} and the observed HB of NGC 6397 in the M_v, log$T_{\rm eff}$ plane.

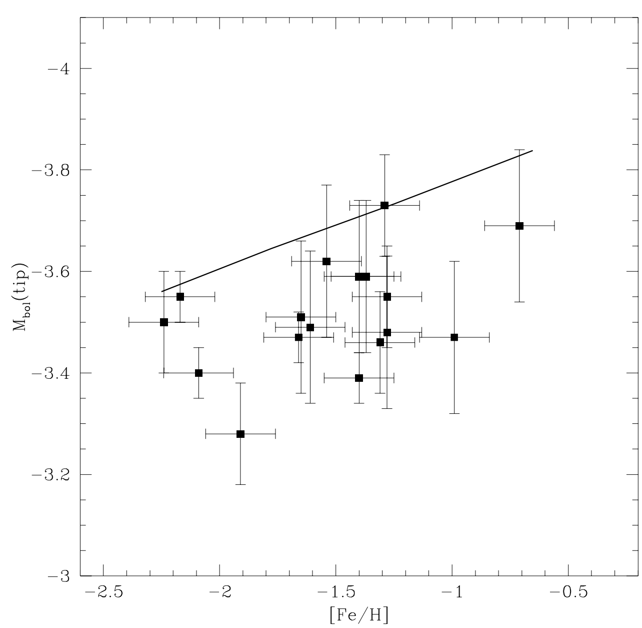

Fig. 8. The bolometric magnitude of the brightest observed star as function of cluster metal content (see Table 4). The solid line shows the theoretical expectation (Table 3).

our models evolved up to the helium flash. We remind that Frogel et al. assumed M_v(HB): = 0.6 for $[Fe/H]< -1$, = 0.7 for $-1 < [Fe/H] < -0.8$, and = 0.8 for $[Fe/H]> -0.8$.

The agreement between the observations and the adopted theoretical scenario appears satisfactory, since the theoretical line represents an upper limit to the luminosity of cluster giants. In any case, the uncertainties involved in a test of this kind are still large, as exemplified by the values of M_{bol}(RG tip) for two clusters – M5 and NGC 288 – with the same metal content: they differ by 0.3 mag, almost all the difference between maximum and minimum M_{bol}(RG tip) in the table.

7. Conclusions

Although many uncertainties still exist in the theoretical models, in the observations and in the conversion procedures, we find strong support for a luminous zero point for RR Lyrae stars as given by Sandage (1993) and Walker (1992). The theoretical slope of the relation M_v(ZAHB) *vs.* $[Fe/H]$ even with the ambiguities discussed above, appears definitively steeper than the one found by Carney et al. (1992) for RR Lyrae stars, and not too far from the one given by Sandage (1993). Low ages for globular clusters (about 12 Gyr) ensue from these results, giving an easy consistency among masses in ZAHB and at the turn–off. We present also theoretical values for the luminosity at the red giant tip, for various heavy element content, and find that they reproduce well the observed behaviour in galactic globular clusters. The case for globular clusters substantially younger than generally accepted until a short time ago is therefore strengthened.

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