

Transport of relativistic nucleons in a galactic wind driven by cosmic rays

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Dedicated to V.L. Ginzburg and J.A. Simpson at the occasion of their 80th Birthdays

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Abstract. For the first time an attempt is made at a self-consistent analytical description of the halo structure and the propagation of energetic particles in late type galaxies like our own. Galactic CR are produced together with hot gas by sources deep in the disk. This leads to a galactic wind and magnetohydrodynamic fluctuations (excited by the cosmic-ray streaming instability) which in turn together determine the transport of these energetic particles into intergalactic space. Wave excitation is balanced locally by nonlinear Landau damping. The cosmic-ray transport equations for the dominant nucleons are solved in an approximate form analytically. Although fully nonlinear, the resulting picture is simple and corresponds to an overall spatial structure that extends to distances considerably greater than the radius of the galactic disk. The inferred source spectrum for cosmic-ray nucleons is a rather hard power law in energy, of index ~ 2.1 . The observed abundances of secondary nuclei are also consistent with this model. The observed disk-halo transition at distances ~ 1 kpc is an important part of the detailed picture in which ion neutral friction damps short-scale magnetic fluctuations below that level.

Key words: cosmic rays – Galaxy: halo – galaxies: halos – turbulence - diffusion

1. Introduction

Among various empirical models suggested for the description of cosmic-ray propagation in our Galaxy and in some other galaxies, there is a class that invokes a Galactic wind (Jokipii 1976; Freedman et al. 1980; Dogiel et al. 1980; Lerche & Schlickeiser 1982 a,b; Berezhinsky et al. 1990; Webber et al.

1992; Bloemen et al. 1993). In this context the cosmic-ray diffusion coefficient (or tensor) $D(\mathbf{r}, p)$ and the wind velocity $\mathbf{u}(\mathbf{r})$ are considered as free parameters fitted by comparison with cosmic-ray, radio-astronomical, and gamma-ray observations.

Our present approach is different. We are seeking, for the first time, to develop a selfconsistent model which simultaneously includes the magnetohydrodynamic calculation of the galactic wind flow sustained by the cosmic-ray pressure and the thermal gas pressure in a rotating galaxy (cf. Zirakashvili et al. 1996, Paper I), and the consideration of transport of the dominant nucleonic cosmic-ray component in this flow. The link between relativistic particles and thermal plasma is established through the cosmic-ray streaming instability.

We shall not consider energetic electrons here since their transport is different, being strongly influenced by radiation losses. Therefore when we use the terms cosmic rays or, equivalently, relativistic particles, they refer to energetic nucleons.

To carry out the above program, we first extrapolate (cf. Breitschwerdt et al. 1993) the MHD galactic wind solutions, calculated in Paper I for heights $|z| > 1$ kpc, down to the galactic midplane in order to establish the velocity field for all $|z| > 0$. The growth rate of the streaming instability is then calculated in terms of the cosmic-ray source spectrum. Assuming local balance between unstable growth and (nonlinear) damping of the waves allows us to determine the cosmic-ray diffusion coefficient $D(\mathbf{r}, p)$ for $|z| \gtrsim 0.5$ kpc. Below that region, in the partially ionized gas disk, ion-neutral friction quenches the streaming instability. Particle scattering is relatively weak in that "internal zone" because it can be only due to large-scale sources like supernova remnants and stellar winds that establish a turbulent spectrum down to those very small scales which lead to particle scattering. Since diffusive particle transport dominates convective transport below a characteristic distance that depends on particle momentum, and since the opposite is true above that distance, approximate analytic solutions can be found for the

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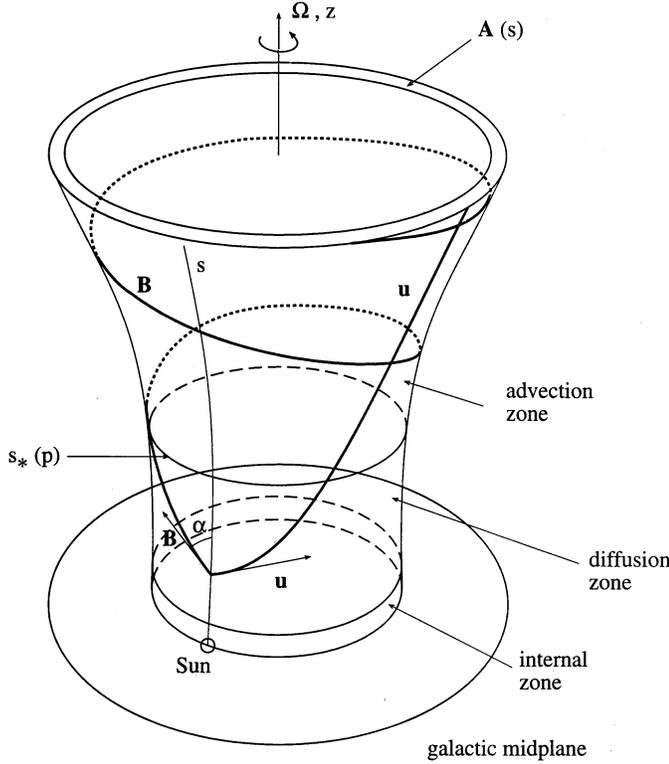


Fig. 1. The structure of the galactic wind flow for a flux tube originating at the position of the Sun. The boundary between the diffusion and advection zones is moving up with energy of the cosmic-ray particle.

particle distribution function $f(\mathbf{r}, p)$ in an assumed steady state. These solutions can then be expanded for weak scattering in the internal zone. From the observationally known cosmic-ray spectrum in the Galactic disk we can then deduce the power-law spectral index of the source spectrum and the cosmic-ray pressure in the disk. It was this pressure that had been used in Paper I to calculate the velocity structure; here, it determines the strength of the cosmic-ray sources in a self-consistent manner. Finally, we solve for the mean matter thickness (“grammage”) as seen by the cosmic rays in the Galaxy before they are ultimately escaping to extragalactic space. Both the magnitude as well as the energy dependence of the matter thickness agree with the observed values for these quantities in our Galaxy and allow a new interpretation of this energy dependence. The same is not true for the resulting cosmic-ray anisotropy which comes out considerably larger than appears to be observed near the Solar system. However this difficult quantity is given by the local structure of the Galactic magnetic field and may not be closely related to the global characteristics described by our model.

In Sect. 2 we calculate the magnetic fluctuation spectrum and the cosmic-ray diffusion coefficient. Sect. 3 contains the solutions for the particle distribution function, whereas in Sect. 4 the grammage constraints are derived. In Sect. 5 we discuss the central question of the conditions that determine the particle diffusion coefficient and finally present our conclusions.

2. Cosmic-ray streaming instability in the diffusion approximation

An essential feature of the stationary galactic wind flow, described in detail in Paper I, consists in its flux-tube structure. The gas velocity $\mathbf{u}(\mathbf{r})$ and the large-scale magnetic field $\mathbf{B}(\mathbf{r})$ are tangents to the same cylindrical surface S whose axis z coincides with the galactic rotation axis, see Fig. 1. Strongly changed cosmic-ray particles diffuse mainly along the magnetic field lines and hence remain on a given surface S . Two close surfaces S and S' form a flux tube with cross section $A(s)$, where s is the meridional coordinate along the surface S .

The form of the flow surfaces chosen in Paper I for the galaxy is described by the following relation

$$\frac{r^2}{r_0^2} - \frac{z^2}{z_0^2 - r_0^2} = 1 \quad (1)$$

which determines s in terms of z through $ds^2 = dz^2 + dr^2$. In a cylindrical coordinate system centered at the center of the galaxy, r_0 is the distance of the cylindrical flux surface at the reference level $s = 1$ kpc above the midplane. For our Galaxy, at the position of the sun, $r_0 = 8.5$ kpc; $z_0 = 15$ kpc is a typical vertical scale of the Galaxy, of the order of its radius. As a consequence, the dimensionless cross section of the flux tube $A(s) \approx \text{const}$ at $s \ll z_0$, and $A(s) \sim s^2$ at $s \gg z_0$. This flux tube geometry corresponds in spirit to that of earlier cosmic-ray driven hydrodynamic wind models that neglect galactic rotation (Breitschwerdt 1991, 1993).

In the steady state and for cylindrical symmetry, the propagation of cosmic rays along the flux tube is described by the following transport equation

$$\begin{aligned} -A^{-1}(s) \frac{\partial}{\partial s} \left(A(s) D_{\parallel}(p, s) \cos^2 \alpha \frac{\partial f}{\partial s} \right) + (u + v_a) \frac{\partial f}{\partial s} \\ - A^{-1}(s) \frac{d(A(s)(u + v_a))}{ds} \frac{p}{3} \frac{\partial f}{\partial p} = q(s, p). \end{aligned} \quad (2)$$

Here $f(s, p)$ is the particle distribution function in position s and (magnitude of the) momentum p . The particle number density is given by the integral $N(> p) = 4\pi \int_p^{\infty} dp p^2 f$, $q(s, p)$ is the source term, and $D_{\parallel}(p, s)$ is the diffusion coefficient along a magnetic field line which makes the angle α with the meridional direction (see Fig. 1). We consider energetic nucleons with momentum $p \gtrsim mc$ (where m is the proton mass, c is the speed of light) and neglect their ionization losses. The diffusion coefficient is defined self-consistently in terms of particle scattering on Alfvén waves generated by cosmic-ray streaming out of the galaxy. The waves propagate along the average magnetic field away from the galactic plane. The average advection velocity for the cosmic rays is $u_w = u + v_a$, where u is the meridional component of the gas velocity, and $v_a = V_a \cos \alpha$ is the meridional component of the total Alfvén velocity $V_a = B/\sqrt{4\pi\rho}$, where ρ is the mass density.

As we shall see below (Eq. 16), the equilibrium wave spectrum has a power-law form close to $W(k) \sim 1/k$, where $W(k)dk$ is the energy density of waves with wave number k

in the interval dk . If we disregard the deviation of the real wave spectrum from a $1/k$ -dependence, then the effective frequency of particle scattering ν does not depend on pitch angle, and the parallel diffusion coefficient is determined by the simple relation

$$D_{\parallel} = v^2/(3\nu), \quad (3)$$

where v is the particle velocity, and

$$\nu = 2\pi^2\omega_B \frac{k_{\text{res}}W(k_{\text{res}})}{B^2}, \quad k_{\text{res}} = 1/(|\mu| r_g), \quad \omega_B = v/r_g \quad (4)$$

(see e.g. Berezhinsky et al. 1990, for details). Eqs. (3) and (4) show the resonant nature of wave-particle interaction in that a particle with charge Ze and Larmor radius $r_g = pc/ZeB$ is scattered by those waves which have $k \sim r_g^{-1}$.

The anisotropic part of the cosmic-ray distribution function in the diffusion approximation is given by the following expression in the wind frame

$$\frac{\partial f}{\partial \mu} = -\frac{v}{\nu} \frac{\partial f}{\partial \ell} - \frac{V_a}{v} p \frac{\partial f}{\partial p}, \quad (5)$$

where μ is the cosine of the pitch angle, and ℓ is the coordinate along the average magnetic field. The second term on the right hand side of Eq. (5) describes the fact that to lowest order the distribution is isotropic in the frame moving with the waves.

The growth rate for the wave energy density of magneto-hydrodynamic waves propagating along the average magnetic field is mainly determined by the proton component of the cosmic rays and is equal to (Lerche 1967, Wentzel 1968, Kulsrud & Pearce 1969, see also Berezhinsky et al. 1990):

$$\Gamma_{\text{cr}} = \frac{\pi^3 e^2 V_a^2}{c^2} \int \int dp d\mu p^2 (1 - \mu^2) v \times \left(\frac{\partial f}{\partial p} + \frac{v}{V_a} \frac{1}{p} \frac{\partial f}{\partial \mu} \right) \delta(kv\mu \pm \omega_B). \quad (6)$$

Here the δ -function reflects the resonant character of the interaction.

Inserting Eq. (5) into Eq. (6) leads to the following equation

$$\Gamma_{\text{cr}}(k) = -\frac{6\pi^3 e^2 V_a}{kc^2} \times \int_{eB/kc}^{\infty} dp p \left[1 - \left(\frac{eB}{pck} \right)^2 \right] D_{\parallel} \cos \alpha \frac{\partial f}{\partial s}. \quad (7)$$

We assume that the amplification of waves is balanced by the nonlinear Landau damping on the thermal ions. According to Völk & Cesarsky (1982), (see also Fedorenko 1992), the non-saturated nonlinear damping for the Alfvén waves propagating in a plasma with parameter $\beta > 0.1$ (where $\beta = 4\pi nT/B^2$, n denotes the plasma number density, and T is the temperature in energy units), is simply

$$\Gamma_{\text{nl}}(k) = \sqrt{\beta} k V_a \int_0^k dk W(k)/B^2 \sqrt{\frac{\pi^3}{2}}. \quad (8)$$

The evolution of the wave spectral energy density $W(k)$ in the reference frame of the gas is governed by the equation (see also the discussion in Paper I):

$$\frac{dW(k)}{dt} = 2W(k)(\Gamma_{\text{cr}} - \Gamma_{\text{nl}}). \quad (9)$$

The total time derivative in Eq. (9) includes a convective transport term $V_a \partial W / \partial \ell$. A comparison of the characteristic convection rate V_a/R , where R is a spatial scale of the wind flow, with the damping rate (8) shows that the damping dominates over convection if

$$\frac{8\pi k W(k)}{B^2} > \frac{1}{kR} \sim 10^{-10} E_{\text{GeV}}. \quad (10)$$

Here we took $R = 3$ kpc, and expressed the value of the wave number k through the approximate resonance condition $k^{-1} = r_g = 10^{12} E_{\text{GeV}} \text{ cm}$ for an ultrarelativistic proton with energy E (in units of GeV) moving in a magnetic field of strength $3\mu\text{G}$. Inequality (10) is fulfilled in our model for the particle energies under consideration. This means that the left-hand side of Eq. (9) may be disregarded. Eq. (9) is then reduced to a local balance between wave growth and damping:

$$\Gamma_{\text{cr}}(k) - \Gamma_{\text{nl}}(k) = 0. \quad (11)$$

Differentiating Eq. (11) with respect to k and using Eqs. (7), (8) yields

$$W(k) = \frac{e^2 B^2 12 \sqrt{2\pi^3}}{\sqrt{\beta} k^3 c^2} \times \int_{eB/kc}^{\infty} dp p \left[1 - 2 \left(\frac{eB}{pck} \right)^2 \right] D_{\parallel} \cos \alpha \frac{\partial f}{\partial s}. \quad (12)$$

We shall see in the next section that cosmic-ray diffusion dominates over advection up to a critical distance $s_*(p)$ above the galactic plane. We will assume that the cosmic rays are produced together with hot gas by sources deep inside the galactic disk and shall ignore the z -extent of this source region. Then, after integration, Eq. (2) gives the following relation above the thin galactic source region

$$-A(s) D_{\parallel} \cos^2 \alpha \frac{\partial f}{\partial s} = Q(p). \quad (13)$$

Here $2Q(p)$ is the cosmic-ray source power density per unit area of the galactic disk (emitted symmetrically in both directions from the galactic plane), and we suppose that the flux tube cross section $A = 1$ at the origin of the flux tube.

At distances larger than the critical distance $s_*(p)$ a precise knowledge of the diffusion coefficient is not important for the considered nucleonic particles since the meridional cosmic-ray transport is dominated by advection with velocity $u + v_a$.

Let us assume for simplicity that the cosmic-ray sources in the disk have a power law spectrum in momentum above a lower cutoff p_0 :

$$Q(p) = \frac{(\gamma - 4)\epsilon}{4\pi c p_0^4} (p/p_0)^{-\gamma} \theta(p - p_0). \quad (14)$$

Here $\theta(p - p_0)$ is the Heavyside function and we choose the value of the minimum momentum as $p_0 = mc$. Finally, ϵ is defined through

$$\epsilon = 4\pi c \int_0^\infty dp p^3 Q(p). \quad (15)$$

In the case of ultrarelativistic particles, the quantity ϵ represents the energy flux density of cosmic rays leaving the galactic disk surface. Confining our interest here to relativistic particles with $p \geq mc$ does not imply that the nonrelativistic cosmic rays do not influence the transport of the relativistic ones. Indeed, they do so indirectly due to their contribution to the nonthermal particle pressure that mainly drives the wind. However, according to Eqs. (7) and (8), only more energetic particles and longer wavelength waves affect the growth and damping of waves with given wavenumber k , respectively. As a result $W(k)$ from Eq. (12) is only determined by particles with momenta exceeding eB/kc . Eqs. (12)-(14) allow us to determine the spectral energy density of waves generated by the streaming of relativistic protons out of the galaxy, at the wave number which is resonant for the particle with the charge Ze and the momentum p :

$$\begin{aligned} & (k \ W(k)) \Big|_{k=ZeB/pc} \\ &= \frac{3(\gamma - 4)^2 \epsilon \sqrt{2\pi}}{\gamma(\gamma - 2) c \sqrt{\beta} A(s) \cos \alpha} \left(\frac{p}{Zmc} \right)^{4-\gamma} \quad \text{for } p > p_0 Z. \end{aligned} \quad (16)$$

Now Eqs. (3) and (4) give the following formula for the cosmic-ray diffusion coefficient (at $p \geq Zmc$):

$$D_{\parallel} = \frac{\gamma(\gamma - 2)}{2\pi^2(\gamma - 4)^2} \frac{c^2 \sqrt{\beta} B^2 v}{\omega_{B_0} \epsilon \sqrt{2\pi}} \left(\frac{p}{Zmc} \right)^{\gamma-3} A(s) \cos \alpha, \quad (17)$$

where $\omega_{B_0} = eB/mc$ is the nonrelativistic proton Larmor frequency.

As the product $BA \cos \alpha = \text{const}$ along a flux tube, Eq. (17) shows that the spatial variation of the diffusion coefficient D_{\parallel} is mainly determined by that of the plasma parameter $\beta^{1/2}$.

3. Cosmic rays in a galactic wind

Using the magnetohydrodynamic solution for the galactic wind flow determined in Paper I, we can find the cosmic-ray distribution function $f(s, p)$ which is governed by the diffusion-convection Eq. (2). It is convenient to introduce a new dimensionless coordinate η instead of s :

$$\eta = A(s) \frac{u(s) + v_a(s)}{u_0 + v_a} = A(s) \frac{u_w(s)}{u_{w0}}. \quad (18)$$

Then Eq. (2) takes the following form:

$$-\frac{\partial}{\partial \eta} \kappa(\eta, p) \frac{\partial f}{\partial \eta} + \eta \frac{\partial f}{\partial \eta} - \frac{p}{3} \frac{\partial f}{\partial p} = q(\eta, p) \frac{A(s)}{u_{w0}} \left(\frac{d\eta}{ds} \right)^{-1} \quad (19)$$

where the effective diffusion coefficient in η is equal to

$$\kappa = \frac{\eta D_{\parallel} \cos^2 \alpha}{u_w} \frac{d\eta}{ds}. \quad (20)$$

Based on the results of Paper I for $s > 1$ kpc for our own Galaxy, and using the considerations for the wind flow at the disk-halo interface (Breitschwerdt et al. 1993), we can extrapolate the solutions from Paper I to heights $s < 15$ kpc and make, more generally, the following approximations for distances $s < 15$ kpc from the Galactic plane: $A(s) \approx \text{const}$, $\cos \alpha \approx 1$, $\beta \approx \text{const}$, $u_w \sim u_{w0} s/s_0$, $u_{w0} = 10^7$ cm/s, $s_0 = 3$ kpc, $D_{\parallel} \approx \text{const}$. The dependence $\eta(s)$ determined by Eq. (18) may then be approximately described as

$$\eta = s/s_0, \quad s_0 = 3 \text{ kpc}, \quad s < 15 \text{ kpc}. \quad (21)$$

The origin of s is chosen to be at the galactic mid-plane.

It is likely that the cosmic-ray streaming instability can not efficiently generate the random wave field in the galactic gas disk itself. At least in partially ionized regions the instability may be suppressed by ion-neutral collisions at $s \lesssim 0.5$ kpc. Alternative sources (supernova explosions and stellar winds) should be responsible for sustaining small-scale turbulence in an "internal zone" with a height of about 0.5 kpc for our Galaxy, i.e. in the region $0 \leq \eta \leq \eta_0$, $\eta_0 \approx 1/6$, see Fig. 1. This turbulence leads only to limited particle scattering and thus to a fairly large diffusion coefficient (e.g. Berezhinsky et al. 1990, Eq. (9.49)) which may significantly exceed the diffusion coefficient due to self-excited waves that is established at the larger distances $s \gtrsim 0.5$ kpc. We shall assume the existence of such an internal zone and denote the value of the diffusion coefficient in this region by κ_i .

The effective diffusion coefficient $\kappa(\eta, p)$ at $\eta > \eta_0$, given by Eqs. (17) and (20), is almost constant for $s < 15$ kpc.

At large distances, $s \gg 15$ kpc, one has approximately $A(s) \sim s^2$, $u_w \approx \text{const}$, $B \sim 1/s$ (the magnetic field is largely azimuthal), $\cos \alpha \sim 1/s$, $\eta \sim s^2$. Neglecting radiative cooling at these low densities, for adiabatic expansion of the gas one would have $\sqrt{\beta} \sim s^{-2/3}$ and hence $D_{\parallel} \sim s^{-2/3}$ (from Eq. (17)), and $\kappa(\eta, p) \sim \eta^{1/6}$. However, the numerical calculations of Paper I show that even at large distances the gas is subject to wave dissipation, although adiabatic cooling dominates. This results in the approximate scaling $\sqrt{\beta} \sim s^{-1/4}$ and, correspondingly, to $\kappa(\eta, p) \sim \eta^{3/8}$.

We may conclude that the introduction of the new coordinate η leads to a modified diffusion-convection transport equation for cosmic rays, Eq. (19), with an effective diffusion coefficient $\kappa(\eta, p)$ that is a rather weak function of distance η , and with an effective convection velocity which is simply equal to η . The dependence of diffusion on momentum is determined by the $D_{\parallel}(p)$ - dependence and is given by Eq. (17).

In the sequel we shall use the following approximation (at $\eta > \eta_0$):

$$\kappa(\eta, p) = K p^{\gamma-3} \eta^a, \quad K = \text{const}, \quad (22)$$

where the parameter a will be treated as a constant (the above estimates showed that $0 \lesssim a \lesssim 3/8$).

It is worth noting that Eq. (17) was derived under the condition that cosmic-ray transport is dominated by diffusion and that

the effect of advection is relatively small, see Eq. (12). Therefore Eq. (22) is valid under the same condition.

Previously cosmic-ray propagation was investigated in the framework of empirical wind models where the wind flow was considered as given and where the cosmic rays were treated as test particles (see references given in the Introduction). The dimensional analysis of the diffusion-convection Eq. (19), supported by the results of numerical calculations in phenomenological models, shows that there is a characteristic distance $\eta_*(p)$ where the diffusion and the convection terms are equal. This characteristic distance may be written as

$$\eta_*(p) = (wKp^{\gamma-3})^{\frac{1}{2-a}}, \quad (23)$$

where w is a numerical factor of order unity.

For $\eta \ll \eta_*$ diffusion dominates over advection in cosmic-ray transport out of the galaxy. For $\eta \gg \eta_*$ advection dominates over diffusion.

With this in mind, we shall find an approximate solution to Eqs. (19), (22), assuming that the cosmic-ray distribution function obeys the equations

$$-\frac{\partial}{\partial \eta} Kp^{\gamma-3} \eta^a \frac{\partial f}{\partial \eta} = 0, \quad (24)$$

for $\eta_0 \leq \eta \leq \eta_*(p)$, i.e. in the diffusion zone outside the internal zone, and

$$\eta \frac{\partial f}{\partial \eta} - \frac{p}{3} \frac{\partial f}{\partial p} = 0, \quad (25)$$

for $\eta > \eta_*(p)$, i.e. in the advection zone, matching these solutions at $\eta = \eta_*(p)$.

Our assumption of a thin source region amounts to a concentration of the sources at $\eta = 0$. Neglecting advection in the internal zone, we come to the following equation, valid for $0 < \eta \leq \eta_*$:

$$-\frac{\partial}{\partial \eta} \kappa_i \frac{\partial f}{\partial \eta} = 0, \quad (26)$$

with the following boundary condition at $\eta = 0$:

$$-\kappa_i \frac{\partial f_i}{\partial \eta} = \frac{Q(p)}{u_{w0}}. \quad (27)$$

Here the notation $f(\eta = 0, p) \equiv f_i(p)$ has been introduced.

Requiring $f(\eta = 0, p) = 0$ as well as continuity of the particle distribution function $f(\eta, p)$ and of the particle fluxes (the diffusion flux $-\kappa \partial f / \partial \eta$, and the convection flux $-\eta p / 3 (\partial f / \partial p)$ at the outer boundaries η_0 and $\eta_*(p)$ of the internal, the diffusion, and the advection zones, the solutions of Eqs. (24) to (27) read as follows:

$$f(\eta, p) = f_i(p) - \frac{\eta Q_0 p^{-\gamma}}{\kappa_i(p) u_{w0}}, \quad \text{for } 0 < \eta \leq \eta_0; \quad (28)$$

$$f(\eta, p) = f_i(p) + \frac{Q_0 p^{-\gamma}}{u_{w0}} \times \left[-\frac{\eta_0}{\kappa_i(p)} + \frac{p^{-\gamma+3}}{(1-a)K} (\eta_0^{1-a} - \eta^{1-a}) \right], \quad (29)$$

for $\eta_0 < \eta \leq \eta_*(p)$;

$$f(\eta, p) = \frac{\gamma + 3 - 3a}{3\gamma - 3 - a\gamma} (wK)^{\frac{\gamma-3}{\gamma+3-3a}} \frac{Q_0}{u_{w0}} (\eta p^3)^{\frac{-3\gamma+3+a\gamma}{\gamma+3-3a}}, \quad (30)$$

for $\eta_*(p) \leq \eta$. Here the source spectrum is taken in the form $Q(p) = Q_0 p^{-\gamma}$.

The cosmic-ray distribution function at the central galactic plane $\eta = 0$ is the following:

$$f_i = \frac{\eta_0 Q_0 p^{-\gamma}}{u_{w0} \kappa_i(p)} + \frac{Q_0 K^{-\frac{1}{2-a}}}{u_{w0}} p^{-\frac{3\gamma-3-a\gamma}{2-a}} \times \left[\frac{\gamma + 3 - 3a}{(3\gamma - 3 - a\gamma) w^{\frac{1}{2-a}}} + \frac{w^{\frac{1-a}{2-a}}}{1-a} - \frac{(\eta_0 (K p^{\gamma-3})^{-\frac{1}{2-a}})^{1-a}}{1-a} \right], \quad (31)$$

for $\eta_*(p) \geq \eta_0$; and

$$f_i(p) = \frac{\eta_0 Q_0 p^{-\gamma}}{u_{w0} \kappa_i(p)} + \frac{3Q_0 p^{-\gamma}}{\gamma \eta_0 u_{w0}}, \quad (32)$$

for $\eta_*(p) < \eta_0$.

The constant $w \approx 1$ remains undetermined in this approach. It could be found in a detailed matching procedure that takes both diffusion and advection into account in the transition regions between the internal, diffusion, and advection zones.

As will be clear from the next section, the data on the abundance of secondary nuclei in cosmic rays of our Galaxy may be naturally explained if the first terms on the right sides of Eqs. (31) and (32) are comparatively small. This gives a flat energy dependence of the mean grammage traversed by cosmic rays at energies below about 2 GeV/nucleon. One can neglect these first terms on the right sides of Eqs. (31), and (32) under the following conditions (which give limits on the diffusion coefficient in the internal zone):

$$\begin{aligned} \kappa_i(p) &\gg \eta_0 \eta_*(p), \quad \text{for } \eta_*(p) \geq \eta_0; \\ \kappa_i(p) &\gg \eta_0^2(p), \quad \text{for } \eta_*(p) < \eta_0. \end{aligned} \quad (33)$$

Let us assume these conditions to hold, following our above discussion on ion-neutral friction.

Then an observer in the disk will measure a power-law momentum spectrum $f_i(p) \sim p^{-\gamma_d}$ with exponent

$$\gamma_d = \gamma + \frac{\gamma - 3}{2 - a}, \quad \text{for } \eta_*(p) \geq \eta_0. \quad (34)$$

The measured cosmic-ray spectrum in the solar neighbourhood exhibits a power law with $\gamma_d = 4.7$. This implies a power-law source spectrum with exponent

$$\gamma = ((2 - a)\gamma_d + 3) / (3 - a) \approx 4.1 \quad (35)$$

for $0 \lesssim a \lesssim 3/8$.

Small variations of the parameter a do not essentially alter the power-law exponent (35).

The expression (34) is obtained using Eqs. (31), (33) and is valid for sufficiently high energy when $\eta_* \geq \eta_0$, above about 2 GeV/n. For smaller energies Eq. (32) together with inequality (33) gives

$$\gamma_d = \gamma \approx 4.1, \quad (36)$$

if a unique power-law source spectrum exists in the entire momentum range. We shall assume this here. The predicted form of the cosmic-ray energy spectrum depends on position above (or below) the galactic plane. Fig. 2 shows the appropriate asymptotic behaviour of the cosmic-ray distribution function $f(\eta, p)$. The spectrum in the advection zone is rather flat, $f \sim p^{-\gamma_a}$, where

$$\gamma_a = \frac{3(3(\gamma - 1) - a\gamma)}{\gamma + 3(1 - a)} \approx 3.9. \quad (37)$$

Here Eqs. (30), (35) and (36) have been used together with $0 \lesssim a \lesssim 3/8$.

The value (23) of $\eta_*(p)$ increases with momentum. At distances $s < 15$ kpc we have $a \approx 0$, $\eta \sim s$, and hence $s_* \sim p^{0.55}$ (the physical distance s_* corresponding to the dimensionless distance η_*). At larger distances $s > 15$ kpc, $a = 3/8$, $\eta \sim s^2$, and hence $s_* \sim p^{0.34}$. The dependence $s_*(p)$ shows how the boundary between the diffusion and the advection zones is moving away from the galactic plane with increasing particle momentum.

In dimensional units the conditions (33) may be rearranged for the diffusion coefficient perpendicular to the Galactic plane $D_i = D_{\parallel i} \cos^2 \alpha$, as follows:

$$D_i \gg s_0 u_{w0} \eta_0 \eta_*(p) \approx 3 \cdot 10^{27} (p/4Zmc)^{0.55} \text{cm}^2/\text{s}, \quad (38)$$

for $p \geq 4Zmc$;

$$D_i \gg 3 \cdot 10^{27} \text{cm}^2/\text{s}, \text{ for } p < 4Zmc.$$

Under the conditions (33), and using the explicit form for D_{\parallel} cf. Eq. (17), the solution (31) reads:

$$f_i(p) = \frac{3(\gamma - 4)^2}{4\sqrt{\gamma(\gamma - 2)}} \left[\sqrt{w} + \frac{\gamma + 3}{3(\gamma - 1)\sqrt{w}} \right] \frac{1}{m^4 c^6} \quad (39)$$

$$\times \sqrt{\frac{\sqrt{2\pi} 2s_0 \omega_{B0} \epsilon^3}{\sqrt{\beta} c u_{w0} B^2 \cos^3 \alpha}} (p/mc)^{-3(\gamma-1)/2},$$

where we have assumed $\eta_*(p) \gg \eta_0$, and $a = 0$.

In order to proceed we must again use empirical results for our Galaxy. This means that expression (39) must be equal to the proton spectrum near the solar system (Perko, 1987):

$$I(E) = 1.93 \frac{v}{c} (0.939 + E_{\text{GeV}})^{-2.7} (\text{cm}^2 \text{s sr GeV})^{-1}. \quad (40)$$

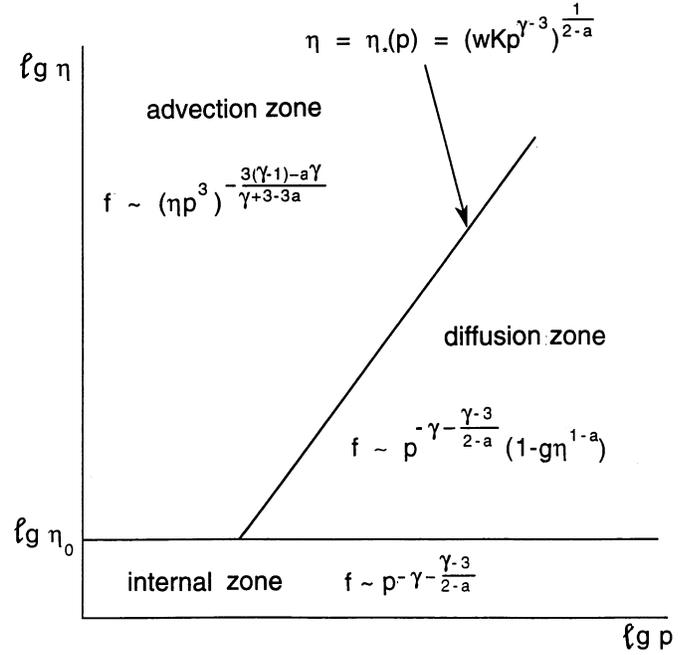


Fig. 2. Cosmic-ray distribution function $f(\eta, p)$. The conditions (33) are assumed to hold. The constant g is determined from Eqs. (29) and (30).

The appropriate momentum distribution function for $p \gg mc$ then reads as

$$f(p) = \frac{7 \cdot 10^{-11}}{(mc)^3} (p/mc)^{-4.7} (\text{cm}^3 \text{sr})^{-1}. \quad (41)$$

Eq. (41) yields the cosmic-ray pressure $P_{\text{cr}} \approx 5 \cdot 10^{-13} \text{erg/cm}^3$ (using mc as minimum momentum) which is in accordance with the value of the cosmic-ray pressure above the gas disk used in our magnetohydrodynamic calculations of the wind flow in Paper I.

Setting the expression (39) equal to expression (41), we determine not only the source index $\gamma = 4.1$, but also the value of the surface cosmic ray emissivity of the Galactic disk

$$\epsilon = 0.63 \cdot 10^{-5} \cos \alpha_0 \quad (42)$$

$$\times \left[\frac{B}{1\mu\text{G}} \frac{u_{w0}}{100\text{km/s}} \frac{3\text{kpc}}{s_0} \left(\frac{\beta}{0.7} \right)^{1/2} \right]^{1/3} (\text{erg/cm}^2\text{s})^{-1},$$

where α_0 is the value of α at the Galactic disk, and we set $w = 1$.

The diffusion coefficient above the internal zone, is defined by Eq. (17) and, for $\eta_0 < \eta \lesssim 5$ is equal to

$$D_{\parallel} = 7.2 \cdot 10^{26} \left[\left(\frac{B}{1\mu\text{G}} \right)^2 \frac{100\text{km/s}}{u_{w0}} \frac{s_0}{3\text{kpc}} \frac{\beta}{0.7} \right]^{1/3} \quad (43)$$

$$\times (p/Zmc)^{1.1} \text{cm}^2/\text{s}.$$

This concludes our program of a self-consistent calculation of cosmic-ray transport properties of a galactic wind, partly driven by the cosmic rays themselves. The steady state level of small-scale Alfvénic turbulence in the wind flow is maintained by the cosmic-ray streaming instability, balanced by the non-saturated nonlinear Landau damping on the thermal ions.

As a next step we have to check how this model fits the data on secondary nuclei in the cosmic rays.

4. The "grammage" constraints

The content of cosmic ray secondary nuclei produced through the spallation of heavier nuclei, and interacting with the interstellar gas, is an important empirical characteristic of any cosmic-ray propagation model (see e.g. Berezhinsky et al. 1990, for a discussion). Nuclear fragmentation is determined by the escape length X which is the mean matter thickness traversed by cosmic rays in the course of their random walk in the Galaxy. By its definition X does not depend on the nuclear cross sections. Experimentally it is found that above a few GeV/n the secondary-to-primary ratio steadily declines (Juliussen et al. 1972; Smith et al. 1973). Specifically, the observations of the B/C-ratio (Engelmann et al. 1990) give the following empirical formula for X :

$$\begin{aligned} X &= 14.0 (v/c) \text{ g/cm}^2, \text{ for rigidity } R < 4.4 \text{ GV}; \\ X &= 34.1 (v/c) R^{-0.6} \text{ g/cm}^2, \text{ for } R \geq 4.4 \text{ GV}. \end{aligned} \quad (44)$$

The particle rigidity is defined as $R \equiv pc/Z$. The measurements by Engelmann et al. (1990) were made in the energy range from 0.6 to 35 GeV/n. The distribution of path lengths x about the mean value X is close to an exponential distribution $\exp(-x/X)$. The above rigidity-dependence continues to hold up to energies of 1 TeV/n (Swordy et al. 1990), a limit basically given by statistics.

We assume for simplicity that the galactic gas distribution has a delta function form, concentrating it in the galactic mid-plane. In this case the inclusion of nuclear fragmentation into our calculations is reduced to a change of the boundary condition (27) for the primary cosmic rays at $\eta = 0$ (see also Ptuskin & Soutoul 1990) This boundary condition now reads:

$$-\kappa_i \frac{\partial f(p, \sigma)}{\partial \eta} \Big|_{\eta=0} = \frac{Q(p)}{u_{w0}} - \frac{nv\sigma}{u_{w0}} f_i(p, \sigma), \quad (45)$$

where σ is the total cross section of nuclear fragmentation, and $2nh$ is the surface gas density of the galactic disk (we take for our Galaxy $n = 1.3 \text{ cm}^{-3}$ as value of the mean density, and as its vertical scale above the Galactic plane $h = 100 \text{ pc}$). The mean matter thickness may then be calculated using the limiting expression (Berezhinsky et al. 1990)

$$X = -m_{\text{ism}} \frac{\partial \ln f_i(p, \sigma)}{\partial \sigma} \Big|_{\sigma=0}. \quad (46)$$

Here, the function $f_i(p, \sigma)$ acts as the moment generating function in probability theory, and m_{ism} is the mean mass of an interstellar atom.¹

Therefore, to calculate X , it is sufficient to find the solution of the set of Eqs. (24)–(26) with the boundary condition (45) in the limit of small cross section σ , and to use Eq. (46) after that. Going through this procedure we finally obtain: the following expression for the matter thickness:

$$\begin{aligned} X &= \frac{m_{\text{ism}} n h v}{u_{w0}} \\ &\times \left[\frac{\eta_0}{\kappa_i} + \frac{\eta_*^{1-a}(p) - \eta_0^{1-a}}{(1-a)Kp^{\gamma-3}} + \frac{3}{f_i(p)} \int_p^\infty \frac{dp f_i(p)}{p \eta_*(p)} \right], \end{aligned} \quad (47)$$

for $\eta_*(p) \geq \eta_0$. Here $f_i(p) = f(\eta = 0, p, \sigma = 0)$ is again given by Eq. (39). In addition

$$X = \frac{3m_{\text{ism}} n h v}{\gamma \eta_0 u_{w0}} \left[\frac{\gamma \eta_0^2}{3\kappa_i(p)} + \left(1 + \frac{\gamma \eta_0^2}{3\kappa_i(p)} \right)^{-1} \right], \quad (48)$$

for $\eta_*(p) < \eta_0$.

A natural interpretation of the low-rigidity dependence $X \sim v/c$, cf. Eq. (44), is given by Eq. (48) if the first term in the square brackets can be neglected. This condition is actually fulfilled if the inequalities (33) and (38) hold. As a consequence, we have approximately

$$X \approx \frac{3m_{\text{ism}} n h v}{\gamma \eta_0 u_{w0}} \approx 10 \frac{v}{c} \text{ g/cm}^2, \text{ for } \eta_*(p) < \eta_0, \quad (49)$$

in remarkable qualitative and in reasonable quantitative agreement with the empirical value (44) for $R < 4.4 \text{ GV}$. Eq. (47) gives at high rigidities

$$X \sim \frac{v}{c} \eta_*^{-1}(p) \sim v(p/Z)^{-\frac{\gamma-3}{2-a}} \sim v(p/Z)^{-0.55}. \quad (50)$$

This is close to the observed dependence of X , cf. Eq. (44). The condition $\eta_* = \eta_0$ is fulfilled for $\kappa = \eta_0^2/w$ or, in dimensional units, for $D = \kappa u_{w0} s_0 = 2.5 \cdot 10^{27} w^{-1} \text{ cm}^2/\text{s}$. Thus, using Eq. (43), we see that the steepening in the $X(p)$ -dependence given by Eq. (44) at 4.4 GV is formally achieved at $w \approx 0.7$. As was discussed above, the value of the parameter w is not determined in our approximate model, but the expected value is $w \sim 1$. The empirical data are consistent with the inequalities (33) which in turn are expected theoretically, as discussed in Sect. 3.

We conclude that the present galactic wind model is in agreement with the "grammage" constraints inferred from the content of secondary nuclei in the cosmic rays for our Galaxy.

¹ To clarify this point, we note that the effect of fragmentation for primary nuclei may be described in terms of the normalized path-length distribution function $G(x)$, so that $f_i(\sigma) = g \int_0^\infty dx G(x) \exp(-\sigma x/m_{\text{ism}})$, where g characterizes the source strength. The mean matter thickness is defined as $X = \int_0^\infty x dx G(x)$, and Eq. (46) follows from these last two equations.

The consideration made so far is valid up to the energy at which the boundary $s_*(p)$ between the diffusion and the advection region reaches the physical outer boundary of the galactic wind (the termination shock), presumably located at about 300 kpc. (The last number is found from the condition that the wind ram pressure is equal to the intergalactic pressure which depends on the intergalactic gas density and temperature and is estimated as $P_{\text{Mg}} = 3 \cdot 10^{-16} (n_{\text{Mg}}/10^{-5} \text{ cm}^{-3})(T_{\text{Mg}}/3 \cdot 10^5 \text{ K}) \text{ dyn/cm}^2$. The value $s_*(p) \approx 300$ kpc corresponds to a particle energy of about 10^{15} eV , where the "knee" in the cosmic-ray spectrum is observed (Kulikov & Christiansen 1959). This is an intriguing coincidence that we wish to note here. Whether the "knee" is caused by the transition between two acceleration mechanisms with different energy dependence, or by a change in particle propagation conditions, given a continuing and constant power-law form of the source spectrum, is unknown. If, for the sake of argument, we took the second position, then the behaviour of the spectrum at higher energies in our model might be determined after having studied the possible termination shock structure and the flow downstream of the shock in the extragalactic medium.

The data on the radioactive isotope ^{10}Be in cosmic rays are also consistent with our model. The surviving fraction of radioactive ^{10}Be is a guide to the cosmic ray age and halo size (see the review by Simpson & Garcia-Muñoz 1988). If in the course of its diffusion a cosmic-ray particle reaches the advection zone, it will not return back to the galactic disk. That is why using ^{10}Be data an observer in the galactic plane actually "measures" not the whole size of the region occupied by cosmic rays, but only the size of the diffusion zone (see also Berezhinsky et al. 1990; Bloemen et al. 1993).

All measurements of the abundance of radioactive isotopes with good statistics were made at low energies $E < 2 \text{ GeV/nucleon}$, where $\eta_*(p) < \eta_0$. Diffusion inside the internal zone is relatively fast for these particles and the particles escape from this zone at its boundary at about 0.5 kpc with a probability determined by the small ratio u_{wo}/c . Hence, for these small energies and for an observer in the galactic plane, the model is reduced to a "leaky box" with an effective halo height of about 0.5 kpc and a mean gas density of 0.26 cm^{-3} which is in agreement with the ^{10}Be data (Simpson & Garcia-Muñoz 1988; Lukasiak et al. 1994). The partial confinement in the leaky box is furnished by the wave barrier at $s = s_0$.

An important information on cosmic-ray propagation is given by the measurements of cosmic-ray anisotropy. The data show a weakly energy-dependent anisotropy with amplitude of the order of 10^{-3} in the energy range $10^{11} - 10^{14} \text{ eV}$. The anisotropy evidently rises with energy at $E > 10^{15} \text{ eV}$ (see Kifune 1990 for a review). The expected anisotropy at the edge of the Galactic disk in our model is

$$\delta_{\perp} = \frac{3D}{v} \frac{1}{f_i} \frac{\partial f_i}{\partial s} \approx 1.3 \cdot 10^{-4} (w + 0.76)^{-1} (p/mc)^{0.55}, \quad (51)$$

for $a = 0$.

Hence, the value of the anisotropy as predicted from Eq. (51) is $5 \cdot 10^{-2}$ at 10^{14} eV (for $w = 1$) which is too high, and we have a difficulty with the explanation of the observations. Formally, the predicted anisotropy may be smaller by a factor of 10 if we take into account the almost central position of the solar system in the Galactic disk.

It should be emphasized that the problem with the predicted high anisotropy arises in any model of cosmic-ray propagation if the dependence $X \sim R^{-0.6}$ holds up to very high energies, e.g. up to the knee at $3 \cdot 10^{15} \text{ eV}$. The solution of this problem that was worked out in detail until now invokes the process of stochastic particle reacceleration on interstellar magnetohydrodynamic turbulence with a Kolmogorov spectrum (Simon et al. 1986; Seo & Ptuskin 1994). Then a weak dependence $X \sim R^{-0.33}$, and an exponent of the source spectrum $\gamma \approx 2.3$ may explain the observations of primary and secondary nuclei. Stochastic reacceleration does not work in the present wind model where Alfvén waves propagate in the direction out of the Galactic plane. The exception is given by the presumably isotropic turbulence in the thin gas disk.

It is also worth noting that by its very nature the cosmic-ray anisotropy is conditioned by the local structure of the Galactic magnetic field and may deviate a great deal from the global Galactic characteristics described by our model. In particular, if the local magnetic field is more stretched out along the Galactic plane compared with the average field in the Galactic disk, then the local anisotropy may be suppressed in the direction perpendicular to the Galactic plane.

5. Discussion and conclusion

The present paper considers cosmic-ray kinetics in the Galaxy under the assumption that the production of cosmic rays in a galactic disk itself largely determines the structure of the galactic wind flow in a rotating galaxy with a frozen-in large-scale magnetic field (see Paper I). The streaming instability of cosmic rays, moving away from the galaxy along the spiral magnetic field, creates small-scale Alfvénic turbulence, in the system. Wave-particle interaction proceeds through the cyclotron resonance. The equilibrium spectrum of turbulence determined by the nonlinear Landau damping of waves on the thermal ions has the suitable form and amplitude to explain the value and the energy dependence of the cosmic-ray diffusion coefficient as known from the empirical modelling of cosmic-ray propagation.

We may ask ourselves, whether the processes of wave dissipation which determine this result are indeed the dominant ones in the halo of a typical late type galaxy. This is a question of the size of the parameter β , i.e. of the ratio of nonresonant (essentially thermal) particle pressure and magnetic field energy density. The competitive process is resonant three-wave coupling of an Alfvén wave with a backward propagating Alfvén wave and a sound wave (Sagdeev and Galeev 1969; Chin and Wentzel 1972; Skilling 1975). The effect occurs only at low values of β : for β approaching unity, the second wave becomes strongly Landau damped linearly, and for $\beta > 1$ the process is unimpor-

tant (Akhiezer et al. 1975). Since the hot halo gas at the disk-halo interface of late type galaxies is presumably shock-heated Supernova Remnant gas, it has almost by definition a high $\beta \gtrsim 1$, and our damping rate (8) should apply. In the accelerating wind flow this gas is subsequently cooling adiabatically and radiatively, while being heated by the dissipation of the scattering Alfvén waves. Thus at large distances s , exceeding the disk radius, β may possibly decrease significantly below unity. In this case our wave dissipation mechanism (8) gives rise to the minimum cosmic-ray diffusion coefficient. A proper calculation of the wave field should then include resonant wave coupling. This question will be investigated in future work.

After many years of studies of plasma effects in cosmic rays (started by Ginzburg, 1965) it is now found how the cosmic-ray streaming instability may work up to very high energies. This has been made possible in the present model because of particle motion along very long, tangled spiral magnetic field lines and, because the effective boundary between cosmic-ray diffusion and advection zones is moving with energy.

Another interesting result of the present considerations is that at energies below about 2 GeV/nucleon cosmic rays are confined in a flat leaky box with a thickness of about 1 kpc. Particles move with a large diffusion coefficient inside this volume (called the internal zone in the present paper) since the considerable density of neutrals in the interstellar gas suppresses the development of the streaming instability. The high level of Alfvénic turbulence outside this region leads to strong scattering of low energy particles and makes them return back to the internal zone. At energies $E \gg 2$ GeV/n and for an observer at the disk our model is almost equivalent to a pure diffusion model with absorbing boundaries, (moving apart with energy). In principle, one can obtain direct information about the outer advection zone using radioastronomical and, possibly, gamma-ray data. We intend to carry out such research in forthcoming work in much the same way as it is usually done in empirical models.

It has been proved by the first satellite measurements of the ^{10}Be abundance in cosmic rays (Garcia-Muñoz et al. 1977) that the cosmic rays observed at the solar system fill a much larger volume than the thin Galactic gas disk. This was an important confirmation of the Galactic model with a halo (Ginzburg & Ptuskin 1976). The continuing research, in particular the first studies of dynamical halo models (Jokipii 1976; Jones 1979), together with an increasingly detailed understanding of the importance of cosmic rays for the dynamics of the interstellar medium (Parker 1969; Ipavich 1975; Breitschwerdt et al. 1991), led to the present attempt to develop an advanced model of cosmic-ray transport in the Galaxy.

The approach used in the present paper was based on a simple approximation of the numerical solution for the magnetohydrodynamic wind flow obtained in Paper I and on approximate analytical solutions of cosmic-ray transport equations. The scenario developed in this way leads to an attractive model where the structure of the galactic halo, cosmic-ray transport coefficients, and boundary conditions for galactic cosmic rays are not free parameters as they are in the usual semi-empirical mod-

els, but are selfconsistently determined by the given power of cosmic-ray sources in the galactic disk, together with the thermal energy input into the interstellar medium and the overall galactic mass and angular momentum structure. This model is in remarkable qualitative and in reasonable quantitative agreement with observations of Galactic cosmic-ray nucleon spectra and of the abundances of secondary nuclei.

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