Astron. Astrophys. 321, 519–522 (1997)

ASTRONOMY AND ASTROPHYSICS

Geometrical effects on radio pulsar profiles and spectra

W. Sieber

Fachhochschule Niederrhein, PF 2850, D-47728 Krefeld, Germany

Received 23 April 1996 / Accepted 10 October 1996

Abstract. Considering the geometry of the hollow-cone model of radio pulsar emission in detail, one is led to the conclusion that several well-known observed effects may be explained quite easily purely on geometrical terms. It can be shown e.g. that the appearance of outriders at high frequencies and a steepening of the whole spectrum (cut-off) must be expected for certain geometrical situations as well as variations in component spectra, e.g. a steepening of the central (core) component. These effects appear in addition to the presumably also existing physical variations with frequency.

Key words: pulsars: general – radiation mechanisms: general

1. Introduction

The hollow-cone model (Radhakrishnan & Cooke 1969; Ruderman & Sutherland 1975) can nowadays be considered as a widely accepted working model for the emission geometry of radio pulsars (see for example Rankin (1983) and Lyne & Manchester (1988)). The model assumes in its basic form that radio emission at a certain frequency is emitted mainly in a circular ring (cone) centred on the magnetic field axis and pointing at latitude α in a reference frame defined by the rotational axis of the neutron star. The radial extent of the cone may be given by the angle ϑ. The chance orientation of the pulsar's rotation axis with respect to the line of sight to the observer determines the so-called "impact angle" β, i.e. the closest angular distance between the line of sight and the magnetic field axis. The (average) intensity distribution along the cut of the line of sight (through part of the hollow-cone) determines in the end the observed average pulse profile (a schematic diagram is given in Fig. 1).

It is furthermore very often assumed that emission at different frequencies is produced at different heights above the star's surface (radius to frequency mapping) and that the detectable emission is directed along the open field lines. One would therefore expect to see larger hollow-cone opening angles at lower frequencies and vice versa.

Send offprint requests to: W. Sieber

2. Model versus observation

Although the predictions of the hollow-cone model appear straightforward and obvious, they are not so easy to verify in the normally given case, where one has to estimate the geometrical parameters, i.e. the angles α, ϑ, and β from observations. The geometry can in principle be deduced from the linear polarization angle swing, observed for most pulsars. There are however in practice two complications: First, many pulsars show orthogonal modes (birefringence) of the polarization angle which may spoil a smooth swing; and, second, even for well defined (one mode) cases with smooth swings, α is normally only poorly determined giving good fits for a wide range of α accompanied by a corresponding variation in β. The exception are pulsars where the polarization can be measured over a wide range, e.g. pulsars with interpulses. This means that either α or ϑ has to be set or estimated from additional arguments (which has been done in several publications). Fortunately, the exact value of α is not needed for the following considerations.

It should be noted also that the true width ΔW_{true} of the pulse profile can be deduced only, when the geometry is known since

$$\Delta W_{\mathrm{true}} = \Delta W_{\mathrm{observed}} \cos(\alpha + \beta) \quad (1)$$

resulting sometimes in very wide measured pulse profiles.

Nearly unpredictable are variations in intensity across the profile (or across the cone), which are observed for many pulsars. A simple minded hollow-cone model would predict equal intensity for the leading and trailing half of the cone. We will just disregard these intensity variations, since no theoretical explanation exists so far.

3. Best information from "central cuts" ($\beta \approx 0^\circ$)

The preceding discussion makes it clear, that the best information about the structure of the emitting cone may be obtained from nearly central cuts ($\beta \approx 0^\circ$), since the intensity distribution across the cone and the width of the cone can be measured directly in this special geometry (independence of β and ϑ, but still dependent on α).

Several investigations of this type were published in the past (Sieber et al. 1975; Thorsett 1991; Phillips & Wolszczan 1992)

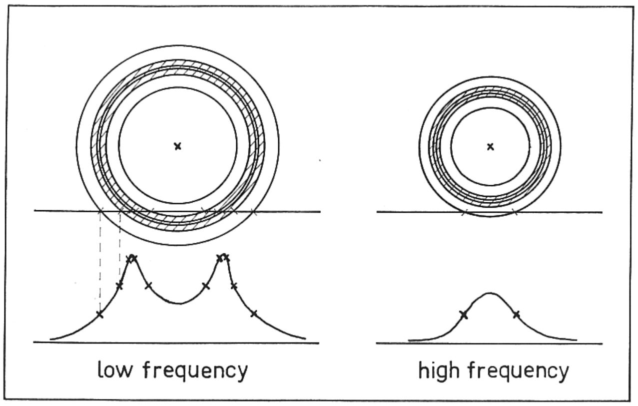

Fig. 1. Cut through the hollow cone defining the intensity distribution over the pulse profile; left: low frequency, right: high frequency

which show that there exists indeed a frequency dependence of the cone width in the sense, that the width increases at lower frequencies (radius to frequency mapping, see above). The separation Δs between the two components may be described as:

$$\Delta s = k f^{-\delta} + s_0 \qquad (2)$$

with δ in the range $0.1 \leq \delta \leq 0.3$ (Thorsett 1991).

This frequency dependence of the component separation is a clearly measured effect, which seems to be directly linked to the cone opening angle and appears therefore to be an intrinsic property of the hollow-cone emission. We will in the following assume, that this property is preserved also for those pulsars, where no clear double structure of the profile due to the chance position of the line of sight cutting through the cone is visible.

4. Grazing cuts ($\beta \neq 0°$)

Given the assumption (see above) that the frequency dependence of the hollow-cone is the same for pulsars with impact angles unequal to zero, one would expect to see a bifurcation of the profile (double profile) at low frequencies due to the widening of the cone opening angle for those pulsars, where the line of sight just touches the outer skirts of the cone at high frequencies (grazing cut, see Fig. 1). Such cases are indeed observed (Sieber et al. 1975), examples being PSR 0950+08 (Hankins et al. 1991) and 2016+28 (Hankins & Rickett 1986).

The question is, if such a transition of the pulse shape is indeed primarily caused by the geometrical effect, i.e. if the geometrical variation is quantitatively large enough to cause such a profile change. One may test this by looking at the scale of frequency variation which occurs for central cuts, the only case where one knows the frequency dependence with certainty. There are well known examples published in the literature (Phillips & Wolszczan 1992), which show that the overall increase even of the pulse width $\Delta W_{0.1}$ (i.e. the width measured at 1/10th of the maximum) is far enough to explain such transitions, e.g. about 100% for PSR 1237+25 and PSR 0525+21 and about 300% for PSR 1133+16 (taking the high-frequency value as a reference). (A better estimate of the cone width ϑ would be given by the component separation Δs, as will be outlined below. But even the less frequency dependent pulse width shows already enough variation.)

We can therefore conclude, that *the bifurcation of profiles at low frequencies is a natural geometrical consequence of the hollow-cone model* (for appropriate geometries of line of sight and magnetic field axis).

5. Outriders, pulse width

The pulse shape may vary even more drastically, when wider impact angles are considered. Interestingly enough, one may get quite considerable profile changes without corresponding changes in pulse width, as can be seen from very simple examples:

One may consider e.g. a gaussian shaped emission beam (a pencil beam) of a pulsar, centred on the magnetic field axis (instead of a hollow cone). Cutting such an emission beam at different impact angles results always in the same pulse width (as demonstrated by Jones (1980), see also our simulation in Fig. 2a). The reason for this perhaps surprising fact is, that the pulse width is computed in pulsar observations always with respect to the maximum of the *received* emission (not to the maximum of the emitted radiation, which is normally not known), which results in a new normalization for each impact angle. The same result applies also, when the impact angle is kept constant and the beam width changes, for example due to the frequency variation of the beam, as observed in reality.[1]

A hollow-cone beam could be simulated by shifting a gaussian curve off-axis and rotating this figure, as shown in Fig. 2b. Here again only minor changes of the pulse width occur, but a clear transition from a simple to a bifurcated profile as discussed in Sect. 4.

It is obvious that the pulse width and even more the observed pulse shape depend critically on the intensity distribution in the outer parts of the beam. This is demonstrated in Fig. 2c, where we start from a slightly distorted gaussian beam with an additional low-intensity ring around the beam as observed e.g. for PSR 2016+28. Such low-intensity "plateau emission" is quite often found for pulsars with double humbed or complex profiles. The plateau emission becomes more and more dominant as the beam width shrinks at higher frequencies, so that apparently "outriders" appear, which may in the end even be stronger than the "core" emission, i.e. the central part of the profile.

We may therefore state that there exists a *natural, purely geometrical explanation for the appearance of outriders*, as observed for some pulsars, e.g. PSR 1642−03, 1749−28, and 1933+16 (Sieber et al. 1975).

[1] The fact that the observed pulse width refers to the *measured* maximum of emission, not to the *radiated* maximum, can corrupt statistical interpretations of pulse width and pulse shape (circular or elongated beam?) if the analysis – which is usually the case – is based on an investigation of measured pulse widths and if an appreciable amount of pulsars shows gaussian-shaped profiles. The influence is less severe, when nearly all pulsars have more or less "box"-shaped profiles.

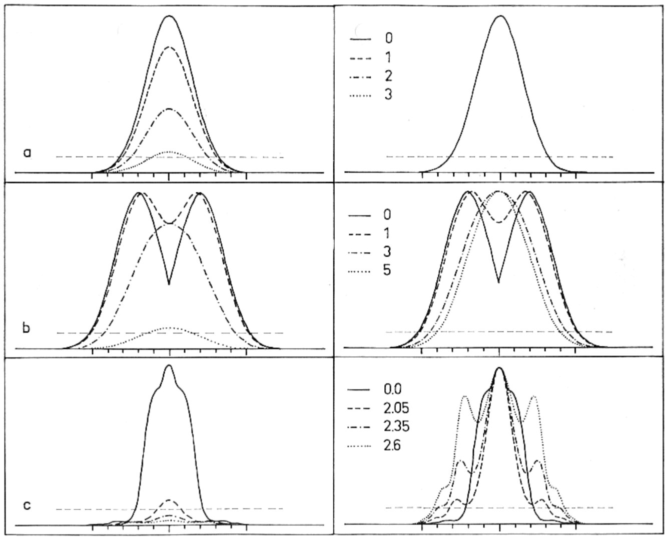

Fig. 2a–c. Cuts through (artificial) pulse profiles, left: not normalized, right: normalized. Angles (here for the corresponding model) are given in degrees. A cutting angle of 0° means that the central cut is shown. For further explanations see the text

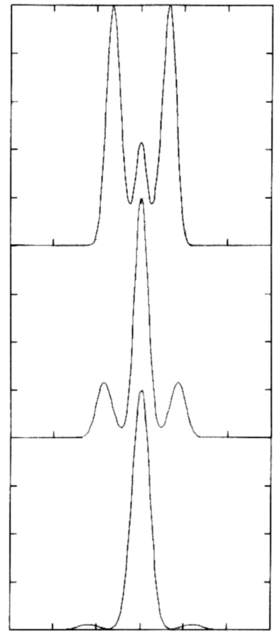

Fig. 3. Frequency dependence of the simulated profile. From top to bottom: 2 GHz, 800 MHz, 200 MHz

It may be speculated also, that we see for some pulsars only these "outskirts" of the beam, i.e. the above mentioned plateau or ring emission around the actual emission beam.

6. Spectrum

The shrinking of the emission beam (hollow cone) at high frequencies is able to influence also the received intensity, i.e. the spectrum of the whole profile or of single components. This can be demonstrated by simulating cuts through an artificial beam and by computing the resulting intensity.

Our artifical beam may consist of a gaussian beam centred on the magnetic field axis (core emission):

$$I_c(\lambda) = a_c \left(\frac{f}{f_0}\right)^{\alpha_c} \exp\left\{-(\beta^2 + \lambda^2)/2b_c^2\right\} \tag{3}$$

with $a_c = 1.0$ intensity of the main (core) component, f observing frequency, $f_0 = 200$ MHz, $\alpha_c = -1.5$ exponent of the core component, β impact angle, λ longitude in the pulsar frame.

The width of the beam is frequency dependent and defined by:

$$b_c = 2.0 \left(\frac{f}{f_0}\right)^{\beta_c} \tag{4}$$

with $\beta_c = -0.2$.

The cone (ring) emission may be given by a shifted gaussian beam:

$$I_r(\lambda) = a_r \left(\frac{f}{f_0}\right)^{\alpha_r} \exp\left\{-(\sqrt{\beta^2 + \lambda^2} - \vartheta)^2/2b_r^2\right\} \tag{5}$$

with $a_r = 0.01$ intensity of the cone emission, $\alpha_r = -1.5$, and frequency dependent beam width and cone radius

$$b_r = 2.0 \left(\frac{f}{f_0}\right)^{\beta_r} \qquad \vartheta = 13.0 \left(\frac{f}{f_0}\right)^{\gamma_r} \tag{6}$$

with $\beta_r = -0.2$ and $\gamma_r = -0.2$, resulting in profiles as shown in Fig. 3 in dependence of frequency. The core I_c, cone I_r, and total intensity

$$I_{tot}(\lambda) = I_c(\lambda) + I_r(\lambda) \tag{7}$$

is presented in Fig. 4.

It is obvious that the spectrum of the whole profile steepens considerably compared to the assumed intrinsic exponent for core and cone emission α_c and α_r of -1.5: We computed on average an observed exponent of -2.5. This means that the "burden" on the physical radiation mechanism to produce steep spectra ($\alpha < -2$) may be – at least for such geometrical arrangements – reduced considerably. It seems sufficient to have intrinsic spectral indices around -1.5; the shrinking of the emission beam can then produce a steepening of the spectrum up to the observed values around ≤ -2 (Sieber 1973).

Fig. 4 shows also, that a much steeper spectrum (compared to the rest of the profile) is observed for the core component, reaching a minimum spectral index of -4.5 for the values used here, purely due to the variations in geometry. It is therefore not necessary to assume that core component and cone component are produced by different physical mechanisms to explain the different spectral behaviour. It is interesting to note that more or less the same conclusions have been derived independently by Kramer et al. (1994), who fitted individual components of integrated profiles by gaussian profiles.

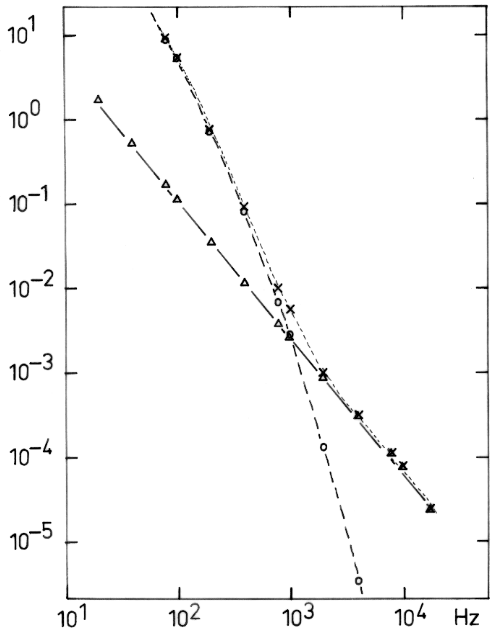

Fig. 4. Spectrum of the simulated profile: Circles – core component, triangles – cone component, crosses – total intensity

The figure suggests furthermore that the whole spectrum may steepen to high frequencies – as is observed in Fig. 4 for the core component alone – when no cone emission, or additional ring emission, exists. This effect might give an explanation for the so-called "cut-off" found in the spectra of many pulsars.

7. Conclusions

Considering the geometry of the hollow-cone model in detail one is led to the following conclusions concerning mean pulse profile shapes, widths, and spectra.

From nearly central cuts one deduces that

1. Emission occurs in one or more nested cones with a central component (core),
2. the width ϑ of the cone is moderately frequency dependent resulting in narrower component separation and observed profile widths at higher frequencies.

Pulsars with grazing cuts can show additional features purely due to geometrical variations:

1. A bifurcation of simple profiles at low frequencies,
2. the appearance of outriders at high frequencies,
3. a steepening of the overall spectrum (cut-off?),
4. a steepening of the spectra of individual components, and
5. a steeper spectrum for the core component compared to the cone components.

References

Hankins T.H., Rickett B.J., 1986, ApJ 311, 684
Hankins T.H., Izvekova V.A., Malofeev V.M. et al., 1991, ApJ 373, L17
Jones P.B., 1980, ApJ 236, 661
Kramer M., Wielebinski R., Jessner A., Gil J.A., Seiradakis J.H., 1994, A&AS 107, 515
Lyne A.G., Manchester R.N., 1988, MNRAS 234, 477
Phillips J.A., Wolszczan A., 1992, ApJ 385, 273
Radhakrishnan V., Cooke D.J., 1969, Astrophys. Lett. 3, 225
Rankin J.M., 1983, ApJ 274, 333
Ruderman M.A., Sutherland P.G., 1975, ApJ 196, 51
Sieber W., 1973, A&A 28, 237
Sieber W., Reinecke R., Wielebinski R., 1975, A&A 38, 169
Thorsett S.E., 1991, ApJ 377, 263

This article was processed by the author using Springer-Verlag LaTeX A&A style file *L-AA* version 3.