Astron. Astrophys. 321, 665–671 (1997)

ASTRONOMY AND ASTROPHYSICS

Standing Rankine-Hugoniot shocks in accretion and wind flows in Kerr geometry

Ju-Fu Lu[1,2], **K.N. Yu**[1], **Feng Yuan**[2], **and E.C.M. Young**[1]

[1]Department of Physics and Materials Science, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong
[2]Center for Astrophysics, University of Science and Technology of China, Hefei, Anhui 230026, P.R.China

Received 31 January 1996 / Accepted 21 October 1996

Abstract. This paper presents a fully relativistic study on the standing shock formation in thin, stationary, axisymmetric, adiabatic fluid flows in Kerr geometry. We find out the energy-angular momentum parameter space for which the flow may develop shocks. The multiplicity of the formal shock location is removed by employing the boundary conditions and stability analysis: the stable shock location is practically unique. The effects of the frame-dragging of a rapid rotating black hole on the shock properties are pointed out. Finally, we discuss the validity of the assumptions made about the flow and conclude that shocks are likely to be common in astrophysical flows.

Key words: accretion, accretion disks – black hole physics – hydrodynamics – relativity – shock waves

1. Introduction

Most previous theoretical works on shocks in accretion or wind flows around black holes were only for the non-rotating black hole case, and were only in terms of the Paczynski & Wiita (1980) pseudo-Newtonian potential (see, e.g. Chakrabarti & Molteni 1995 for references). Recently, Yang & Kafatos (1995) made a fully relativistic treatment of isothermal shocks in accretion flows onto a Schwarzschild black hole. Astrophysically realistic black holes, however, are generally believed to possess considerable angular momentum (e.g. Rees 1984). Perhaps because of the complexity of the Kerr metric components, a few papers published so far in the shock study have been devoted to the Kerr black hole case: Chakrabarti (1990) dealt analytically with standing Rankine-Hugoniot shocks in rotating accretion and wind flows in Kerr geometry, but the work was rather initial, in the sense that only a few examples of the shock solutions were presented; Sponholz & Molteni (1994) gave a plenty of numerical results on shocks in a thin, adiabatic accretion disk around a rotating black hole, but using another pseudo-Newtonian potential proposed by Chakrabarti & Khanna (1992).

Send offprint requests to: Ju-Fu Lu (second address)

In the present paper we discuss the problem of shock formation in stationary, axisymmetric, adiabatic flows of perfect fluids in, or very close to, the equatorial plane of Kerr geometry. It is known that the formation of shocks is based on the existence of multi-sonic points in the flow, because the flow is supposed to be originally subsonic and terminally supersonic (when an accretion flow crosses the black hole horizon, or a wind flow goes far away), it must pass through a sonic point on each side of the shock. It is therefore necessary to introduce some background knowledge of sonic points in the following paragraphs, which is contributed by many authors, e.g. Liang & Thompson 1980; Lu 1985; Lu & Abramowicz 1988; Anderson 1989; Abramowicz & Kato 1989.

For the flow considered here, there exist two intrinsic constants of motion along a fluid world line, namely the specific total energy, $E = -hu_t$, and the specific angular momentum, $l = -u_\varphi/u_t$, where u_μ's are the four velocity components obeying the normalization condition $u_\mu u^\mu = -1$ (we use the Boyer-Lindquist coordinates together with $G = c = 1$ units and $-+++$ signature), h is the specific enthalpy, i.e. $h = (P+\varepsilon)/\rho$, with P, ε and ρ being the pressure, the mass-energy density and the rest mass density, respectively. Other useful relations are the conservation of rest mass along the flow (we assume a conical shape flow), $\dot{M} = r^2 \rho u^r$ = const., and an equation of state which is assumed to be a polytropic one, i.e. $P = K\rho^{(1+1/n)}, \varepsilon = \rho + nP$, where n is the constant polytropic index, and K is a measurement of the specific entropy of the flow, which is a constant in a shock-free flow, but can increase across the shock. As Chakrabarti (1989) noticed, it is wise to define an 'entropy related' mass flow rate as $\dot{\mu} = K^n \dot{M}$, which is a conserved quantity for a shock-free flow, but can become larger at the shock due to the generation of entropy.

Under such circumstances, it can be shown that the properties of the critical (sonic) point in the radial motion of the flow, i.e. the location of the point, r_c, and the sound speed of the flow (defined as $b = (dP/d\varepsilon)^{1/2}$) at the point, b_c, which is equal to the local radial three velocity of the flow measured by a corotating observer, and accordingly the value of $\dot{\mu}$ are all determined by the two physical parameters, E and l. In particular (see Fig. 1), in the parameter space spanned by E and l, there is a strictly defined region bounded by four lines: the vertical line $E = 1$ and three characteristic functional curves $l_{\rm k}(E)$, $l_{\rm max}(E)$, and $l_{\rm min}(E)$, where $l_{\rm k}$ is the Keplerian angular momentum of the fluid (Lu, Yu & Young 1995), $l_{\rm max}$ and $l_{\rm min}$ are, respectively, the maximum and the minimum values of the function $l = l(E, r_{\rm c})$, i.e. the values of l satisfying $\partial l/\partial r_{\rm c}|_{\rm E} = 0$. Only such a flow with its parameters located within this region can have two physical sonic points, the inner one $r_{\rm in}$, and the outer one $r_{\rm out}$; in between there is still one more, but unphysical, sonic point $r_{\rm mid}$. The two physical sonic points are corresponding to the same pair of E and l, but are distinct in the radius and in the local sound speed, thus determining two different values of $\dot{\mu}$. The multi-sonic point region is divided by another characteristic functional curve, $l_{\rm c}(E)$ into two parts: in region I (= Ia + Ib) only $r_{\rm out}$ is realized in a shock-free global solution (i.e. that joining the black hole horizon to the large distance), while in region II (= IIa + IIb) only $r_{\rm in}$ is; in either case the global solution corresponds always to the smaller one of the two potential values of $\dot{\mu}$, while the larger value of $\dot{\mu}$, which is connected with the unrealizable sonic point, cannot be realized in shock-free solutions.

However, when shocks are taken into account it seems that an accretion flow with parameters in region I may develop a shock in its radial motion after passing through $r_{\rm out}$, the value of $\dot{\mu}$ jumps from the smaller one to the larger one at the shock, then the flow becomes supersonic again by passing through $r_{\rm in}$; while a wind flow belonging to region II may also have a shock, although taking exactly the opposite way: $r_{\rm in} \longrightarrow$ a shock $\longrightarrow r_{\rm out}$, across the shock the value of $\dot{\mu}$ increases too. Although the formation of these shocks is only a possibility for the moment, it is certain that neither accretion flows in region II, nor winds in region I could possibly produce shocks, because such processes would require the value of $\dot{\mu}$, i.e. require the entropy of the flow to decrease, violating the second law of thermodynamics.

Based on the above knowledge we go a step further in the present paper, examining the possibilities and the properties of the standing Rankine-Hugoniot shock in terms of the intrinsic physical parameters of the flow.

2. Multiplicity of the shock location

The Rankine-Hugoniot shock conditions for a relativistic fluid flow are continuities for rest-mass, energy, and momentum flux densities(Landau & Lifshitz 1987):

$$
\begin{aligned}
&[\rho u^r] = 0,\\
&[T^{tr}] = [(\varepsilon + P)u^t u^r] = 0, \qquad (1)\\
&[T^{rr}] = [(\varepsilon + P)u^r u^r + P g^{rr}] = 0,
\end{aligned}
$$

where $T^{\mu\nu}$'s are the energy-momentum tensor components, $g^{\mu\nu}$'s are the metric components, and the square brackets denote the difference between the values of a quantity on the two sides of the shock. For flows with properties described in Sect.1 we transform Eq.(1) into the following working set of the jump conditions across the shock:

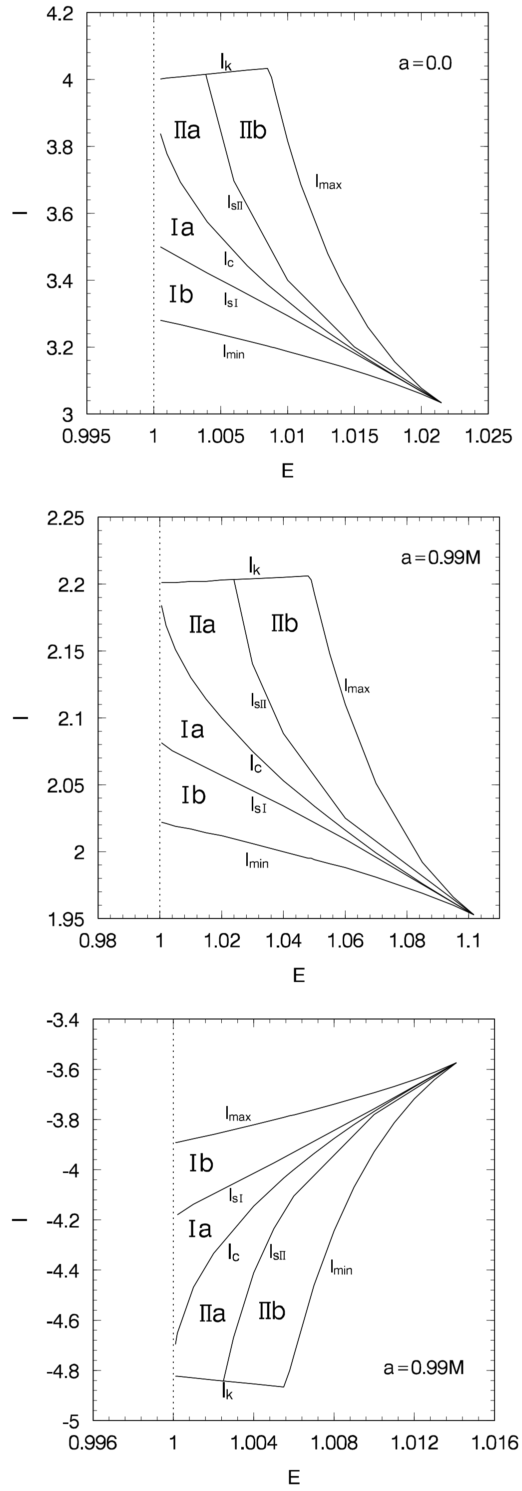

Fig. 1a–c. Energy-angular momentum parameter space of standing shock formation in adiabatic flows around a black hole. **a** (upper) is for a Schwarzschild black hole ($a = 0$), **b** (middle) for a rapid Kerr hole ($a = 0.99$) with prograde flows, and **c** (lower) for a rapid Kerr hole with retrograde flows. Only for an accretion flow with its parameters E and l located within region Ia (bounded by lines $l_{\rm c}, l_{\rm sI}$ and $E = 1$), or a wind flow with E and l belonging to region IIa (bounded by lines $l_{\rm k}, l_{\rm sII}, l_{\rm c}$ and $E = 1$), there exists an unique stable shock. See text for details.

$$E = (\frac{1 + g_{rr}u^r_- u^r_-}{-L})^{1/2}/(1 - nb^2_-), \quad (2a)$$

$$E = (\frac{1 + g_{rr}u^r_+ u^r_+}{-L})^{1/2}/(1 - nb^2_+), \quad (2b)$$

$$\dot{\mu}_- = r_s^2(\frac{nb^2_-}{(n+1)(1 - nb^2_-)})^n u^r_-, \quad (2c)$$

$$\dot{\mu}_+ = r_s^2(\frac{nb^2_+}{(n+1)(1 - nb^2_+)})^n u^r_+, \quad (2d)$$

$$\frac{1}{\dot{\mu}_-}(\frac{b^2_-}{1 - nb^2_-})^{n+1}(\frac{(n+1)u^r_- u^r_-}{b^2_-} + ng^{rr}) = \frac{1}{\dot{\mu}_+}(\frac{b^2_+}{1 - nb^2_+})^{n+1}(\frac{(n+1)u^r_+ u^r_+}{b^2_+} + ng^{rr}), \quad (2e)$$

where L is the effective potential of the flow motion defined as $L(r, l) = g^{tt} - 2g^{t\varphi}l + g^{\varphi\varphi}l^2$ (Lu et al. 1995), r_s is the location of the shock, the subscripts - and + denote the values before and after the shock, respectively.

As stated in Sect.1, once the values of the two constants, E and l, are given, the value of $\dot{\mu}$ is accordingly obtained, so it is not another independent constant, but is the eigenvalue of the problem, $\dot{\mu} = \dot{\mu}(E, l)$. A flow with E and l located within the multi-sonic point region of the parameter space (bounded by lines l_k, l_{max}, l_{min} and $E = 1$ in Fig. 1) can have two different values of $\dot{\mu}$, the smaller one, $\dot{\mu}_-$, is realized either in a shock-free global solution, or in the preshock part of a shock-included solution, while the larger one, $\dot{\mu}_+$, cannot be realized in a shock-free solution, it can be realized only in the postshock part of a shock solution. Therefore, with a suitably given pair of E and l, and two accordingly determined quantities, $\dot{\mu}_-$ and $\dot{\mu}_+$, the five Eqs. (2a-e) enable us to solve for five unknowns: $r_s, u^r_-, b_-, u^r_+, b_+$. The radial three-velocity measured by a corotating observer, $v_{(r)}$, is obtained by a transformation (Lu 1986): $v^2_{(r)} = u_r u^r/(1 + u_r u^r)$, and the Mach number is defined as $M = v_{(r)}/b$. The shock solution of the flow, as in the shock-free case, is completely determined by the two constants of motion, E and l.

The first thing is the location of the shock. The computing results show that there exists a new strictly defined function, $l_s(E)$, drawn in Fig. 1 as two curves l_{sI} and l_{sII}, for an accretion flow with its parameters E and l located within region Ia (bounded by lines l_c, l_{sI}, and $E = 1$), or a wind flow with E and l belonging to region IIa (bounded by lines l_k, l_{sII}, l_c and $E = 1$), there are four formal shock locations, denoted by r_{s1}, r_{s2}, r_{s3} and r_{s4}, which are related to the three formal locations of the sonic point, r_{in}, r_{mid} and r_{out}, as

$$r_{s1} < r_{in} < r_{s2} < r_{mid} < r_{s3} < r_{out} < r_{s4}; \quad (3)$$

while for an accretion flow belonging to region Ib (bounded by lines l_{sI}, l_{min} and $E = 1$ in Figs. 1a and 1b, and by lines l_{sI}, l_{max} and $E = 1$ in Fig. 1c), or a wind flow belonging to region IIb (bounded by lines l_k, l_{max} and l_{sII} in Figs. 1a and 1b, and by lines l_k, l_{min} and l_{sII} in Fig. 1c), there are only two formal shock locations, denoted by r_{s1} and r_{s4}, satisfying

$$r_{s1} < r_{in}, \quad r_{out} < r_{s4}. \quad (4)$$

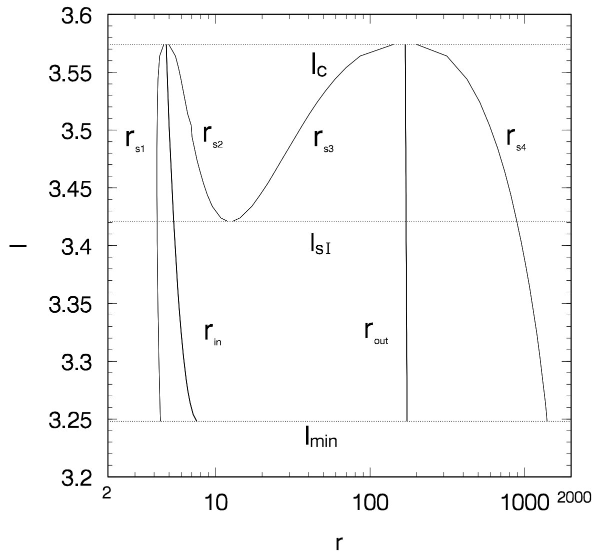

Fig. 2. An example of the multiplicity of the formal shock location in accretion flows, where $a = 0$ and $E = 1.004$. For $l_c > l > l_{sI}$ there are four formal shock locations satisfying relation (3), while for $l_{sI} > l > l_{min}$ there are only two satisfying relation (4).

An example of the multiplicity of the formal shock location, taken from region I of Fig. 1a, is shown in Fig. 2, where E = 1.004, and the corresponding l_c = 3.574 (hereafter l, r and the black hole's specific angular momentum a are all in units of the black hole mass), l_{sI} = 3.421, l_{min} = 3.248, it is clearly seen that for $l_c > l > l_{sI}$ there are four formal shock locations satisfying relation (3), while for $l_{sI} > l > l_{min}$ there are only two satisfying relation (4). These results are qualitatively similar to those for adiabatic flows around a non-rotating black hole in the context of Paczynski & Wiita (1980) potential (Chakrabarti 1989), as well as those for isothermal accretion flows in Schwarzschild geometry (Yang & Kafatos 1995). A preliminary discussion on the nature of these formal shock locations can be made by employing the boundary conditions: for an accretion flow onto a black hole both r_{s1} and r_{s4} can be ruled out, because the flow must be supersonic when crossing the black hole horizon and subsonic when starting at the large distance, and no sonic point exists inside r_{s1} or outside r_{s4} (however for accretion onto a neutron star r_{s1} is a possible shock location, because the flow is terminally subsonic); while for a wind flow r_{s1} is not possible either, but r_{s4} is possible if the flow is terminally subsonic. The possibilities of shocks at r_{s2} and r_{s3}, however, cannot be judged in this way, on which we focus our attention from now on.

3. Stability analysis

The ambiguity of the shock locations r_{s2} and r_{s3} was first removed by Chakrabarti & Molteni (1993) for adiabatic flows around a non-rotating black hole in the context of Paczynski & Wiita (1980) potential. By checking the variation of the total pressure of the post-shock flow as the shock is perturbed, they argued that the shock at r_{s3} is stable for accretion, and unstable for winds, but did not give a definite conclusion about the shock at r_{s2}. More recently, Yang & Kafatos (1995) made a fully relativistic analysis on the stability of shocks in isothermal accretion flows onto a Schwarzschild black hole, and found that the shock at r_{s2} is unstable, while the shock at r_{s3} is stable except when r_{s3} is very close to the point at which $du^r_-/dr = 0$. Here we follow Yang & Kafatos' (1995) method, extending to the case of adiabatic accretion and wind flows in Kerr geometry.

Across the shock, the momentum flux density, defined as (different from T^{rr} only by a constant factor)

$$F = \frac{1}{\dot{\mu}}(\frac{b^2}{1-nb^2})^{n+1}(\frac{(n+1)u^r u^r}{b^2} + ng^{rr}), \tag{5}$$

is conserved, resulting exactly in Eq.(2e). If due to some perturbation, the shock location is moved from r_s to $r_s + \delta r$, the momentum flux density may not be in balance, the resulting difference across the shock is

$$\delta F = F_+ - F_- = (\frac{dF_+}{dr} - \frac{dF_-}{dr})\delta r \equiv \Delta \delta r. \tag{6}$$

The stability of the shock depends on the sign of Δ. If $\Delta > 0$, for accretion flows (i.e. along decreasing r), when $\delta r > 0$ (or $\delta r < 0$), the momentum flux just after the shock is larger (or smaller) than that just before the shock, so the shock should be put to shift towards further increasing (or further decreasing) r, thus the shock is unstable; but for wind flows (i.e. along increasing r) the shock is stable, because the imbalance of the momentum flux due to δr would always cause the shock to move back towards its unperturbed location. On the contrary, $\Delta < 0$ implies that the shock in accretion flows is stable, and the shock in winds is unstable.

The present case is mathematically more complex than that studied by Yang & Kafatos (1995), so that it is not possible to obtain an analytic criterion of the sign of Δ, and the only way to estimate Δ is that through numerical calculations. It is seen from Eqs (5) and (2) that to evaluate dF_-/dr, the values of du^r_-/dr and db_-/dr are needed, which can be solved out by combining the differential form of Eqs (2a) and (2c), i.e. $dE(r, u^r_-, b_-)/dr = 0$ and $d\dot{\mu}_-(r, u^r_-, b_-)/dr = 0$; similarly, the values of du^r_+/dr and db_+/dr, needed to evaluate dF_+/dr, can be solved out by combining the differential form of Eqs (2b) and (2d), i.e. $dE(r, u^r_+, b_+)/dr = 0$ and $d\dot{\mu}_+(r, u^r_+, b_+)/dr = 0$. In this way, we have performed calculations for the Schwarzschild black hole and the Kerr hole, as well as for prograde flows and retrograde ones, and have been able to reach the following conclusion: *for accretion flows the shock at* r_{s3} *is always stable, while the shock at* r_{s2} *is unstable except when the value of* l *is very close to that of* l_s*; for wind flows the shock at* r_{s3} *is always unstable, while the shock at* r_{s2} *is stable except when* l *is very close to* l_s. It is seen from Fig. 2 that when l is close to l_s the values of r_{s2} and r_{s3} become close to each other. To show more precisely the exceptional case when l is very close to l_s, we

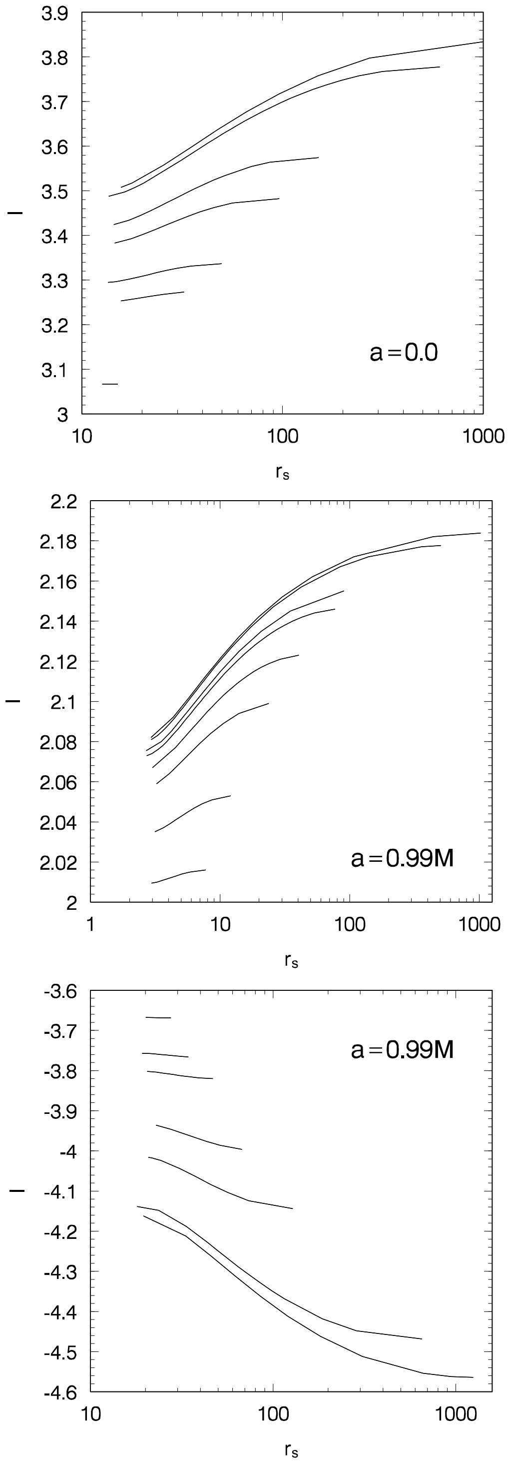

Fig. 3a–c. Stable shock locations for accretion flows, i.e. r_{s3}, as functions of E and l. **a** (upper) is for $a = 0$, where the seven lines are, from the top to the bottom, for $E = 1.0005, 1.001, 1.004, 1.006, 1.01, 1.012, 1.02$, respectively; **b** (middle) is for $a = 0.99$ and prograde flows, the eight lines are, from the top to the bottom, for $E = 1.0005, 1.001, 1.004, 1.006, 1.012, 1.02, 1.04, 1.06$, respectively; **c** (lower) is for $a = 0.99$ and retrograde flows, the seven lines are, from the top to the bottom, for $E = 1.012, 1.01, 1.009, 1.006, 1.004, 1.001, 1.0005$, respectively.

give a numerical example: for the black hole's specific angular momentum a = 0.99 and prograde accretion flows (cf. Fig. 1b), when E = 1.004 the corresponding $l_{\rm s}$ is slightly smaller than 2.0754, in the numerical calculation when the parameter l is taken to be 2.0754 both the shocks at $r_{\rm s2}$ and $r_{\rm s3}$ are stable (in fact $r_{\rm s2}$ and $r_{\rm s3}$ are so close to each other now, so that it is practically not possible to distinguish them), but when l = 2.08 the shock at $r_{\rm s2}$ becomes unstable, and the value 2.08 is still not the lower limit of l to which the resulting $r_{\rm s2}$ keeps being an unstable shock location.

4. Locations and strengths of stable shocks

The resulting stable shock location for accretion flows, i.e. $r_{\rm s3}$, is shown in Fig. 3 as a function of E and l. The range of E and l values in Fig. 3 is identical with region Ia in Fig. 1. It is seen from Fig. 3 that there is always an almost uniform lower limit of stable shock locations for different values of E and l: in the case of a rapid Kerr black hole with prograde flows (Fig. 3b) this limit is about three gravitational radii, which is considerably closer to the hole than that in the case of a rapid Kerr hole with retrograde flows (about twenty gravitational radii, see Fig. 3c) as well as that in the Schwarzschild black hole case (somewhere between ten and twenty gravitational radii, see Fig. 3a). On the other hand, the shock location can always extend outwards to almost the same distance for flows with the same total energy, no matter what kind the central black hole is of, for example, for E = 1.0005 the shock location $r_{\rm s3}$ extends to about a thousand gravitational radii in all the three cases shown in Figs. 3a, b, c. It is also observed from Fig. 3 that the shock location for retrograde flows around a rapid Kerr black hole (Fig. 3c) behaves similarly as that for flows around a Schwarzschild hole with higher values of l (Fig. 3a, here we mean the absolute value of l), reflecting the fact that in the former case some centrifugal force is 'spent' to fight against the frame-dragging; for prograde flows around a rapid Kerr hole (Fig. 3b, cf. also Fig. 1), however, the frame-dragging helps to strengthen the centrifugal barrier, so that only those with lower angular momenta and higher energies are allowed to go through the barrier, i.e. to form shocks. Astrophysical implications of this distinction need further studying.

The corresponding shock strength, defined as the ratio of Mach numbers just before and just after the shock, M_-/M_+, is drawn in Fig. 4 as functions of E and l. It is seen that the strength generally increases with decreasing energy and/or decreasing angular momentum (in absolute value), and that the shock in prograde flows around a rapid Kerr black hole is stronger than that in flows around a Schwarzschild hole as well as that in retrograde flows around a rapid Kerr hole.

For the sake of completeness, we show the location of the stable shock in winds, i.e. $r_{\rm s2}$, in Fig. 5 as functions of E and l, the range of E and l values is identical with region IIa in Fig. 1a. The corresponding shock strength is given in Fig. 6.

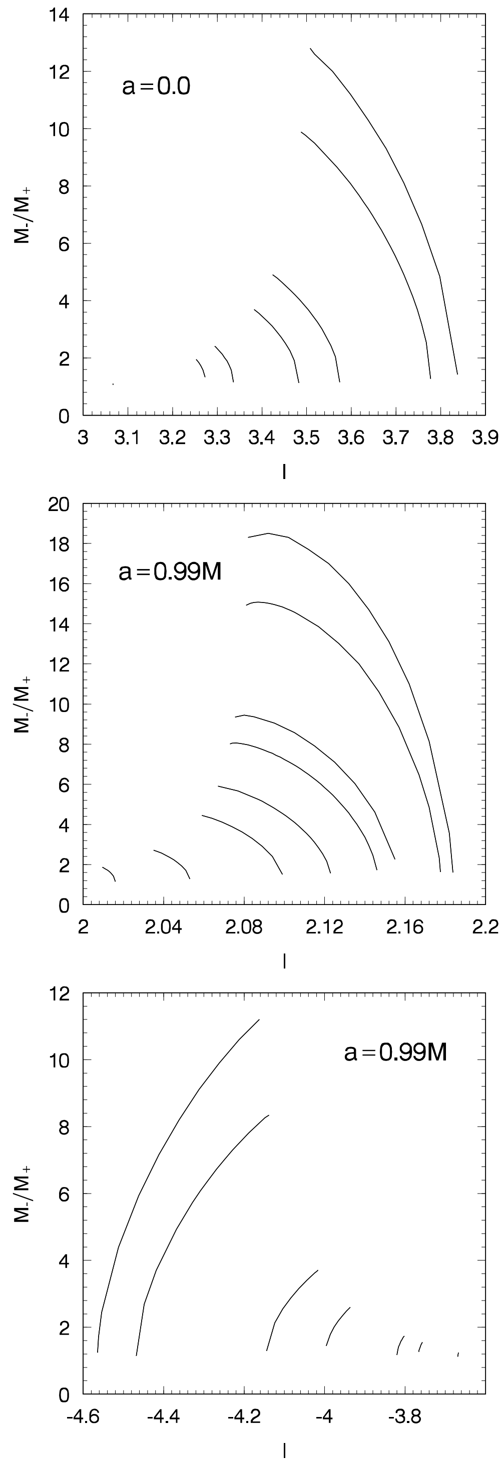

Fig. 4a–c. Strengths of the stable shocks for accretion flows as functions of E and l. **a–c** are for a = 0, a = 0.99 and prograde flows, and a = 0.99 and retrograde flows, respectively. The lines in the three figures are, from the rightmost to the leftmost, corresponding to those from the top to the bottom in Figs. 3a, 3b, and 3c, respectively.

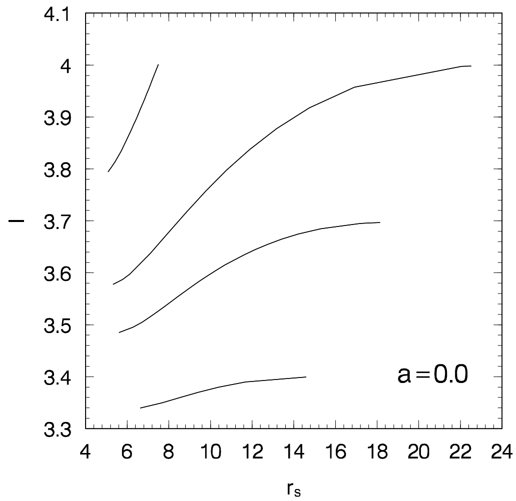

Fig. 5. Stable shock locations for $a = 0$ and wind flows, i.e. r_{s2}, as functions of E and l. The four lines are, from the top to the bottom, for E = 1.001, 1.004, 1.006, 1.01, respectively.

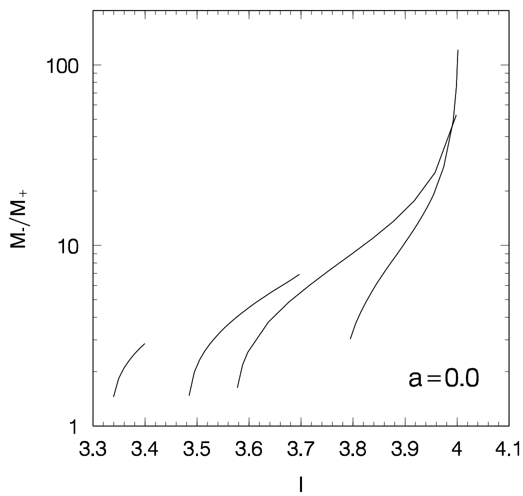

Fig. 6. Strengths of the shocks in Fig. 5. The four lines are, from the rightmost to the leftmost, corresponding to those from the top to the bottom in Fig. 5, respectively.

5. Discussion

In this paper, we have studied theoretically the nature of Rankine-Hugoniot shocks in thin, axisymmetric accretion and wind flows in Kerr geometry. We have presented the energy-angular momentum parameter space for which the flow may develop shocks. The ambiguity in regard of the shock location is removed by stability analysis: the stable shock location is practically unique. We have also shown that the frame-dragging effect of a rapid Kerr black hole causes shocks in prograde flows to locate closer to the hole, and to be stronger than those in retrograde flows as well as those in the Schwarzschild hole case. These fully relativistic results agree qualitatively, thus confirm those obtained in terms of a pseudo-Newtonian black hole model, e.g. Chakrabarti & Molteni (1993) for a non-rotating black hole, and Sponholz & Molteni (1994) for a rotating one, further suggesting that shocks are likely to occur in astrophysical disks and winds.

We now make some comments on the assumptions for the flow made in this paper, i.e. that the flow has a conical shape, and that the flow motion is adiabatic. Certainly, none of these is strictly valid in realistic situations: at the shock the flow is likely to expand in the vertical direction, and to loss energy into its surroundings, and a sophisticated theory of the shock process taking all these effects into account is definitely awaited. However, from the existing literature it seems that the major conclusion made about the shock in the flow ignoring these detailed effects is still meaningful. Fukue (1987) constructed the first example of the standing shock in the adiabatic, conical accretion flow. The above mentioned works of Chakrabarti & Molteni (1993) and Sponholz & Molteni (1994) studied, both analytically and through numerical simulation, the nature of Rankine-Hugoniot shocks in a flow with constant thickness. The assumption that the flow has a constant thickness should not be virtually different from that the flow keeps a conical shape, since in both cases the flow has a fixed rigid surface and the vertical equilibrium in the flow is not assumed. On the other hand, the Rankine-Hugoniot shock in a flow whose shape is not fixed but the vertical equilibrium holds has been studied by, e.g. Chakrabarti (1989), and the relevant result is similar with that obtained using the other model, i.e. the flow has a fixed shape and no vertical equilibrium. In fact, as Chakrabarti (1992) showed, if the vertical motion is ignored, then there is virtually no difference in the transonic properties of axisymmetric, polytropic disks whether they are of constant thickness, or of conical shape, or in vertical equilibrium. Extending further, when the assumption of adiabatic flow is replaced by that of isothermal flow, as done by Yang & Kafatos (1995), the major conclusion reached accordingly, i.e. the multiplicity of the formal shock location and the uniqueness of the stable shock location, remains qualitatively the same as that in the adiabatic case. Altogether, the efforts of all the above authors and others seem to have indicated that shocks can be common in inviscid flows, independent of the flow's shape, and whether the flow motion is adiabatic or it is isothermal.

One factor that affects seriously the shock formation is viscosity. As Chakrabarti & Molteni (1995) showed for isothermal accretion flows, viscosity is repulsive to shocks in the sense that a large enough viscosity can cease shocks, and that the flow remains subsonic throughout the accretion disk and becomes supersonic only at the inner sonic point located very close to the black hole; for flows with a small viscosity shocks can still form, but they are weaker and located farther away as compared with those in inviscid flows. These results are sound in the view that the viscous process is to convert gravitational potential energy to thermal energy of the accreting matter continuously and gradually, thus reducing kinetic energy (converted from gravitational energy too) that an inviscid flow could gather, and weakening the shock, or even ceasing it if the viscosity is large. We believe that viscosity has similar effects on the flow studied in the present paper, then our conclusions made here should hold for a small viscosity, but not for a large one.

Recently Narayan & Yi (1994) constructed self-similar, advection-dominated accretion flows in which most of the viscously dissipated energy is carried in by the accreting gas as entropy rather than being radiated. Shock-included accretion flows studied in the present paper have similar properties of advection: the entropy at the inner sonic point is higher than that at the outer sonic point, this means that the flow is advecting all the entropy generated at the shock without radiating it, and all the energy is also advected along with the flow downstream into the black hole. It is also interesting to note that our flows all have the relativistic Bernoulli constant $E > 1$, identical with the positive value of the non-relativistic Bernoulli constant for flows of Narayan & Yi (1994). Accordingly, our flows are expected to occur in physical situations similar to that for Narayan & Yi(1994) flows, i.e. when the optical depth is either very small (the radiative cooling is unable to keep up with the viscous energy generation), or very large (the cooling time of the disk is longer than the accretion time). Astrophysically, both the Narayan & Yi flows and ours have very sub-Eddington luminosities. However this does not necessarily mean that these flows are difficult to detect observationally, as adequate accretion rates may result in detectable luminosities even for a very low efficiency. For this reason Abramowicz & Lasota (1995) described advectively dominated disks as *secret guzzlers*, of which several astrophysical objects have been regarded as candidates (see references in Abramowicz & Lasota 1995).

Apart from these similarities, the self-similar solution of Narayan & Yi (1994)has a limitation that it cannot be applied to the inner region of the disk due to the restrictions of the boundary, and it is in the inner region where the accretion flow eventually becomes supersonic. More precisely, as Abramowicz & Lasota (1995) analyzed, it is the difficult enterprise of solving the problem of transonic flows that masked for a long time the real physical issues of advection dominated flows. However, a complete and self-consistent theory for the black hole accretion flow must take into account the flow's transonic nature, as done very recently by Igumenshchev, Chen & Abramowicz (1996) who incorporated both the advective cooling and the transonic motion in the flow. To sum up, there have been two kinds of advection-dominated solutions for black hole accretion, in both of which the flow passes the inner sonic point near the black hole. The difference between them is: in the Narayan & Yi solution the dissipation of gravitational energy to thermal energy in the accretion disk is by some continuous viscous processes, and the flow remains subsonic throughout the disk except in the innermost region, while in the solution of the present paper the dissipation is via shocks, the additional outer sonic point in the flow is required. A detailed comparison between the two kinds of solutions will be a subject of our future work.

References

Abramowicz M.A., Kato S., 1989, ApJ 336, 304
Abramowicz M.A., Lasota J.-P., 1995, Comments Astrophys. 18,141
Anderson M., 1989, MNRAS 239, 19
Chakrabarti S.K., 1989, ApJ 347, 365
Chakrabarti S.K., 1990, ApJ 350, 275
Chakrabarti S.K., 1992, MNRAS 259, 410
Chakrabarti S.K., Khanna R., 1992, MNRAS 256, 300
Chakrabarti S.K., Molteni D., 1993, ApJ 417, 671
Chakrabarti S.K., Molteni D., 1995, MNRAS272, 80
Fukue J., 1987, PASJ 39, 309
Igumenshchev I.V., Chen X., Abramowicz M.A., 1996, MNRAS 278, 236
Landau L.D., Lifshitz E.M., 1987, Fluid Mechanics 2nd ed. Pergamon Press, Oxford
Liang E.P.T., Thompson K.A., 1980, ApJ 240, 271
Lu J.F., 1985, A&A 148, 176
Lu J.F., 1986, Gen. Rel. Grav. 18, 45
Lu J.F., Abramowicz M.A., 1988, Acta Ap. Sin. 8, 1
Lu J.F., Yu K.N., Young E.C.M., 1995, A&A 304, 662
Narayan R., Yi I., 1994, ApJ 428, L13
Paczynski B., Wiita P.J., 1980, A&A, 88, 23
Rees M., 1984, ARA&A, 22, 471
Sponholz H., Molteni D., 1994, MNRAS 271, 233
Yang R., Kafatos M., 1995, A&A 295, 238

This article was processed by the author using Springer-Verlag LaTeX A&A style file *L-AA* version 3.