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The Hall effect and the decay of magnetic fields

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Abstract. We consider the influence of the Hall effect on the evolution of a strong magnetic field which can magnetize plasma. The Hall current couples various multipole field components and provides a possibility of an energy exchange among modes. Due to this, the evolution of strong magnetic fields is substantially more complicated than the ohmic decay of weak magnetic fields. Non-linear effects can create a complex magnetic configuration even from the simplest one corresponding originally to the pure dipole field outside the star. The energy transfer among field components leads to a non-monotonic behaviour of individual modes. Variations of the large scale magnetic field are of the same origin as the so called helicoidal oscillation of magnetized plasma. The period of variations is determined by the time scale associated with the Hall effect and may be sufficiently short for a strong magnetic field. The non-linear phenomena may be important for the magnetic evolution of white dwarfs and neutron stars.

Key words: magnetic fields – MHD – stars: magnetic fields – stars: neutron – stars: white dwarfs

1. Introduction

Observational data on the surface magnetic field of many white dwarfs and neutron stars provide evidence for the complex character of their magnetic configurations. In the atmospheres of white dwarfs, the field strength can be deduced from Zeeman broadened absorption lines while the field direction can be inferred from polarization measurements (see, e.g., Chanmugam 1992 for a general review). It turns out that the surface magnetic configurations of many white dwarfs are likely to consist of a mixture of different modes rather than a simple dipole structure. Thus, Meggitt & Wickramasinghe (1989) and Ferraro & Wickramasinghe (1989) have found the presence of the quadrupole mode in atmospheres of some white dwarfs, Achilleos et al. (1992) have concluded that the field of the magnetic white dwarf Feige 7 can be represented as the sum of the dipole and quadrupole components with the strengths approximately 2/3 and 1/3 of the total field strength, respectively. There is also evidence for a complex magnetic field structure of white dwarfs in AM Her like systems (see Chanmugam 1992 for details). Probably, the magnetic configurations of at least some neutron stars differ from the pure centered dipole as well. For instance, it has been pointed out by Krolik (1991) that the fields of millisecond pulsars may consist of few multipole components, and an admixture of higher order modes can account for the unusual pulse morphology of these objects. Thus, it seems that departures from the simplest dipole configuration are widespread both in white dwarfs and neutron stars.

The problem of the origin and evolution of the magnetic field in white dwarfs and neutron stars is still far from being solved. Therefore, one cannot exclude that a complex distribution of the magnetic field originates from the initial conditions which are subject to many uncertainties. The decay of magnetic eigenmodes in white dwarfs has been first considered by Chanmugam & Gabriel (1972) who argued that the decay time scale of the lowest multipoles is of the order of 10^{10} yr but high order modes can evolve on a shorter time scale. Wendel et al.(1987) have analysed in detail the influence of the cooling of white dwarfs on the time evolution of individual modes, and confirmed that the decay of the fundamental eigenmode proceeds on a time scale $\sim 10^{10}$ yr. These calculations showed that it is more likely to observe complex magnetic configurations in relatively young white dwarfs if high order modes are generated during the initial evolutionary stages.

However, a complex magnetic configuration can be created also from a simple one due to non-linear magnetohydrodynamic effects in the course of the evolution of magnetized stars. One possibility is associated with a dependence of transport processes on the magnetic field. Thus, the conductive properties of magnetized plasma are anisotropic and the electric resistivities along and across the field may differ. Besides, the presence of the magnetic field induces the so called Hall current which is perpendicular both to the electric, $\boldsymbol{E}$, and magnetic, $\boldsymbol{B}$, fields. This current is non-dissipative since it does not contribute di-

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rectly to an increase in the density of entropy, $\dot{Q}$,

$$\dot{Q} = \boldsymbol{j} \cdot \boldsymbol{E} = R_{\|} j_{\|}^2 + R_{\perp} j_{\perp}^2 \; , \quad (1)$$

where $\boldsymbol{j}$ is the current density, $R_{\|}$ and $R_{\perp}$ are the components of the resistivity tensor; the subscripts $\|$ and $\perp$ mean the component of the corresponding quantities along and across the magnetic field. However, the Hall current couples different modes and alters a magnetic configuration by redistributing the energy among modes. In this way, the Hall current can affect indirectly the rate of the magnetic field dissipation and, in principle, can increase this rate substantially for strongly magnetized stars. Numerical simulations first done by Urpin & Shalybkov (1991) for the simplest toroidal magnetic configuration confirmed this conclusion. More recently several attempts have been made to consider more complex magnetic configurations including a poloidal field (see, e.g., Naito & Kojima 1994; Muslimov 1994; Muslimov et al.1995). Naito & Kojima (1994) studied the effect of the Hall current expanding the vector potential of the magnetic field by a set of Legendre polynomials up to $l = 5$, but they restricted their calculations to the case when the Hall effect is small in comparison with the ohmic dissipation. Muslimov (1994) and Muslimov et al.(1995) analysed a transfer of the energy among the modes assuming that the magnetic configuration consists originally of dipole, quadrupole and lowest order toroidal components. They argued that this configuration is the simplest one influenced by the Hall effect. In both these papers, the authors neglected the influence of the poloidal components on the toroidal one in order to simplify the numerical calculations. This simplification causes some doubts, however, since the obtained results do not even reproduce well known phenomena (like helicoidal oscillations of the magnetic field) associated with the Hall effect.

In the present paper, we focus on the most important phenomena caused by the Hall effect keeping in mind applications to particular astrophysical objects (white dwarfs and neutron stars) for the forthcoming paper. The non-linear magnetic evolution is complicated, and to understand its physical content it is helpful to consider initially a very idealized example which, nevertheless, describes qualitatively the main features of more realistic models. Our adopted model is therefore maximally simplified: we treat the decay of a strong magnetic field in a uniform conducting sphere. The original magnetic configuration is also chosen to be the simplest one: it is assumed that only the lowest order poloidal mode, corresponding to the dipole field outside the sphere, is presented at $t = 0$. Our numerical calculations illustrate the process of the generation of higher order poloidal and toroidal modes from the simple original configuration and the evolution of these modes.

The paper is organized as follows. Sect. 2 presents the main equations governing the evolution of the magnetic field in the presence of the Hall effect. Sect. 3 describes the numerical method and results of our calculations. Principal conclusions are summarized in Sect. 4.

2. Basic equations

Faraday's induction equation and Ampere's equation governing the evolution of the magnetic, $\boldsymbol{B}$, and electric, $\boldsymbol{E}$, fields read in the magnetohydrodynamic approximation

$$\frac{\partial \boldsymbol{B}}{\partial t} = -c \nabla \times \boldsymbol{E} \quad (1)$$

and

$$\nabla \times \boldsymbol{B} = \frac{4\pi}{c} \boldsymbol{j} \; , \quad (2)$$

where $\boldsymbol{j}$ is the current density. Neglecting fluid motions, the generalized Ohm's law may be written in the form (see, e.g., Lifshitz & Pitaevskii 1981)

$$\boldsymbol{E} = R_{\|} \, j_{\|} + R_{\perp} \, j_{\perp} + R_{\wedge} \, \boldsymbol{j} \times \boldsymbol{b} \; , \quad (3)$$

where $R_{\wedge}$ is the so called Hall component of the resistivity tensor, $\hat{R}$; $\boldsymbol{b} = \boldsymbol{B}/B$.

The conductive properties of the internal region of white dwarfs and the neutron star crust are determined by degenerate electrons. With a sufficient accuracy, the electron scattering can be described in the relaxation time approximation. Then, if the magnetic field is non-quantizing, the parallel and perpendicular to the magnetic field components of the resistivity tensor are related by $R_{\|} = R_{\perp} = R_0$ where R_0 is the electric resistivity at $B = 0$ (see Urpin & Yakovlev 1980). Ohm's law (3) can then be rewritten as

$$\boldsymbol{E} = R_0 (\boldsymbol{j} + \alpha \, \boldsymbol{j} \times \boldsymbol{b}) \; , \quad (4)$$

where $\alpha = R_{\wedge}/R_0 = \omega_B \tau$ is the Hall parameter, ω_B and τ are the gyrofrequency and relaxation time of electrons, respectively. Note that the Hall parameter depends linearly on the magnetic field. At $\alpha \leq 1$, the evolution of the magnetic field is weakly influenced by the Hall effect whereas at $\alpha > 1$ this influence is especially pronounced. The magnetization condition, $\alpha > 1$, can be fulfiled for the relatively wide range of the magnetic fields detected in white dwarfs and neutron stars (see for more details Urpin & Yakovlev 1980).

For the sake of simplicity, we assume the conducting sphere to be uniform thus both R_0 and α/B are independent of coordinates. Then, the induction equation (1) transforms to

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\frac{c^2 R_0}{4\pi} \nabla \times [\, \nabla \times \boldsymbol{B} + \alpha \, (\nabla \times \boldsymbol{B}) \times \boldsymbol{b} \,] \; . \quad (5)$$

One can represent the magnetic field as the sum of poloidal, $\boldsymbol{B}_p$, and toroidal, $\boldsymbol{B}_t$, field components. The present paper is addressed to the evolution of the axisymmetric magnetic configuration thus one has in this case (see, e.g., Chandrasekhar & Prendergast 1956)

$$\boldsymbol{B} = \boldsymbol{B}_p + \boldsymbol{B}_t = \nabla \times [A(r, \theta) \boldsymbol{e}_{\varphi}] + B_{\varphi}(r, \theta) \boldsymbol{e}_{\varphi} \; , \quad (6)$$

where $\boldsymbol{e}_{\varphi}$ is the unit vector in the azimuthal direction; r, θ, φ are spherical coordinates. At $\alpha = 0$, the toroidal and poloidal components evolve independently whereas at $\alpha \gg 1$ they are strongly coupled. If $\alpha = 0$, the eigenmodes decay exponentially with a slower decay for a lower order mode. The decay time scale of the fundamental poloidal mode is (see, e.g., Landau & Lifschitz 1960)

$$\tau_0 = \frac{4a^2}{\pi c^2 R_0} \ , \qquad (7)$$

where a is the radius of the sphere.

The induction equation (5) yields two equations governing the evolution of the poloidal and toroidal magnetic fields,

$$\frac{\partial \boldsymbol{B}_p}{\partial t} = -\frac{c^2 R_0}{4\pi}\{\nabla\times(\nabla\times\boldsymbol{B}_p)+\beta\nabla\times[(\nabla\times\boldsymbol{B}_t)\times\boldsymbol{B}_p]\} \ , \quad (8)$$

$$\frac{\partial \boldsymbol{B}_t}{\partial t} = -\frac{c^2 R_0}{4\pi}\{\nabla\times(\nabla\times\boldsymbol{B}_t)+\beta\nabla\times[(\nabla\times\boldsymbol{B}_p)\times\boldsymbol{B}_p + (\nabla\times\boldsymbol{B}_t)\times\boldsymbol{B}_t]\} \ , \quad (9)$$

where $\beta = \alpha/B$. Note that $\nabla\times\boldsymbol{B}_p$ is the toroidal vector thus the cross-product $(\nabla\times\boldsymbol{B}_p)\times\boldsymbol{B}_t$ is zero. It is seen from Eq. (9) that even if the original magnetic configuration is purely poloidal, a toroidal field will be generated due to the Hall current associated with the term proportional to $(\nabla\times\boldsymbol{B}_p)\times\boldsymbol{B}_p$. In its turn, the toroidal field will generate high-order poloidal modes due to the non-linear term on the right hand side of Eq. (8). However, the Hall current can not provide a generation of poloidal modes if the original magnetic configuration is purely toroidal. Thus, in order to analyse the influence of the Hall effect on the evolution of the magnetic field there is no need to consider a complicated original configuration consisting at least of three modes as was pointed out by Muslimov (1994) and Muslimov et al.(1995). Even the simplest dipole magnetic configuration is influenced by the Hall current, and the evolution of this configuration may illustrate the main qualitative features of the non-linear field decay. That is why we suppose the initial field to be poloidal with the φ-component of the vector potential, $A(r,\theta)$, given by

$$A = \frac{1}{2}\pi B_0 \sin\theta \left[\frac{\sin(\pi r/a)}{(\pi r/a)^2} - \frac{\cos(\pi r/a)}{(\pi r/a)}\right] \ . \qquad (10)$$

In the uniform conducting sphere, this distribution corresponds to the fundamental mode of the dipole field with the surface strength B_0 at the magnetic pole.

The boundary condition requires the magnetic field to be continuous at the surface, $r = a$. Since we assume that the sphere is isolated in a vacuum, it implies that the toroidal component vanishes at the surface. The poloidal field, $\boldsymbol{B}_p$, should be matched to the magnetic multipole moments at $r = a$. Besides, both the toroidal field, B_φ, and the potential, $A(r,\theta)$, go to zero at $r \to 0$ from a symmetry.

3. Numerical results

If we introduce the dimensionless time variable $\tau = (c^2 R_0/4\pi a^2)t$ into Eqs. (8) and (9) then the solution is characterized by only one dimensionless parameter $\alpha_0 = \beta B_e$ which is equal to the surface value of the Hall parameter at the magnetic equator, B_e is the initial strength of the magnetic field at $r = a$ and $\theta = \pi/2$. We calculated the field decay for $100 \geq \alpha_0 \geq 25$. To solve Eqs. (8) and (9) with the corresponding boundary and initial conditions we use the standard finite difference method. The explicit scheme was applied with the time step restricted by the Courant-Friedrichs-Lewy condition (see, e.g., Potter 1973). Unfortunately, our computational facilities do allow us to study the non-linear decay for a very high magnetization, $\alpha_0 \gg 100$.

The behaviour of the magnetic field for all considered values of α_0 is qualitatively similar. Already aduring the early evolutionary phase, the Hall current generates higher order modes from the original dipole one. Amplitudes of these newly generated modes reach a quasi-steady state value on a relatively short time scale associated with the Hall effect, $\tau_H = \tau_0/\alpha_0$. Both toroidal and poloidal modes are generated but the strength of the mode decreases rapidly with its number (see also Naito & Kojima 1994). Thus, among poloidal modes, only those with $l \leq 5$ give an appreciable contribution to the external magnetic field (in our calculations, we follow the evolution of multipoles up to $l = 9$). Since the Hall current provides coupling among the multipole moments, the energy of the original dipole field is partly converted to that of higher order modes and, due to this, the strength of the dipole field decreases faster than in the case of a linear decay. One more important point, characterizing the non-linear field decay, is a non-monotonic behaviour of individual multipole components. All modes including the original dipole one undergo oscillations with the period of the order of τ_H. These oscillations are caused by a non-dissipative exchange of energy between modes and are of the same origin as well known helicoidal oscillations of a magnetized plasma (see, e.g., Kingsep et al.1990). The oscillating behaviour is especially pronounced for the newly generated modes which even can change sign. The original dipole mode evolves non-monotonously as well but the amplitude of oscillations is small in comparison with the average strength of the dipole field, at least, during the considered evolutionary phases.

Figs. 1 and 2 show the time dependence of the toroidal component of the magnetic field for different radii and for $\theta = 0.8\pi$. Symmetry of the original poloidal magnetic configuration implies that the toroidal field generated due to the Hall effect is anti-symmetric to the equatorial plane thus $B_\varphi = 0$ at $\theta = \pi/2$. The oscillating character of the evolution is seen in particular for the toroidal field component. At the initial stage while the dipole field does not decay appreciably, the period of oscillations is approximately equal to τ_H but dissipation of the field in the course of the evolution leads to an increase of this period. The amplitude of the oscillations depends on r and θ, reaching its maximum value at $r \simeq 0.7a$ for all considered values of the parameter α_0. Near the surface and in the vicinity of the centre where the toroidal field tends to zero, oscillations are evidently less pronounced. As a function of θ, the strength of the toroidal field has its maximum at $\theta \approx 40°$ and $140°$ but, as was mentioned, B_φ is oppositely directed in the upper and low hemispheres. Note that for the considered values of α_0, the maximum strength of the generated toroidal field is $\sim 20\%$ of B_0. Due to the ohmic dissipation, the strength of the toroidal component decreases slowly on a time scale of the order of τ_0.

The time dependence of the poloidal field components at the surface are plotted in Figs. 3 and 4 for $\alpha = 25$ and 50, respec-

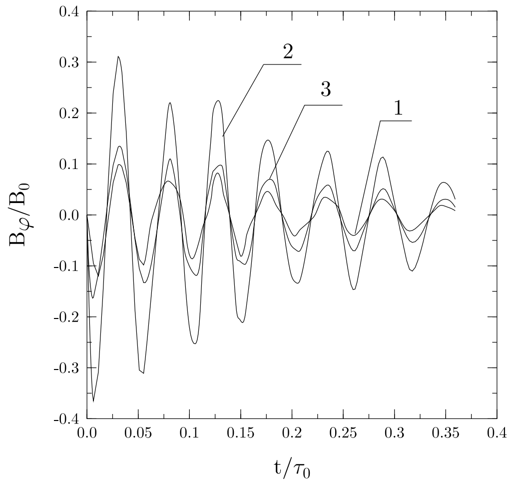

Fig. 1. The time dependence of the toroidal magnetic field for α_0 = 25 and θ = 140°. The toroidal field is normalized to the initial surface value of the poloidal field at the magnetic pole. The curves correspond to the different radii: $r = 0.3a$ (curve 1), $0.7a$ (2) and $0.9a$ (3).

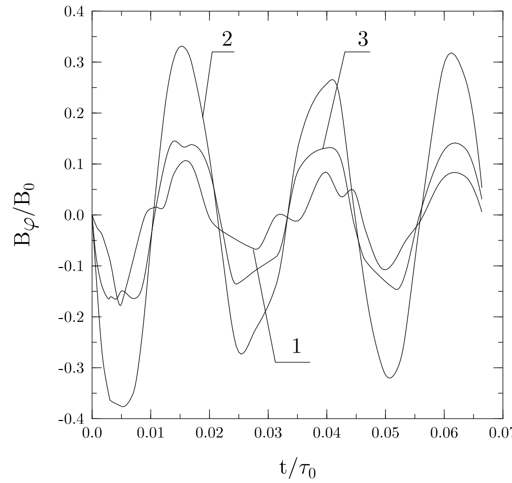

Fig. 2. The same as in Fig. 1 but for α_0 = 50.

tively. We followed the evolution of multipoles with $l \leq 9$ but already the components with $l > 5$ give a negligible contribution to the total magnetic configuration in all cases considered. That is why only the multipoles with $l = 1,\ 3$ and 5 are represented in Figs. 3 and 4. Like the toroidal field, the poloidal components show oscillations as well. An oscillating behaviour is typical not only for newly generated modes but for the dipole mode as well although the amplitude of the dipole variations is relatively small ($\sim$ 10% of the non-oscillating component of the dipole field). Due to these variations, there exist some phases in the course of the evolution during which the strength of the dipole field increases. The duration of these phases is $\approx \tau_H/2$, and the amplification of the dipole moment may reach $\sim$ 10%. Note that this value depends on α_0, being higher for a larger initial Hall parameter. The modes with l = 3 and l = 5 undergo stronger variations than the dipole one: the oscillating fraction of the l = 3 mode is of the order of its average strength, whereas the l = 5 mode evolves even with sign reversals. However, the strength of both these components is substantially weaker than the dipole field. Thus, the l = 3 mode contributes only $\sim$ 20% of the total field for $\alpha_0 = 50$ and $\sim 10 - 15\%$ for $\alpha_0 = 25$. The contribution of the l = 5 harmonic is appreciably smaller ($\sim 2 - 3\%$). The period of poloidal variations is obviously the same as for toroidal ones since an oscillating behaviour is caused by the energy transfer among modes. Of course, a non-monotonic decay is typical only for individual modes but the total magnetic energy evolves monotonously. The decay of the field caused by the ohmic dissipation decreases the amplitude of the oscillations and increases their period.

4. Discussion

We have presented calculations of the non-linear field decay influenced by the Hall current. Our study has been limited to an illustrative case of the field decay in a uniform conducting sphere. The adopted model is maximally simplified but it describes well the main characteristic features of a field dissipation process in the case when the magnetic field is strong enough to magnetize plasma. In contrast to previous studies (see, e.g., Muslimov 1994, Muslimov et al. 1995) which used unjustified simplifications to solve the problem, our analysis is based on direct 2D-simulations. Calculations show that even the evolution of the simplest initial magnetic configuration corresponding to the dipole field outside the sphere is rather complicated. The Hall current changes drastically the behaviour of the magnetic field due to coupling and energy transfer among modes. As a result, a complex magnetic configuration can be created from the simplest one in the course of evolution. Starting from a pure dipole field (l = 1), the generated magnetic configuration consists of the poloidal harmonics with $l = 3$ and $l = 5$ (other multipoles are substantially weaker) and a toroidal component which is anti-symmetric to the equatorial plane.

The most remarkable feature of the decay is an oscillating behaviour of individual modes. Both the newly generated harmonics and the dipole mode undergo oscillations associated with a non-dissipative energy exchange among modes. The nature of these oscillations can be easily understood if one consider as the example of linear waves in a magnetized plasma. For illustration, consider the behaviour of small perturbations of the toroidal and poloidal magnetic fields, $\delta \boldsymbol{B}_t$ and $\delta \boldsymbol{B}_p$, respectively, in a conducting sphere in the presence of an unperturbed poloidal magnetic field, $\boldsymbol{B}_{p0}$. For the sake of simplicity, we assume that the wavelength of perturbations is short in comparison with both the radius of the sphere and the length scale of $\boldsymbol{B}_0$, thus perturbations can be described in the local approximation. A behaviour of small perturbations is governed by the linearized Eqs. (8) and (9) which read

$$\frac{\partial \delta \boldsymbol{B}_p}{\partial t} = -\frac{c^2 R_0}{4\pi} \{\nabla \times (\nabla \times \delta \boldsymbol{B}_p) + \beta \nabla \times [(\nabla \times \delta \boldsymbol{B}_t) \times \boldsymbol{B}_{p0}]\} , \tag{11}$$

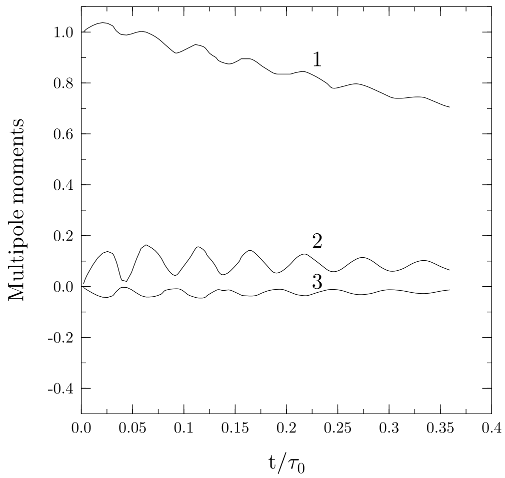

Fig. 3. The time dependence of the poloidal field components with l = 1 (curve 1), l = 3 (2) and l = 5 (3) at the surface for α_0 = 25. The poloidal multipoles are normalized to the initial magnitude of the dipole component.

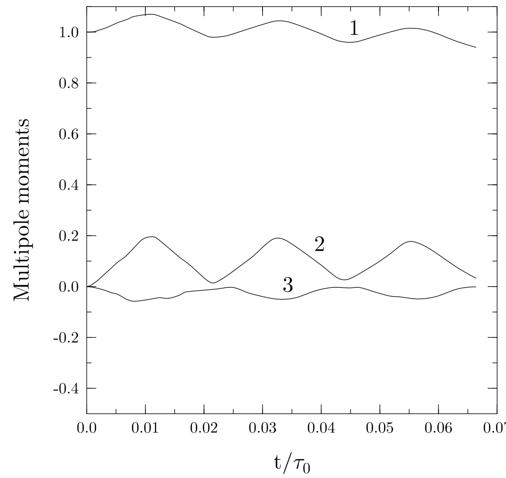

Fig. 4. The same as in Fig. 3 but for α_0 = 50.

$$\frac{\partial \delta \boldsymbol{B}_t}{\partial t} = -\frac{c^2 R_0}{4\pi}\{\nabla\times(\nabla\times\delta\boldsymbol{B}_t)+\beta\nabla\times[(\nabla\times\delta\boldsymbol{B}_p)\times\boldsymbol{B}_{p0}]\} \ . \tag{12}$$

Like in the case of a large scale field, the Hall current associated with the terms proportional to β couples the poloidal and toroidal components of the magnetic field. In the local approximation, one can assume that both $\delta\boldsymbol{B}_p$ and $\delta\boldsymbol{B}_t$ are proportional to exp($i\omega t - i\boldsymbol{k}\cdot\boldsymbol{r}$) where ω and $\boldsymbol{k}$ are the frequency and wave vector of the wave, respectively. For the plane wave, Eqs. (11) and (12) yield a set of algebraic equations,

$$(i\omega+\omega_\sigma)\delta\boldsymbol{B}_p = -\frac{c^2 R_0 \beta}{4\pi}(\boldsymbol{k}\cdot\boldsymbol{B}_{p0})\boldsymbol{k}\times\delta\boldsymbol{B}_t \ , \tag{13}$$

$$(i\omega+\omega_\sigma)\delta\boldsymbol{B}_t = -\frac{c^2 R_0 \beta}{4\pi}(\boldsymbol{k}\cdot\boldsymbol{B}_{p0})\boldsymbol{k}\times\delta\boldsymbol{B}_p \ , \tag{14}$$

where $\omega_\sigma = c^2 R_0 k^2/4\pi$ is the inverse ohmic decay time of the wave. Eqs. (13) and (14) illustrate most clearly the coupled behaviour of the field components. The Hall current associated with $\delta\boldsymbol{B}_t$ changes the strength and, hence, the energy of the poloidal field. In its turn, the toroidal field is influenced by the Hall current produced by the poloidal field. Since the energy exchange among modes is non-dissipative, the Hall currents give a contribution only to Re ω whereas Im ω is completely determined by the ohmic dissipation. The dispersion relation corresponding to the set of Eqs. (13) and (14) is

$$\omega = \pm\omega_\sigma \ \beta \ (\boldsymbol{k}\cdot\boldsymbol{B}_{p0}) \ /k + i\omega_\sigma \ . \tag{15}$$

In a strong magnetic field with $\beta B_{p0} \gg 1$, this equation describes waves with the period $P = 2\pi k/\omega_\sigma\beta(\boldsymbol{k}\cdot B_{p0})$ and with an amplitude slowly decreasing due to the ohmic dissipation on a time scale $\sim \omega_\sigma^{-1}$. These short wavelength modes (sometimes called helicoidal) are determined by the Hall effect and can exist only in a strongly magnetized plasma. They are completely analogous to the oscillations considered above in a large scale magnetic field.

Like small scale modes, oscillations of the total magnetic field have a period determined by the Hall time scale, $\tau_H = \tau_0/\alpha_0$. Both the newly generated modes and the original dipole undergo oscillations, thus the dipole moment can even increase during some evolutionary phases. The amplitude of the oscillations may reach a relatively high value depending on the initial Hall parameter. For example, a varying fraction of the dipole component may be as strong as ~ 0.1 of the total field strength for the considered range of α_0. Higher order modes may also contain an appreciable fraction of the total field. Thus, the poloidal mode with l = 3 can provide about 20% of the field strength at the surface, the toroidal field which is concentrated in the interior reaches $\sim$ 20% of the total field strength. Other field components are substantially weaker and do not in practice influence on the magnetic evolution. The ohmic decay of the magnetic field diminishes the influence of the Hall current which is linearly dependent on the field. Due to this, the period of magnetic variations becomes longer whereas their amplitude becomes smaller. At the late evolutionary stage when the magnetic field is reduced by dissipation to such an extent that $\alpha \leq 1$, the Hall timescale is comparable with the ohmic one (or longer) and oscillations are smoothed. During the further evolution, the behaviour of the field is weakly influenced by the Hall effect and does practically not differ from the linear decay case.

As was mentioned, the Hall current does not contribute directly to the rate of the ohmic dissipation. However, the Hall drift changes the distribution of the magnetic field and makes its configuration more complex. The dissipation rate is strongly sensitive to a curvature of the magnetic field lines and, therefore, a generation of higher order modes associated with the Hall current may accelerate the dissipation of the field. Calculations first done by Urpin & Shalybkov (1991) confirm this qualitative conclusion. It was argued that the behaviour of the magnetic energy integrated over the volume departs notably from the standard ohmic dissipation if the initial magnetic field is sufficiently strong ($\alpha \geq 20$). For weaker initial fields, departures from the linear decay are not significant.

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