

Substructure effects on the collapse of density perturbations

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Abstract. We solve numerically the equations of motion for the collapse of a shell of baryonic matter, made of galaxies and substructure of $10^6 M_\odot - 10^9 M_\odot$, falling into the central regions of a cluster of galaxies taking account of dynamical friction. We calculate the evolution of the expansion parameter, $a(t)$, of the perturbation using a coefficient of dynamical friction, η_0 , calculated for a perturbation in which clustering is absent and a coefficient η_{cl} obtained from a clustered one. The effect of the dynamical friction is to slow down the collapse (V. Antonuccio-Delogu & S. Colafrancesco 1994, hereafter AC) producing an observable variation of the parameter of expansion of the shell. The effect increases with increasing η and with the increasing of clustering. Finally, we show how the collapse time depends on η_0 and η_{cl} .

Key words: cosmology: theory – galaxies: formation

1. Introduction

In the most promising cosmological scenarios, structure formation in Universe is generated through the growth and collapse of primeval density perturbations originated from quantum fluctuations (Guth & Pi 1982; Hawking 1982; Starobinsky 1982; Bardeen, Bond, Kaiser and Szalay 1986, hereafter BBKS) in an inflationary phase of early Universe. Density fluctuations originated in the inflationary era are Gaussian distributed and their statistics can be expressed entirely in terms of the power spectrum of density fluctuations:

$$P(k) = \langle |\delta_{\mathbf{k}}|^2 \rangle \quad (1)$$

where

$$\delta_{\mathbf{k}} = \int d^3k \exp(-i\mathbf{k}\mathbf{x}) \delta(\mathbf{x}) \quad (2)$$

$$\delta(\mathbf{x}) = \frac{\rho(\mathbf{x}) - \rho_b}{\rho_b} \quad (3)$$

and ρ_b is the mean background density.

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In biased structure formation theory it is assumed that cosmic structures of linear scale R form around the peak of density field, $\delta(\mathbf{x})$, smoothed on the same scale. Density perturbations evolve towards non-linear regime because of gravitational instability which breaks away from general expansion at a time, t_m , given by:

$$t_m = \left[\frac{3\pi}{32G\rho_b} (1 + \bar{\delta}) \right]^{1/2} (1+z)^{3/2} \quad (4)$$

where z is the redshift, $\bar{\delta}$ is the overdensity within r . When $\bar{\delta} \simeq 1$ the density perturbation begins to collapse. The collapse time depends on the characteristic of initial fluctuation field such as the average overdensity and on the environment in which the perturbation is embedded. This last feature depends on the cosmological scenario.

In this paper, we consider the CDM model (Liddle & Lyth 1993) based on a scale invariant spectrum of density fluctuations growing under gravitational instability. The modality of the collapse of a perturbation depends on the chosen model. A very simple model for accretion of matter in cluster of galaxies was firstly investigated by Gunn & Gott (1972). Gunn & Gott's model is an oversimplification of the perturbation evolution. In their model they considered only the collapse of spherical uniform overdense shells of matter surrounding an isolated top-hat perturbation embedded within a homogeneous background without including the tidal interactions among the shell of matter and the external density perturbations and excluding the presence of substructure (collapsed objects whose length is shorter than that of the main perturbation). This does not take into account the provisions of CDM models according to which there is an abundant production of substructure during the evolution of the Universe.

In this scenario structure formation proceeds bottom-up through gravitational clustering, merging and violent relaxation of small scale substructure (White & Rees 1978). So the matter inside a given region is clumped in a hierarchy of objects of various dimensions. The substructure acts as a source of stochastic fluctuations in the gravitational field of the protostructure inducing dynamical friction (AC) and eventually producing a modification of the motion of shells of matter in a density perturbation. The galaxies inside a shell of matter are subject to

the stochastic gravitational field produced by the substructure and their motion undergoes a preferential deceleration in the direction of motion. In particular, dynamical friction produces a delaying effect in the collapse of the regions with $\bar{\delta} \leq 10^{-2}$ inside the perturbation as shown in (AC). The final result is that Gunn & Gott's model is inadequate to describe the collapse of a spherical perturbation and it requires a revision.

To this aim we used the modified equation of motion of a shell of baryonic matter given by AC, in which a frictional force that takes into account dynamical friction effects is introduced (we study only the component of the dynamical friction due to the galaxies belonging to the shell and to the substructure, neglecting gravitational bound to the shell) and we solve it by numerical integration.

In this paper we show:

a) how the expansion parameter, $a(t)$, of the shell is changed by the presence of substructure both in the case of an unclustered and in a clustered system; b) how clustering increases the effects of dynamical friction; c) the changes produced by dynamical friction on the collapse time of the perturbation and the role of clustering in this process.

2. Modification of Gunn & Gott's equation

In a hierarchical structure formation model, the large scale cosmic environment can be represented as a collisionless medium made of a hierarchy of density fluctuations whose mass, M , is given by the mass function $N(M, z)$, where z is the redshift. In these models matter is concentrated in lumps, and the lumps into groups and so on. In what follows, we consider a shell of matter outside the main body of a perturbation which collapsed to form a cluster of galaxies.

The equation of motion of a shell of matter (as previously said composed of galaxies and substructure) around a maximum of the density field (neglecting tidal interactions and the substructure), can be expressed in the form:

$$\frac{d^2 r}{dt^2} = -\frac{GM}{r^2(t)} \quad (5)$$

(Peebles 1980, Eq. 19.9), where M is the mass enclosed with the proper radius $r(t)$. Using Gunn & Gott's notation the proper radius can be written as:

$$r(r_i, t) = a(r_i, t)r_i \quad (6)$$

where r_i is the initial radius and $a(r_i, t)$ is the expansion parameter of the shell. At the initial time t_i the initial condition is given by

$$a(r_i, t_i) = 1 \quad (7)$$

In the presence of substructure it is necessary to modify the equation of motion, Eq. (5), because the graininess of mass distribution in the system induces dynamical friction that at last introduces a frictional force term in Eq. (5).

In a material system, the gravitational field can be decomposed into an average field, $\mathbf{F}_0(r)$, generated from the smoothed

out distribution of mass, and a stochastic component, $\mathbf{F}_{stoch}(r)$, generated from the fluctuations in number of the field particles. The stochastic component of the gravitational field is specified assigning a probability density, $W(\mathbf{F})$, (Chandrasekhar & von Neumann 1942). In an infinite homogeneous unclustered system $W(\mathbf{F})$ has been given by Holtmark distribution (Chandrasekhar & von Neumann 1942) while in inhomogeneous and clustered systems $W(\mathbf{F})$ has been given by Kandrup (1980) and Antonuccio-Delogu & Barandela (1992) respectively. The stochastic force, \mathbf{F}_{stoch} , in a self-gravitating system modifies the motion of particles as it is done by a frictional force. In fact a particle moving faster than its neighbours produces a deflection of their orbits in such a way that the average density is greater in the direction opposite that of traveling causing a slowing down in its motion.

Following Kandrup (1980) in the hypothesis that there are no correlations among random force and its derivatives the frictional force is given by:

$$\mathbf{F} = -\eta\mathbf{v} = -\frac{\int W(F)F^2T(F)d^3F}{2 \langle v^2 \rangle} \mathbf{v} \quad (8)$$

where η is the coefficient of dynamical friction, $T(F)$ the average duration of a random force impulse, $\langle v^2 \rangle$ the characteristic speed of a field particle having a distance $r \simeq (\frac{GM}{F})^{1/2}$ from a test particle (galaxy). This formula permits to calculate the frictional force for inhomogeneous systems when $W(F)$ is given. If the field particles are distributed homogeneously the dynamical friction force is given by:

$$F = -\eta v = -\frac{4.44G^2 m_a^2 n_a}{[\langle v^2 \rangle]^{3/2}} \log \left\{ 1.12 \frac{\langle v^2 \rangle}{G m_a n_a^{1/3}} \right\} \quad (9)$$

(Kandrup 1980), where m_a and n_a are respectively the average mass and density of the field particles. Using the virial theorem we can written the dynamical friction force as follows:

$$F = -\eta v = -\frac{4.44[G m_a n_a^3]^{1/2}}{N} \log \left\{ 1.12 N^{2/3} \right\} \frac{v}{a^{3/2}} \quad (10)$$

where N is the total number of field particles. In this last equation it is assumed that the field particles generating the stochastic field are virialized. This is justified by the previrialization hypothesis (Davis & Peebles 1977).

To calculate the dynamical evolution of the galactic component of the cluster it is necessary to calculate the number and average mass of the field particles generating the stochastic field.

The protocluster, before the ultimate collapse at $z \simeq 0.02$, is made of substructure having masses ranging from $10^6 - 10^9 M_\odot$ and of galaxies. We suppose that the stochastic gravitational field is generated from that portion of substructure having a central height ν larger than a critical threshold ν_c . This latter quantity can be calculated (following AC) using the condition that the peak radius, $r_{pk}(\nu \geq \nu_c)$, is much less than the average peak separation $l_{av} \equiv n_a(\nu \geq \nu_c)^{-1/3}$ (see AC and BBKS). The condition $r_{pk}(\nu \geq \nu_c) < 0.1 n_a(\nu \geq \nu_c)^{-1/3}$ ensures that

the peaks of substructure are point like. Using the radius for a peak (AC Eq. 13) that is:

$$r_{pk} = \sqrt{2}R_* \left[\frac{1}{(1 + \nu\sigma_0)(\gamma^3 + (0.9/\nu)^{3/2})} \right]^{1/3} \quad (11)$$

where γ , R_* are parameters related to moments of the power spectrum (BBKS Eq. 4.6A), we obtain a value of $\nu_c = 1.3$ and then we have $n_a(\nu \geq \nu_c) = 50.7 Mpc^{-3}$ ($\gamma = 0.4$, $R_* = 50 Kpc$) and m_a is given by:

$$m_a = \frac{1}{n_a(\nu \geq \nu_c)} \int_{\nu_c}^{\infty} m_{pk}(\nu) N_{pk}(\nu) d\nu = 10^9 M_{\odot} \quad (12)$$

(in accordance with the result of AC), where m_{pk} is given in Peacock & Heavens (1990) and N_{pk} is the average number density of peak (BBKS Eq. 4.4). Now we study the evolution of a shell of matter, made of substructure and galaxies, as modified by dynamical friction.

As previously told, we suppose that the substructure has a Maxwellian distribution of velocity and that the galaxy moves in the substructure background. The modified equation of motion for each galaxy of a shell of matter can be written in the form:

$$\frac{d^2 r}{dt^2} = -\frac{GM}{r^2(t)} - \eta v \quad (13)$$

(Langevin 1908, Kandrup 1980, Saslaw 1985, Kashlinsky 1986, AC) In the spherical collapse model, it is supposed that the infall is radial for every galaxy in the protocluster. Eq. (13) describes the motion of each galaxies, the only parameter that describes the collapse is the radial distance, r , or the parameter $a(t)$ previously defined. Remembering that the average density is given by:

$$\bar{\rho}(r_i, t) = \frac{3M}{4\pi a^3(r_i, t)r_i^3} \quad (14)$$

and that mass conservation requires:

$$\bar{\rho}(r_i, t_i) = \frac{\bar{\rho}(r_i, t_i)}{a^3(r_i, t)} \quad (15)$$

equation (13) in terms of $a(r_i, t)$ can be written as:

$$\frac{d^2 a}{dt^2} = -\frac{4\pi G \rho_{ci}(1 + \bar{\delta}_i)}{3a^2(t)} - \eta \frac{da}{dt} \quad (16)$$

where ρ_{ci} is the background density at a time t_i and $\bar{\delta}_i$ is the overdensity within r_i . Using the parameter $\tau = \frac{t}{T_{c0}}$, where $T_{c0}/2$ is the collapse time in absence of dynamical friction (Gunn & Gott Eq. 16), Eq. (16) may be written in the form:

$$\frac{d^2 a}{d\tau^2} = -\frac{4\pi G \rho_{ci}(1 + \bar{\delta}_i) T_{c0}^2}{a^2(\tau)} - \eta_0 T_{c0} \frac{da}{d\tau} \left(1 + \frac{\eta_{cl}}{\eta_0}\right) \quad (17)$$

Equation (17) is obtained remembering that the probability density $W(F)$ depends linearly on the correlation function (AC)

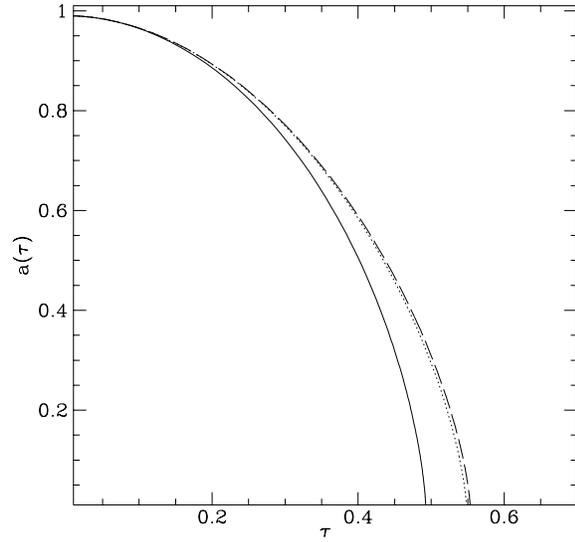


Fig. 1. Temporal evolution of the expansion parameter of a shell of matter made of galaxies and unclustered substructure. The solid line is $a(\tau)$ when dynamical friction is absent while the dotted line is $a(\tau)$ when it is taken into account ($\eta_0 \neq 0$ and $\eta_{cl} = 0$). The dashed line shows the effect of dynamical friction when $\eta'_0 > \eta_0$. We assume a cluster radius of $R_{cl} = 5h^{-1} Mpc$, a central overdensity $\bar{\delta} = 0.01$ and a total number of peaks of substructure $N_{tot} = 10^3$.

and so it is possible to decompose the coefficient of dynamical friction, η , as follows:

$$\eta = \eta_0 + \eta_{cl} \quad (18)$$

where η_0 is the coefficient of dynamical friction of an unclustered distribution of field particles, while η_{cl} takes into account clustering.

3. Results and discussions

The time evolution of the expansion parameter, $a(\tau)$, can be obtained solving Eq. (17).

There are two ways to solve equation (17). The first one is to look for an asymptotic expansion, the second one is numerical. In AC the authors gave only an expression for the collapse time, T_c , taking into account dynamical friction. We solved Eq. (17) numerically using a Runge-Kutta integrator of 4th order and we studied the motion of a shell of matter of low density, $\bar{\delta} = 0.01$, typical of a perturbation present in the outskirts of a cluster of galaxies. We chose the initial conditions remembering that at the maximum of expansion the initial velocity is zero. We show in fig. 1 the results of our calculation when the dynamical friction is present but considering only the case of an unclustered distribution ($\eta_0 \neq 0$ and $\eta_{cl} = 0$).

In this case the expansion parameter of a shell, $a(\tau)$, is plotted versus τ , for different values of η_0 , the coefficient of dynamical friction for an unclustered system (stochastic force generators randomly distributed).

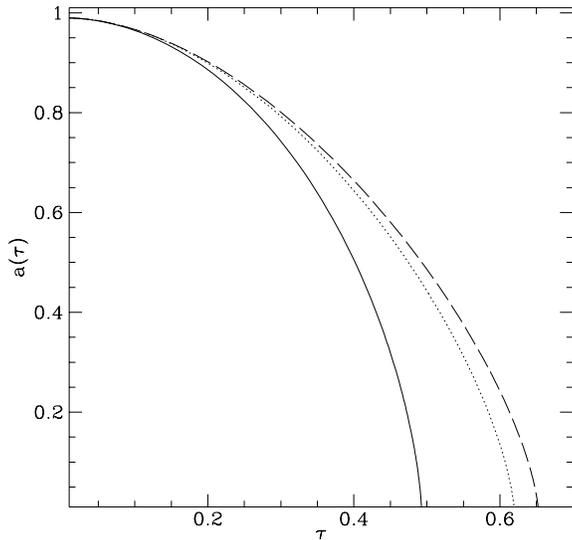


Fig. 2. Temporal evolution of the expansion parameter of a shell of matter made of galaxies and clustered substructure. The solid line is $a(\tau)$ when dynamical friction is absent while the dotted line is the same when it is taken into account ($\eta_{cl} \neq 0$). The dashed line shows the effect of dynamical friction when $\eta'_{cl} > \eta_{cl}$. We assume a cluster radius of $R_{cl} = 5h^{-1}Mpc$, a central overdensity $\bar{\delta} = 0.01$, and a total number of peaks of substructure $N_{tot} = 10^3$.

The effect of dynamical friction is to make the decrease of $a(\tau)$ less steep and consequently to slow down the collapse of the shell in agreement with the analytic calculation performed by AC. The evolution of the parameter of expansion, $a(\tau)$, is more and more modified as η_0 increases, in comparison to Gunn & Gott's model, as one can see from Fig. 1.

In Fig. 2 we show the same parameter, $a(\tau)$, for different values of η_{cl} , the coefficient of dynamical friction of a clustered system (clustering of substructure is required, for example, by the minihalo model (Mo et al. 1993)).

Clustering produces a sensible slowing down of the shell collapse with respect to unclustered systems ($\eta_{cl} = 0$). This result is perfectly understandable since the introduction of positive clustering produces a greater probability for a test particle to be scattered during its motion, increasing the role of dynamical friction.

The characteristic values for η are between $0 < \eta < 3$, they depend strongly on the value of many parameters like ν_c and others (Gambera et al. 1996, in preparation). For a typical protocluster configuration like that of Coma, that can be considered almost as a lower limit having an initial overdensity, at $z \simeq 10^3$, given by $\bar{\delta} \simeq 8 \cdot 10^{-3}$ and a total mass estimated in $M \simeq 10^{15}M_\odot$ (Hughes 1989), supposing that small scale substructure is made of objects of $M \simeq 10^9M_\odot$, we obtain $\eta \simeq 0.2$.

Finally, we computed the time of collapse of a shell, T_c , versus both η_0 and of η_{cl} . To this aim we solved Eq. (17) varying η_0 and for each value of this parameter we obtained the zero of

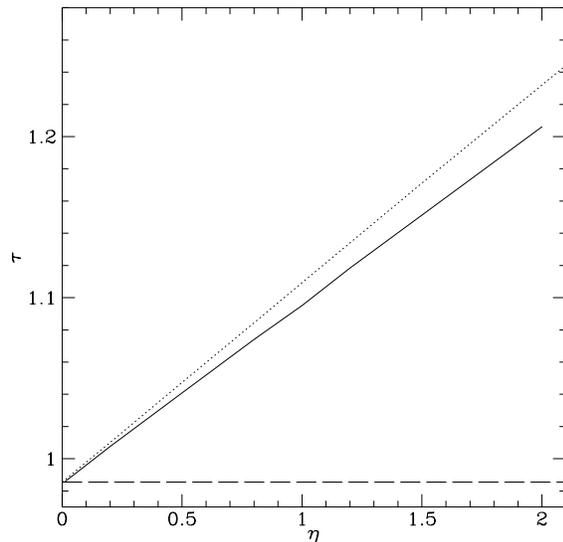


Fig. 3. The variation of time of collapse $\tau = \frac{t}{T_{c0}}$ with η . The solid line is the time of collapse of an unclustered system in which dynamical friction is present while the dotted line is the same for a clustered system and the dashed line is the time of collapse for a system in which dynamical friction is absent

$a(\tau)$, that gives us T_c . We repeated the same procedure for η_{cl} . In Fig. 3 we show the results of the computation.

It is interesting to observe that T_c is very sensitive to the changes of the coefficient of dynamical friction for a clustered system η_{cl} and increases with increasing η_{cl} while it is less sensitive to the changes of the coefficient of dynamical friction for an unclustered system. Dynamical friction changes the dynamical evolution of low density perturbations ($\bar{\delta} \simeq 0.01$) and clustering contributes to increase this effect.

4. Conclusions

In this paper we have discussed the dynamics of the infall of a shell of baryonic matter of a spherical perturbation, when the effects of dynamical friction are taken into account, using the time evolution of the parameter of expansion, $a(\tau)$. In particular, we showed that the parameter $a(\tau)$ of a shell of matter of a protocluster decreases less steeply with increasing η_0 and that clustering produces a further increase in this effect. This effect, then, could be the cause because many clusters of galaxies are not yet relaxed. Finally, we showed how the collapse time, T_c , varies in presence or absence of dynamical friction (both in the case in which $\eta_{cl} = 0$ and when $\eta_{cl} \neq 0$).

The collapse time, T_c , of an infalling shell increases with increasing values of η_0 (see Fig. 3). Clustering enlarges the change in the dynamical evolution of the perturbation produced by dynamical friction. An interesting point on which we are in progress (Gambera et al. 1996, in preparation) is the determination of how the growth of the collapse time depends on ν_c and on others parameters. Besides, we want to find an analytic relation that links τ with the parameters on which it depends.

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