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Studies of the Centaurus cluster

III. Luminosity functions of individual Hubble-types as compared to Virgo and Fornax

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Abstract. Deep ($B_T < 21.5$) overall and type-specific luminosity functions (LFs) are presented for the galaxies populating the central region of the Centaurus cluster. Assuming the "same distance" hypothesis for the spatial distribution of the two Centaurus velocity components Cen30 and Cen45, these LFs are discussed and compared with those found in the two nearby clusters in Virgo and Fornax. Although the three overall LFs have similar appearance in general, clear variations exist due to differences in the mixtures of Hubble types. The Schechter parameters $M^*_{B_T}$ and α are used for a quantitative discussion of the LFs. We present new evidence against their universality. Both parameters change from one cluster to the other and depend for a single cluster on the cut-off in absolute magnitude.

In contrast to the overall LF, previously derived shapes of the type-specific LFs (Sandage et al. 1985b) could be fully confirmed. Bright galaxies of type E, S0, and spiral have bell-shaped, Gaussian-like LFs, whereas the LF of early-type dwarfs (dE&dS0) rises exponentially with decreasing luminosity. These results strongly support the idea that the shapes of the *type-specific* LFs are invariant at least among clusters and have a physical origin. Furthermore, statistical evidence is given for a skewness towards the faint end in the LF of E galaxies. The LF of the Im&BCD's is disturbed by the magnitude completeness limit of our survey and is not well defined.

As an application of the close similarity in form we used the type-specific LFs of S0's, spirals and dE&dS0's in the Centaurus and the Virgo clusters as distance indicators. The results are consistent and give a mean relative distance modulus of $\Delta(m-M)^0_{Cen-Virgo}$ = 1.63(±0.14).

Key words: galaxies: clusters: individual: Centaurus cluster – galaxies: general – galaxies: luminosity functions, mass function – galaxies: evolution

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1. Introduction

This is the third paper in a series on the Centaurus cluster and its population of dwarf galaxies. The general scope of this work is outlined in Paper I (Jerjen & Dressler 1996a) which gives all technical details of the survey, classification, and photometry of Centaurus dwarf galaxies and also contains the outcoming new list of cluster members (the Centaurus Cluster Catalogue, CCC) on which the results of the remaining Papers II–IV are based. The present Paper III is a study of the overall and type-specific B-band LFs of Centaurus cluster galaxies, aiming at an intercomparison of the LFs of the three nearby clusters in Virgo, Fornax, and Centaurus. In the following we give a brief general introduction to the topic of LFs, in order to show where the present work fits in.

The (differential) luminosity function $\Phi(M)dM$ (or LF) of galaxies gives the number of objects of a complete, volume-limited galaxy sample having their total magnitudes within the interval $[M, M+dM]$. Knowledge of the LF is central to many problems in extragalactic astronomy. For instance, the LF is presumably connected to the mass function of galaxies. In that case it reflects both the characteristics of the primordial density fluctuation field (the power spectrum) and the astrophysical processes responsible for the formation and evolution of galaxies, i.e. collapse, cooling, star formation, and feedback that decide how effectively gas is converted into light at the chosen wavelength range. Furthermore, the LF for galaxies of individual Hubble types become more and more important for studies of the large-scale peculiar velocity field. It is needed to predict the redshift distribution of a galaxy type in various magnitude intervals which is essential to correct for the Malmquist bias in observational data.

Up to now, the knowledge of the LF is mainly restricted to the optical wavelength range, mostly to the B-band, due to the spectral sensitivity of photographic plates primarily used for such studies. This is also the case for our study. First investigations in other wavelengths were undertaken e.g. in the infrared by Lawrence et al. (1986) and Isobe & Feigelson (1992).

The early work on the luminosity function of galaxies was marked by a well-known controversy. Hubble and Humason investigated distant, high surface-brightness field galaxies of elliptical and spiral type with known redshifts. They found a bell-shaped magnitude distribution for these galaxies (Hubble & Humason 1931; Hubble 1936a, b). On the other hand, Zwicky (1942) used thermodynamical arguments to postulate an exponential increase of the LF towards fainter magnitudes. His idea was supported by the continuously increasing number of faint dwarf irregulars and dwarf ellipticals discovered in the Local Group and its near neighbourhood (e.g. Baade & Hubble 1939; Baade 1944).

From the discussion in Binggeli et al. (1988) it becomes clear what the crucial differences are between the two samples, (a) the LFs of individual morphological types T have different forms, (b) the counted galaxies belong to environments of distinct local density ρ, and (c) the completeness limit in absolute magnitude M_{lim} is much deeper for Zwicky's sample. Hubble's photographic plates did not allow him to detect dwarf galaxies with their characteristic low surface brightness. Hence, he established the LF of the giant galaxies only, which indeed has a near-Gaussian profile. Zwicky focussed his view on the Local Group and slightly beyond. At that time, this region was the only part of the Universe for which an almost complete (or actually rather deep) volume-limited sample of galaxies was known. The study of the Local Group galaxies thus implicitly took into account the importance of the dwarfs as galaxy types which are dominating the Universe in numbers. In contrast to the giants, the dwarf LF starts at the faint end of the giant distribution and steeply increases towards fainter magnitudes. This makes clear that both Hubble and Zwicky were right.

The most popular and often used analytical expression to parameterise an observed LF was introduced by Schechter (1976):

$$\Phi(L) \propto (L/L^*)^\alpha \cdot e^{(-L/L^*)}$$

$$\Phi(M) \propto 10^{0.4(M^*-M)(\alpha+1)} \cdot e^{-10^{0.4(M^*-M)}}$$

$$\log(\Phi(M)) = (M^* - M)(\alpha+1) - 1.086 \cdot 10^{0.4(M^*-M)} + c$$

$$\log(\Phi(M)) = \begin{cases} (M^* - M)(\alpha+1) + c & : \quad M \gg M^* \\ -10^{0.4(M^*-M)} + c & : \quad M \ll M^* \end{cases}$$

where M (L) is the total absolute magnitude (luminosity) of a galaxy in a given wavelength range and c is a constant. The parameter α defines the faint-end slope of $\log(\Phi(M))$ ($M \gg M^*$). For $\alpha > -1$, $\Phi(M)$ is bound to the bright *and* the faint ends and is roughly bell-shaped in appearance. However, for $\alpha < -1$ the function increases continuously. M^* (L^*) defines the "characteristic" magnitude (luminosity) also known as the "knee" of the function. It is located at the transition between the two above given asymptotic functions of $\log(\Phi(M))$.

1.1. $\Phi(M, \rho)$

First Schechter and other authors (e.g. Felten 1977) found good agreement between the analytic function, with $M^*_{B_T} \sim -21.0$ ($H_0 = 50\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ here and throughout this paper) and $\alpha \sim -1.25$, and the observed overall LF in clusters as well as in the field. The LF appeared independent of the environmental density ρ and one claimed the "universality" of the parameter set. However, with an increasing number of investigated clusters it became obvious that this hypothesis does not hold, e.g. Dressler (1978) found several clusters with very flat faint end slopes ($\alpha \sim -1$) confirming earlier results by Oemler (1974). Evidence of shape variations as a function of ρ could be found by comparing clusters with groups (Turner & Gott 1976; Binggeli et al. 1988; Ferguson & Sandage 1991) and the study of field galaxies (Choloniewski 1986; Phillipps & Shanks 1987; Loveday et al. 1992; Marzke et al. 1994; Ellis et al. 1995). All these newer results pointed to a shallow faint-end slope of the LF with typically $\alpha \sim -1$ for low-density regions. The parameter $M^*_{B_T}$ was scattered about $-21.0(\pm 0.5)$.

An important contribution to the clarification of the differences between cluster and field LFs came from the morphology-density relation for giant galaxies (Dressler 1980a) and dwarfs (Binggeli et al. 1990). Roughly speaking high-density regions are dominated by the early-type galaxies, ellipticals (E), lenticulars (S0), and dwarf ellipticals (dE&dS0), whereas low-density regions are mainly populated by spirals (S) and the late-type dwarfs, irregulars (Im) and blue compact dwarfs (BCD). Hence, environmental effects on the *overall* LF must be expected because it is a composition of many galaxy types. As we will see below, the most significant variation in shape has its origin in the different characteristics of the LF of the two dwarf families.

1.2. $\Phi(M, T)$

The idea of a universal shape of the LF of galaxies was mainly due to the lack of detailed morphological information on the galaxies. In the eighties this missing parameter T has been explored by several authors. Thompson & Gregory (1980) used large-scale photographic plates to explore the separate LFs of E+dE+dS0, S0, and Sp+Irr galaxy types in the Coma cluster. A similar study was presented by Kraan-Korteweg (1981) for the Virgo cluster. It followed the comprehensive surveys of the Virgo cluster (Binggeli et al. 1985), the Fornax cluster (Ferguson 1989), and five nearby groups (Ferguson & Sandage 1990). Like the present study they were based on the extraordinary 20-inch photographic plates in B of the Las Campanas 100-inch du Pont telescope. The high angular resolution of these plates (11 arcsec/mm) made it possible to extend the Hubble classification scheme down to the very faint dwarf galaxies (Sandage & Binggeli 1984) and to investigate the LFs for all individual Hubble types and subtypes in great details (Sandage et al. 1985a; Sandage et al. 1985b, hereafter SBT85; Ferguson & Sandage 1988, hereafter FS88; Ferguson & Sandage 1991). Based on large and complete galaxy samples, SBT85 demonstrated that the LFs of all *classical* Hubble types are bound at the bright and at the faint end. The distributions of the S0's and spirals could be well approximated by Gaussian functions. In contrast, the number of dwarfs takes off where the LF of the bright galaxies comes to zero. Both dwarf families obey Schechter functions but with very different parameters. Going to fainter luminosities the early-type dwarfs increase exponentially in number ($\alpha \sim -1.3$), whereas the LF of the late-type dwarfs flattens and possibly decreases ($\alpha \geq -1$) after an increase over the first two magnitudes. These trends for the dwarf types are valid down to the very faint magnitude completeness limit near about $M_{B_T} = -13.2$.

1.3. The Hubble-Zwicky controversy

Two conclusions result from the dependency of Φ from ρ and T: (a) the shape parameter α strongly depends on the completeness limit in absolute magnitude M_{lim} of a galaxy sample. For bright cut-offs the derived LF appears near-Gaussian ($\alpha \geq -1$) because no dwarfs are considered. However for fainter cut-offs the LF is governed by the dwarfs. In the presence or absence of a large dE&dS0 population (high versus low-density environment) the LF steeply increases ($\alpha \sim -1.25$) or remains about flat ($\alpha \gtrsim -1$). Only for sufficient faint luminosity limits there is the hope to measure a stable value for α. (b) the overall LF cannot be "universal" because of the different contribution of galaxy types. Thompson & Gregory (1980) pioneered this idea by modelling LFs for different mixtures of galaxies (E+dE+dS0:S0:Sp+Irr) using observed distributions in the Coma cluster. This idea was recently extended to higher morphological resolution by Jerjen et al. (1992) in order to estimate environmental effects on the overall LF. Typical variations of $\Delta M^* \sim 1$ mag and $\Delta\alpha \sim 0.3$ were found. Similar trends for the observed LFs of seven groups and clusters were reported by Ferguson & Sandage (1991).

In view of this multi-parameter-dependency of the luminosity function of galaxies, it seems highly desirable to have more volume-limited samples with high morphological resolution and faint completeness limits, covering a broad range in environmental density. This is best achieved by doing deep, wide-field imaging of nearby groups and clusters of galaxies. The uniqueness of the du Pont 100-inch telescope with this respect, in terms of angular, and hence *morphological* resolution, has already been mentioned (for a full account cf. Paper I). The present study of the Centaurus cluster is again based on this instrument and is aiming to explore cluster-to-cluster variations of the LFs. Although this new cluster population is about two magnitudes fainter than Virgo and Fornax due to a larger distance and Galactic extinction (Paper II and below), it is still sufficiently nearby to allow a very detailed intercomparison of the LFs of these three clusters. What we reveal is essentially the confirmation of the trends found in the Virgo and Fornax clusters, i.e. a near invariance of the type-specific LFs and, in consequence, a failure of the Schechter function to explain the overall LFs.

2. Centaurus cluster galaxies

In our study we work with the galaxy sample listed in the Centaurus Cluster Catalogue (CCC) which is the result of a deep photometric survey of the Centaurus cluster (Paper I). Details of the survey can be found there and shall be briefly summarised. The central region of the Centaurus cluster ($12^h44^m40^s <$ RA(1950) $< 12^h52^m40^s$ and $-41°40' <$ Dec(1950) $< -40°12'$) was visually inspected on a film copy of a photographic plate taken with the Las Campanas du Pont telescope (Dressler 1980b). More than 1100 galaxies were detected and assigned a cluster membership probability of 100% (definite), 75% (likely), 50% (possible), or 0% (background), following the criteria worked out by Binggeli et al. (1985). This means that there was taken into account the surface brightness, the resolution of the galaxy in stellar content, and - if available - the radial velocity. That this method is a powerful tool to decide on cluster membership was demonstrated convincingly for the Virgo cluster recently. In a compilation of new velocities for 144 Virgo Cluster Catalogue galaxies more than 95 percent of the morphological guesses agree with the redshift data (Binggeli et al. 1993). Furthermore, Drinkwater et al. (1996) confirmed a sample of 291 "background" galaxies in the Virgo cluster region.

After the morphological classification of the 296 Centaurus cluster galaxies (membership probability of 50% or higher) several photometric and structure parameters have been determined for each of them, e.g. total apparent B magnitude, mean effective surface brightness, exponential central surface brightness and scale length. The total magnitude has a typical error of ≤ 0.2 mag for bright galaxies and ≤ 0.3 mag for the faint galaxies, respectively. This is small enough to have negligible effect on the LFs, which will be binned into units of 0.5 mag.

3. Luminosity functions

The B_T magnitudes and morphological types of all galaxies of the CCC are used to derive LFs of different subsamples. We consider the two velocity components of the Centaurus cluster, Cen30 and Cen45 (Lucey & Carter 1988, Dressler 1994 and references therein), as belonging to one cluster. This procedure is further justified in Jerjen & Dressler (1996b = Paper II). The LFs of the Centaurus cluster galaxies will be compared with the LFs derived from the Virgo Cluster Catalogue (VCC, Binggeli et al. 1985) and the Fornax Cluster Catalogue (FCC, Ferguson 1989). The three data sets are based on very similar precepts and should therefore be directly comparable. A comparison in absolute magnitudes requires, however, the relative distances of the three clusters. We assume for the Centaurus cluster an *apparent* distance modulus of $\Delta(m-M)_{Cen-Virgo} = 2.10$ mag (Paper II) relative to a true modulus of $(m-M)^0_{Virgo} = 31.70$ for the Virgo cluster (Sandage & Tammann 1990), including here 0.42 mag of differential Galactic extinction (Burstein & Heiles 1984). The distance difference between the Virgo and Fornax clusters is notoriously uncertain. We use the modulus $\Delta(m-M)_{Fornax-Virgo} = 0.20$ mag as an average value for Fornax relative to Virgo (Sandage & Tammann 1990 and references therein) which is also in good agreement with recent determinations based on the surface-brightness–magnitude relation for dE&dS0s (0.28 mag, Jerjen & Binggeli 1996a), on the surface-brightness-fluctuation method (0.26 mag, Tonry 1996), and on Cepheids (Mould 1996).

3.1. Overall luminosity function

We show the overall LF of the Centaurus cluster in the form of a histogram in Fig. 1a. Definite and likely cluster members are marked in dark grey, possible members are in white. The dashed line at B_T = 18.5 (equivalent to $M_{B_T} = -15.3$) indicates the completeness limit of our survey as discussed in Jerjen & Binggeli (1996b).

The Centaurus LF cannot be naively compared to the LFs of the *complete* Virgo (Fig. 3 in SBT85) or Fornax cluster sample (Fig. 13, FS88). This is because the LF of a cluster is varying for survey areas of different extent due to the well-known population gradients as a function of the local projected galaxy density (Dressler 1980; Binggeli et al. 1990) or the "clustercentric" radius (e.g. Whitmore et al. 1993). For instance, fewer spirals are expected in a survey focussed on the inner part of a cluster than in a survey also included the low-density outer cluster region. In order to overcome this problem to a first approximation we confine the three cluster samples to equal cluster-specific radii. Strictly speaking, we take cluster galaxies of the VCC and FCC into account within the radial area of 2.4 core radii ($r_{c,Vir}$ = 1°.40 and $r_{c,For}$ = 0°.77 see Ferguson & Sandage 1990) which is the largest area covered by all three surveys ($r_{c,Cen}$ = 0°.32, Paper II). The galaxy surface density at the border is about 15% of the central value. Further properties of these so-called adapted cluster samples are discussed in Paper II.

The LFs of our selected Virgo and Fornax samples are shown in Figs. 1b and c. Possible cluster members (class 2 in VCC, class 3 in FCC) are again indicated with white bars. All three samples have roughly the same completeness limit in apparent magnitude at B_T = 18.5. The dashed line at $M_{B_T} = -15.3$ represents the completeness limit in absolute magnitude of the Centaurus survey. It is repeated in the other panels to guide the eye.

The overall LFs show the same general appearance: a steep increase in numbers of galaxies from brighter to fainter magnitudes up to the completeness limit. Nevertheless, they are very different in detail. A characteristic point of the overall LF seems to be at $M_{B_T} \sim -17.5$ where the transition from giant to dwarf galaxies takes place. For Virgo this transition is inconspicuous. But it is much more pronounced for the Centaurus and Fornax clusters, and recent studies on the LF of the Coma cluster again observed a prominent gap at this luminosity (Bernstein et al. 1995; Biviano et al. 1995; Kashikawa et al. 1995). It is obvious that with such a feature in the LF the Schechter function can not provide a good fit to the data. In spite of this problem we force-fitted Schechter functions to the three cluster LFs in Figs. 2a-c. Thereby, the fit limits were taken at three different cut-off luminosities $M_{B_T,lim} = -19$, -17, and -15.3 where the last value corresponds to the Centaurus survey completeness limit. A fourth fit was carried out for the deeper Virgo and the Fornax cluster surveys with cut-offs at their respective completeness limits -13.2 and -13.4. The error of each bin was taken Poissonian as $\sqrt{n}$ with n being the sum of definite and likely members and half-weighted possible members. All fits were normalised by the real number of observed galaxies up to the applied cut-off. The best fitting Schechter parameters $M^*_{B_T}$ and α are listed in Table 1 together with the goodness of the fit χ^2/ν (ν the number of degrees of freedom). For illustration the corresponding Schechter functions are shown in Figs. 2a-c.

Table 1. Best fitting Schechter parameters for the LFs of the three cluster populations as a function of absolute magnitude cut-offs.

$M_{B_T,lim}$	$M^*_{B_T}$	$\Delta M^*_{B_T}$	α	$\Delta\alpha$	χ^2/ν
Centaurus					
-19.0	-18.27	0.55	1.29	1.63	0.72
-17.0	-19.18	0.33	-0.53	0.25	0.67
-15.3	-20.22	0.41	-1.18	0.08	1.33
Virgo					
-19.0	-21.86	1.22	-1.42	0.32	0.51
-17.0	-21.71	0.68	-1.32	0.11	0.69
-15.3	-21.80	0.60	-1.35	0.05	0.64
-13.2	-21.56	0.43	-1.32	0.02	1.04
Fornax					
-19.0	-25.50	86.1	-1.67	0.74	0.13
-17.0	-21.60	1.7	-1.14	0.28	0.24
-15.3	-25.19	32.2	-1.48	0.11	0.51
-13.4	-25.28	35.6	-1.42	0.05	0.54

The stable behaviour of the derived Schechter parameters of the Virgo cluster in dependence of different cut-off magnitudes appears rather coincidental. In the case of the Centaurus and Fornax clusters the values of $M^*_{B_T}$ and α vary widely. Bright cut-offs provide for the Centaurus cluster even α values larger than -1, suggesting a spurious decline of the LF at the faint end. The Fornax cluster takes an excessively steep rise with $\alpha = -1.67$ if cut at $M_{B_T,lim} = -19$. Without further information one could misinterpret it as the expected increase of dwarf galaxy numbers for a fainter sample, which obviously is not true. The real rise occurs $\sim$ 3 mag fainter. The very low values of $M^*_{B_T}$ derived for the Fornax LF are meaningless because of the large errors. $M^*_{B_T}$ even jumps by almost 4 mag if cut at $M_{B_T,lim} = -17$. This random variation of the Schechter parameters as $M_{B_T,lim}$ is changed is not reflected in the goodness-of-fit parameter χ^2/ν. Therefore, Schechter parameters derived from only the bright population of clusters must be interpreted with great caution. For samples going deep into the cluster population the Schechter parameters become more stable due to the dominating effect of the dwarf galaxies. But a firm set of Schechter parameters can only be expected for $M_{B_T,lim} > -14$ which can hardly be reached for clusters beyond $\sim$ 4000 km s^{-1}. The price of a stable Schechter fit to the observed LF is that the fine structure of the LF is wiped out. This fine structure is, however, physically meaningful as discussed below. It goes without saying that in view of the unstable behaviour of $M^*_{B_T}$ it cannot be used in practice as a distance indicator.

3.2. Type-specific luminosity functions

As outlined in the introduction, a full understanding of the characteristics of a LF requires a study of its subsamples according to the Hubble type. We believe that the types given in the Centaurus Cluster Catalogue are sufficiently reliable such that a

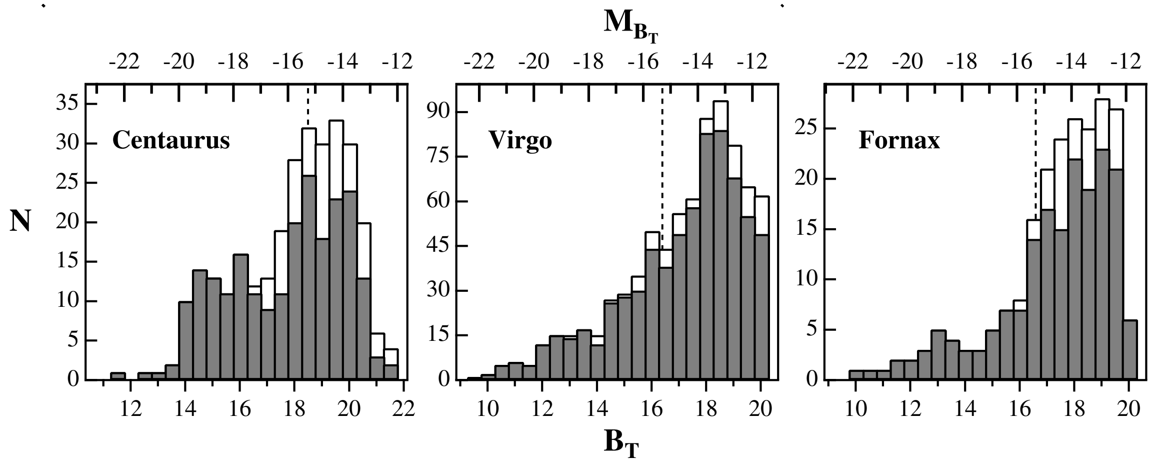

Fig. 1. (left panel) The LF of all galaxies of the Centaurus cluster. The apparent B_T magnitudes have been divided into half magnitude bins. For each bin, definite and likely members are indicated in grey, possible members by white bars. For a comparison, the corresponding LFs of the Virgo and Fornax cluster galaxies, restricted to an equivalent cluster area, are given in the central and right panel, respectively. The dashed line at $M_{B_T} = -15.3$ indicates the completeness limit of the Centaurus cluster survey; it is repeated in the other panels to guide the eye. The actual completeness limits are -13.2 for the VCC and -13.4 for the FCC.

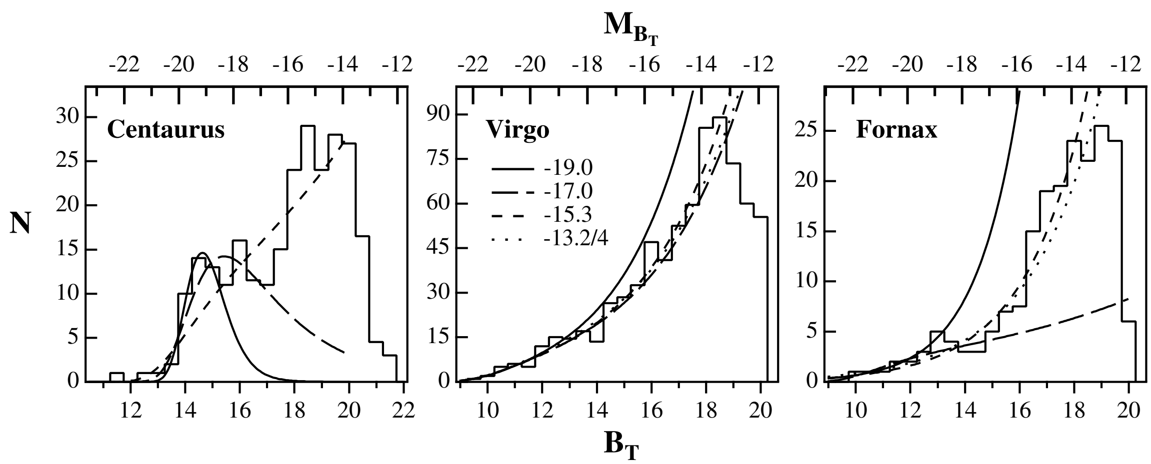

Fig. 2. Schechter functions best fitting into the binned LFs of the three clusters. Different cut-offs in absolute magnitude were applied as coded in the central diagram. In particular, the dotted lines are the results from cut-offs of the Virgo and Fornax samples at their respective completeness limits -13.2 and -13.4

determination of the type-specific LFs is warranted. The results for 16 Hubble type bins are shown in the left panels of Figs. 3 and 4. At this level of morphological resolution no dependence of the individual LFs on the environmental density ρ is to be expected and we assume the shape invariance of these LFs for different *volume-limited* samples. This allows us to study the individual LFs without the restriction on the adapted Virgo and Fornax cluster samples.

For comparison we show the corresponding type-specific LFs of the Virgo cluster in the right panels of Figs. 3 and 4 taken from SBT85[1]. For each cluster, definite and likely members with a bar or a nucleus are shown in black. Intermediate types such as E/S0, S0/a, Sxy, Im/BCD, dE or dS0 etc. have been split up with half weight to each neighbouring type. Further, the small numbers of types BCD?, Im?, and dE(dS0)? were added with half weight. Virgo galaxies of type "?" have been omitted.

The type-specific LFs of the Centaurus and Virgo clusters are qualitatively very similar. Those of the non-dwarf galaxies (Fig. 3) are all clearly bell-shaped being bound at the bright and at the faint end. For a single cluster the LF of E galaxies is not very well defined due to the small number of only 1 or 2 galaxies per half-magnitude bin distributed over 6 magnitudes. It always looks fairly shallow at the bright end but with a sharp cut-off at the faint end.

Just the opposite seems to be true for the LF of the S0 galaxies. The Gaussian-like profile has a clear maximum at $M_{B_T} \sim -19.7$ and is slightly skewed towards the bright side. Similar behaviour can be found in Virgo (right panel of Fig. 3)

[1] In Fig. 1 of SBT85, the LFs of the galaxies from type Sa to Sd were displaced by 0.5 mag (1 bin) to the bright side by accident. This has been corrected in our Fig. 3.

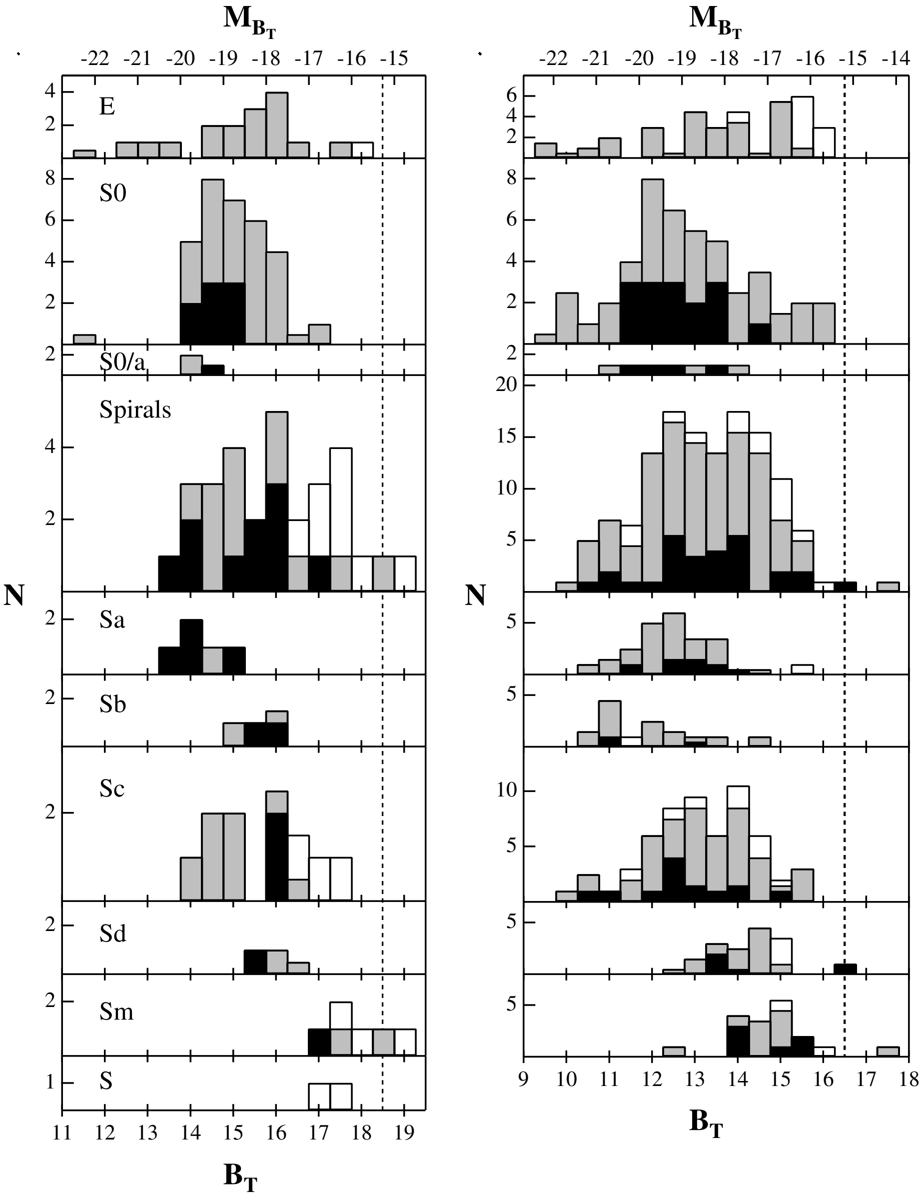

Fig. 3. The magnitude distribution of the high-luminosity galaxies E, S0, and spirals of Centaurus (left) and Virgo (right). For each bin, definite and likely members are indicated in black or in grey, depending on whether the galaxy is barred or not. All possible members are marked with white bars. The dashed line indicates the completeness limit of the Centaurus cluster survey.

and Fornax (FS88, Fig. 8). However, the LF of the Centaurus S0's does appear to be narrower than those of the other clusters. As emphasised in Paper II, another difference is the large type fraction of 25% S0 galaxies in the Centaurus cluster as compared to only 10% and 13% in Virgo and Fornax, respectively. This property is mainly responsible for the pronounced bump in the overall LF of the Centaurus cluster (Fig. 1).

The LF of the spiral galaxies is shown in Fig. 3 as a whole and as subdivided into the subsamples Sa, Sb, Sc, Sd, and Sm. It is obvious that the Centaurus cluster has no spirals as bright as the Virgo cluster. This is reflected also by the fact, independent of any assumed distance between the two clusters, that the brightest Virgo spirals are only 0.5 mag fainter than the brightest elliptical galaxy. In contrast, this magnitude difference is about 2 mag in the Centaurus cluster. Moreover, a correlation between subtypes and mean absolute magnitudes can be found for Centaurus, i.e. the later the spiral the fainter the galaxy. In details, the Sb and Sc galaxies are fainter than the Sa galaxies while the opposite is true for the Virgo spirals (cf. SBT85). The late-type spirals, Sd and Sm, are particularly faint, but this is the same as in the Virgo cluster. Their faintness has long been known from van den Bergh's (1960a, b) luminosity classification. The spirals of very late type with their poorly developed spiral structure are the transition between the bright Sc spirals and the Im galaxies which are on average still fainter than the Sd's and Sm's (cf. the Sd and Sm panels in Fig. 3 and the Im panel in Fig. 4).

The galaxy types of lower luminosity, i.e. the Im and BCD galaxies and the early-type dwarfs, have again similar luminosity distributions in both clusters. The number of early-type dwarfs increases well beyond the completeness limit of the Centaurus cluster and continues to increase in the case of the Virgo

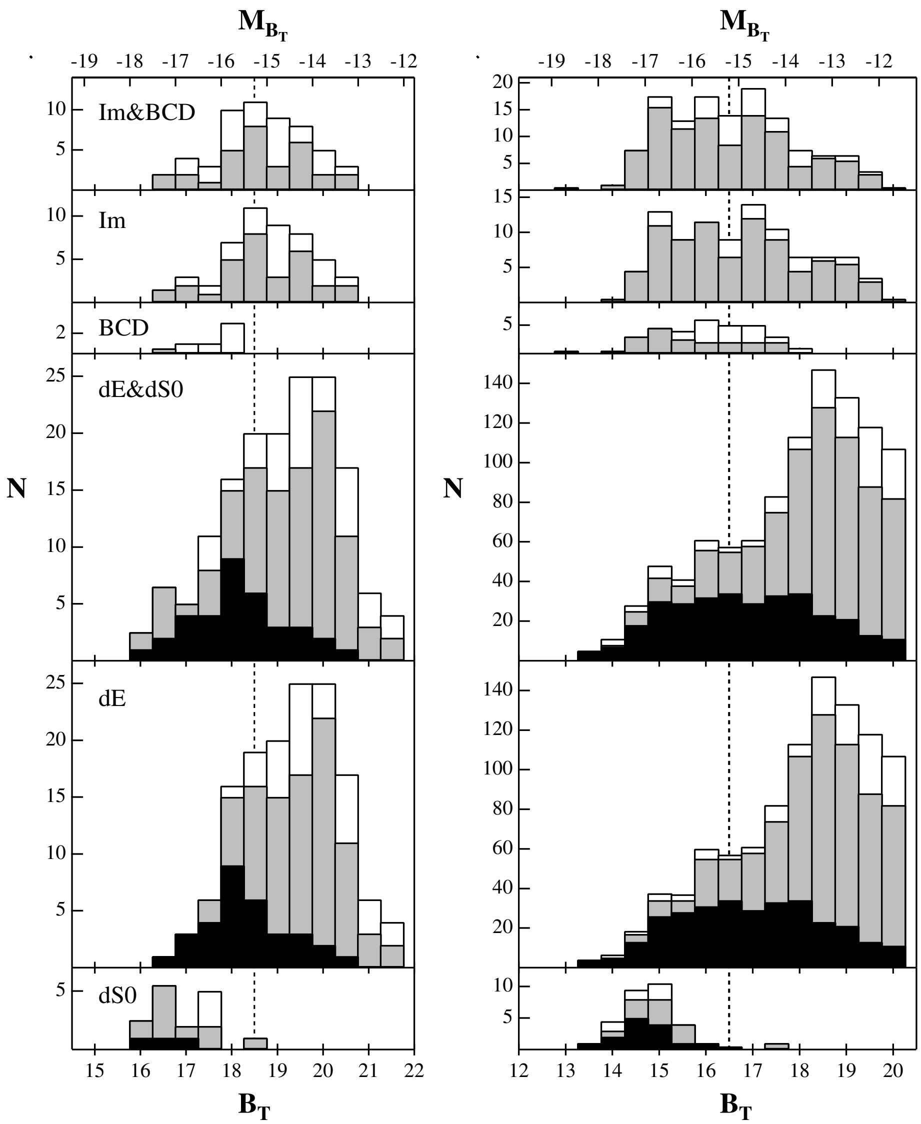

Fig. 4. The magnitude distribution of the early-type dwarfs dE and dS0 as well as the irregulars Im and BCD in Centaurus (left) and Virgo (right). For each bin, definite and likely members are indicated in black or in grey, depending on whether the galaxy has a nucleus or not. All possible members are marked with white bars. The dashed line indicates the completeness limit of the Centaurus cluster survey.

cluster down to at least -13.2 (right panel of Fig. 4). An exception are the dS0 galaxies: they disappear before the completeness limit is reached. The number of nucleated dwarfs probably decreases with decreasing luminosity, too. In any case their relative contribution diminishes.

A somewhat controversial point is the behaviour of the current star-forming galaxies Im's and BCD's. Their number decreases in the Centaurus cluster only faintwards of the completeness limit. For the deeper Virgo sample the faint end of the Im&BCD LF decreases as well and it was shown recently (Drinkwater et al. 1996) that the feature is real and not due to incompleteness. But the true shape - exponentially rising, flat, or decreasing - of this type-specific luminosity function remains debatable. It might be there is no "universal" trend (at least not in the B-band). Such an idea is getting additional credit from the again very different LF found for these Hubble types in Fornax (FS88, Fig. 12). The reason why the Im&BCD LF could be strongly related to an individual cluster is the detection criterion valid for these particular galaxy types. They are detected by their star formation activity and not by their surface brightness. Therefore the appearance of the LF probably reflects how effective the star formation mechanisms take place in the considered galaxy aggregate.

4. Best LF parameters

As a first step to a more quantitative comparison of the different type-specific LFs of the three clusters we have fitted analytical functions to the histograms of S0, Sp, Im&BCD, and dE&dS0 galaxies. Following SBT85, the distributions of S0 and spiral galaxies are approximated by Gaussians with a mean magnitude μ and a dispersion σ. On the other hand, the distributions of Im&BCD and dE&dS0 galaxies are fitted by Schechter functions characterised by M^* and α. All fits were restricted to galaxies brighter than the respective completeness limit. The best-fitting apparent magnitude parameters were transformed into absolute magnitudes using the distance moduli discussed in Sect. 3. All values with their errors are collected in Table 2.

Table 2. Parameters of best fitting analytical profiles of individual Hubble types.

Type	**Centaurus** ($M_{B_T} \leq -15.3$)		**Virgo** ($M_{B_T} \leq -13.2$)		Function
S0	$\mu_{M_{B_T}}$ = −19.04(±0.22)	σ_{B_T} = 0.83(±0.16)	$\mu_{M_{B_T}}$ = −18.97(±0.20)	σ_{B_T} = 1.38(±0.17)	Gauss
spirals	$\mu_{M_{B_T}}$ = −18.53(±0.45)	σ_{B_T} = 1.71(±0.43)	$\mu_{M_{B_T}}$ = −18.49(±0.12)	σ_{B_T} = 1.36(±0.09)	Gauss
Im&BCD	$M^*_{B_T}$ = −17.55(±3.42)	α = −1.35(±0.79)	$M^*_{B_T}$ = −16.16(±0.24)	α = −0.31(±0.18)	Schechter
dE&dS0	$M^*_{B_T}$ = −18.67(±4.06)	α = −1.68(±0.56)	$M^*_{B_T}$ = −17.79(±0.32)	α = −1.33(±0.06)	Schechter

The decisive question is whether the parameters are the same within the errors for a given galaxy type in the Centaurus and the Virgo cluster. For the S0 and the spirals the centres of the distributions are very well defined. The cluster-to-cluster variation is less than 0.1 mag for each Hubble type. The dispersion is consistent for the spirals but slightly different for the S0's. The derived values of the Schechter parameter $M^*_{B_T}$ agree for each dwarf family. But the errors of the Centaurus dwarf samples are large. For the early-type dwarfs a flat or even decreasing LF can be ruled out. Our results suggest a weighted mean value of $\alpha = -1.35$. A very different result we get for this parameter in the case of the Im&BCD's. Reasons could be the small number of magnitude bins which were available for the fit to the Centaurus dwarfs and a possible cluster-to-cluster variations of the LF for these galaxy types as mentioned before.

An obvious consequence of the existence of bell-shaped LFs on the one hand and of unbound or even exponentially rising LFs on the other hand is that the shape of the *overall* LF must depend on the mixture of galaxy types of a particular sample. The observed variety of the overall LFs in the Centaurus, Virgo and Fornax clusters (Fig. 1) are just the result of different type mixtures. It is also evident that the superposition of different type-specific LFs will generally not result in a smooth overall LF as found in Virgo. As a consequence, the Schechter function is generally not flexible enough to provide a good fit to the overall LFs. The effect of different type mixtures on the overall LF is discussed in more detail by Binggeli et al. (1988) and Jerjen et al. (1992).

In a second step the possible skewness of the early-type giant LFs was analysed statistically. For this purpose we transformed the apparent magnitudes of the three E and S0 cluster samples into luminosities according to our assumed cluster distance moduli (Sect. 3). This yields reference galaxy samples for each Hubble-type whose LFs are shown in Fig. 5. The expression that quantifies the skewness of a data set is defined as $\beta_1 = c_3/c_2^3$ where c_i are its i-th central moments. A symmetric distribution would give $\beta_1 = 0$. Our two membership-weighted reference samples of 70.5 E and 107 S0 galaxies have values of $\beta_1 = -0.28 \pm 0.10$ and $\beta_1 = -0.06 \pm 0.20$, respectively. The given error corresponds to the HWHM of the skewness value distribution of 5000 bootstrap samples (Diaconis & Efron 1983) generated for each data set. The negative value we get for the E galaxies deviates from zero by 2.8σ and confirms the visual impression of a significantly skewed distribution towards fainter magnitudes. However, in the case of the S0's the error of β_1 does not exclude a zero value. Thus, no evidence for an asymmetric distribution could be found for this Hubble type.

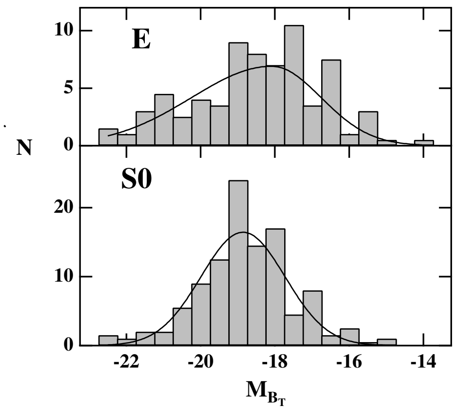

Fig. 5. LFs of the E and S0 galaxies composed of three cluster samples. The curves illustrate the best fitting Gaussian profiles. In the case of the elliptical galaxies, a Gaussian function with different wings (dispersions) on the bright and faint end was used.

Moreover, we fitted reliable functions to the LFs of the combined samples in order to find the best parameter sets. A Gaussian fit with independent bright and faint wings (dispersions) was applied to the E distribution to take into account its asymmetry. As the most robust centre of the distribution we chose the sample median value $M_{B_T} = -18.33$. The best fit gives $\sigma_{B_T,bright}$ = 2.15(±0.36) and $\sigma_{B_T,faint}$ = 1.32(±0.21). For the S0 sample a standard Gaussian fit gives the parameters $\mu_{M_{B_T}}$ = −18.90(±0.12) and σ_{B_T} = 1.13(±0.10). All fits were normalised by the numbers of sample galaxies.

Our result for the elliptical galaxies stands in contradiction to the bright end skewed LF for this galaxy type reported by Muriel et al. (1995). These authors used a magnitude-limited subsample of the APM Bright Galaxy Survey (Loveday 1989) which is based on scanned Schmidt plates. Their result is not very surprising, given bias of magnitude-limited samples in general. Such samples consist of very luminous objects and are not comparable to the more compact ellipticals populating the fainter end of the type-specific LF. Becoming more and more star-like in appearance, the latter are hardly detectable even on a high resolution Las Campanas du Pont plate of the nearby Centaurus cluster (cf. Paper I). They must simply be missed in the APM Bright Galaxy Survey.

We take our result to suggest that the observed skewness towards *fainter* luminosity in the LF of E galaxies is real and has a possible physical origin. In the next section an approach from the theoretical point of view will be discussed supporting this idea. However, we have to emphasise that our data on the three E populations may also be incomplete at the faint luminosity end

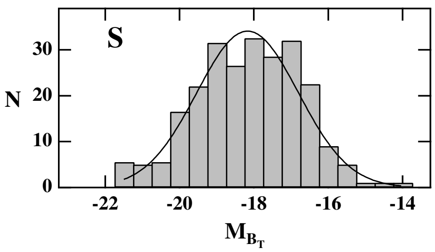

Fig. 6. The LF of spiral galaxies composed of three cluster samples. The curve is the best fitting Gaussian profile.

for the same reason as mentioned before. Hence, it is not possible here to quantitatively characterise the skewness by e.g. a typical number ratio between faint and bright ellipticals in clusters.

Following the same procedure as for the E and S0 galaxies we generated a reference sample of 245.5 spiral galaxies. The LF is shown in Fig 6. Fitting again a Gaussian profile to the histogram yields a well defined mean value $\mu_{M_{B_T}} = -18.20(\pm 0.09)$ and a standard deviation $\sigma_{B_T} = 1.37(\pm 0.07)$.

5. What the theory tells us

It is generally assumed that the present-day galaxy distribution results from primordial density fluctuations in the early universe that became gravitationally unstable. Although the number of dark matter haloes that collapse and virialize can be predicted once the power spectrum is chosen (Press & Schechter 1974; White & Rees 1978; Bond et al. 1990), the physical processes that govern galaxy formation are far from being understood. Detailed discussions of the problems are given for gas cooling (White & Frenk 1991), evolution of the stellar population in galaxies (Guiderdoni & Rocca–Volmerange 1987; Rocca–Volmerange & Guiderdoni 1988), star-formation (Lacey & Silk 1991; Kauffmann et al. 1993), feedback from star formation (Larson 1974; White & Rees 1978; Dekel & Silk 1986; White & Frenk 1991), and the merger rate of galaxies (Katz et al. 1993; Katz & White 1993). As a consequence, theoretical results come mainly from models which are based on more or less plausible assumptions about the physics of galaxy formation. In the framework of these theories the observed luminosity function of galaxies serves as an important test object. The quality of a model of structure formation is often measured by how well it can approximate the LF. Unfortunately, all existing models are still far away from being convincing. But at least some progress can be noted.

The inferred mass of the most luminous galaxies (e.g. Blumenthal et al. 1984) agrees well with the maximum mass ($\sim 10^{12} M_\odot$) below which gas haloes can collapse by dissipation and form stars. This theoretical upper mass limit is given by setting the cooling time-scale equal to the dynamical time-scale for the gas at the virial temperature of the galaxy. It explains the observed bright-end cut-off of the LF. More massive haloes form groups and clusters by merging.

There is also a minimum mass below which the equilibrium between star-formation and its feedback, i.e. heating due to the energy input by supernovae, breaks down. It was shown (Wyse & Silk 1985; Dekel & Silk 1986; Schaeffer & Silk 1988) that for galaxies with a virial velocity below $v_c \leq 100\,\mathrm{km\,s^{-1}}$ the gravitational potential is too shallow to retain the interstellar gas and consequently star formation becomes inefficient.

Taking into account these two mass limits, Schaeffer & Silk (1988) were able to divide their theoretical LF into those of the high (E&S0) and low (dE&dS0) surface brightness galaxies. The transition between the two LFs is roughly at $M_{B_T} = -17.5$ which can be identified with the observed transition gap between giants and dwarfs in the overall LFs of the Centaurus, Virgo, and Fornax clusters. Furthermore, a clear skewness to fainter luminosity appears in the theoretical E&S0 LF (Fig. 1 in Schaeffer & Silk). If their model in fact produces mainly E-like galaxies, the skewness in the derived LF can be understood as the theoretical counterpart of our LF for ellipticals (Fig. 5).

The understanding of the observed LFs of E, S0, and spiral galaxies may require separate luminosity (or mass) functions of their disk and bulge components. The smooth transition from spirals to Im galaxies as one goes to fainter luminosities, i.e. the non-existence of dwarf spirals, is probably explained by the fact that faint low-mass galaxies have simply too little rotation and hence too little *differential* rotation to form a spiral pattern (Sandage & Binggeli 1984).

6. Luminosity functions as distance indicators

The luminosity of the "knee" M^* of the overall LF has received much attention as a promising standard candle (Bautz & Abell 1973, Austin et al. 1975). However, as was shown by many authors and also in this paper, there are clear variations of M^* depending on many observational and physical parameters, e.g. the survey area as a function of core radius r/r_c, the morphological type, and the local galaxy density ρ. Obviously there is no universal value for M^* and consequently M^* is an inappropriate distance indicator. However the *type-specific* LFs of the four main Hubble types E, S0, Sp, and dE&dS0, appear to be stable for different clusters. This offers an interesting compensating possibility which will be tested in the following.

Historically, the use of morphologically selected galaxies as relative distance indicators goes back to Hubble & Humason (1931), Humason (1936), and Humason et al. (1956) who used the n-th ranked cluster galaxy to establish their "Hubble diagrams". Sandage & Hardy (1973) used strictly the first ranked E galaxy to extend the Hubble diagram to even larger redshifts; they corrected the magnitude of the brightest cluster galaxy for cluster richness and for the Bautz-Morgan class (Bautz & Morgan 1970), which estimates the magnitude contrast of the brightest member galaxy over the remaining cluster members. This raises the question whether the LF of *all* cluster ellipticals might provide an even more stable standard candle. The answer is negative, because E cluster members are scarce and cover a range of 6 mag. This causes the mean of their presumably slightly skewed Gaussian LF to be poorly defined.

However, the more abundant S0 and spiral galaxies and their bell-shaped LFs which come to zero before the completeness limit is reached, provide an obvious opportunity. Even the suit-

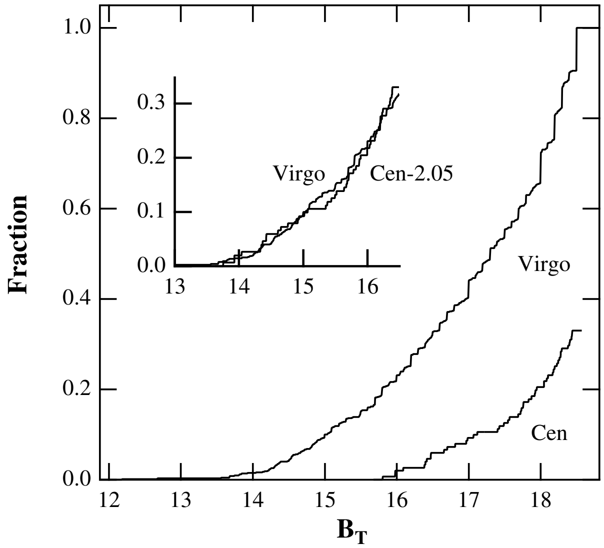

Fig. 7. The unbinned cumulative and normalised luminosity functions of the dE&dS0 populations in Centaurus and Virgo. That of Centaurus is downscaled to a factor of 0.33. The inner graphic shows the overlap of the adjusted LF of Centaurus after a magnitude shift of 2.05.

ability of LFs of different spiral types as distance indicators was discussed again recently (Sandage 1996a, b). In our case, we fitted Gaussian profiles to the LFs separately to the S0 and spiral galaxies in the Centaurus and Virgo clusters. After being corrected for differential Galactic extinction (A_B = 0.42 mag, Burstein & Heiles 1984) the differences of the mean magnitudes $\mu_{B_T,Cen} - \mu_{B_T,Virgo}$ (cf. Table 2) are then expected to be the modulus difference between the two clusters.

It goes without saying that the Im&BCD galaxies cannot be used in the same way because the known cluster samples have different properties. Yet another opportunity is offered by the early-type dwarfs in Centaurus. We assume that their LF is intrinsically the same as that of the Virgo dwarfs, i.e. the observed LFs in the clusters differ only due to unequal population sizes and a distance difference. A simultaneous solution for these two parameters by the χ^2-method using the unbinned cumulative LFs of the dE&dS0 gives a factor of 0.33($\pm$0.05) by which the Centaurus cluster is poorer in early-type dwarfs than the Virgo cluster in the average down to the completeness limit B_T = 18.5. This value is in good agreement with the pure number ratio of 0.42 found for the two dwarf populations (Paper II). The derived distance modulus difference is 2.05($\pm$0.16) when extinction is not taken into account, or 1.63($\pm$0.16) after correction. The fitting procedure is illustrated in Fig. 7.

All three derived relative distances between the Centaurus and Virgo clusters, based on the LFs of the S0, spirals, and dE&dS0 galaxies are compiled in Table 3. The consistency of the three independent values is very satisfactory. Furthermore, the mean weighted modulus difference agrees perfectly with other relative distances from the literature as discussed in Paper II. We conclude that the three type-dependent LFs are useful as distance indicators in the case of clusters of galaxies. The LFs

Table 3. Relative distances of Centaurus to Virgo.

Hubble type	$\Delta(m-M)^0_{Cen-Virgo}$ [A_B = 0.42]
S0	1.61($\pm$0.30)
spirals	1.64($\pm$0.47)
dE&dS0	1.63($\pm$0.16)
mean:	1.63($\pm$0.14)

of the bright S0 and spiral galaxies should be useful in the future also for more distant clusters. Clusters out to $\sim$10 000 km s^{-1} should be accessible for this method with classical photographic plate material. Moreover, the resolution power of the HST could be used to study these type-specific LFs in very distant young galaxy clusters.

7. Summary and conclusions

The main results are summarized as follows.

The overall LF of galaxies in clusters is clearly governed by the type-mix of different Hubble types. Above all the smoothness of a cluster LF is determined by the transition from the giant to the dwarf LF taking place at about $M_{B_T} = -17.5$. Hence, all kinds of shapes of overall LFs have to be expected within the prediction of the morphology-density relation. Which set of Schechter parameters M^* and α are best fitting an overall LF depends on the individual cluster population. No *universal* set is able to reproduce all LF features. Moreover, strong parameter variations are found in dependence of the cut-off in absolute magnitude for all specific cluster galaxy samples. Only in the case of the exceptionally smooth LF observed in Virgo the parameters remain relatively stable. There is no way to correct the Schechter parameters for these kind of effects which makes it obvious why M^* can only be a poor distance indicator.

The LF for ellipticals is poorly defined for a single cluster due to the small number of objects. Nevertheless, the analysis of a larger combined cluster sample reveals evidences for a skewness in the distribution to fainter magnitudes. A similar feature is also predicted by theory (Schaeffer & Silk 1988). If an analytical description of the LF of E galaxies is needed, we recommend a modified Gaussian profile with centre at $\mu_{M_{B_T}} = -18.3$ and two different wings quantified by the standard deviations $\sigma_{B_T,bright}$ = 2.2 and $\sigma_{B_T,faint}$ = 1.3.

Our complete samples of S0 and spiral galaxies exhibit LFs with bell-shaped profiles in all three clusters. The LFs of each specific Hubble type appear cluster-independent which gives evidence for the invariance of the LF shape and supports the idea of an underlying physical origin. The best-fitting Gaussian parameters for the S0 are $\mu_{M_{B_T}} = -18.9$ and σ_{B_T} = 1.13. For the spirals we derived $\mu_{M_{B_T}} = -18.2$ and σ_{B_T} = 1.37.

The LFs of the early-type dwarfs dE&dS0 are very similar in Centaurus, Virgo, and Fornax and consistent with a Schechter parameter $\alpha \sim -1.35$. This means that all known dE&dS0 LFs increase exponentially to all known sample completeness limits.

The late-type dwarf LF is not very well known. Several reasons make it difficult to follow the question whether there is an individual or an invariant LF shape for different cluster populations, e.g. the influence of the magnitude completeness limit of a galaxy survey and the detection criterion valid for these particular galaxy types. A cluster-dependent LF for the Im&BCD's seems to be more likely.

As the Schechter parameter M^* is not reliable as a distance indicator, we propose instead to use the mean magnitudes of the Gaussian LFs of S0's and spirals as standard candles. Furthermore the relative shift of the *empirical* dE&dS0 galaxy LF might be useful as well for distance determinations. As a first application we derived an average relative distance modulus of $\Delta(m-M)^0_{Cen-Virgo}$ = 1.63($\pm$0.14) for Centaurus and Virgo, which is in excellent agreement with the best available value 1.68($\pm$0.14) for these two clusters (Paper II).

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