

Geometric differences between the gaseous and stellar warps in the Milky Way

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Abstract. Careful modelling of the stellar and gaseous distributions in the Milky Way disc has been carried out in order to study the stellar warp. We have analysed the observations in the near infrared made by the "Diffuse Infrared Background Experiment" (DIRBE) on board COBE as reported by Freudenreich et al. (1994). These authors found a dependence of the displacement of the maximum emission on galactic longitude, which is however less than that observed at 21 cm. Our model enables us to reject the interpretation of this displacement as being due to the fact that the Sun does not exactly lie in the galactic plane. Either the stellar disc is large but is less warped than the gaseous disc, or it has a cutoff at $r \lesssim 15$ kpc. Preliminary results from DENIS (Ruphy et al. 1996) indicate a cutoff at 15 kpc, favouring the latter interpretation.

Key words: Galaxy: general – Galaxy: structure – infrared: stars

1. Introduction

Surface brightness data of DIRBE (Diffuse Infrared Background Experiment) were analysed by Freudenreich et al. (1994) (hereafter F94) in near infrared bands 1.25, 2.2, 3.5 and 4.9 μm , whose emission comes mainly from K and M giant stars in our galaxy.

They found a deviation of the maximum emission plane with respect to the mean plane of the Galaxy, with $l \sim 90^\circ$ – 100° and $l \sim 240^\circ$ – 250° being the longitudes corresponding to maximum deviation in the Northern and Southern Hemispheres respectively. A tempting interpretation of this effect could be the warp of the galactic disc of the Milky Way, found in other observations corresponding to other objects and wavelengths (Pandley et al. 1990; Fick & Blitz 1982; Wouterloot et al. 1990; Sodroski et al. 1993). The stellar warp for young stars has already been detected by Miyamoto et al. (1989) and by Porcel & Battaner (1995). However there was a lack of precise observations of this phenomenon in the case of giant red stars, and so the mapping of F94 covered an important observational gap.

F94 point out that the deviation of the maximum emission plane in the near-infrared bands has a lower amplitude than that of the gas; they therefore suggest that the stellar disc might be less warped than the gas one in the warp. This fact seems to confirm the early observations by Guibert, Lequeux & Viallefond (1978) that no trace of a red warp was found in the Milky Way. However, in their analysis of the observations, F94 point out that there are other possible explanations besides a smaller warp. Other possibilities are: a) a displacement of the Sun with respect to the symmetry plane of the disc, and b) a disc truncation at relatively small galactocentric radii. Both effects can explain, at first glance, the asymmetry in the maps of F94, and their lower amplitude with respect to the gas.

To distinguish between these three possibilities, we develop a model which describes the stellar distribution and that of the gas in HI and H₂, taking into account extinction. The model has several adjustable parameters which enable us to obtain the synthetic maps and the curve $[l,b]$ giving the latitude of peak brightness for the IR emission, with the object of comparing them with the real data of DIRBE. This procedure enables us to obtain the $[l,b]$ curve we would expect if the stellar disc were warped with a similar geometry to that found for young objects. In the same way, it enables us to find the $[l,b]$ curve expected if the stellar disc were warped and truncated, and if it were warped but with the observer's position (that of the Sun) displaced from the plane.

If the hypothesis of a stellar warp smaller than that of gas is right (this interpretation will be favoured in our conclusions) these observations could introduce restrictions that should be explained by the different theoretical models (Binney 1992; Combes 1994; Battaner 1995 and others). If the warp was originated by magnetic forces (as suggested by Battaner et al. 1990, 1991) acting on the gas but not on the stars, even though they were born from the gas, we would expect a differential behaviour between the stellar and gas warps. The stellar warp, specially that of old stars, would be much smaller than that of the gas. Other theories about warps (Ostriker & Binney 1989; Binney 1992; Sparke & Casertano 1988 and others) suppose that this distortion is due to gravitational forces. The predictions of these gravitational models about the differential warp are less clear,

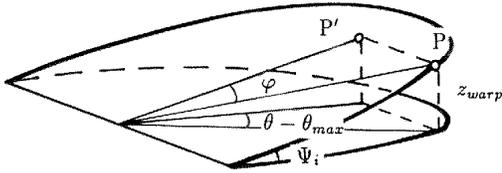


Fig. 1. Ring of radius R_i raised at a tilt angle Ψ_i over the mean galactic plane .

but as gravitation acts in the same way on every kind of particle and thus on every kind of star, we would expect, from these models, stellar and gas warps to be more similar. They might be different because of viscosity effects in both the gas and the stars (dynamic viscosity) but in any case, the distribution of old stars in the warp region constitutes an observational restriction for the different theoretical models.

Therefore, the F94 analysis of DIRBE observations constitutes an interesting tool to restrict the theoretical possibilities of understanding a longstanding problem, which as yet does not have a unique and clear explanation.

2. A stellar warp model

We use galactocentric cylindric coordinates (R, θ, z) , where R is the distance from the galactic centre measured on the mean galactic plane, θ is the galactocentric azimuth starting from $l = 0^\circ$ counterclockwise, and z is the elevation over the plane $b = 0^\circ$.

Let us suppose that the gas and the stars corotate following the tilted ring model of Rogstad et al. (1974) in circular trajectories which are raised over the mean galactic plane in a given direction θ_{\max} . This model assumes that the disc is composed of a fixed number of circular, concentric rings. Each ring, with radius R_i , has a different tilt angle Ψ_i , with respect to the central rings that are not warped and that determine an interior plane. We consider a given orbit of radius R_i which forms an angle Ψ_i with the mean galactic plane (Fig. 1).

The elevation z_{warp} of an arbitrary point P of the above orbit is obtained in the following way. Let P' be its projection over the maximum warp plane (perpendicular to the galactic plane and which contains the maximum warp direction). The points P and P' have the same elevation over the mean plane, z_{warp} . The point P is determined by its azimuth φ inside the circle (measured starting from the maximum warp plane) and by its galactocentric coordinates $(R, \theta, z_{\text{warp}})$. From Fig. 1 we obtain:

$$\frac{R_i \cos \varphi}{R_i} = \frac{z_{\text{warp}}}{z_c}$$

where z_c is the maximum elevation of the orbit or maximum warp

$$z_{\text{warp}} = z_c \cos \varphi \quad (1)$$

As the angle Ψ_i is very small, the azimuth over the orbit of point P can be made equal to the galactic azimuth (over the mean galactic plane) measured in the same way, starting from

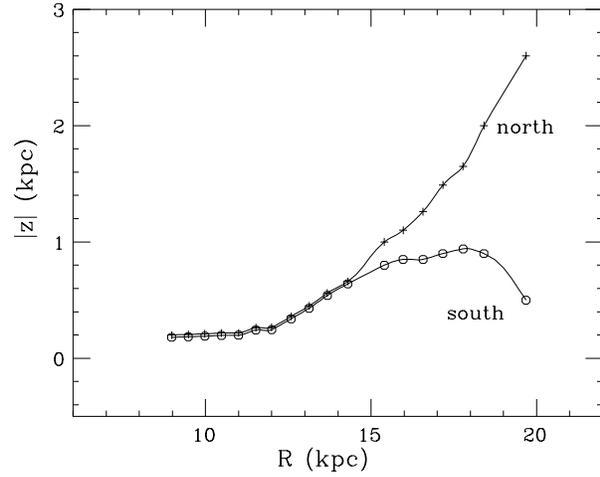


Fig. 2. Galactic warps in both Hemispheres (from Burton 1988).

the maximum warp direction, i.e. $\varphi \simeq (\theta - \theta_{\max})$. Then Eq. (1) becomes

$$z_{\text{warp}} = z_c \cos(\theta - \theta_{\max}) \quad (2)$$

The value of z_c is obtained from the data of Burton (1988) and Diplás & Savage (1991). Fig. 2 shows the mean displacement of the HI layer with respect to the inner plane of the galaxy. Distances have been converted using the values recommended by the IAU, i.e. $R_\odot = 8.5$ kpc and $V_\odot = 220$ km s $^{-1}$, and a flat rotation curve. We see that up to 15 kpc the warps in the Northern and Southern Hemispheres match. From this galactocentric radius the warp in the Southern Hemisphere returns to the mean plane.

As shown by Burton (1988) the direction θ_{\max} is approximately the same for all the orbits. This author considers this direction to be $\theta_{\max} = 80^\circ$ while Diplás & Savage (1991) consider it to be between 90° and 110° .

In Sect. 3.1 we develop a model for the density of atomic hydrogen in our galaxy, with the object of calculating its extinction. Once this density is known it is possible to obtain the HI column density for any direction (l, b) as $N(\text{HI})(l, b) = \int n(r, l, b) dr$, where r is the distance along the line of sight in the direction (l, b) , and $n(r, l, b)$ is the HI density at point (r, l, b) .

If we consider profiles which are perpendicular to the galactic mean plane $b = 0^\circ$, we can obtain the mean latitude. Fig. 3 shows the observational data analysed by Freudenreich et al. (1993) and that obtained with the warped gas model for angles $\theta_{\max} = 80^\circ, 90^\circ$ and 100° . The best result is obtained for $\theta_{\max} = 90^\circ$, so we will take this value for the model.

3. Near IR emission

As the warp in our galaxy is a phenomenon which takes place outside the solar radius and at large scales, we limit ourselves to considering the emission due to the exponential disc, without considering the influence either of the spiral arms or of the corrugations that may be present in the stellar disc.

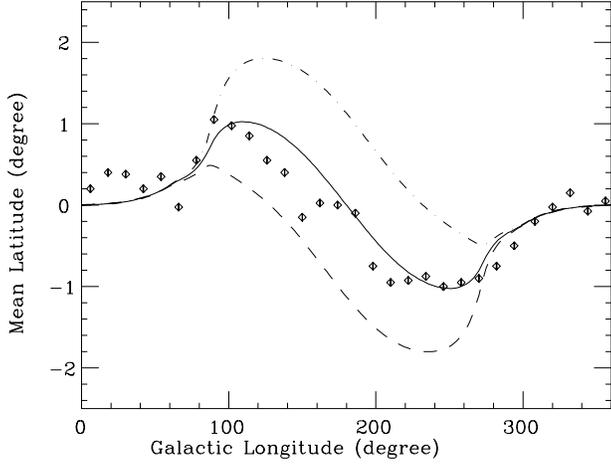


Fig. 3. The symbol \diamond stands for the centroids of a velocity-integrated HI map (from Freudenreich et al. 1993). The solid line shows the HI column density averaged on latitude given by the model with $\theta_{\max} = 90^\circ$, the dashed line for $\theta_{\max} = 80^\circ$ and the dotted-dashed line for $\theta_{\max} = 100^\circ$.

To obtain the surface brightness of the galaxy we use spherical heliocentric coordinates (r, l, b) with r being the distance from the Sun, l the galactic longitude and b the galactic latitude. The stellar warp is described in the preceding section in galactocentric coordinates, which are related to these ones by

$$R = \left[(R_\odot - r \cos b \cos l)^2 + (r \cos b \sin l)^2 \right]^{1/2} = \left(R_\odot^2 + r^2 \cos^2 b - 2rR_\odot \cos b \cos l \right)^{1/2} \quad (3)$$

$$\theta = \arctan \frac{r \cos b \sin l}{r \cos b \cos l - R_\odot} \quad (4)$$

$$z = r \sin b \quad (5)$$

The Differential Count Function $\mathcal{A}(m, l, b) dm$ is defined as the number of stars projected over a fixed surface given by the solid angle ω in the direction (l, b) , and whose magnitude is in the interval $(m, m + dm)$. Its contribution to the surface brightness is

$$d\mathcal{F} = \mathcal{A}(m, l, b) f_\nu(m) dm \quad (6)$$

where $f_\nu(m)$ is the flux of a star with apparent magnitude m . As $f_\nu(m) = 10^{-0.4m} f_\nu(0)$, we have

$$\mathcal{F} = \int_0^\infty \mathcal{A}(m, l, b) 10^{-0.4m} f_\nu(0) dm \quad (7)$$

For the K band, $f_\nu(0) = 665$ Jy and for the L band $f_\nu(0) = 277$ Jy (Koornneef 1983). The Differential Count Function $\mathcal{A}(m, l, b)$ has been treated by several authors with Stellar Count Models in the near IR (Jones et al. 1983; Ruelas-Mayorga 1990; Wainscoat et al. 1992) who, for the exponential disc, use

essentially the same database. Following these authors, the stellar count or the flux can be expressed as a function of spectral type as:

$$\mathcal{A}(m, l, b, S) = \omega \int_0^{r_{\max}} \phi(M, S) n_s(r) r^2 dr \quad (8)$$

$$\phi(M, S) = \frac{\phi_\circ}{(2\pi)^{1/2} \sigma} e^{-\frac{(M-M_\circ)^2}{2\sigma^2}} \quad (9)$$

i.e. the stars of the same spectral type peak around a central value M_\circ following a gaussian distribution with a standard deviation σ (Trumpler & Weaver 1957). The parameter ϕ_\circ is the number of stars per pc^3 in the solar neighbourhood for a given spectral type. The values of the distribution $\phi_\circ(S)$, M_\circ and σ are obtained from observational data. We use the colours compiled by Koornneef (1983) in the K, L band. $\phi(M) = \sum \phi(M, S)$ is the luminosity function, which we consider to be constant for the whole galaxy. $n_s(r, l, b)$ stands for the density of spectral type S, normalized to the corresponding density in the solar neighbourhood.

All the models, both in the visible range and in the IR, use an exponential disc in such a way that its dependence on galactocentric distance R is of the form $n(R) = \exp(-(R - R_\odot)/H)$, taking the stellar density in the solar neighbourhood as the unit. Even though the exponential decrease is universally accepted, the value of the disc scale length H , is not. Kent et al. (1991) analysed the different values for H appearing in the bibliography, extending from 1 to 5.5 kpc. They obtained an intermediate value of 3 kpc from an analysis of the Spacelab data. The lower values for H correspond to observations in the IR. Mikami & Ishida (1981) obtained a value of $H=1$ kpc by analysing the data of the TMSS (Two Micron Sky Survey), which has a limit magnitude of 3 mag in the K band and hence observes only in the solar neighbourhood. Other values for H are 2 kpc (Maihara et al. 1978; Jones et al. 1981), 2.5 kpc (Ruelas-Mayorga) and 3 kpc (Eaton et al. 1984). All of these obtained their values from star counts in the IR. We assume here an intermediate value $H=2.5$ kpc according to recent measurements by Robin et al. 1992 and Ruphy et al. 1996.

Therefore, the normalized density for each spectral type is:

$$n_s(R, z) = e^{-\frac{(R-R_\odot)}{H} - \frac{|z-z_{\text{warp}}|}{h_z(S)}} \quad (10)$$

where $h_z(S)$ stands for the scale height for spectral type S. The values for $h_z(S)$, ϕ_\circ , M_\circ y σ are taken from Wainscoat et al. 1992. Here there is an implicit hypothesis: that h_z is independent of R . This is probably not true, h_z increases with R due to the decrease in k_z , the vertical force, but the inclusion of this flaring cannot significantly modify our results.

We consider as the limit magnitude for integration $m = 15$ mag, from which its contribution to Eq. (7) is negligible. Fig. 4 shows the percentage contribution to the flux from the different spectral types. We see that the main contribution comes essentially from K/M giant stars, in agreement with the analysis performed by Arent et al. 1994 with the experimental data of DIRBE.

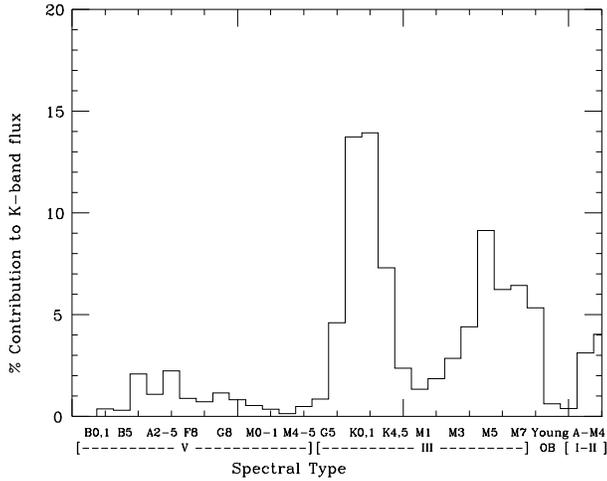


Fig. 4. Contribution to flux in the K band from the different spectral types following the model developed in the text for $l = 70^\circ$ and $b = 0^\circ$.

3.1. Extinction in the Galaxy

Though the extinction in the Galaxy for the K band is 10 times lower than that of the V band, it is convenient to take it into account, particularly for the region close to the galactic plane. We assume that the ratio dust/gas is constant and depends only on the total hydrogen density, in its atomic form as well as in its molecular one. We take $A_V = 3E(B - V)$, with $E(B - V) = N(\text{HI} + \text{H}_2)/58 \times 10^{20}$ (Bohlin et al., 1987) where $N(\text{HI} + \text{H}_2)$ is the hydrogen column density measured in cm^{-2} . Let A_λ be the extinction, measured in magnitudes, for wavelength λ .

We assume here the extinction variation for the different bands given by Rieke & Lebofski (1986).

For the extinction between two points along the line of sight with coordinates (r_1, l, b) and (r_2, l, b) , we have, therefore :

$$A_K = 5.95 \times 10^{-23} \int_{r_1}^{r_2} [n(\text{HI}) + 2n(\text{H}_2)] dr \quad (11)$$

$$A_L = 3.08 \times 10^{-23} \int_{r_1}^{r_2} [n(\text{HI}) + 2n(\text{H}_2)] dr \quad (12)$$

For HI we consider a gaussian distribution with $z_{1/2}=110$ pc (Dickey & Lockman, 1990) and a radial surface density as per Burton (1988). For H_2 we assume another gaussian distribution perpendicular to the galactic mean plane, but with $z_{1/2}=60$ pc (Combes et al. 1991; Bronfman et al. 1988), the radial density being adopted from Bronfman et al. (1988) within the solar circle, and from Grabelsky et al. (1987) outside the solar circle. In addition, the flaring of HI and H_2 is taken from Diplas et al. (1991) and Grabelsky et al. (1987), respectively.

The role of dust associated with H_2 is problematic due to the very clumpy nature of H_2 . Perhaps it would also be reasonable to forget H_2 . However our model without extinction indicates that extinction effects are not decisive. This is shown in Fig. 5. Though the flux values are higher when no extinction is taken into account, the curve $[l, b_0]$ remains virtually the same. Here

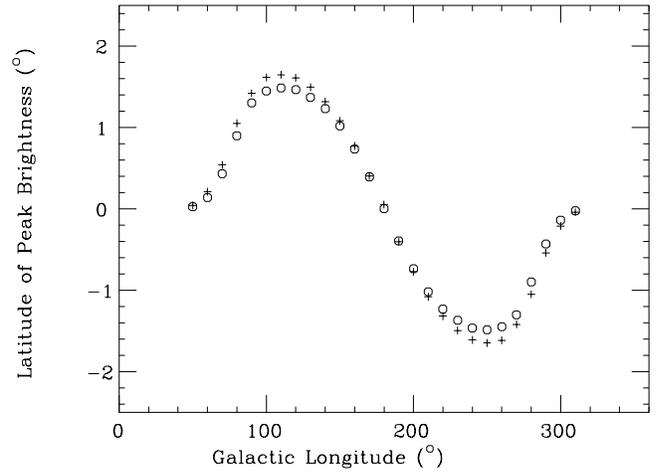


Fig. 5. Effects of extinction on the latitude of peak brightness. Open circles stand for the latitude of peak brightness in the K band with extinction and + without extinction.

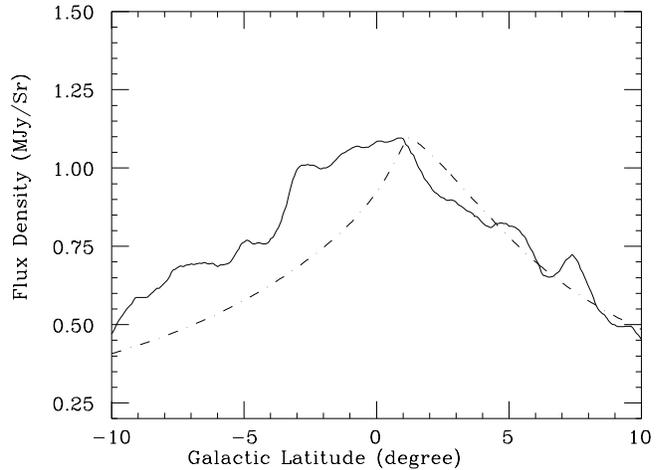


Fig. 6. Latitude profile for $l = 120^\circ$ in the $2.2 \mu\text{m}$ band of DIRBE. The dashed line shows the model prediction for the same longitude.

b_0 is the latitude of peak brightness at a given galactic longitude l , with the same definition made by Freudenreich et al. (1994).

Once the extinction is known, it is easy to integrate Eqs. (7) and (8) by means of $m_K - M_K = 5 \log r - 5 + A_\lambda(r)$. Fig. 6 shows the latitude profile for the flux and its comparison with the DIRBE data for K band in the direction $l = 120^\circ$.

4. Comparison of observations with the model

At present, the most complete database concerning surface brightness of our Galaxy is probably that of DIRBE, which presents the surface brightness of the galactic plane for $|b| \leq 10^\circ$.

It is very unfortunate that the north warp is just behind the Cygnus arm. The extinction must be very high in the arm and the emission by young stars is also higher. This is an almost unavoidable problem that largely prevents the clear observation

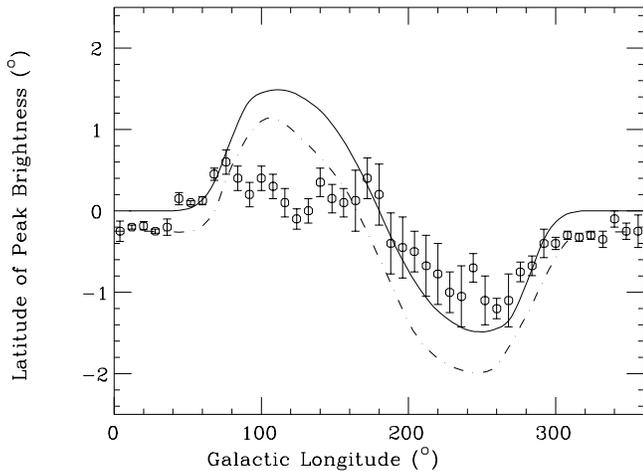


Fig. 7. Open circles stand for the latitude of peak surface brightness v.s. Galactic longitude of DIRBE $2.2 \mu\text{m}$, data taken from Freudenreich et al. (1994). The solid line shows the same magnitude given by the model for an elevation of the Sun $z_{\odot} = 0$ pc and the dashed line for an elevation of the Sun $z_{\odot} = 15$ pc.

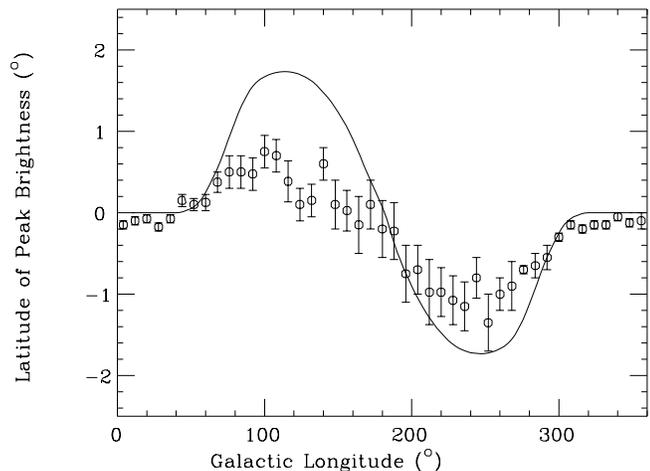


Fig. 8. Open circles stand for the latitude of peak surface brightness of DIRBE $3.5 \mu\text{m}$ data taken from Freudenreich et al. (1994). The solid line shows the same magnitude given by the model with the Sun in the plane of the Galaxy.

of the north warp. No attempt to include the Cygnus arm in the model was made. However this arm has an extension of only $\sim 30^\circ$ while we are considering a large scale phenomenon. The north warp has an extension higher than 90° . Note also no similar arm affects the observation of the south warp. The clumpiness of the molecular clouds and the presence of local arms must introduce noise into the observational curve $[l, b_0]$, but the 360° large-scale variation due to warps or other macroscopic effects is appreciated.

Fig. 7 and Fig. 8 show b_0 for DIRBE data taken from F94, as well as the model prediction with the same estimator. The peak to peak amplitude for DIRBE data is $\sim 1.8^\circ$ for the K band and

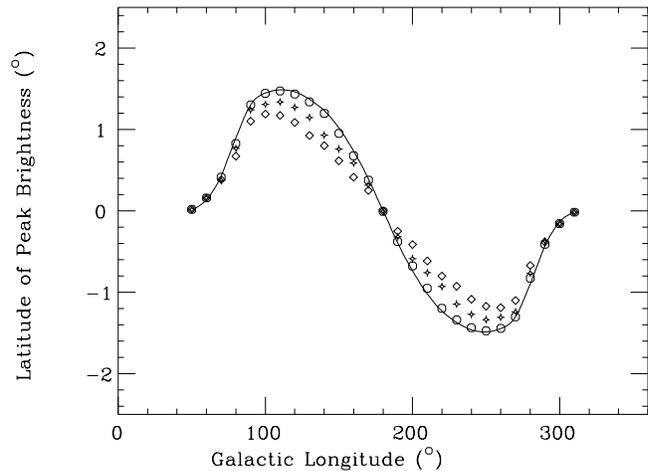


Fig. 9. Effect of stellar disc truncation. The solid line shows the contribution of a disc with 25 kpc radius. \circ shows the same for a disc truncated at 16 kpc, $+$ for a disc truncated at 14 kpc and \diamond for a disc truncated at 12 Kpc.

$\sim 2.2^\circ$ for the $3.5 \mu\text{m}$ band, while the model predicts higher values, 2.8° and 3.2° respectively for the same magnitude.

When the distribution is asymmetric, the mean and maximum values are different. The mean latitude flux averaged can be used as well as the estimator, as made by Freudenreich et al (1993), and in this case we obtain the same result i.e. the model predicts a warp amplitude which is higher than that of the observed one.

It is easy to show the systematic effect of the elevation of the Sun over the mean galactic plane. The best present day values are $z_{\odot} \sim 15$ pc (Hammersly et al. 1995; Cohen 1995). The normalized density in Eq. (10) would become

$$n_s(R, z) = e^{-\frac{(R-R_{\odot})}{H}} - \frac{|z - z_{\text{warp}} + z_{\odot}|}{h_z(S)}$$

An elevation of 15 pc produces a global shift of -0.2° for every longitude as shown in Fig. 7.

A stellar disc truncation might be an explanation of the lower amplitude of the latitude of peak brightness in experimental data. It is possible to reproduce this effect in our model and we perform it for stellar disc cutoff, r_c at 12, 14 and 16 Kpc. This is shown in Fig. 9 which shows how b_0 decreases as the stellar disc becomes smaller. We see that for $r_c > 16$ kpc from Galactic Centre, the contribution to the flux is almost negligible.

5. Conclusions

We have built a model for the surface brightness of our Galaxy in the K, L band. It shows that the emission is mainly due to K/M giant stars. Its contribution takes place within a galactocentric radius of 16 kpc ($R_{\odot} = 8.5$ kpc).

The brightness due to these stars, calculated for an exponential disc of scale length 2.5 Kpc assuming they are warped in the same way as gas or as the young population, shows a distribution of the latitude of peak brightness versus longitude

with a higher amplitude than observed. This difference cannot be due to an offset of the Sun with respect to the mean galactic plane.

If the smaller stellar warp is due to a truncation in the stellar distribution, this sudden truncation would take place at a radius smaller than 15 Kpc. This seems to be very small. As a recent comparison Lequeux, Dantel-Fort and Fort (1995) found a truncation radius of ~ 20 Kpc for the warped galaxy NGC 7814. Nevertheless truncation is an interesting possibility.

For our galaxy, it is specially interesting to compare our results with those recently obtained in the DENIS project (Ruphy et al. 1996), in which a cutoff radius of 15 ± 2 kpc was found. As shown in our Fig. 9, this small value render the cutoff hypothesis very plausible. Under this interpretation, the stellar system would show a smaller warp than the gas, because there are no stars in the region of interest.

Summarizing, the apparent lower IR warp with respect to the gaseous warp, as has been reported by Freudenreich et al. (1994) cannot be explained by a nonvanishing height of the Sun over the galactic plane. Two possible explanations remain: Either the stellar disc extends to large galactocentric radii but is not as warped as the gaseous disc, or the stellar disc is truncated at a radius less than about 15 kpc. If the truncation takes place at 15 kpc, as actually obtained by Ruphy et al. (1996), the second interpretation should be favoured.

In this case, a search for a different dynamic behaviour of stars and gas in the periphery of the disc cannot be carried out in the Milky Way, because of the absence of stars in the region of interest.

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