Astron. Astrophys. 322, 127–130 (1997)

ASTRONOMY
AND
ASTROPHYSICS

On the velocity distribution of radio pulsars at birth

J.W. Hartman

Astronomical Institute, Utrecht University, P.O.Box 80000, 3508 TA Utrecht, The Netherlands

Received 2 August 1996 / Accepted 29 October 1996

Abstract. We investigate the transverse velocity distribution of young (< 3 Myr) radio pulsars, using pulsar population synthesis techniques. We find that, after taking into account errors in proper motion measurements, an initial velocity distribution that has many more pulsars at low velocities than the distribution proposed by Lyne & Lorimer (1994) can produce a transverse velocity distribution that describes the observed distribution equally well.

Key words: stars: neutron, pulsars: general, celestial mechanics, stellar dynamics

1. Introduction

Radio pulsars are born in violent supernova events. During these events, a newborn pulsar might get an appreciable kick velocity. Some pulsars are observed to have transverse velocities of over 1000 km s^{-1}. The shape of the radio pulsar kick velocity distribution is still under debate (Lorimer 1996). Apart from the intrinsic interest for pulsar studies, it is important for the evolution of binary systems that are the progenitors for X-ray binaries (e.g. van den Heuvel & Rappaport 1987, Portegies Zwart & Verbunt 1996) or for those models that explain Gamma-Ray-Bursters as a phenomenon linked with a population of old neutron stars in the galactic halo (Li & Dermer 1992).

Recently, Lyne & Lorimer (1994) argued that the birth velocities of radio pulsars are much higher than was previously thought. They based their conclusion on proper motion data of 29 radio pulsars with characteristic ages smaller than 3 Myr. They did however not take into account any errors in proper motion measurements, and how they propagate into the transverse velocity distribution.

Recent radio pulsar population synthesis calculations showed that observed distributions of pulsar properties (e.g. magnetic field strength, period, luminosity and dispersion measure) are also adequately described with a birth velocity distribution of pulsars with many more pulsars at low velocities than the Lyne & Lorimer (1994) velocity distribution (Hartman et al. 1997). Such a velocity distribution has shown to be compatible with proper motion measurements by Hansen & Phinney (priv. comm.). To be noted here is that Hartman et al. (1997) did not fit the velocities.

In this paper we present detailed simulations regarding the birth velocity distribution of radio pulsars and the resulting transverse velocity distribution, and the effect of errors in proper motion measurements on the transverse velocity distribution. In Sect. 2 we describe how we calculate the transverse velocity distribution. In Sect. 3 we compare our calculations with the observations and in Sect. 4 we give our conclusions.

2. Method

To simulate the observed transverse velocity distribution that arises from different assumed initial velocity distributions we use pulsar population synthesis. We do this to take into account selection effects: contrary to the the velocity distribution, the proper motion distribution is sensitive to selection effects for young pulsars.

The radio pulsar population synthesis techniques are described in detail in Bhattacharya et al. (1992) and Hartman et al. (1997). The calculations as presented in this paper are obtained using basically the same code. We limit ourselves here to a brief overview, and only go into detail about those parts of the calculations that directly deal with the velocities and proper motions of radio pulsars.

In short, we calculate a simulated population of radio pulsars using a Monte Carlo technique. We assume initial distributions in age, position, velocity and magnetic fields. From these distributions, we randomly pick a pulsar. We compute the magnetic field and period at the age the pulsar is given and we calculate the orbit of the pulsar in the galactic potential during its age. We check whether the pulsar is still above the death-line (below which a radio pulsar is not visible as such anymore) and whether the pulsar is beamed towards us. From the period and period derivative we calculate a model luminosity. We choose the actual luminosity from a distribution of luminosities around the model luminosity. If then the pulsar is potentially observable, we check whether this pulsar meets the detection criteria of one of four major pulsar surveys. This is repeated until we have a large enough sample of simulated pulsars.

For the purposes of this paper, we also calculate the proper motion of the pulsar. This is done by computing the transverse

Send offprint requests to: J.W. Hartman velocity in the galactic longitude l and latitude b direction. We calculate the proper motion (in galactic coordinates) from

$$\mu = \frac{v_t}{4.74\ d} \qquad [mas\ yr^{-1}] \tag{1}$$

where μ is the proper motion in mas yr^{-1} , d the distance in kpc and v_t the transverse velocity in km s^{-1}. For comparing our simulated proper motions with the observed ones as listed in the Princeton pulsar catalogue (Taylor et al. 1993) as well as studying the errors in the observed proper motions, we convert them to proper motions in equatorial coordinates, and add a 'de-correction' for differential galactic rotation and solar motion. For galactic rotation we use a flat rotation curve of $\Theta = 225$ km s^{-1}. This results in the following expression for correction due to galactic rotation (see e.g. Mihalas & Binney 1981):

$$\Delta\mu_l = \frac{1}{4.74\ d}\left(\frac{\Theta}{R}(R_0 \cos l - d \cos b) - \Theta \cos l\right) \tag{2}$$

where R and R_0 is the galactic radius of the pulsar and the Sun, respectively. We take Solar motion to be 16.5km s^{-1} in the direction $l = 53°, b = +25°$. In rectangular galactic coordinates this is $V_X = -9$ km s^{-1}, $V_Y = 12$ km s^{-1} and $V_Z = 7$ km s^{-1} and we can write the correction in the proper motion due to solar motion as

$$\Delta\mu_l = \frac{1}{4.74\ d \cos b}(V_X \sin l - V_Y \cos l) \tag{3}$$

$$\Delta\mu_b = \frac{1}{4.74\ d}(V_X \sin b \cos l + V_Y \sin b \sin l - V_Z \cos b) \tag{4}$$

The initial velocity distributions we use in these calculations to derive the simulated observed transverse velocity distribution are the Lyne & Lorimer (1994) velocity distribution and the Hansen-Phinney modification of the Paczyński (1990) velocity distribution (Hereafter called 'Model A' and 'Model B', respectively. These notations are in accordance with those used in Hartman et al. 1997). D. Lorimer kindly supplied to us a table of numbers respresenting the the differential distribution which we used to derive the Model A velocity distribution, as well as the pulsars and their velocities on which Lyne & Lorimer (1994) based their velocity distribution. The Model B velocity distribution is similar to the Paczyński (1990) distribution, but with a much larger width:

$$p(u)du = \frac{4}{\pi}\frac{du}{(1+u^2)^2}, u = \frac{v_\mathrm{i}}{\sigma_v}, \quad \sigma_v = 600 \text{ km s}^{-1} \tag{5}$$

Both these assumed initial 3D velocity distributions are shown in Fig. 1. It can be seen that the Hansen-Phinney-Paczyński distribution has a similar high velocity tail, but has maximum probability at zero velocity. It is not very likely that the real velocity distribution has maximum probability at zero, since pulsar progenitors already have typical velocities of the order $10 - 50$km s^{-1}, but we use it as a convenient velocity distribution to investigate the effect of a velocity distribution with many more low velocity pulsars with respect to the Lyne & Lorimer (1994) distribution.

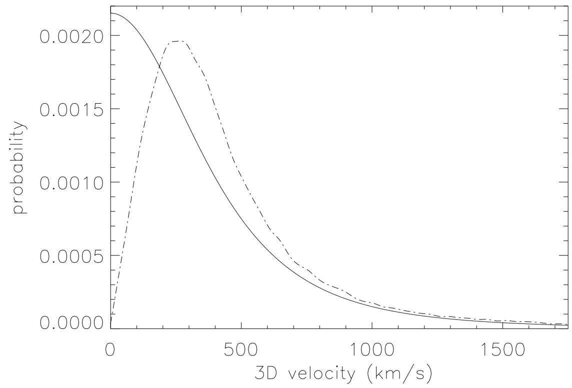

Fig. 1. The distribution of initial velocities of radio pulsars according to the Lyne & Lorimer (dashed-dotted line) and to the Hansen-Phinney-Paczyński equation(solid line).

The parameters we use in these calculations are the best fit parameters as found in Hartman et al. (1997). We find that the results as presented in this paper are not sensitive to changes of these parameters, as we are mainly discussing dynamical effects. Also the decay time of the magnetic field is not important since we only take into account pulsars younger than 3 Myr.

3. Comparison with the Lyne & Lorimer sample

We want to compare our simulated transverse velocity distributions with the sample from which Lyne & Lorimer derive their velocity distribution. A list of these pulsars and their transverse velocities was kindly supplied to us by D. Lorimer. For this paper we will use transverse velocities as can be calculated from the proper motions as found in the Princeton catalogue (Taylor et al. 1993) and after applying the necessary corrections. (In comparing these velocities with those used by Lyne & Lorimer (1994), we found differences up to 100 km/s for individual pulsars. This is due to the fact that Lyne & Lorimer did not correct for differential galactic rotation and for the solar peculiar motion in deriving the velocity distribution of radio pulsars. The mean and rms of the corrected and uncorrected velocity distribution however are about the same. We thank D. Lorimer for help in clarifying this issue.)

To be consistent with our population synthesis model, we had to remove some pulsars from the sample of Lyne & Lorimer. Since we only simulate pulsars within a distance projected onto the galactic plane of 4 kpc, we applied the same criterion to the 29 pulsars Lyne & Lorimer used. This means we had to remove 3 pulsars. Also, another 4 pulsars have fluxes below the minimum flux of all of the 4 surveys we have modelled. However, we decided to keep those because their distances are evenly distributed within the sample.

Further, we have 3 pulsars that have periods less than the initial period in our simulations (0.1 s). Since it is unlikely that their kinematical properties will change in the short time before they have spun down to .1 s (about a few thousand years), we also kept these in our sample. Finally, we removed pulsar B1951+32

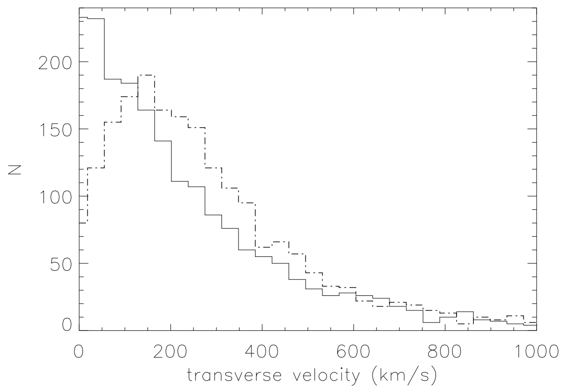

Fig. 2. The transverse velocity distribution of the simulated sample of detected radio pulsars that have a characteristic age less than 3 Myr. Both distributions show 2000 pulsars. The dashed-dotted line represents the distribution that results using the Model A distribution as initial velocity distribution, the solid line using the Model B distribution.

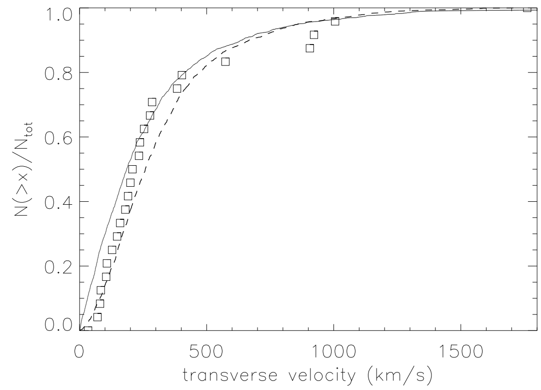

Fig. 3. The cumulative transverse velocity distribution of 25 real pulsars from the study by Lyne & Lorimer (1994) (open squares), together with the simulated sample using the Model B distribution (solid line) and the Model A distribution (dashed line).

from the list since no proper motion measurements are available for this pulsar. This leaves us with a total sample of 25 pulsars.

The remainder of this section is divided in two subsections. In the first subsection, we do not take into account any effect of errors in proper motion measurements on the transverse velocity distribution. In the second subsection we do take this into account by perturbing the proper motion distribution with an error distribution.

3.1. No proper motion measurement errors

In Fig. 2 we show the transverse velocity distribution of pulsars with simulated detection in Model A and Model B, only counting those pulsars that have characteristic ages less than 3 Myr.

As can be seen in Fig. 2, the transverse velocity distributions are substantially different. This is of course no surprise, since the low space velocity pulsars also should be observed as having low transverse velocity. In Fig. 3 we show the cumulative transverse velocity distribution for 25 pulsars from Lyne & Lorimer (1994). These are the real pulsars that have proper motions measurements and characteristic ages less than 3 Myr. In the same plot we also have drawn the cumulative distributions for the simulated samples. It shows that in Model B many pulsars with low transverse velocities are generated.

In Table 1 we list the numerical properties of the distributions. We also compare both the simulated samples with the real sample by means of a Kolmogorov-Smirnov test. The outcome from these tests is listed in Table 1. Both models seem to fit the data equally well.

In our Model A distribution, the calculated numbers of $\langle v_T \rangle$ and $\langle v_{3D} \rangle$ are lower than the observed values quoted by Lyne & Lorimer (1994). For the transverse velocity this can be explained as due to 3 pulsars in the observed sample having transverse velocities close to 1000 km s^{-1}.

Table 1. Numerical properties for the transverse velocity distributions for 25 observed real pulsars and for the simulated transverse velocity distributions for Model A and Model B. The numbers given are the means of 40 runs consisting of simulated samples of 2000 pulsars each, all with characteristic ages less than 3 Myr. The KS values indicates the probability that the distributions that are tested are drawn from the same mother distribution.

	$\langle v_T \rangle$ (km s^{-1})	$\langle v_{3D} \rangle$ (km s^{-1})	K-S test
observed	345±70	450±90	—
A	328	426	0.44
B	286	371	0.40
A, σ_μ = 5	343	426	0.23
B, σ_μ = 5	309	371	0.88

3.2. Perturbing the proper motion measurements

We now investigate the effect of measurement errors in proper motion observations in our simulations. Harrison et al. (1993) (who are responsible for 10 proper motions from the Lyne & Lorimer (1994) sample) have looked at 44 pulsars to measure their proper motions. They obtained 31 determinations and 13 upper limits. So it is not inconceivable that the underlying 3-dimensional velocity distribution contains a significant number of pulsars with proper motions equal or less to their measurement errors. We simulate the errors on the proper motion by adding a perturbation to the simulated proper motions. From the Princeton pulsar catalogue (Taylor et al. 1993), we find that the errors in the proper motions in right ascension and declination have median absolute errors of 7 mas yr^{-1} and 5 mas yr^{-1}, respectively. Since the observed proper motions are measured in a number of surveys, the total error distribution is not simply Gaussian. Lorimer (1994) models the errors from four surveys with 4 different distributions with widths ranging from 7.5 mas yr^{-1} to 30 mas yr^{-1}. In order to also model the long

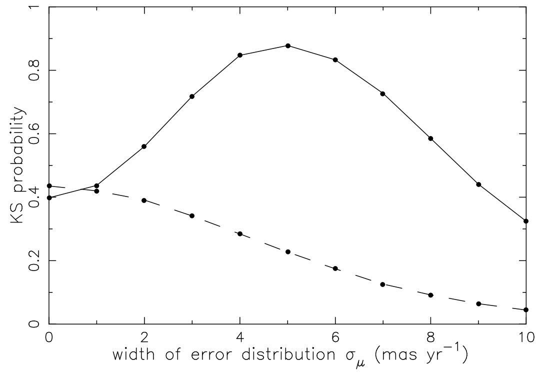

Fig. 4. The KS-probablities that the simulated and observed transverse velocity distribution are drawn from the same mother distribution as a function of the width of the proper-motion error distribution. The values are the means of 40 runs, each consisting of 2000 simulated pulsars. The dashed line represents Model A, the solid line Model B.

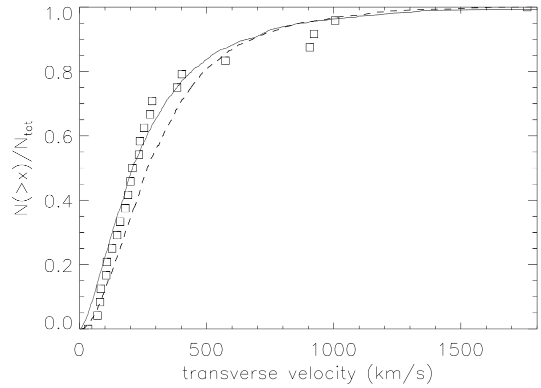

Fig. 5. The cumulative transverse velocity distribution of 25 real pulsars from Lyne & Lorimer (1994) (open squares), together with the simulated sample using the Model B distribution (solid line) and the Model A distribution (dashed line), after including errors in the proper motion. Both samples contain 2000 pulsars.

tail that is visible in the total error distribution, we choose our errors σ from an exponential distribution with width σ_μ:

$$P(\sigma)\mathrm{d}\sigma \propto \exp(-\frac{\sigma}{\sigma_\mu})\mathrm{d}\sigma \tag{6}$$

These errors are applied to the proper motions in right ascension and declination. These perturbed proper motions are converted to proper motions in galactic coordinates and after correcting for differential galactic rotation and solar motion we calculate the transverse velocity. To investigate the effect of errors on the resulting transverse velocity distribution we have varied σ_μ. The result of this is shown in Fig. 4. It shows that the Model A distribution only gets worse with increasing errors, while the Model B distribution agrees very well with the observed distribution for $\sigma_\mu = 5$ mas yr^{-1} (see Table 1) and less well again with larger errors. This can be understood since introducing errors will generally shift low proper motions to higher values, while higher proper motions will be scattered more isotropically. As a result of this, on average the transverse velocities will be higher. Looking at Fig. 3, for Model A this means that the fit only can get worse, while for Model B our simulated distribution will first coincide with the observed one before the fit becomes worse again. The transverse velocity distribution for both models for $\sigma_\mu = 5$ mas yr^{-1} is shown in Fig. 5.

4. Conclusion

We have shown that, when taking into account that small proper motions are hard to measure and that the errors in these measurements are substantial, the observed transverse velocity distribution of young radio pulsars can very well be described with an initial velocity distribution that has a large component at low velocities, like our Model B distribution. It is clear that on basis of the calculations presented in this paper, we cannot draw any conclusions as to which birth velocity distribution is to be preferred, since for both acceptable Kolmogorov-Smirnov probabilities can be obtained. First of all more proper motion measurements of (young) pulsars are needed to be able to give a conclusive answer. Very accurate measurements will be obtained by several VLBI programs are currently carried out to observe parallaxes and proper motions of pulsars (e.g. Campbell et al. 1996) , and this should also provide a check on previous measurements and their errors. That these errors can be large, is recently illustrated with results on PSR B2021+51. Campbell et al. (1996) show that their VLBI measurements disagree at a 3σ level with the proper motion as measured by Lyne et al. (1982), and the authors write that similar disagreements are found with other pulsars in the Lyne et al. (1982) sample.

Acknowledgements. I'd like to thank Dipankar Bhattacharya and Frank Verbunt for useful comments and discussion.

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