Astron. Astrophys. 322, 197–201 (1997)

ASTRONOMY
AND
ASTROPHYSICS

Constraints on the magnetic configuration of Ap stars from simple features of observed quantities

M. Landolfi[1], **S. Bagnulo**[2], **M. Landi Degl'Innocenti**[3], **E. Landi Degl'Innocenti**[4], **and J.L. Leroy**[5]

[1] Osservatorio Astrofisico di Arcetri, Largo E. Fermi 5, I-50125 Firenze, Italy (landolfi@arcetri.astro.it)
[2] Armagh Observatory, College Hill, Armagh BT61 9DG, Northern Ireland (sba@star.arm.ac.uk)
[3] C.N.R., Gruppo Nazionale di Astronomia, Unità di Ricerca di Arcetri, Largo E. Fermi 5, I-50125 Firenze, Italy (mlandi@arcetri.astro.it)
[4] Dipartimento di Astronomia e Scienza dello Spazio, Università degli Studi di Firenze, Largo E. Fermi 5, I-50125 Firenze, Italy (landie@arcetri.astro.it)
[5] Observatoire Midi-Pyrénées, 14 Avenue Edouard Belin, F-31400 Toulouse, France (leroy@srvdec.obs-mip.fr)

Received 28 October 1996 / Accepted 3 December 1996

Abstract. According to the oblique rotator model, the time variations of the quantities usually employed to investigate the magnetic configuration of Ap stars (mean longitudinal field, mean surface field, broad band linear polarization) are described by simple laws. For each quantity, certain typical features can easily be identified. We show that these features set definite constraints on the magnetic configuration.

Key words: polarization – stars: magnetic fields – stars: chemically peculiar

1. Introduction

The study of the magnetic field of Ap stars is based on various phenomena related to the Zeeman effect: the splitting of spectral lines (yielding the so-called mean magnetic field modulus or mean surface field, B_s), the circular polarization in individual lines (yielding the so-called mean longitudinal field, B_l), the broad band linear polarization (BBLP) resulting from the differential saturation of σ and π components in all the magnetic lines contained in the passband. The first measurements of B_l and B_s go back to several years ago (Babcock 1947, 1960), while BBLP measurements have been undertaken more recently (Kemp & Wolstencroft 1974; Leroy 1995 and references therein), probably because of the small BBLP degree – usually, a few parts in 10^{-4} – which requires sophisticated instrumentation.

B_s measurements are mainly sensitive to the intensity of the magnetic field, while they are not very sensitive to its geometry. BBLP measurements, on the contrary, have a large sensitivity to the field geometry[1] and a small sensitivity to the field intensity. B_l measurements are something intermediate. Obviously, the maximum information is obtained by using together, when available, all kinds of observations. In particular, it has been shown (Leroy et al. 1995, Appendix) that BBLP and B_l measurements are truly complementary in determining the magnetic field geometry.

The combined use of observational data of different kinds (and, in many cases, obtained at different times) requires careful fitting techniques (see, e.g., Bagnulo et al. 1995). Here we just want to emphasize how it is possible, under certain assumptions, to establish a number of constraints on the magnetic configuration of Ap stars on the basis of a few features that can easily be recognized in each kind of measurement. Some of the following arguments are scattered in the literature (see, e.g., Landolfi et al. 1993, hereafter referred to as Paper I; Leroy et al. 1993, 1995; Hensberge et al. 1977); only a unified treatment leads, however, to the simple and significant picture presented in this paper – see Fig. 2 and Table 1.

Recent investigations have shown that the observations of some Ap stars can adequately be explained in terms of the dipolar oblique rotator model (Bagnulo et al. 1995), while more complex models are required in other cases (Leroy et al. 1994, 1996). The interpretative scheme derived in this paper can only be applied to stars of the former group, and – even in that case – cannot replace the rigorous fitting methods mentioned above. On the other hand, the scheme provides a useful tool to check the consistency of a series of observations with the simple dipolar oblique rotator model. When such consistency exists, the scheme can be used to derive approximate values for the geometrical parameters of the star even from low-accuracy data, since the simple features considered here can directly be deduced from a qualitative inspection of the observational material.

Send offprint requests to: M. Landolfi

[1] Incidentally, only BBLP measurements provide information on the orientation of the stellar rotation axis projected on the plane of the sky. Note that the dependence on the orientation of particular directions, related to some kind of anisotropy of a celestial body, is a quite general property of linear polarization; see, e.g., Brown et al. (1978).

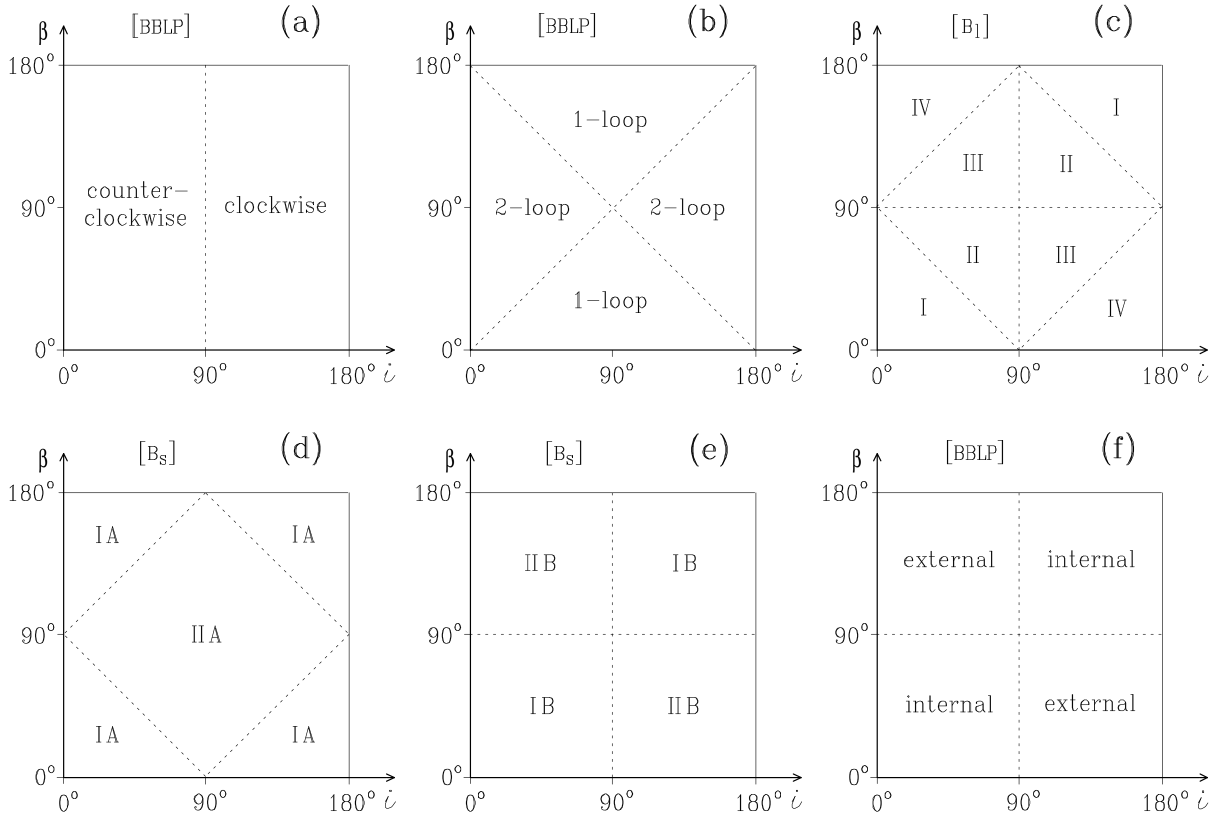

Fig. 1a–f. Limitations on the possible values of the angles i and β imposed by certain features of magnetic field measurements

2. Basic features of the different kinds of measurements

Let us consider the classical dipolar oblique rotator model: the magnetic field at the stellar surface can be considered as generated by a 'frozen' magnetic dipole located at the star center, whose axis is tilted with respect to the stellar rotation axis. The magnetic configuration on the visible hemisphere varies with the rotation phase, and this leads, in general, to a variation of the intensity and polarization of spectral lines, having the same period as the stellar rotation. The fundamental parameters characterizing the magnetic configuration are the inclination angle i between the rotation axis and the line of sight and the angle β between the magnetic and rotation axes.

For a precise definition of the geometry of the oblique rotator model, we refer the reader to Fig. 1 of Paper I. Here we just point out that – contrary to the conventions that are often used in the literature – the angles i and β are allowed to vary in the range

$$0^\circ \le i \le 180^\circ\,, \qquad 0^\circ \le \beta \le 180^\circ \tag{1}$$

($i = 0^\circ$ means that the *positive* rotation pole lies at the center of the visible hemisphere: the star is seen to rotate in the *counterclockwise* direction; $\beta = 0^\circ$ means that the positive magnetic pole coincides with the positive rotation pole). The definition in Eqs. (1) allows one to fully specify the geometry of the model, including the handedness of the rotation and the sign of the magnetic poles.

First we consider the BBLP originated by this model. A theory of the phenomenon has been presented in Paper I, under the basic assumptions that the atmosphere is chemically homogeneous and described by the Milne-Eddington model, and that the spectral lines contained in the passband are unblended and characterized by the same (average) properties. As apparent from Figs. 2 and 4 of Paper I, the stellar rotation produces typical curves in the plane Q-U (the frequency-integrated, disk-averaged Stokes parameters) which strongly depend on the values of the i and β angles. The U vs. Q curves (or 'polarization diagrams') just quoted were obtained by using the simple analytical expressions for Q and U resulting from the further assumption of weak magnetic field. However, the thorough investigation of Bagnulo et al. (1995) has shown that this assumption has negligible effects on the shape of the polarization diagrams.

The first outstanding feature of these diagrams, which can be deduced even from moderately accurate observations, is their direction (clockwise or counterclockwise). From Fig. 2 and the symmetry properties (15) and (16) – or from Fig. 4 and the symmetry properties (21) and (22) – of Paper I, we see that the diagrams are described in the counterclockwise direction for $0^\circ < i < 90^\circ$ and in the clockwise direction for $90^\circ < i < 180^\circ$.[2] This simple result allows one to establish which rotation

[2] Obviously, the direction of the observed polarization diagrams depends crucially on the definition of the Q and U Stokes parameters. We adopt the operational definition given by Shurcliff (1962).

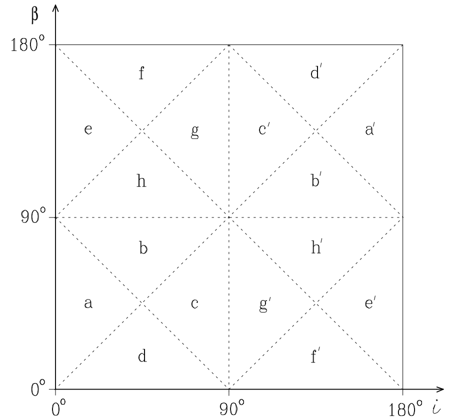

Fig. 2. Partition of the domain (i, β) produced by the set of features considered

pole (positive or negative) lies on the visible hemisphere; note that this piece of information cannot be derived from B_l or B_s measurements. In other words, the knowledge of the direction of the polarization diagram is, by itself, sufficient to restrict to one half the parameter-space domain defined in Eqs. (1) – see Fig. 1a. It should be noticed that for $i \approx 90°$ the diagrams are 'knotted' because of magneto-optical effects, so that their direction may be difficult to ascertain (see Paper I, Fig. 4).

Another evident feature of the polarization diagrams is that they can consist either of a single loop or of two loops, depending on the values of i and β. This entails another partition in two regions of the domain defined in Eqs. (1). According to Paper I (Sect. 5), the two regions are those shown in Fig. 1b.

Let us now turn to the mean longitudinal field B_l. It is well-known that the dipolar oblique rotator model leads to the simple relation

$$B_l = k \cos l = k\,(\cos i \cos\beta + \sin i \sin\beta \cos f)\ , \tag{2}$$

where k is a positive constant (proportional to the polar magnetic field intensity and dependent on the limb-darkening coefficient), l is the (time-dependent) angle between the magnetic axis and the line of sight, and f is the phase angle describing the stellar rotation (the reader is again referred to Fig. 1 of Paper I for the precise definition of l and f). Excluding the limiting values $i = 0°$ or $180°$, $\beta = 0°$ or $180°$, Eqs. (1) imply $\sin i \sin\beta > 0$, so that the curve $B_l(f)$ has a maximum for $f = 0°$ and a minimum for $f = 180°$, i.e.,

$$\begin{aligned} B_l^{\max} &= B_l(f=0°) = k\,\cos(i-\beta)\ , \\ B_l^{\min} &= B_l(f=180°) = k\,\cos(i+\beta)\ . \end{aligned} \tag{3}$$

According to the values of i and β, the function $B_l(f)$ can be everywhere positive, or everywhere negative, or partly positive and partly negative. More precisely, we can distinguish the four following cases, which in general can easily be discriminated by inspection of the observational data

$$\begin{array}{ll} \text{I)} & B_l^{\max} > B_l^{\min} > 0\ ; \\ \text{II)} & B_l^{\max} > 0 > B_l^{\min}\ , \quad \left|B_l^{\max}\right| > \left|B_l^{\min}\right|\ ; \\ \text{III)} & B_l^{\max} > 0 > B_l^{\min}\ , \quad \left|B_l^{\max}\right| < \left|B_l^{\min}\right|\ ; \\ \text{IV)} & 0 > B_l^{\max} > B_l^{\min}\ . \end{array} \tag{4}$$

These four cases yield a partition in four regions of the domain defined in Eqs. (1). Using Eqs. (3) and (4), it can be seen that the regions are those illustrated in Fig. 1c.

Next, consider the mean surface field B_s. For the dipolar oblique rotator model, a good approximation to the exact expression of B_s is given by the simple formula (Hensberge et al. 1977, Eqs. (6), (8), and (21))

$$B_s = k' \cos^2 l + k'' \sin^2 l\ , \tag{5}$$

where k' and k'' are positive constants (proportional to the polar magnetic field intensity and dependent on the limb-darkening coefficient), and l is the angle appearing in Eq. (2). Substitution of the expression of $\cos l$ given by Eq. (2) yields

$$\begin{aligned} B_s = \ & k''\,(1 + h\,\cos^2 i \cos^2\beta + h\,\sin^2 i \sin^2\beta \cos^2 f \\ & + \tfrac{1}{2} h\,\sin 2i \sin 2\beta \cos f)\ , \end{aligned} \tag{6}$$

where $h = (k' - k'')/k''$ is a positive number ranging from 0.20 to 0.33 according to the value of the limb-darkening coefficient. Thus the function $B_s(f)$, everywhere positive, has a simple dependence on the rotation phase. In a rotation period it has (excluding the limiting values $i = 0°$ or $180°$, $\beta = 0°$ or $180°$) either one maximum and one minimum or two maxima and two minima, so that we can distinguish between the two cases

$$\begin{array}{ll} \text{I A)} & B_s(f)\ \text{ has 2 extrema}\ , \\ \text{II A)} & B_s(f)\ \text{ has 4 extrema}\ . \end{array} \tag{7}$$

In both cases, there is an extremum at $f = 0°$ and another one at $f = 180°$; in case II A, the two additional extrema are symmetrical about $f = 0°$ (or $f = 180°$) and correspond to the same value of B_s. It is easily seen that cases I A and II A take place when $|\tan i \tan\beta|$ is less than unity or greater than unity, respectively. We thus have a partition of the (i, β) plane as illustrated in Fig. 1d.

The preceding arguments show that certain simple features of BBLP, B_l, and B_s measurements contain different and complementary information on the magnetic geometry of the oblique rotator. In principle, the three features summarized in Figs. 1a, 1b, and 1c allow one to restrict the possible values of the fundamental angles i and β to a region whose size is $1/16$ of the complete domain (see Fig. 2; note that two stars characterized by (i_0, β_0) and $(180° - i_0, 180° - \beta_0)$ are identical except for the rotation direction). On the other hand, the three features summarized in Figs. 1a, 1b, and 1d restrict the values of i and β to a region whose size is $1/8$ of the complete domain.

Up to now BBLP, B_l, and B_s measurements have essentially been regarded as independent. However, a reliable interpretation in terms of the dipolar oblique rotator model requires their

Table 1. The six asterisks in each column characterize the properties of the region marked in Fig. 2 by the letter in the header

		a	b	c	d	e	f	g	h	a′	b′	c′	d′	e′	f′	g′	h′
[BBLP]	counterclockwise	∗	∗	∗	∗	∗	∗	∗	∗								
[BBLP]	clockwise									∗	∗	∗	∗	∗	∗	∗	∗
[BBLP]	1-loop			∗	∗		∗	∗				∗	∗		∗	∗	
[BBLP]	2-loop	∗	∗			∗			∗	∗	∗			∗			∗
$[B_l]$	I	∗			∗					∗			∗				
$[B_l]$	II		∗	∗							∗	∗					
$[B_l]$	III							∗	∗							∗	∗
$[B_l]$	IV					∗	∗							∗	∗		
$[B_s]$	I A	∗			∗	∗	∗			∗			∗	∗	∗		
$[B_s]$	II A		∗	∗				∗	∗		∗	∗				∗	∗
$[B_s]$	I B	∗	∗	∗	∗					∗	∗	∗	∗				
$[B_s]$	II B					∗	∗	∗	∗					∗	∗	∗	∗
[BBLP]	internal	∗	∗	∗	∗					∗	∗	∗	∗				
[BBLP]	external					∗	∗	∗	∗					∗	∗	∗	∗

phase-consistency. As apparent from Eq. (3), the epoch corresponding to $f = 0°$ is univocally determined by the condition $B_l = B_l^{max}$. According to the discussion following Eqs. (7), one of the extrema of the function $B_s(f)$ should occur at that epoch.[3] The existence of a significant phase shift should be regarded as an indication of inconsistency of the measurements with the oblique rotator dipolar model.

Under the assumption of phase-consistency, it can easily be seen that the behavior of the $B_s(f)$ curve yields another partition of the (i, β) domain. In fact, we can distinguish the following cases

$$\begin{aligned} &\text{I B)} \quad B_s(f = 0°) > B_s(f = 180°) \ , \\ &\text{II B)} \quad B_s(f = 0°) < B_s(f = 180°) \ . \end{aligned} \tag{8}$$

As apparent from Eq. (6), the corresponding regions are those shown in Fig. 1e.

Finally, the position of the zero-phase point (still determined from the condition $B_l = B_l^{max}$) on the polarization diagrams leads to a further partition of the (i, β) domain. As apparent from Figs. 2 and 4 of Paper I, when $0° < i < 90°$ and $0° < \beta < 90°$ the zero-phase point lies on the apex of the internal loop for 2-loops diagrams; for 1-loop diagrams, where the internal loop reduces to a 'cusp' or a 'maximum concavity' point, it lies on that cusp. Denoting by *internal* both these situations, and using again the symmetry properties (15) and (16) – or (21) and (22) – of Paper I, we obtain a partition of the (i, β) plane as shown in Fig. 1f. In the regions labelled *external*, the zero-phase point lies on the apex of the external loop or on the point 'opposite to the cusp' for 2-loop and 1-loop diagrams, respectively. As already noticed for the direction of the diagrams, the position of the zero-phase point may be difficult to ascertain for $i \approx 90°$. Obviously, the above argument assumes phase-consistency of BBLP and B_l measurements.

Clearly the three last properties, illustrated in Figs. 1d, 1e, and 1f, are redundant and do not lead to a partition of the (i, β) domain finer than in Fig. 2. It should also be noticed that, owing to the small value of the parameter h in Eq. (6), it may be difficult in practice to distinguish between cases I A and II A of Eqs. (7).

Table 1 summarizes the characteristics of the various kinds of measurements associated with each of the 16 regions shown in Fig. 2. It provides a quick way to test the consistency of a set of observations with the dipolar oblique rotator model, and – if consistency is found – to get an approximate estimate of the values of the fundamental angles i and β.

3. Conclusion

Measurements of the time variations of the mean longitudinal field, mean surface field, and broad band linear polarization contain complementary information on the magnetic configuration of Ap stars. This appears clearly if appropriate features, easily deduced from the observations, are considered (see Fig. 1). Fig. 2 and Table 1 show the considerable amount of information that can be obtained from these features alone, when measurements of different kinds are available for the same star.

Acknowledgements. S. Bagnulo was supported in this work by the UK PPARC grant GR/L21259.

[3] Accurate knowledge of the rotation period is of course required when considering observations taken at distant times.

References

Babcock H.W., 1947, ApJ 105, 105
Babcock H.W., 1960, ApJ 132, 521
Bagnulo S., Landi Degl'Innocenti E., Landolfi M., Leroy J.L., 1995, A&A 295, 459
Brown J.C., McLean I.S., Emslie A.G., 1978, A&A 68, 415
Hensberge H., van Rensbergen W., Goossens M., Deridder G., 1977, A&A 61, 235
Kemp J.C., Wolstencroft R.D., 1974, MNRAS 166, 1
Landolfi M., Landi Degl'Innocenti E., Landi Degl'Innocenti M., Leroy J.L., 1993, A&A 272, 285
Leroy J.L., 1995, A&AS 114, 79
Leroy J.L., Landolfi M., Landi Degl'Innocenti E., 1993, A&A 270, 335
Leroy J.L., Landstreet J.D., Bagnulo S., 1994, A&A 284, 491
Leroy J.L., Landolfi M., Landi Degl'Innocenti M., et al., 1995, A&A 301, 797
Leroy J.L., Landolfi M., Landi Degl'Innocenti E., 1996, A&A 311, 513
Shurcliff W.A., 1962, Polarized light, Harvard University Press, Cambridge

This article was processed by the author using Springer-Verlag LaTeX A&A style file *L-AA* version 3.