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Computation of the distance moduli of RR Lyrae stars from their light and colour curves

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Abstract. We use B and V data of globular cluster variables to derive a formula for the distance moduli of RRab stars. The method employs the Fourier decomposition of the V light curve and the average $B-V$ colour index. By using our former result for the V_0 absolute magnitude, we also obtain an expression for the dereddened colour index. With the aid of the new formulae, the relative distance moduli can be estimated within an error of $<$ 0.03 mag. Although we also make an absolute calibration, it is cautioned that this may be more affected by possible systematic errors originating mostly from the Baade−Wesselink magnitudes. On the basis of the scatter of the individual distance moduli computed with and without reddening correction, it is shown that inhomogeneous reddening plays a role in several clusters. By using our formulae we derive new expressions for the I_c and K absolute magnitudes on a sample of stars which contains mostly field stars with accurate photometry. As a by-product of this derivation we also give optimum estimations for the selective absorption coefficient R_V. We show that the K absolute magnitude contains important contribution also from the Fourier parameters, besides the well known dependence on the period. The I_c absolute magnitude is superbly correlated with the Fourier parameters, which implies that this colour is a very good candidate for the accurate estimation of the absolute magnitude.

Key words: stars: variables – stars: distances – stars: oscillations – stars: horizontal-branch – globular clusters: general

1. Introduction

In a previous paper (Kovács and Jurcsik 1996, hereafter KJ), we have shown that the intensity averaged absolute V magnitude of the RRab stars can be well approximated by a linear formula which contains the period and the A_1, φ_{31} Fourier parameters of the V light curve. The methodology of the derivation was *purely empirical* which is a great advantage of our approach, since it eliminates the ambiguities of the semi-empirical (i.e. Baade−Wesselink) and theoretical (i.e. numerical hydrodynamical, see Simon and Clement 1993) considerations. Once the absolute magnitude is computed, the true distance modulus can be obtained through a reddening correction. Although there are several formulae for the estimation of the interstellar reddening from the observed colour index, light curve and physical parameters (e.g. Blanco 1992 and references therein), here we derive an expression for the true distance modulus which is automatically reddening free and more accurate than the ones obtained by any former approach.

The structure of the paper is as follows. In Sect. 2 we describe our method, followed by the details of the data-sets and the derivation of the formulae for the distance modulus, the reddening and for the V absolute magnitude (Sects. 3 and 4). In the subsequent section we employ our formulae to mostly field I_c (Cousins I) and K data, in order to derive new expressions for the absolute magnitudes in these colours. In closing, Sect. 6 summarizes the main conclusions of the paper.

2. The method

The apparent brightness can be written in the following form

$$V = V_0 + A_V + d \ , \tag{1}$$

where V_0 is the absolute magnitude, d is the distance modulus and A_V is the interstellar absorption in the V band. The latter can be expressed through the selective extinction E_{B-V}

$$A_V = \alpha E_{B-V} \ , \tag{2}$$

with a properly chosen selective absorption coefficient, which we denote here by α. Similarly to Madore and Freedman (1991), we use the definition of $E_{B-V} (\equiv (B-V) - (B_0 - V_0))$, and write Eq. (1) in the form

$$W = W_0 + d \ , \tag{3}$$

with

$$W = V - \alpha(B - V) \ . \tag{4}$$

The function W_0 depends *solely* on the physical parameters of the star. Consequently, for distant clusters, where the $d = const.$

assumption is valid with a high accuracy, one can estimate the function W_0 up to an additive constant by applying our standard assumption about the existence of relations between the light curve and the physical parameters. Once the dependence of W_0 on the Fourier parameters has been determined with the aid of cluster variables, one can estimate the relative distance modulus of *any* variable (of the same class) simply by computing $W - W_0$. In principle, α can also be determined through cluster fitting if there is enough variation in the internal reddening within the clusters. Since this is usually not significant enough for the reliable computation of α, it is better to fix it to some 'standard' value (e.g. to 3.1). In extreme cases α may deviate from its standard value substantially (e.g. M4, see Liu and Janes 1990 and references therein). In Sects. 4 and 5 we shall discuss how one can test this possibility and correct for its effect.

In KJ we used Eq. (1) to derive a formula for the absolute magnitude. We see that this approach may carry large errors if differential reddening plays a role. On the other hand, using two colour observations, we can largely eliminate this effect and get an expression for W_0, which in turn can be used to compute the distance modulus as already mentioned.

The true colour index $B_0 - V_0$ can also be determined. Eqs. (1) and (2) yield

$$V - V_0 - \alpha(B - V) = -\alpha(B_0 - V_0) + d \quad . \tag{5}$$

Here V_0 denotes the *magnitude averaged* absolute brightness, which can be obtained in the same way as the intensity averaged value (see KJ).

An application of these formulae utilizes the available data on field stars with I_c and K photometry. The absolute magnitudes in these colours can be computed by the use of our formulae for the distance modulus and for the reddening. These absolute magnitudes are then fitted to the Fourier parameters of the V light curves. We shall see from the quality of the fits that the new calibrations should be fairly accurate.

3. The data

The basis of our analysis is the CCD and photographic B and V data published for several galactic and Magellanic Cloud (LMC) globular clusters. This set is basically the same as in KJ, except that here the very recently published data on M5 (Brocato et al. 1996) are also included. Table 1 lists the clusters, the number of stars and the sources used for their light curves in the different colours. We include all RRab stars with reliable and stable V light curves. The restrictions for the data in the other colours are less severe, since we use them only to compute the average brightnesses in those colours. In an application of the distance modulus formula, we use cluster I_c data. These are also listed in Table 1. For the determination of the $B_0 - V_0$ zero point and to find the expressions for the I_c and the infrared K absolute magnitudes, we use B, V, I_c and K observations of mostly field (Table 2) and some globular cluster stars (Table 1). Data published in I colour systems other than that of the Cousins, have been transformed to the latter with the aid of standard methods.

Table 1. Multicolour data of the globular cluster stars

	V		B		I_c		K	
Cluster	N	Ref.	N	Ref.	N	Ref.	N	Ref.
M4	4	1,2,3,4	4	1,2,3,4	3	1,2	3	2
M5	23	5,6,7,8	10	5,8,9	22	6,7,9	8	6,10
M68	5	11	5	11	5	11	–	
M92	6	6,12	5	12	3	6	2	6,10
M107	8	13	8	13	–		–	
NGC 3201	12	14	12	14	–		–	
Rup. 106	10	15	10	15	–		–	
NGC 1466	8	16	8	16	–		–	
NGC 1841	9	17	9	17	–		–	
Reticulum	8	18	8	18	–		–	

References: (1) Clementini et al. 1994; (2) Liu & Janes 1990; (3) Cacciari 1979; (4) Sturch 1977; (5) Brocato et al. 1996; (6) Cohen & Matthews 1992; (7) Reid 1996; (8) Storm et al. 1991; (9) Cohen & Gordon 1987; (10) Storm et al. 1992; (11) Walker 1994; (12) Carney et al. 1992; (13) Dickens 1970; (14) Cacciari 1984; (15) Kaluzny et al. 1995; (16) Walker 1992b; (17) Walker 1990; (18) Walker 1992a.

4. Derivation of the formulae: distance modulus, reddening, absolute magnitude

Our goal is to derive a compatible set of equations which is able to give an accurate representation of the apparent magnitudes and colours of the cluster variables (Table 1). The primary quantity we deal with is the reddening-free brightness W (Eq. 4) which is directly connected with the distance modulus (Eq. 3). The colour index $B_0 - V_0$ will be determined through Eq. (5) by the use of the absolute magnitude V_0, obtained in the same way as in KJ. It follows from the derivation that these formulae and the one for W_0 are compatible.

The value of the selective absorption coefficient (R_V) is usually a matter of dispute. As we have already mentioned, throughout this paper we take the most often quoted value of 3.1. The only exception is M4, which is obscured by the Scorpius-Ophiuchus complex of dark nebulae. According to Liu and Janes (1990) a better value for this cluster is 3.8. In Sect. 5 we show that, indeed, this is true, although we get a somewhat larger value. Therefore, if not stated otherwise, we use $R_V = 4.1$ for the stars of M4, but in any other case we stick to $R_V = 3.1$. In general, a change in the selective absorption coefficient from α to R_V yields a change in the distance modulus, namely

$$d_{R_V} = d_\alpha + (\alpha - R_V)E_{B-V} \quad . \tag{6}$$

In this way one can always convert the distance modulus to the correct value which corresponds to the appropriate R_V.

As in KJ, we use the *sine* Fourier decompositions of the V light curves in searching for the best representation of any quantity which is assumed to depend solely on the stellar parameters. In this process we examine *all linear* combinations of

Table 2. Multicolour data of the field stars

	V	B	I_c	K
Star	References			
SW And	1,2	1	1	1,3
XX And	4,5,6	4,5,6	–	–
AT And	4,7	4,7	–	–
WY Ant	4,8,9	9	9	10
X Ari	8,11,12	4,6,12	12	11,12
V Cae	13	13	13	–
SS Cnc	4,6,14	4,6,	–	–
V499 Cen	8	15	–	–
RR Cet	1,16	1	1	1
UU Cet	8,17	17	17	18
S Com	4,6,7	4,6	–	–
W Crt	8,9,19,20	9	9	10
DX Del	2,16	2	2	21
SU Dra	1	1	1	1
SW Dra	22	22	23	22
BK Dra	24	24	–	–
BT Dra	24	24	–	–
RX Eri	1	1	1	1
SX For	20	20	–	–
RR Gem	1,7	1	1	1
UW Gru	25	25	–	–
TW Her	7,26	26	–	26
SV Hya	8	20	–	–
RR Leo	1,4,14	1	1	1
RX Leo	4,6	4,6	–	–
SS Leo	8,27	27	27	27
VY Lib	8,20	20	–	–
TT Lyn	1,2,5	1	1	1
V445 Oph	2,8,27	2	2	27
AV Peg	1	1	1	1
AR Per	1	1	1	1
RV Phe	8,23,28	23,28	23,28	18
BB Pup	8,9	9	9	10
V440 Sgr	8	23	23	23
VY Ser	8,27	27	27	26,27
W Tuc	17	17	17	18
TU UMa	1,2	1	1	1
UU Vir	1,2,26	1,2,26	1,2	1,26
AT Vir	4,6,20,29	4,6,20	–	–

References: (1) Liu & Janes 1989; (2) Barnes et al. 1988; (3) Jones et al. 1992; (4) Fitch et al. 1966; (5) Penston 1973; (6) Sturch 1966; (7) Stępień 1972; (8) Lub 1977; (9) Skillen et al. 1993a; (10) Skillen et al. 1993b; (11) Jones et al 1987a; (12) Fernley et al. 1989; (13) Hansen & Petersen 1991; (14) Epstein 1969; (15) Warren 1966; (16) Meylan et al. 1986; (17) Clementini et al. 1990; (18) Cacciari et al. 1992; (19) Bookmeyer et al. 1977; (20) Clube et al. 1969; (21) Skillen et al. 1989; (22) Jones et al. 1987b; (23) Cacciari et al 1987; (24) Piersimoni et al. 1993; (25) Bernard 1982; (26) Jones et al. 1988; (27) Fernley et al. 1990; (28) Cacciari et al. 1989; (29) Eggen 1994.

the Fourier amplitudes and phases. The maximum number of parameters is 8, and the optimum parameters are chosen from a set which includes the first 6 Fourier components and the period. In the expressions tested, nonlinear relations seem to play *no role*.

In the cluster fitting, the procedure automatically yields also the distance moduli. We minimize the following expression

$$\mathscr{D} = \sum_{j=1}^{N_c} \sum_{i=1}^{N_j} [W^j(i) - W_0(P^j(i), A_1^j(i), ...) - d^j]^2 \quad , \qquad (7)$$

where N_c is the number of clusters, N_j is the number of stars in the j-th cluster, d^j is the corresponding distance modulus. For any fixed set of Fourier parameters the solution can be easily found by standard linear least squares methods.

As a final introductory remark it is stressed that throughout this paper we use *magnitude averages* rather than intensity averages as some researchers prefer. Although the intensity averages – at least for the luminosity in V – are closer to the static model values (Bono et al. 1995), for the present purpose the magnitude averages are simpler to use and give equally good correlations with the Fourier parameters.

4.1. The distance indicator W_0

As listed in Table 1, we have altogether 79 stars in the galactic and LMC globular clusters with B and V photometry. In the following we use these stars in the fit of W (Eqs. 3 and 4).

At each step of the fitting process we discard the star which deviates the 'most substantially' from the overall trend determined by the bulk of the sample. The outlying stars always show up clearly, and their selection is unambiguous. We mention that the reason for the discrepancy of a given star can be manifold, e.g. crowding, hidden Blazhko behaviour, photometric errors, etc. Since it is often not possible to decide which one of these possible reasons causes the defect of the data, we do not wish to discuss the problems of the individual stars left out from the analysis. The same is true for the stars to be omitted in the various regressions to be discussed in the rest of the paper.

We use $\alpha = 3.1$ for *all clusters*, including M4, too. Our tests have shown that changing α has essentially no effect on the selection of the outlying stars. At the same time, there is a slight indication that the above 'standard' value of α yields the best overall fitting accuracy.

We find that it is necessary to omit 13 stars (M5 V59; M68 V22; M92 V4; M107 V8, 11; NGC 3201 V14, 22, 34, 73; Ruprecht 106 V10, 14; NGC 1841 V9, 12) to reach a situation when no other obviously outlying star can be found. We refer to the remaining 66 stars as *basic data-set*.

The parameters and standard deviations of the various fitting formulae are given in Table 3. In the left part of the table the unbiased estimations of the standard deviations of the best fitting formulae are listed together with the number of parameters used. We see that σ levels off when more than 3 parameters are used. Therefore, in the right part of the table the best 3 parameter fits are compared. It is seen that there are quite a number of

Table 3. Fitting properties of W_0 for the basic data-set

n par. fits		3 par. fits	
n	σ	params.	σ
1	0.0368	P, A_2, φ_{41}	0.0287
2	0.0329	P, A_1, φ_{41}	0.0298
3	0.0287	P, A_3, φ_{41}	0.0298
4	0.0286	P, A_4, φ_{41}	0.0309
5	0.0288	P, A_2, φ_{51}	0.0313
6	0.0290	P, A_2, φ_{31}	0.0314
7	0.0292	P, A_5, φ_{41}	0.0314

Table 4. Correlation coefficients $K_{ij}(= K_{ji})$ in the error formula Eq. (9)

i	j	K_{ij}	i	j	K_{ij}
1	1	0.003455	2	3	0.002583
1	2	−0.003669	2	4	−0.000155
1	3	−0.003355	3	3	0.004528
1	4	−0.000167	3	4	0.000285
2	2	0.005308	4	4	0.000112

compatible formulae but all of them contain the period and some combinations of an amplitude and a phase. The coexistence of many formulae of similar accuracy is a result of the interrelations among the Fourier parameters as discussed in Jurcsik and Kovács (1996). We refer to Kovács (1996) for further details on the statistical properties of the W_0 fit.

Since the lowest order amplitude is relatively more accurately estimated, we prefer the formula for W_0 which contains the amplitude A_1. Anticipating the zero point to be determined in Sect. 4.3, we obtain the following formula

$$W_0 = 0.676 - 1.943P + 0.315A_1 + 0.068\varphi_{41} \quad . \tag{8}$$

We recall that the phase refers to a *sine* decomposition and it should be chosen as the closest value to 1.6. Similarly to KJ, the formal error can be computed from the following expression

$$\sigma^2_{W_0} = 0.0992\sigma^2_{A_1} + 0.0046\sigma^2_{\varphi_{41}} + \sum_{i,j=1}^{4} K_{ij}p_ip_j \quad , \tag{9}$$

where $p_1 = 1$, $p_2 = P$, $p_3 = A_1$, $p_4 = \varphi_{41}$, and the correlation matrix K_{ij} is given in Table 4. We remark that the period is assumed to be error-free in the above formula.

In Fig. 1 we plot W_0 vs. $W - d_\alpha$ in order to demonstrate the quality of the fit by Eq. (8). To illustrate that the individual cluster relations overlap in this diagram, and therefore, that the good fit is not a mere consequence of the proper choice of the distance moduli, we highlight the stars of M5 (open circles).

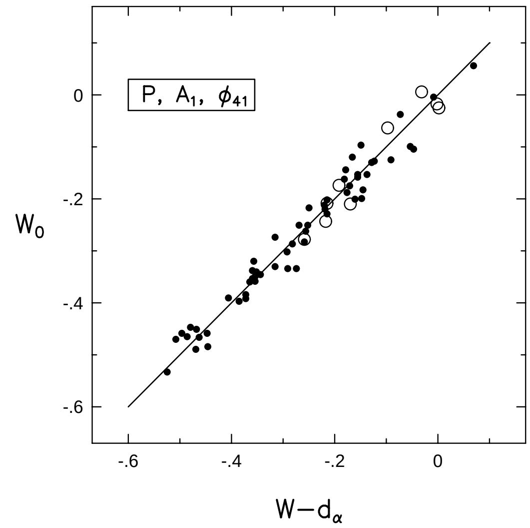

Fig. 1. Correlation of the 'observed' and fitted W_0 for the basic data-set. The stars of M5 are shown by open circles. The 45° line is drawn for reference

4.2. V_0 and $B_0 - V_0$

In KJ we derived a formula for the intensity averaged absolute magnitude M_V. By using basically the same data-set (193 stars of the globular clusters, the Sculptor dwarf galaxy and a sample of Baade−Wesselink stars), we fit the *magnitude averages* with the Fourier parameters. As we have already mentioned in Sect. 2, this direct fit of V_0 is influenced by any inhomogeneity in the individual cluster reddenings. We hope that this effect is minimized by the omission of the outlying stars and by the large amount of data. Then, the so-obtained formula for V_0 is used in computing $B_0 - V_0$ by fitting Eq. (5) to the basic data-set of this paper. In the computation of V_0 we experience an ambiguity in choosing among the best 3 parameter fits which all contain the period and some combinations of an amplitude and a phase. In order to be compatible with the formula of W_0, we choose the one with the parameters P, A_1, φ_{41}. Finally, we get the following set of equations (again, the zero point of $B_0 - V_0$ is anticipated from the next subsection)

$$V_0 = 1.630 - 1.415P - 0.273A_1 + 0.062\varphi_{41} \quad , \tag{10}$$

$$B_0 - V_0 = 0.308 + 0.163P - 0.187A_1 \quad . \tag{11}$$

It is possible to iterate Eqs. (10), (11) by applying one of them to the data and compute the other through Eq. (5). Our tests have shown that the solution converges to a statistically equivalent set of equations without any appreciable improvement in the fitting accuracy. Therefore, we accept the above solution obtained through the simple direct fit.

To get an insight into the significance of the above expression of $B_0 - V_0$, in Table 5 we show the dependence of the fitting accuracy on the number and combination of parameters. We think that on the basis of the behaviour of the dispersion, the above choice of $B_0 - V_0$ is well justified.

To visualize the goodness of the fits we show the 'observed' vs. fitted quantities in Fig. 2. In the upper panel, open circles show the stars of the Sculptor, whereas the same symbols in the lower panel indicate the variables of M5. We see that the stars of the Sculptor exhibit a considerable scatter, a part of which

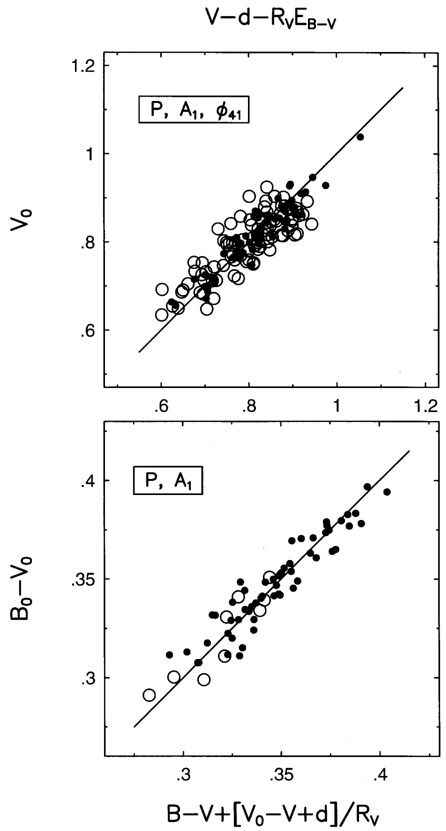

Fig. 2. Correlation of the 'observed' and fitted values for V_0 and $B_0 - V_0$. The stars of the Sculptor and of M5 are shown by open circles (upper and lower panels, respectively). Dots are for the stars of the basic data-set. The 45° lines are shown for reference

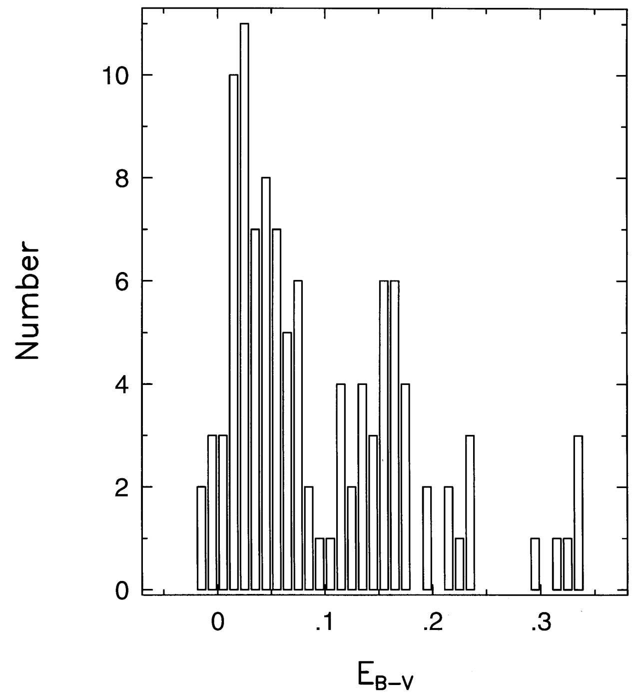

Fig. 3. Distribution of the reddening for the stars of Tables 1 and 2

Table 5. Fitting properties of $B_0 - V_0$ for the basic data-set

n par. fits		2 par. fits	
n	σ	params.	σ
1	0.0108	P, A_1	0.0095
2	0.0095	P, A_2	0.0096
3	0.0093	P, A_3	0.0103
4	0.0093	A_1, φ_{21}	0.0104
5	0.0093	A_1, φ_{51}	0.0106
6	0.0093	A_1, φ_{61}	0.0106
7	0.0094	A_1, φ_{31}	0.0106

is presumed to originate in the inhomogeneous reddening. The M5 variables span a substantial fraction of the range of $B_0 - V_0$, similarly to what we have seen in the case of W_0.

4.3. The zero points of $B_0 - V_0$ and W_0

We calibrate $B_0 - V_0$ and W_0 separately. The calibration of W_0 relies partially on the Baade–Wesselink (BW) luminosities, whereas that of $B_0 - V_0$ utilizes the distribution of the observed reddenings and is completely *empirical*, i.e. independent of any theoretical assumptions.

We start with the calibration of the colour index. Since the colour excess E_{B-V} is a non-negative quantity, we require that the E_{B-V} values calculated by Eq. (11) on a 'reasonably' large sample of stars should satisfy this property. Of course, one should take into consideration the fact that our formula for $B_0 - V_0$ is an approximate one and that there is observational noise, too. Therefore, it is also possible to get 'slightly' negative values if the extinction is small.

By using the B, V data of Tables 1, 2 and Eq. (11), we compute the distribution function of E_{B-V} on a sample of 118 stars (Fig. 3). The cutoff at around $E_{B-V} = 0$ is nicely exhibited. The zero point is fixed by this cutoff. It is seen that there is some ambiguity in the accurate definition of this cutoff. However, it is clear that even for this limited sample the error of the zero point of $B_0 - V_0$ cannot be larger than $0.01 - 0.02$ mag. We think that more accurate calibration of the zero point is only a matter of time, when the massive photometric surveys will supply us with a large amount of two color data.

Turning to the calibration of W_0, it is cautioned that at the moment it is not possible to make an accurate calibration of W_0, because of the lack of good distance or absolute luminosity etalons. The purpose of the present calibration is only to bring our formulae into agreement 'on the average' with the results of the recent BW analyses. In our approach the random errors of the BW luminosities are cancelled out, but of course we are still affected by any possible systematic errors (e.g. Walker 1992c).

The simplest way to compute the zero point of W_0 is the direct combination of the zero points of Eqs. (10) and (11) via Eq. (4). In this way we get a value of 0.675. This approach is not entirely correct, because Eq. (8) is not an exact linear combination of Eqs. (10) and (11) but rather a least squares

Table 6. Distance moduli obtained from the B, V colours

Cluster	N	d	σ_d
M4	4	11.03	0.03
M5	9	14.06	0.03
M68	4	14.83	0.02
M92	4	14.43	0.03
M107	6	13.65	0.02
NGC 3201	8	13.34	0.03
Rup. 106	8	16.48	0.02
NGC 1466	8	18.39	0.03
NGC 1841	7	18.14	0.03
Reticulum	8	18.20	0.03

solution as described in Sects. 2 and 4.2. However, the difference is very small, which can be checked very easily.

Denoting the coefficients of Eq. (8) by c_i, we calculate the zero point c_0 with the aid of the following equation

$$c_0 = -\frac{1}{N}\sum_{i=1}^{N} c_1 P(i) + c_2 A_1(i) + c_3 \varphi_{41}(i) - W_s(i) \quad , \tag{12}$$

where $W_s(i)$ is the 'synthetic' value of $W_0(i)$, computed from Eqs. (10) and (11). The index may run through any data-set with accurate Fourier decompositions. In our case this data-set is a sample of about 300 stars. Finally, with this method we get a value of 0.676 for the zero point. This is practically equal to the value obtained directly from Eqs. (10) and (11).

4.4. Compatibility of the distance moduli

Here we would like to test the consistency of the distance indicator W_0. Since W_0 was derived by the use of (B, V) data, it is interesting to ask whether the so-derived distance moduli are consistent with the ones obtained through a similar formula using V and I_c colours.

First, as an application of the (B, V) distance modulus formulae (Eqs. (3), (4) and (8)), we compute the distance moduli of the clusters entering in our analysis. We use $R_V = 3.1$ for all clusters, except for M4, for which $R_V = 4.1$ is applied. The result is given in Table 6. We remark that due to the compatibility of Eq. (8) and Eqs. (10), (11), our results do not change if we switch between the two sets of equations. Column 4 gives the standard deviation computed from the individual distance moduli. The number of stars used in each cluster is given in the second column. We think that the σ_d values give a good estimation of the expectable accuracy of the formulae.

Now we turn to the consistency test. Unfortunately, the cluster (V, I) data alone do not permit to perform this test, because they are too few and relatively too noisy. Therefore, we combine the cluster and field data with the aid of the (B, V) distance indicator W_0.

In calculating the distance moduli, we proceed as in Sect. 4.1 but this time we use the $X \equiv I_c - \beta(V - I_c)$ reddening-free quantity, where $\beta = 1.5$ from Eqs. (15) and (16) with $R_V = 3.1$.

In performing the test, the field B, V, I_c data (Table 2) are used to compute X_0 for these stars. In this procedure we apply Eq. (8) to compute the distance moduli for the field stars. The so-obtained X_0 values of the field stars are then considered as 'observed' X values of one cluster and added as the fifth cluster to the other four galactic clusters. Then, the data are fitted in the same way as in the case of the cluster (B, V) data. After leaving out the 6 outlying stars (M5 V19, 27, 32, 59; M68 V22; SS Leo), we get a sample of 52 stars. The best fits contain P, φ_{41} and some amplitude. To be compatible with the other formulae, we choose the fit with the P, A_1, φ_{41} parameters. They yield the following formula

$$X_0 = 0.833 - 2.804P + 0.345A_1 + 0.156\varphi_{41} \quad . \tag{13}$$

The standard deviation of the above fit is 0.034 mag. The distance modulus is computed in the same way as in the case of the W_0 indicator, i.e. $d = X - X_0$. We remark that all computed distance moduli remain the *same* within $0.01 - 0.02$ mag by choosing a sample with fewer or larger number of stars omitted or with another choice of parameters (i.e. using some other amplitude in the formulae with P and φ_{41}).

The distance modulus for M4 should be corrected, because of the larger selective absorption coefficient applicable to this cluster. Using the extinction formula of Cardelli et al. (1989) and the transformation between the Johnson and Cousins I (e.g. Clementini et al. 1995), we compute the distance modulus with the aid of the following formulae

$$d = d_{1.5} + (1.5 - \beta)E_{V-I_c} \quad , \tag{14}$$

$$\beta = R_{I_c}/(R_V - R_{I_c}) \quad , \tag{15}$$

$$R_{I_c} = 0.751R_V - 0.485 \quad , \tag{16}$$

$$E_{V-I_c} = (R_V - R_{I_c})E_{B-V} \quad , \tag{17}$$

where $R_{I_c} = A_{I_c}/E_{B-V}$, $R_V = 4.1$ and the E_{B-V} values are computed by Eq. (11) from the B, V data. With the above correction the distance modulus of M4 decreases by 0.11 mag compared with the one ($d_{1.5}$) obtained directly from Eq. (13).

As can be seen from Table 7, the agreement between the (B, V) and the (V, I_c) distance moduli can be regarded as very good if we consider all the possible error sources (limited size of the data sets, relatively poorer quality of the cluster I data, accuracy of the representation of the I_c system, etc). It is especially worth mentioning the importance of the accurate zero point of the V magnitude. Because of the large coefficients of the colour terms and of the opposite signs of V in the expressions of W and X, an error of ΔV in V causes a difference of $5.6\Delta V$ between the distance moduli derived from W and X. Although a sizable average zero point error does not seem to be present in the data-sets used here, it is worth to remember the importance of the accurate reproduction of the standard photometric systems when deriving precise distance moduli.

4.5. Inhomogeneous cluster reddenings

It has been known for some time that even globular clusters show inhomogeneities in the properties of the interstellar matter (e.g.

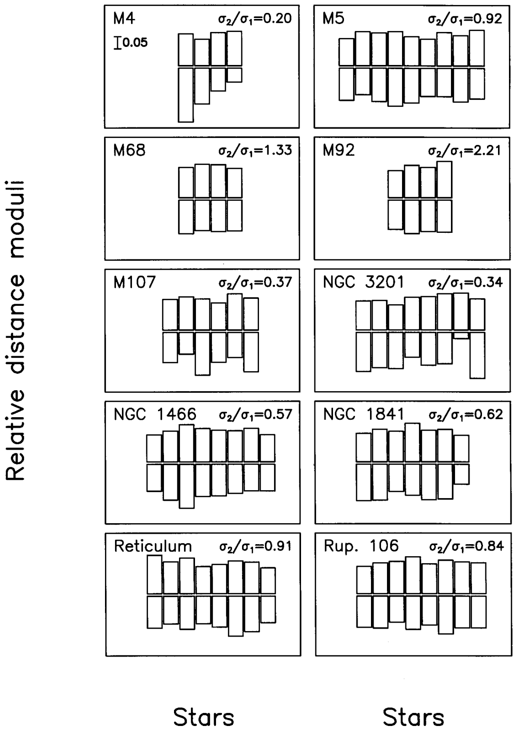

Fig. 4. Individual distance moduli computed by Eq. (8) (upper rectangles) and Eq. (10) (lower rectangles). The ratios of the standard deviations of the distance moduli are shown in each panel

Table 7. Distance moduli computed from different colours

Cluster	N	$d_{B,V}$	N	$d_{V,I}$
M4	4	11.03	3	11.04
M5	9	14.06	18	14.12
M68	4	14.83	4	14.83
M92	4	14.43	3	14.40

Cacciari 1984; Liu and Janes 1990 and references therein). The best example is M4, where the star-to-star differences in E_{B-V} could reach 0.07 mag. With the aid of the distance modulus formulae (Eqs. (3), (4), (8)) here we show that inhomogeneous reddening plays a role in several clusters. On the one hand we calculate the individual distance moduli with the above mentioned formulae, which yield the *true* (i.e. reddening-free) distance moduli. These will then be compared with the *reddened* distance moduli computed by the formula for the absolute magnitude (Eq. (10)). Inhomogeneous reddening will be exhibited as a larger scatter in the individual distance moduli computed from the V_0 absolute magnitudes. A smaller scatter of the W_0 distance moduli will indicate that, indeed, the larger scatter of the V_0 moduli is attributed to inhomogeneous reddening. As we shall see at the end of this subsection, the so computed degree of inhomogeneity is in a strong correlation with the variation of the directly computed reddening values.

Fig. 4 shows the individual distance moduli (shifted to an arbitrary zero point) for each globular cluster entering in our analysis. The upper and lower rows display the W_0 and V_0 distance moduli, respectively. The corresponding ratios of the standard deviations of the distance moduli are also shown. We see that except for M68 and M92, the dispersions of the distance moduli decrease when using the reddening-free formula of W_0. The large increase in the case of M92 indicates the size of the statistical error in the estimation of the ratio of the standard deviations for small data-sets and for low level of inhomogeneity in the cluster reddening. For the clusters M4, M107 and NGC 3201 a decrease by a factor of 3 in the standard deviations, clearly shows that *inhomogeneous* reddening in these clusters are *significant*. For NGC 1466 and NGC 1841 the decrease is 60%, which again indicates inhomogeneity, although in a less degree than in the former cases. The other clusters show no or very little sign of inhomogeneity.

Table 8. Globular cluster reddenings

Cluster	N	E_{B-V}	σ_E
M4	4	0.34	0.04
M5	9	0.06	0.01
M68	4	0.03	0.01
M92	4	0.01	0.01
M107	6	0.37	0.02
NGC 3201	8	0.19	0.04
Rup. 106	8	0.16	0.01
NGC 1466	8	0.06	0.01
NGC 1841	7	0.15	0.02
Reticulum	8	0.03	0.01

In concluding this subsection, the average reddenings and their standard deviations for the clusters discussed above are shown in Table 8. The reddenings are calculated according to Eq. (11). We see that the data shown in the table confirm our results obtained by the comparison of the reddened and dereddened distance moduli.

5. The I_c and K absolute magnitudes

The purpose of this section is to apply our distance modulus and reddening formulae to mostly field I_c and K data to derive the relations between the V light curves and the respective absolute magnitudes. The larger part of the data-set consists of field stars which implies good observational accuracy. The method also enables us to estimate the values of the selective absorption coefficient.

5.1. The I_c absolute magnitude

In order to apply our distance modulus and reddening formulae (Eqs. (8) and (11)) to derive the I_c absolute magnitudes (I_0), we need B, V and I_c data. These are listed in Tables 1 and 2. We fit the data in three steps: (1) field stars only, (2) field + clusters (without M4), (3) field + clusters (with M4). In all cases we use R_V = 3.1, except for M4, where we set R_V = 4.1. We

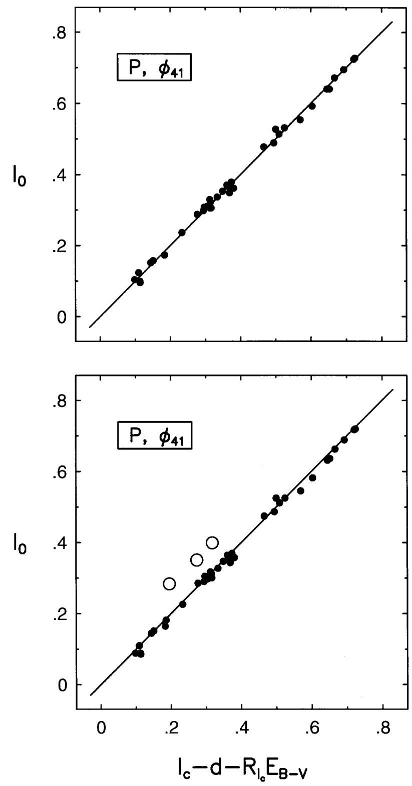

Fig. 5. *Upper panel:* Fitted vs. 'observed' I_0 for the 38 stars of the field and of M5, M68, M92. *Lower panel:* as above, but including the stars of M4 (open circles). The reddening for all stars (including also those of M4) is computed with R_V = 3.1

Table 9. Fitting properties of I_0 for the sample of 24 field stars

n par. fits		2 par. fits	
n	σ	params.	σ
1	0.0368	P, φ_{31}	0.0094
2	0.0094	P, φ_{41}	0.0106
3	0.0093	P, φ_{21}	0.0146
4	0.0086	P, φ_{51}	0.0163
5	0.0086	P, A_5	0.0199
6	0.0082	P, A_6	0.0224
7	0.0083	P, A_3	0.0240

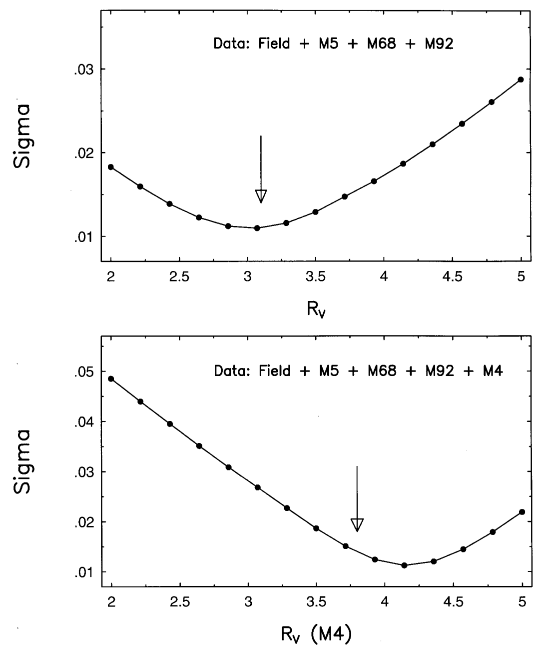

Fig. 6. Fitting accuracy of I_0 vs. selective extinction coefficient R_V. Arrows indicate the 'standard' values of R_V for 'ordinary' interstellar matter and for the special case of M4

justify these parameter values in the course of the derivation, and especially in step (3).

Starting with the field stars only, we have 25 stars altogether. Although this unedited sample of stars already shows the tight correlation between the I_0 magnitudes and the Fourier parameters, the omission of only one star (SS Leo) further improves the situation. Table 9 contains the more detailed information of the properties of the different fits for the remaining 24 stars. We think that the two parameter description seems to be very well justified. Since there is practically no difference between the (P, φ_{31}) and the (P, φ_{41}) combinations, because of the result of step (2), we prefer the latter combination, which yields

$$I_0 = 1.301 - 2.007P + 0.113\varphi_{41} \quad , \tag{18}$$

Progressing to step (2), we add M5, M68 and M92 to the field stars. This way we start with 41 stars. Omitting only SS Leo and two stars from M5 (V19 and 28), we get the following formula

$$I_0 = 1.299 - 1.989P + 0.110\varphi_{41} \quad , \tag{19}$$

which is in excellent agreement with Eq. (18). In addition, as it is seen from Table 10, the above parameter combination is the best among the two parameter fits. The 'observed' ($I_c - d - R_{I_c}E_{B-V}$) vs. the fitted absolute magnitudes are plotted in the upper panel of Fig. 5. The quality of the fit is very good, indeed.

Turning to the last step of the analysis of the I_c data, we also include the 3 stars of M4. By using our 'standard' value of R_V = 3.1 for all stars (including also those of M4), we compute the new regression shown in the lower part of Fig. 5.

The discordance of the stars of M4 is attributed to the incorrect R_V value used for this cluster. In the following we show how the method of relating the Fourier parameters to the absolute magnitude enables us to estimate the optimum value of R_V. We shall see that, indeed, the optimum value of R_V for M4 is around 4.1, close to the values used in the literature (e.g. Liu and Janes 1990).

In finding the optimum value of R_V, we simply scan the possible range of R_V and plot the fitting accuracy of I_0. We apply Eq. (16) to compute the corresponding value of R_{I_c}. For the stars of the field, M5, M68 and M92, we get the result shown in the upper panel of Fig. 6. We see that the value of 3.1 for the parameter R_V is well justified. In the next step we fix R_V to 3.1 for all stars, except for those of M4. By changing R_V only for

Table 10. Fitting properties of I_0 for the sample of 38 stars (field + clusters)

n par. fits		2 par. fits	
n	σ	params.	σ
1	0.0421	P, φ_{41}	0.0110
2	0.0110	P, φ_{31}	0.0125
3	0.0108	P, φ_{51}	0.0163
4	0.0099	P, φ_{21}	0.0207
5	0.0096	P, φ_{61}	0.0229
6	0.0097	P, A_5	0.0267
7	0.0098	P, A_6	0.0277

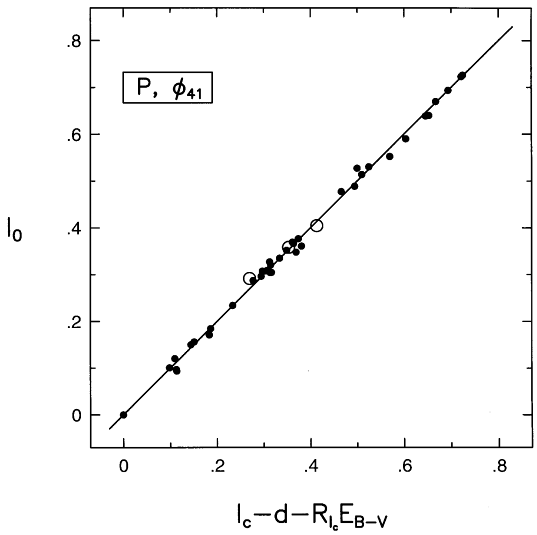

Fig. 7. Fitted vs. 'observed' I_0 for the 41 stars of the field and of M4, M5, M68 and M92. Except for M4, the reddening for all stars is computed with R_V = 3.1. For M4 (open circles) we use R_V = 4.1

this cluster, we get the plot shown in the lower panel of Fig. 6. It is clear that the optimum value of R_V for M4 is around 4.1, in a good agreement with the usually applied value.

Finally, in searching for the best representation of the I_c absolute magnitude, we proceed by including the 3 stars of M4 with the proper extinction coefficient. Again, the same stars should be omitted as in step (2). We get 41 stars which yield our final formula

$$I_0 = 1.303 - 1.993P + 0.108\varphi_{41} \quad , \tag{20}$$

with a fitting accuracy of 0.011 mag. Fig. 7 illustrates the quality of the fit.

5.2. *The K absolute magnitude*

The advantage of the infrared colours is the small effect of the interstellar reddening and the larger range of RR Lyrae luminosities in these bands. However, as we shall see, the relatively larger noise level of the K magnitudes makes these advantages less valuable as one might think at the first sight.

Table 11. Dependence of the standard deviations on the number of Fourier parameters (n) for various fits of the K_0 magnitudes

n	Params.	Field	Field + Clusters
1	P	0.059	0.053
2	P, φ_{31}	0.024	0.030
3	P, φ_{31}, ...	0.020	0.027
4	P, φ_{31}, ...	0.020	0.027
5	P, φ_{31}, ...	0.020	0.028

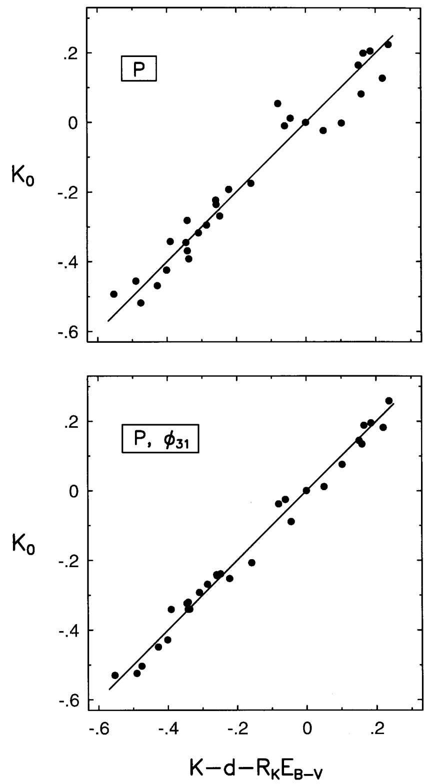

Fig. 8. Single and two parameter regressions for the K absolute magnitude in the calculated vs. observed planes. The 45° lines are shown for reference

Using the B, V, K data of Tables 1 and 2, we search for the Fourier expression of the K absolute magnitude. For the selective absorption coefficient we use the formula of Cardelli et al. (1989)

$$R_K = 0.162R_V - 0.148 \quad , \tag{21}$$

with R_V = 4.1 for the stars of M4 and R_V = 3.1 for all the others.

As in the case of the I_c data, first we analyse only the field stars. After leaving out V445 Oph and VY Ser, for the remaining 22 stars we obtain the standard deviations shown in the third column of Table 11. The preference toward the *two parameter* fit is clearly demonstrated. This result *invalidates* the use of the simple *period–luminosity* relation which has been applied up to now (e.g. Longmore et al. 1990).

In the second step of the analysis we add cluster K data to the field stars. Discarding the 5 most outlying stars M5 V28, 32; M92 V1; V445 Oph and VY Ser, for the rest we get the standard deviations as given in the fourth column of Table 11. Again, the preference toward the two parameter fit is demonstrated, although the contrast is less striking, because of the higher noise level. We see that all samples are essentially consistent with the assumption that there is basically a two parameter dependence of the K_0 magnitudes. By using this last set of 28 stars, we get

$$K_0 = 0.045 - 2.672P + 0.234\varphi_{31} \quad , \tag{22}$$

where the phase should be chosen as the closest value to 5.1. By comparing the standard deviations and the ranges of the I_0 and K_0 absolute magnitudes (Tables 10, 11 and Figs. 7, 8) we see that the I_0 magnitudes correlate better with the Fourier parameters. Therefore, despite of the lower absorption in K, this result suggests that with a proper reddening correction the I_c observations are preferred over the K observations, at least for the purpose of the absolute magnitude determination.

6. Conclusions

It has been demonstrated that multicolour observations of cluster RR Lyrae stars are extremely useful in calculating reddening-free quantities with the aid of the Fourier parameters of the V light curves. The method applies the idea that we have already used in the determination of the formulae for the iron abundance (Jurcsik and Kovács 1996) and for the V absolute magnitude (Kovács and Jurcsik 1996). The basic idea is simply that the light curve depends somehow on the physical parameters of the star. Once we have an observed or calculated quantity which depends solely on the stellar parameters, we can attempt to find the relation between this quantity and the Fourier parameters of the light curve. So far, all the relations we studied have proven to be linear in terms of the period, amplitudes and phases.

The first quantity we dealt with was the reddening-free brightness $W \equiv V - \alpha(B-V)$ with a properly chosen selective extinction coefficient α. The use of cluster variables enabled us to determine the functional dependence of W on the Fourier parameters. Once this function is known, one can determine the *true* (i.e. reddening-free) distance modulus simply by subtracting the observed and calculated values of W (Eqs. (3), (4) and (8)).

Using again cluster variables, we applied our result for the V_0 absolute magnitude to derive the relation for the intrinsic colour index $B_0 - V_0$ (Eq. (11)). These formulae are compatible with the expression of W_0.

The distance modulus and the intrinsic colour formulae were used to derive expressions for the I_c and K absolute magnitudes (Eqs. (20), (22)). We used data-sets which contained mostly field stars with accurate photometry. It has been shown that besides the period, the Fourier parameters also play important role in the representation of these quantities. The very tight correlation found for the I_c absolute magnitude suggests the preference of this colour over the infrared K for absolute magnitude determinations — assuming that we have a fair estimation on the reddening.

Based on the individual distance moduli of cluster variables, we have shown that inhomogeneous reddening is important in several clusters.

We have demonstrated that the method is capable to give optimum estimations for the selective absorption coefficient R_V. In our sample of stars with B, V and I_c magnitudes, only the variables of M4 proved to be peculiar in this respect with R_V = 4.1, whereas the 'standard' R_V = 3.1 was shown to be the optimum for the other stars.

In conclusion, with the aid of the formulae presented in this paper, it is possible to compute distance moduli, absolute magnitudes and intrinsic colour indices of RRab stars. For the application of the formulae the only requirement is the existence of accurate V light curve and colour index. Although the absolute calibration might still suffer from possible systematic errors due to some problems which can occur mostly in the calibrating Baade−Wesselink stars, the accuracy of the formulae for the relative quantities is quite good, perhaps the best among the presently available methods for RR Lyrae stars.

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