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Resonant absorption and the spectrum of 5-min oscillations of the Sun

V.I. Zhukov

Central Astronomical Observatory Russian Academy of Sciences, 65 Pulkovo, 196 140 St. Petersburg, Russia

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Abstract. Magnetic field influence on 5-min solar oscillations spectrum is investigated. Proper oscillations spectrum is calculated for the model of the shell with linear temperature profile in the bottom layer while the top layer is exemplified by homogeneous magnetic field and sharp temperature increase, reaching the coronal value at the height about $\sim$ 2500km. It is shown, that taking into account of acoustic energy absorption at Alfven resonant levels in the chromosphere one can provide an explanation for existing discrepancies between the observed and theoretically calculated frequencies of the 5-min oscillations.

Key words: Sun: oscillations – chromosphere – MHD

1. Introduction

Recent researches lead to suggestion that the resonant absorption of the 5-min oscillations in the chromosphere of the Sun may be responsible both for sharp temperature increase in the upper chromosphere, and the existing discrepancies between theoretically calculated and observed frequencies of 5-min oscillations (Zhukov 1992).

In present paper we performed the calculation of a spectrum of 5-min oscillations (taking into account their resonant absorption in the "canopy" region of magnetic field) for realistic model of the shell of the Sun. It is shown, that the frequencies of 5-min oscillations obtained with regard to the resonant absorption differ from proper frequencies, calculated for the model without "canopy" magnetic field, by the values about several μHz.

In Sect. 2 the description of used model is given and the basic equations are indicated.

The results of numerical calculations are discussed in Sect. 3.

2. The basic equations

As it has been shown in (Zhukov 1992), partial escape of acoustic energy from a waveguide due to tunnel leakage of waves (for example, into a region of Alfven resonances or merely into a transparency region) should result in a change of the waveguide proper frequencies. The values of these changes naturally depend on various reasons and primarily on a structure of the waveguide.

To derive the proper frequencies of 5-min oscillations, which could be to some extent suitable for comparison with observations, in the present paper we calculate proper frequencies of a waveguide with

1. the bottom layer without magnetic field; temperature falls with height linearly;
2. the top layer with the homogeneous magnetic field and sharp increase of temperature up to coronal value at the height about 2500km.

2.1. The convective zone

As it is known, to describe adequately the 5-min oscillations with sufficiently large ℓ as a first approximation it is possible to restrict the consideration to the plane-layer model with linear temperature change in upper layers of the convective zone.

In a cartesian system of coordinates with Z-axis directed upwards, the basic equation for vertical components of velocity v_z for a layer ($z < 0$) with a linear profile of temperature has the form (Nye & Thomas 1976; Evans & Roberts 1990)

$$c^2\left(\omega^2 - c^2k^2\right)\frac{\mathrm{d}^2 v_z}{\mathrm{d}z^2} + \left[c^2k^2\frac{\mathrm{d}c^2}{\mathrm{d}z} - g\gamma\left(\omega^2 - c^2k^2\right)\right]\frac{\mathrm{d}v_z}{\mathrm{d}z} + \left[\left(\omega^2 - c^2k^2\right)^2 + \frac{(\gamma-1)g^2k^2}{\omega^2}(\omega^2 - c^2k^2) - gk^4c^2\frac{\mathrm{d}c^2}{\mathrm{d}z}\right]v_z = 0 \quad (1)$$

And pressure fluctuations in acoustic wave are described by the equation

$$p = i\frac{\rho_0\omega}{\omega^2 - c^2k^2}\left(c^2\frac{\mathrm{d}v_z}{\mathrm{d}z} - gv_z\right) \quad (2)$$

Henceforward we choose for all perturbed values a dependence from x, y and t in the form $\exp i(k_x x + k_y y + \omega t)$ and use the following notations: ρ_0 – is the undisturbed density, c – is the speed of sound, g – is the acceleration due to gravity, which is considered constant, γ – is the adiabatic index and $k^2 = k_x^2 + k_y^2$.

In general, the Eq. (1) is valid for any dependence of speed of sound on height, however in this work, as it was already marked above, we shall restrict our consideration to the case, when for $z < 0$

$$c^2 = c_{01}^2 \left(1 - \frac{z}{z_1}\right)$$

2.2. The chromosphere – corona

We take for the upper layer ($z > 0$) the dependence of the squared speed of sound from z in the form

$$c^2 = c_{02}^2 \left(1 + \delta \left[\tanh\left(\alpha\left(\frac{z}{z_2} - \beta\right)\right) - \tanh(-\alpha\beta)\right]\right)$$

and that in this layer there is a homogeneous horizontal magnetic field $\boldsymbol{H}_0$. Then in a cartesian system of coordinates with Z-axis directed upwards, and X-axis along the magnetic field, the basic equation for vertical component of velocity v_z is (Nye & Thomas 1976; Rae & Roberts 1982; Zhukov 1988)

$$\begin{aligned}
&\frac{d}{dz}\left[\rho_0(c^2 + v_A^2)\frac{(\omega^2 - \omega_A^2)(\omega^2 - \omega_T^2)}{(\omega^2 - \omega_+^2)(\omega^2 - \omega_-^2)}\frac{dv_z}{dz}\right] + \\
&\frac{d}{dz}\left[g\rho_0\left(1 - \frac{\omega^2(\omega^2 - \omega_A^2)}{(\omega^2 - \omega_+^2)(\omega^2 - \omega_-^2)}\right)\right] v_z + \\
&\rho_0(\omega^2 - \omega_A^2)\left[1 - \frac{g^2 k^2}{(\omega^2 - \omega_+^2)(\omega^2 - \omega_-^2)}\right] v_z = 0
\end{aligned} \tag{3}$$

Fluctuations of total pressure in a wave are given by the equation

$$\begin{aligned}
p + \frac{H_0 h_x}{4\pi} &= \frac{i}{\omega}\frac{\rho_0(\omega^2 - \omega_A^2)}{(\omega^2 - \omega_+^2)(\omega^2 - \omega_-^2)} \\
&\cdot\left[(c^2 + v_A^2)(\omega^2 - \omega_T^2)\frac{dv_z}{dz} - g\omega^2 v_z\right]
\end{aligned} \tag{4}$$

here are denoted

$$\omega_A^2 = v_A^2 k_x^2, \ \omega_T^2 = c_T^2 k_x^2,$$

where $v_A^2 = H_0^2/4\pi\rho_0$, $c_T^2 = c^2 v_A^2/(c^2 + v_A^2)$ and $\omega_\pm^2$ are the roots of the following equation

$$\omega^4 - \omega^2(c^2 + v_A^2)k^2 + c^2 v_A^2 k^2 k_x^2 = 0$$

and h_x is perturbation of x-component of the magnetic field in a wave. The other notations are the same as above.

It is easily to see, that the Eq. (3) has two types of singular points. Singular points $\omega^2 = \omega_A^2$ refer to as by points of Alfven resonance, and the points $\omega^2 = \omega_T^2$ refer to as by points of cusp resonance.

The Eq. (3) is much simplified at $k_y = 0$. In this case

$$(\omega^2 - \omega_+^2)(\omega^2 - \omega_-^2) = (\omega^2 - c^2 k_x^2)(\omega^2 - \omega_A^2)$$

and Eq. (3) takes the form

$$\begin{aligned}
&\frac{d}{dz}\left[\rho_0\left(c^2 + v_{Ax}^2\right)\frac{\omega^2 - \omega_T^2}{\omega^2 - c^2 k_x^2}\frac{dv_z}{dz}\right] + \\
&\left\{\frac{d}{dz}\left[g\rho_0\left(1 - \frac{\omega^2}{\omega^2 - c^2 k_x^2}\right)\right] + \right. \\
&\left.\rho_0\left(\omega^2 - \omega_A^2\right)\left[1 - \frac{g^2 k_x^2}{(\omega^2 - c^2 k_x^2)(\omega^2 - \omega_A^2)}\right]\right\} v_z = 0.
\end{aligned} \tag{5}$$

Thus, for waves, propagating in plane, formed by vectors of intensity of a magnetic field and gradient of inhomogeneousty of the medium is present only a cusp resonance.

In approximation of an incompressible medium and with $g = 0$ the Eq. (5) further simplified and takes the form

$$\frac{d}{dz}\left[\rho_0\left(\omega^2 - \omega_A^2\right)\frac{dv_z}{dz}\right] - k_x^2\rho_0\left(\omega^2 - \omega_A^2\right) v_z = 0. \tag{6}$$

It is this equation that was used in the vast majority of works on resonant absorption of waves (see f.e. Tataronis 1975; Kappraff & Tataronis 1977; Ionson 1978; Steinolofson 1984; Mok & Einaudi 1985). It must be emphasized, however, that in Eq. (6) singular point $\omega^2 = \omega_A^2$ is the cusp resonance, rather than Alfven, since here $\omega^2 = \omega_A^2$ is dispersion relation for Alfven p-waves, rather than usual (or v-type) transverse Alfven waves (Banõs 1955; Lee & Roberts 1986; Zhukov 1986).

Investigation of propagation of waves in non-uniform medium with the magnetic field has shown, that in the vicinity of a resonant levels occurs effective absorption of waves, and amount of energy, absorbed on resonant level, does not depend on mechanism of a dissipation of waves (it can be Joule dissipation, attenuation because of viscosity or radiative exchange) and from value of coefficients of a dissipation (provided that they are rather small).

Since in a region of the "canopy" of a magnetic field are located Alfven resonances for 5-min oscillations, it is naturally that the absorption of energy of 5-min oscillations on Alfven resonances should result with on the one hand to heating of a chromosphere, and on the other hand, to some change of proper frequencies of 5-min oscillations. To account of shift proper frequencies p-modes of oscillations has been the objective of the present paper.

2.3. The dispersion relation

As it was mentioned, the considered two-layers model of the shell of the Sun provides reasonable accuracy in determination of five-minute oscillations with large ℓ. The reason is that one can use for these modes the plane-layer model and a linear temperature profile in the convective zone. From the physical point of view it is quite obvious (see f.e. Kahn 1961) that when there is no magnetic field in an upper layer, there should be an acoustic waveguide in the vicinity of z=0. Given the magnetic field in an upper layer, waves from the capture region would partially tunnel leakage into Alfven resonance region.

In works (Evans & Roberts 1990; Wright & Thompson 1992) it was accepted, that $k_y = 0$ and, besides the parameters of model were chosen in such a way, that in chromosphere there were no cusp resonances for p-modes oscillations and, hence, were no radiations of waves from the region of capture and theirs resonant absorption and consequently $\omega_i = 0$. In this case shift of frequencies for p-modes of oscillations, roughly speaking, is ceased only by change of the position of the top point of turn for p-modes of oscillations, which can be caused, for example, the presence in chromosphere of a magnetic field. However in general case with $k_y \neq 0$ for p-modes of oscillations in chromosphere there are Alfven and cusp resonances and, hence, the energy of the p-modes of oscillations will partially leave from bottom layer ($z < 0$) in the region of resonances. And if from waveguide it is possible the radiation of waves, by a consequence it is change of proper frequencies of waveguide and attenuation of oscillations in waveguide ($\omega_i \neq 0$) (see f.e. Wright & Thompson 1992). This is the main reason of shift of proper frequencies of p-modes of oscillations in considered here model.

To calculate the spectrum of acoustic waves captured in the shell we are needed the dispersion relation for the frequency of the captured wave ω. This frequency (due to a presence of resonant layers in the region with the magnetic field) is generally complex ($\omega = \omega_r + i\omega_i$).

Formally, the dispersion relation is derived from the boundary conditions at z=0

$$v_{z1} = v_{z2}$$

$$\frac{\partial p_1}{\partial t} - g\rho_{01} v_{z1} = \frac{\partial}{\partial t}\left(p_2 + \frac{H_0 h_x}{4\pi}\right) - g\rho_{02} v_{z2} \qquad (7)$$

and from the condition that the sum of kinetic and magnetic energy density of a wave is approaching zero at $|z| \to \infty$.

On the assumption, that temperature is continuous at $z = 0$ (i.e. $c_{02} = c_{01} = c_0$), the rather complicated system of equations is found from Eqs. (1), (3) and the system (7) for the determination of ω_r and ω_i (see Appendix A1).

3. Calculation of the proper frequencies

The system of Eqs. (A1) was solved numerically with the same parameters of the convective zone, as in (Evans & Roberts 1990, Table 1), (i.e. $c_0 = 6.6829$km s^{-1}, $g = 274$m s^{-1}, $\gamma = 5/3$) and $z_1 = 250$km. For the top layer it was taken $H_0 = 30$G, $z_2 = 500$km, while in the formula which describes the speed of sound variation with height the parameters ($\delta = 120$ and $\alpha = 3$, $\beta = 4$) were matched in such a way that $T \sim 4600$K at $z = 0$ and $T \sim 7000$K at $z = 1500$km. The general trend of temperature as a function of height in the chromosphere – corona region (in the layer $z > 0$) is shown on Fig. 1.

At Fig. 2 the diagnostic diagram is presented. It is calculated for the case, when there is no magnetic field in the upper layer($H_0 = 0$) and, hence, there is no resonant absorption of wave energy.

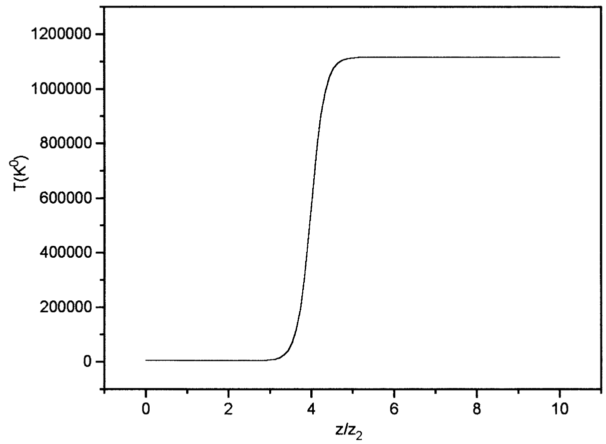

Fig. 1. Dependence of temperature on height in the chromospere – corona

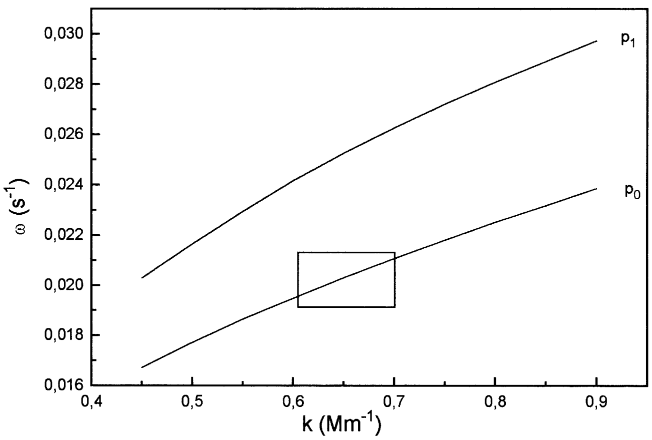

Fig. 2. The diagnostic diagram for the shell without magnetic field

In paper (Zhukov 1992) it was shown, that the tunnel leakage of wave energy into the region of Alfven resonances should result in proper frequencies change. To determine the shift of the frequency it is essential to find a solution of the system (A1) with very high accuracy. This requires to spend too much computer time. So here we were restricted the consideration to the narrow range of frequencies and wave numbers which is represented by the small rectangle on Fig. 2. The results of calculations are shown at Fig. 3 (where is denoted $\delta\nu = (\omega_r - \omega_0)/2\pi$, ω_r – being the real part of proper frequency for the shell with magnetic field, and ω_0 – being the proper frequency for the shell without magnetic field).

As it was expected, the tunnel leakage of wave energy into the region of Alfven resonance leads to change of proper frequency ω_r for several μHz, that is, for a value of difference between observed and theoretically calculated frequencies. For wave modes which are presented at Fig. 3 the Alfven resonant levels are located in the chromosphere at heights $z_A \approx (1.19 \div 1.21)z_2$, and cusp levels are located within a rather thin layer close to $z_c = 3z_2$ and ω_i for them being in the order of $5\,10^{-7}$s^{-1}.

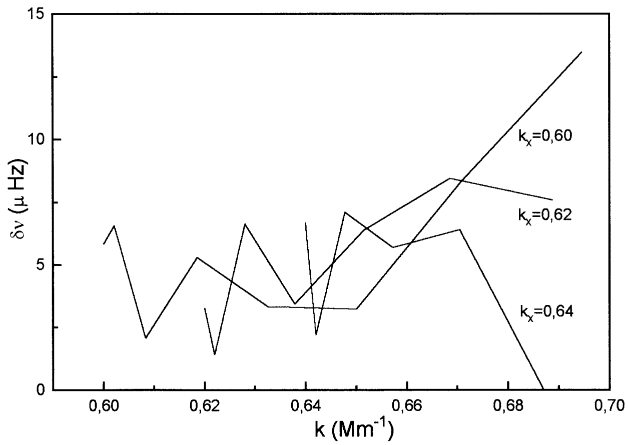

Fig. 3. Shift of the frequencies due to resonant absorption

An important feature of a spectrum of proper oscillations in presence of Alfven resonances is the fact, that the spectrum of fluctuations will be continuous. The point is that generally speaking value of shift of frequency is determined by a position z_{A} ($v_{\mathrm{A}}(z_{\mathrm{A}}) = \omega_{\mathrm{r}}/k_{\mathrm{x}}$) of Alfven resonance in the chromosphere and, hence, at fixed k ($k = (k_{\mathrm{x}}^2 + k_{\mathrm{y}}^2)^{1/2}$) at continuous change k_{x} continuously varies proper frequency.

To provide some indication of an extent of magnetic field impact on the shell oscillations spectrum in Table 1 proper frequencies $\omega = \omega_{\mathrm{r}} + i\omega_{\mathrm{i}}(\mathrm{s}^{-1})$ the calculated at $k_{\mathrm{x}} = 0.6\mathrm{Mm}^{-1}$ are present.

4. Conclusion

The proper frequencies calculation of spectrum of oscillations, for the shell which consist of the bottom layer with the linear temperature profile and the top layer with sharp temperature increase up to million of degrees at the height about 2500km and homogeneous horizontal magnetic field has shown, that the absorption of acoustic energy at Alfven resonant levels in the chromosphere of the Sun essentially influences on the spectrum of 5-min oscillations and should be taken into account, if we want to achieve matching between the theoretically calculated and observed frequencies of oscillations.

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Appendix A

$$\begin{aligned} F_1(v_{\mathrm{r}1}v_{\mathrm{r}2} - v_{\mathrm{i}1}v_{\mathrm{i}2}) - F_2(v_{\mathrm{r}1}v_{\mathrm{i}2} + v_{\mathrm{r}2}v_{\mathrm{i}1}) &- \\ C(v_{\mathrm{r}1}\phi_2 - v_{\mathrm{i}1}\psi_2) + D(v_{\mathrm{r}1}\psi_2 + v_{\mathrm{i}1}\phi_2) &- \\ G_1(v_{\mathrm{r}2}\phi_1 - v_{\mathrm{i}2}\psi_1) - G_2(v_{\mathrm{i}2}\phi_1 + v_{\mathrm{r}2}\psi_1) &= 0 \\ F_2(v_{\mathrm{r}1}v_{\mathrm{r}2} - v_{\mathrm{i}1}v_{\mathrm{i}2}) + F_1(v_{\mathrm{r}1}v_{\mathrm{i}2} + v_{\mathrm{r}2}v_{\mathrm{i}1}) &- \\ D(v_{\mathrm{r}1}\phi_2 - v_{\mathrm{i}1}\psi_2) - C(v_{\mathrm{r}1}\psi_2 + v_{\mathrm{i}1}\phi_2) &+ \\ G_2(v_{\mathrm{r}2}\phi_1 - v_{\mathrm{i}2}\psi_1) - G_1(v_{\mathrm{i}2}\phi_1 + v_{\mathrm{r}2}\psi_1) &= 0 \end{aligned} \tag{A1}$$

Table 1. Proper frequencies of oscillations of the shell for various intensity of a magnetic field values

k_{y} (Mm^{-1})	$H_0 = 0$	$H_0 = 15\mathrm{G}$	$H_0 = 30\mathrm{G}$
0.0	$\omega_0 = 0.019472$	$\omega_{\mathrm{r}} = 0.019489$	$\omega_{\mathrm{r}} = 0.019509$
		$\omega_{\mathrm{i}} = 4.58\,10^{-7}$	$\omega_{\mathrm{i}} = 5.33\,10^{-7}$
0.1	$\omega_0 = 0.019615$	$\omega_{\mathrm{r}} = 0.019612$	$\omega_{\mathrm{r}} = 0.019628$
		$\omega_{\mathrm{i}} = 4.98\,10^{-7}$	$\omega_{\mathrm{i}} = 4.86\,10^{-7}$
0.2	$\omega_0 = 0.020003$	$\omega_{\mathrm{r}} = 0.020001$	$\omega_{\mathrm{r}} = 0.020024$
		$\omega_{\mathrm{i}} = 4.90\,10^{-7}$	$\omega_{\mathrm{i}} = 5.03\,10^{-7}$
0.3	$\omega_0 = 0.020601$	$\omega_{\mathrm{r}} = 0.020646$	$\omega_{\mathrm{r}} = 0.020654$
		$\omega_{\mathrm{i}} = 4.66\,10^{-7}$	$\omega_{\mathrm{i}} = 4.54\,10^{-7}$

In a dispersion relation (A1) are denoted

$$v_{\mathrm{z}n} = v_{\mathrm{r}n} + iv_{\mathrm{i}n}$$

$$\phi_n = \frac{\mathrm{d}v_{\mathrm{r}n}}{\mathrm{d}z},\ \psi_n = \frac{\mathrm{d}v_{\mathrm{i}n}}{\mathrm{d}z},\ n = 1, 2$$

where $v_{\mathrm{z}n}$ – is the vertical component of velocity for the bottom and the top layer correspondingly.

$$G_1 = \frac{\rho_{01}}{\rho_{02}} \frac{c^2\left[\Omega^2(\Omega^2 - c^2k^2) + W^2\right]}{(\Omega^2 - c^2k^2)^2 + W^2}$$

$$G_2 = \frac{\rho_{01}}{\rho_{02}} \frac{c^4k^2W}{(\Omega^2 - c^2k^2)^2 + W^2}$$

$$\begin{aligned} F_1 \ = \ & g\frac{G_1}{c^2} + \\ & g\frac{\Omega^2[C(\Omega^2 - \omega_{\mathrm{T}}^2) + DW] - W[D(\Omega^2 - \omega_{\mathrm{T}}^2) - CW]}{(c^2 + v_{\mathrm{A}}^2)[(\Omega^2 - \omega_{\mathrm{T}}^2)^2 + W^2]} \end{aligned}$$

$$\begin{aligned} F_2 \ = \ & -g\frac{G_2}{c^2} + \\ & g\frac{\Omega^2[D(\Omega^2 - \omega_{\mathrm{T}}^2) - CW] + W[C(\Omega^2 - \omega_{\mathrm{T}}^2) + DW]}{(c^2 + v_{\mathrm{A}}^2)[(\Omega^2 - \omega_{\mathrm{T}}^2)^2 + W^2]} \end{aligned}$$

$$C = \frac{A(\Omega^2 - \omega_{\mathrm{A}}^2) + BW}{A^2 + B^2}$$

$$D = \frac{WA - B(\Omega^2 - \omega_{\mathrm{A}}^2)}{A^2 + B^2},$$

here

$$A = \frac{(\Omega^2 - \omega_{\mathrm{A}}^2)a + Wb}{(c^2 + v_{\mathrm{A}}^2)[(\Omega^2 - \omega_{\mathrm{T}}^2)^2 + W^2]}$$

$$B = \frac{W[b(\Omega^2 - \omega_{\mathrm{A}}^2) - a]}{(c^2 + v_{\mathrm{A}}^2)[(\Omega^2 - \omega_{\mathrm{T}}^2)^2 + W^2]},$$

where

$$a = \Omega^4 - \omega_{\mathrm{cv}}^2\Omega^2 + c^2v_{\mathrm{A}}^2k^2k_{\mathrm{x}}^2 - W^2,\ b = 2\Omega^2 - \omega_{\mathrm{cv}}^2,$$

$\omega_{\rm cv}^2 = (c^2 + v_{\rm A}^2)k^2,$

here are designated

$\Omega^2 = \omega_{\rm r}^2 - \omega_{\rm i}^2,\ W = 2\omega_{\rm r}\omega_{\rm i}$

In a dispersion relation (A1) values of these coefficients calculated in the point $z = 0$ are used.

References

Banõs, A. 1955, Proc. Roy. Soc. London A.233, 350
Evans, D.J., Roberts, B. 1990, ApJ 356, 704
Ionson, J.A. 1978, ApJ 226, 650
Kahn, F.D. 1961, ApJ 134, 343
Kappraff, J.M., Tataronis, J.A. 1977, J. Plasma Phys. 18, 209
Lee, M.A., Roberts, B. ApJ 301, 430
Mok, Y., Einaudi, G. 1985, J. Plasma Phys. 33, 199
Nye, A.H., Thomas, J.H. 1976, ApJ 204, 573
Nye, A.H., Thomas, J.H. 1976, ApJ 204, 582
Rae, C., Roberts, B. 1982, MNRAS 201, 1171
Steinolofson, R.S. 1984, Phys. Fluids 27, 781
Tataronis, J.A. 1975, J. Plasma Phys. 13, 87
Wright, A.N., Thompson, M.J. 1992, A&A 264, 701
Zhukov, V.I. 1986, Fizica Plazmy 12, 101
Zhukov, V.I. 1988, Fizica Plazmy 14, 871
Zhukov, V.I. 1992, Solar Phys. 139, 201

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