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The rate of supernovae from the combined sample of five searches

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Abstract. With the purpose to obtain new estimates of the rate of supernovae we joined the logs of five SN searches, namely the Asiago, Crimea, Calán-Tololo and OCA photographic surveys and the visual search by Evans. In this way we improved the SN statistics (the sample counts 110 SNe) and hence, reduced the uncertainties.

The computation was based on the control time method which allowed the proper merging of the observations of each galaxy in the various searches. In addition to discussing the choice for the various input parameters, we verified the existence of two biases against SN discoveries, one in the nuclear regions of distant galaxies, most severe for deep photographic surveys, and a second in inclined spirals, in particular late spirals.

After correction for these two biases we obtained the rates of each SN type in the different types of galaxies. We found that the most prolific galaxies are late spirals in which most SNe are of type II (0.88 SNu). SN Ib/c are rarer than SN Ia (0.16 and 0.24 SNu, respectively), ruling out previous claims of a very high rate of SNIb/c. We also found that the rate of SN Ia in ellipticals (0.13 SNu) is smaller than in spirals, supporting the hypothesis of different ages of the progenitor systems in early and late type galaxies.

Finally, we estimated that even assuming that separate classes of faint SN Ia and SN II do exist (SNe 1991bg and 1987A could be the respective prototypes) the overall SN rate is raised only by 20-30%, therefore excluding that faint SNe represent the majority of SN explosions. Also, the bright SNIIn are intrinsically very rare (2 to 5 % of all SNII in spirals).

Key words: supernovae: general – surveys – galaxies: general – galaxies: stellar contents of

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1. Introduction

The rate of supernovae (SNe) is a key parameter regulating the chemical evolution of galaxies, the kinematics of the interstellar medium, the production of cosmic rays, in addition to being a fundamental constraints for stellar evolution theories.

In recent years there has been a renewed interest in the search for SNe and in fact the number of discoveries in the last decade almost equaled those of the previous century. Despite these efforts, published estimates of the rate of SN still bring large uncertainties.

In order to obtain a direct measurement of the rate of SNe in a given galaxy sample one has to divide the number of SN discoveries by the period of surveillance. The latter can be estimated either by making some "reasonable" assumptions (e.g. Tammann et al. 1994 and reference therein) or by computing the *control time* through the detailed analysis of the log of a given SN search (Zwicky 1938, 1942). Since only the SNe discovered in the particular search enter the computation, in general the main problem with this approach is the small statistics. This can be improved by joining into a single database the data of different SN searches but, so far, this has been done only for the Asiago and Crimea SN searches (Cappellaro et al. 1993a, 1993b hereafter PI and PII).

In this paper we present new estimates of the rate of SNe based on the joint efforts of five SN searches, namely the Asiago and Crimea searches, the search by Evans (van den Bergh et al. 1987, Evans et al. 1989), the OCA search (Pollas 1994) and the Calán/Tololo search (Hamuy et al. 1993). In this way we collected the largest SN sample ever used for SN rate calculation.

In the following, after introducing the individual searches, we describe the recipe used in the computation with emphasis on the updating and improvements with respect to previous works. We make an effort to explicitly mention all the critical assumptions and parameters involved in the calculation and to discuss in detail the biases of SN searches. Finally, the computed SN rates are reported and compared with previous estimates.

2. The SN searches

In the following we give a brief description of the main characteristics of each SN search. More details can be found in the references indicated below.

Asiago: the SN search was conducted at Asiago from 1959 to 1990, initially with the 40/50 cm Schmidt telescope (S40) and, after 1967, also with the larger 67/92 cm Schmidt (S67). During the search 31 SNe were discovered (mostly by L. Rosino) and about 20 more, first discovered by others, were recorded in the survey plates (PI).

Crimea: this search started in 1961 using the 40 cm astrograph of the Sternberg Institute in Crimea. It announced the discovery of 21 SNe and 18 more were recorded (Tsvetkov 1983, PI). The search is still active but, due to temporary difficulties, few plates were obtained after 1991.

Evans: aimed at the prompt discovery of nearby SNe, this is the most successful visual SN search. At the beginning, in 1980, a 25 cm telescope was employed which was substituted by a 41 cm telescope in November 1985 (Evans et al. 1989). Whereas the search is still active, here we use only the observations performed to the end of 1988 (almost 100000 individual observations). The SN sample counts 24 objects (van den Bergh et al. 1987; Evans et al. 1989).

OCA: the search began in 1987 based on the 90/152 cm Schmidt telescope of the Observatoire de la Côte d'Azur (OCA, Pollas 1994). Differently from the others, the OCA search is not systematic but makes use of plates obtained for other purposes. In particular, there were not predefined sky fields and observing strategy. Most of the plates were very deep, with limiting magnitude up to 21-22 and therefore many faint SNe were found. On the about 500 plates examined to the end of 1994, 68 SNe have been found (6 were first discovered by others).

Calán/Tololo (C&T): this search began in 1990. The scientific rationale was to produce a sample of SNe at moderate distances ($0.01 < z < 0.1$) suitable for cosmological studies. A 60/90 cm Schmidt telescope was employed for the regular monitoring of selected fields resulting in the discovery of 49 SNe (Hamuy et al. 1993) with 5 more first discovered by others.

With the exception of the visual search by Evans, the searches listed above use photographic plates and wide field telescopes. The Asiago and Crimea surveys were aimed at the long term monitoring of relatively nearby galaxies and were able to discover SNe only up to the distance of the Coma cluster whereas both the OCA and C&T searches could discover SNe up to $z = 0.1 - 0.15$.

In Fig. 1 we report, for each search, the distributions of $m(SN)$, the apparent magnitudes of the SNe **at discovery**. The cut-off of the distributions at faint magnitudes is imposed by $m_{\rm lim}$, the limiting magnitudes for SN discovery of the searches

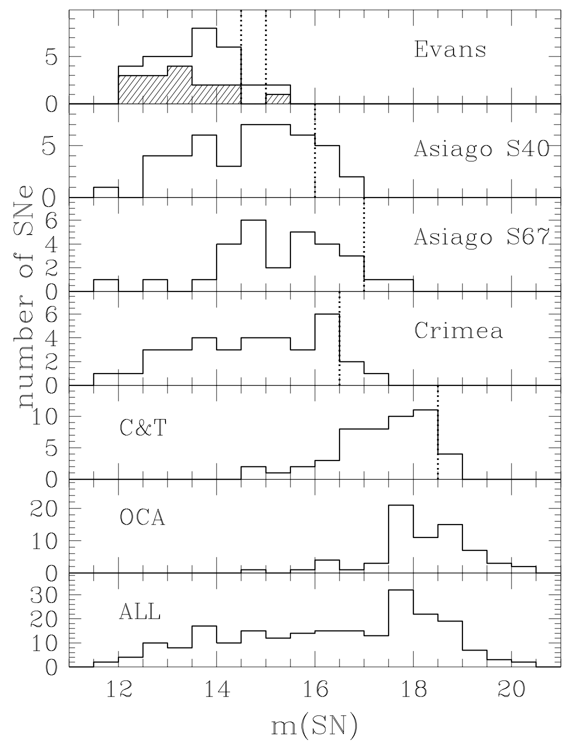

Fig. 1. Distributions of the apparent magnitudes at discovery, $m(SN)$, of the SNe found in the different searches. For Asiago, Crimea and C&T the magnitudes are B, for the Evans' search they are visual whereas for the OCA search they are mixed. The dashed area in the Evans' search histogram refers to SNe discovered before 1985 November with the 25 cm telescope. The dotted lines indicate the adopted limiting magnitudes for each search (for the OCA search read text).

which are, in general, 1-2 mag brighter than the usual definition of plate limit. In the bottom panel of Fig. 1 is the distribution of $m(SN)$ for the combined SN sample showing that most SNe have been found at apparent magnitudes between 13 to 19 with a relatively flat distribution.

In principle, even for a given telescope and observing strategy (i.e. plate type, exposure time, etc.) $m_{\rm lim}$ changes from plate to plate depending on weather conditions but also within the same plate because of the background variations. For instance, it is most difficult to discover a SN when it appears projected on the high surface brightness, inner regions of a galaxy rather than in its outskirts. However, because of the huge number of observations involved, we cannot analyze each single case and we are therefore forced to a statistical approach. For the systematic surveys adopting a fixed observing strategy we determined a unique $m_{\rm lim}$, which is the average of the limiting magnitude for SN discovery in different plates and plate positions. It must be stressed that even if the dispersion of the limiting magnitudes of individual observations can be quite large (of the order of 0.5-1 mag) the assumption of a unique value for the $m_{\rm lim}$ of a systematic search is not critical as long as this value is well determined. A conservative estimate of the uncertainty of this crucial parameter is $\Delta m_{\rm lim} = \pm 0.5 mag$.

Both in the Asiago and Evans searches two different telescopes have been used, hence two different $m_{\rm lim}$ have been adopted for each of the two searches. In particular, for the Evans' search with the 41 cm telescope the value adopted here, $m_{\rm lim}$ = 15.0, is 0.4 mag brighter than in van den Bergh & McClure (1994; see also van den Bergh and Tammann 1991).

The adopted limiting magnitudes for the systematic searches are indicated by the dotted vertical lines in the different panels of Fig. 1. Note that, allowing for better than average observing conditions, in each search a few SNe are found at magnitudes fainter than the average $m_{\rm lim}$.

Because of the lack of a fixed observing strategy, the limiting magnitude for the OCA observations greatly change from plate to plate. Fortunately, an estimate of the limiting magnitude for each plate of this search is available, varying in the range 16 to 21 mag, and has been used in the calculation.

3. Computational recipe

The present determination of the rate of SNe is based on the *control time* method which has been introduced by Zwicky (1942) and subsequently revisited by several authors (e.g. Cappellaro & Turatto 1988, PI, PII, van den Bergh et al. 1987, van den Bergh & McClure 1994). In the following we discuss the assumptions and the algorithm adopted in the calculation.

3.1. Galaxy sample

With the exception of the Evans' search, the SN searches that we are considering are based on wide field plates which, in a single shoot, allow the surveillance of many galaxies. Therefore the galaxy sample is not defined *a priori* but was selected by extracting from a suitable list those galaxies which appear in at least one of the survey plates. As we will describe later on, for the computation we need to know for each galaxy the recession velocity, the morphological type, the luminosity and, for spirals, the axial ratio. Hence we need a list of galaxies for which these data are available and homogeneous.

In analogy to PII we use as input list the Third Reference Catalogue of Bright Galaxies (de Vaucouleurs et al. 1991, hereafter RC3). It turns out that, among the about 23000 galaxies listed in the RC3, over 10000 have been observed at least once by our surveys. Unfortunately, for many of them some of the required parameters are missing in the RC3. To reduce this problem, we consulted the Leda extragalactic database[1] and integrated missing data when available. After that, galaxies still with incomplete data were excluded from the sample (7773 galaxies remaining).

Obviously in the computation of the SN rate only the SNe discovered in one of the galaxies of the sample have to be considered. This reduces the number of SNe of our sample from 211, the total number of SNe discovered in the five searches, to 110. The reduction is especially severe for the OCA and C&T searches whose targets are distant galaxies most of which are not listed in the RC3 or for which the required data are not available, and instead has no effect for the Evans' search.

[1] The Lyon-Meudon Extragalactic Database (LEDA) is supplied by the LEDA team at the CRAL-Observatoire de Lyon (France).

3.2. SN light curves

For each galaxy of the sample, we need to compute $m_{\rm sn}(t)$, the "apparent" light curve of a possible SN in that galaxy. This depends on the SN type and on the galaxy distance or, more precisely, we can write:

$$m_{\rm sn}(t) = M_{0,\rm sn} + \Delta m_{\rm sn}(t) + \mu + A_g + < A_i >$$

where $M_{0,\rm sn}$ is the intrinsic (extinction corrected) absolute magnitude at maximum of the SN, $\Delta m_{\rm sn}(t)$ describes the light curve evolution relative to maximum, μ is the galaxy distance modulus and A_g the galactic absorption. Finally, $< A_i >$ is the average extinction of the SN population in that galaxy due to the internal dust as seen from our particular line of sight.

Similar to PI and PII, μ was derived from the Hubble flow velocity ($v3k$ in RC3) if this was larger than 1500 km s^{-1}, otherwise the distance modulus from Tully's catalog (Tully 1988) was adopted[2]. For the galactic absorption, A_g, we used the Burstein & Heiles estimates as reported in the RC3.

The light curve shapes, $\Delta m_{\rm sn}(t)$, of the different types of SNe were obtained by selecting templates from literature (Leibundgut et al. 1991, Patat et al. 1993, 1994). Because of the different kinds of searches involved we needed templates for B, V and R bands.

In the present work we calculate the rates for the three basic types of SNe namely Ia, Ib/c and II. At present the statistics are not large enough to separate Ib from Ic that, for this reason, were lumped together. In PI the adopted templates for SN Ia and SNIb/c were the same (although the absolute magnitudes at maximum were different). Instead in the present work we adopt the light curves of SN 1990I (Della Valle et al., in preparation) as template for SN Ib/c. Concerning SN II, it is well known that they exhibit quite heterogeneous photometric behaviors with different light curve shapes and maximum luminosities (cf. Patat et al. 1993, 1994). To account for this we calculate separately the rates for the two photometric classes of IIP (plateau) and IIL (linear) and derive the total rate of SNII by summing the two contributions. It turns out that the results are very similar to those which are obtained by adopting, in the calculation of the control time, a light curve which is intermediate between Plateau and Linear. No account is made for the rare class of SN IIn or for the possible existence of a separate class of faint SNII similar to SN 1987A. Some comments about the relative contribution of these classes of SNe will be made in the discussion.

More tricky is the problem of determining $M_{0,\rm sn}$ and $< A_i >$, the latter being, in general, unknown. Let us discuss this important point in some detail.

In a first approximation, we can assume that ellipticals are dust free, that is $< A_i >= 0$ mag. Therefore by taking the average absolute magnitude of SN Ia in ellipticals (Ia are the

[2] Throughout this paper H_0 = 75 km s^{-1} Mpc^{-1} is assumed.

only type of SNe found in these galaxies) we obtain directly $M_{0,\mathrm{Ia}}$. In particular, we adopt the value reported by Miller & Branch (1990), $< M_\mathrm{B} >= -18.95$ mag.

Dust extinction is certainly important in spiral galaxies. For the moment let us neglect the dependence of A_i on the inclination of the disk along the line of sight, which will be discussed in Sect. 5.2. Direct estimates of the average absorption suffered by SN Ia in face-on spirals range from $< A^B_{i,0} >= 0.4$ mag (Miller & Branch 1990) to 0.7 mag (Della Valle and Panagia, 1992), that is the average observed magnitude of SN Ia is 0.4-0.7 fainter than the intrinsic value.

On the other side there are indications that the intrinsic magnitude of SN Ia correlates with the Hubble type of the parent galaxies. Evidence are still preliminary but it seems that in spirals $M_{0,\mathrm{Ia}}$ is 0.3–0.5 mag brighter than in ellipticals (Branch et al. 1996, Hamuy et al. 1995). Given the uncertainties, we will assume that the two effects cancel out and that the "observed" absolute magnitudes ($< M_B >= M_0+ < A_i >$) of the SN Ia in ellipticals and face-on spirals are the same.

For type II (either P or L) and Ib/c there are no estimates of the average absorption in the parent galaxies. We simply assume $< M_B >$ from Miller & Branch (1990) as representative of the "observed" absolute magnitudes for these classes of SNe in face-on spirals, $< M_B >= -17.11$, -16.53 and -17.05 for Ib/c, IIP and IIL, respectively. The above assumption is probably reasonable given that, although the Miller & Branch sample includes SNe in spirals of all inclinations it certainly suffers a bias against heavily absorbed SNe.

Similar to PI we allowed for a Gaussian distribution of the SN magnitudes with variance σ corresponding to the observed ones. We stress that, as demonstrated in PI, the calculated SN rate is not very sensitive to changes in the shapes or width of the SN luminosity distribution, but instead it strongly depends on the mean value.

The need for an analytical approximation of SN light curves (cf. PI) has been eliminated and observed light curves can be readily input to the program. The adopted templates for the B band are shown in Fig. 2.

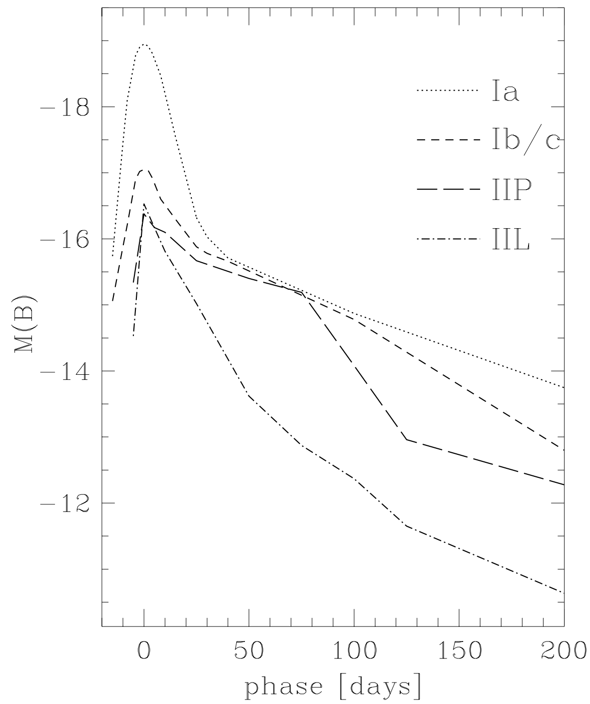

Fig. 2. B template light curves. For the calculation of the control time, the light curves are truncated 400 days after maximum.

3.3. Control time and SN rate

Once we compute for each galaxy of the sample the expected light curves, $m_{sn}(t)$, of all SN types, we can obtain the control time $ct^{\mathrm{sn}}_{\mathrm{i,j}}$, that is the interval of time during which a possible SN in the j^{th} galaxy stays brighter the m_lim of the i^{th} observation. Next we compute $tct^{\mathrm{sn}}_{\mathrm{j}}$, the total control time for the series of n observations of the galaxy (sorted on the epoch t_i) according to the following recipe:

$$tct^{\mathrm{sn}}_{\mathrm{j}} = \sum_{\mathrm{i=1}}^{\mathrm{n}} \Delta t_\mathrm{i} \times L_j c_i \qquad (1)$$

where:

$$\Delta t_\mathrm{i} = \begin{cases} ct^{\mathrm{sn}}_{\mathrm{i,j}} & \text{if } t_\mathrm{i} - t_{\mathrm{i}-1} \geq ct^{\mathrm{sn}}_{\mathrm{i,j}} \text{ or } i = 1 \\ t_\mathrm{i} - t_{\mathrm{i}-1} & \text{otherwise} \end{cases},$$

L_j is the galaxy blue luminosity and c_i is a correcting factor introduced to account for the bias against SN discovery in the nuclear regions of galaxies which will be discussed in Sect.5.1. The galaxy luminosity is introduced as a normalization factor because it has been demonstrated that the SN rate is proportional to the galaxy luminosity (Tammann 1974; PII). L_j is computed from the B^0_T magnitude reported in the RC3 and expressed in unit of solar luminosity adopting $M^B_\odot = 5.48$.

Finally, we compute ν_sn, the rate of a given SN type in each galaxy sample as:

$$\nu_\mathrm{sn} = \frac{N_\mathrm{sn}}{\sum_{\mathrm{j=1}}^{\mathrm{N_G}} tct^{\mathrm{sn}}_{\mathrm{j}}}$$

where N_sn is the number of SNe of that type discovered during the search(es) in the N_G galaxies of the sample. The rate ν_{sn} is expressed in SNu, i.e. SNe per 10^{10} $\mathrm{L}_\odot$ per century and, because of the normalization to galaxy luminosity, scales as $(H/75)^2$.

For about 1/4 of the SNe of our sample no detailed classifications are available. Most of them are of type I but for a few (10 out of 110) not even this broad classification is available. The unclassified SNe have been redistributed among the three basic types according to the observed distribution in the merged sample, that is in E-S0 100% type Ia, in spirals 35% type Ia, 15% type Ib, 50% type II.

4. The SN rates from individual SN searches

It is a useful exercise to compare the rate of SNe calculated using separately the data of each search. The results are shown in Table 1 where for each SN search we report the number of RC3 galaxies included in the search (col. 2), the average redshift of the galaxies of the sample (col. 3), the total control time for SN Ia (which gives an indication of the weight of the search on the combined sample) (col. 4), the number of SNe discovered in the RC3 galaxy sample (col. 5), and the overall SN rate (col. 6) before corrections for the biases which will be discussed in Sect. 5. Note that the numbers in the last row giving the results for the combination of the five searches are not the plain sums of the values in the columns since several SNe and galaxies are in common between individual searches and because, according to the definition, the cumulative control time is smaller (or equal if the searches do not overlap) than the sum of the control times of the individual searches.

The raw rates of the different searches reported in col. 6 differs by more than a factor 2. In the next section we will show that observational biases affect more heavily photographic surveys and, among these, those aimed to reach fainter limiting magnitudes. Therefore, the fact that the Evans' visual search gives the highest raw SN rate whereas the deep OCA and C&T searches the lowest values, is not unexpected.

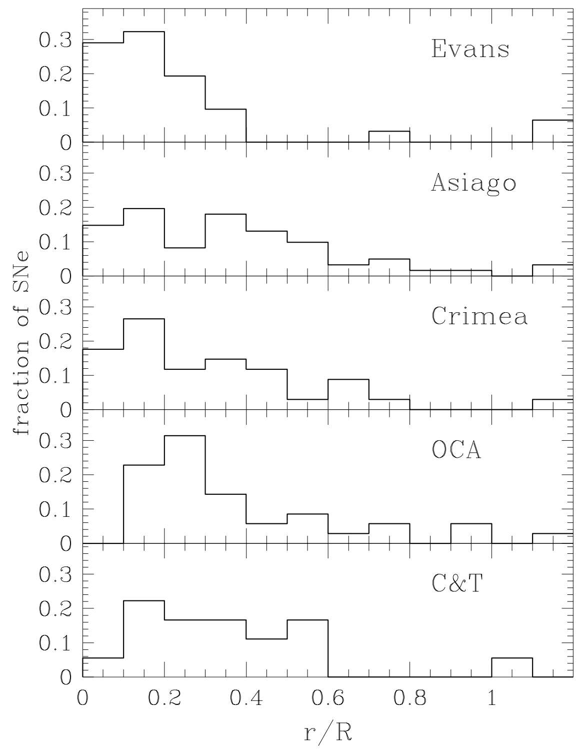

Fig. 3. In the different panels we report for each search the distributions of r/R, the ratio of the distances from the nucleus of the SNe discovered to the semi-major axis of the parent galaxies. To improve the statistics, for the Evans sample we included SNe discovered after 1989.

5. Biases on SN discovery

5.1. The nuclear regions of galaxies

It was first pointed out by Shaw (1979) that in the general list of SNe there is an apparent deficiency of objects in the inner regions of more distant galaxies compared with nearby ones. To some extent this is due to a bias in the parent galaxy sample, more distant SN parent galaxies being in the average bigger (Cappellaro & Turatto, 1996), but even after this dependence has been removed by normalizing the SN radial distances to the galaxy radius, it results that at least 40% of the SNe which explode in galaxies with recession velocities larger than 3000 km s^{-1} are lost because of this effect. The bias is found to be more important for deep photographic searches and negligible for visual and CCD searches in nearby galaxies.

To verify this effect for the search we are considering, in Fig. 3 we compare the relative radial distributions of the SNe found in the different searches. As can be seen the peak of the observed distribution shifts from the galaxy nucleus to an outer radius moving from the Evans' visual search to the photographic Asiago and Crimea surveys and even more outward for the deep OCA and C&T surveys.

In some cases, especially for very deep exposures, the nuclear bias can be due to the over-exposure on the photographic plates of the high surface brightness, inner regions of the galaxies. Even if the saturation regime is not reached, as was normally the case for the searches we considered, it is most difficult to identify a new object if it appears projected on regions of high photographic density. Also, the small scale of the telescopes usually employed for photographic surveys hampers the discovery of a SN when it projects near the galaxy nucleus. Visual and CCD searches are less biased because of a better dynamic range and, typically, larger scale.

For our purposes the consequence of this bias is that a portion of each galaxy cannot be probed by photographic SN searches. We accounted for it by introducing in equation (1) an appropriate correcting factor, c_i, measuring the fraction of the galaxy luminosity, L_j, actually surveyed by the i^{th} observation.

The value of the correcting factor, c_i, and its dependence on the search and galaxy type and distance were estimated in two independent ways:

1. we selected from the RC3 sample a few galaxies of different types, luminosities and recession velocities for which detailed surface photometry was available (e.g. Caon et al. 1990, Jørgensen et al. 1992). Then we retrieved from the archives of the Asiago, Crimea and OCA searches a few typical survey plates on which these galaxies appear. Via artificial star experiments (cf. Turatto et al. 1994) and direct examination of the plates we estimated for each galaxy the limiting radial distance within which SNe cannot be detected. With this, and knowing the luminosity profiles, we estimated the fraction of the galaxy which is unaccessible to the search. The results of this analysis can be summarized as follows: i) whereas we found significant differences from case to case, there is no clear evidence of an increase

Table 1. The average rate of SNe [SNu] for each SN search.

search	number of galaxy	$< v >$ km s^{-1}	TC(Ia) $100yr \times 10^{10}$L$_\odot$	number of SNe	SN rate [SNu]		
					raw	− nuclear bias	− inclination bias
Evans	1377	1805	72	24	0.54	0.54	0.80 (0.69 - 0.88)
Asiago	2412	3960	197	51	0.39	0.50 (0.42 - 0.58)	0.76 (0.65 - 0.86)
Crimea	2697	3603	112	33	0.41	0.50 (0.45 - 0.53)	0.76 (0.65 - 0.86)
OCA	4352	6200	72	16	0.29	0.46 (0.35 - 0.63)	0.58 (0.54 - 0.61)
C&T	1775	5040	60	12	0.21	0.34 (0.26 - 0.44)	0.44 (0.40 - 0.47)
All	7773	4320	425	110	0.35	0.45 (0.39 - 0.52)	0.66 (0.58 - 0.74)

of the bias with the galaxy distance; ii) in the case of early type galaxies the fraction of the galaxy unaccessible to the search is in the average 10-20% for the Asiago and Crimea searches, 20-40 % for the OCA search; iii) the bias seems less severe in spirals than in ellipticals (roughly by a factor 2) .

2. we can estimate the importance of the nuclear bias also by assuming that the bias is negligible for the visual search in nearby galaxies. In particular we assume that the radial distribution of the Evans' sample gives the intrinsic radial distribution of the SNe. Instead the radial distribution of the SNe found in photographic searches can be considered unbiased only outside a given radius which we adopted r/R = 0.3. Thus the fraction of SNe which is lost in photographic surveys can be estimated by normalizing the radial distributions of the different searches for $r/R > 0.3$.
   With this method we obtain correcting factors which are significantly larger that those obtained with the former one. Among the photographic surveys, the least affected appears the Crimea search (with an estimated SN deficiency of $\sim$ 25%), more biased the Asiago search ($\sim$ 35%) and the OCA and C&T searches ($\sim$ 50%). There is no apparent difference between early and late type galaxies but the SN statistics for early type galaxies are very poor. It appears that there is a small trend with the distance, the bias in galaxies with recession velocities greater than 3000 km s^{-1} being 20 − 30% more severe than in nearby ones.

Since we have no reasons to favor either result, we decided to adopt as correcting factors the averages of those estimated in the two ways described above. The values of c_i range from 0.85 for the Crimea observations of nearby spiral galaxies to 0.55 for the OCA and CTIO observations of distant ellipticals. No corrections was introduced for the Evans observations (c_i = 1).

The values of the total SN rates for each SN search corrected for the nuclear bias are reported in col. 7 of Table 1 where in parenthesis are reported the extreme values obtained using alternatively the correction factors of either method. These give an indication of the uncertainties of the correction.

As expected, after this correction the average SN rates of the different searches become very similar with the exception of the C&T rate for which the small statistics may be a problem.

5.2. The inclination of spirals

The presence of a strong bias in the discovery of SNe in inclined spiral galaxies has been first mentioned by Tammann (1974). Subsequently several authors (cf. van den Bergh & Tammann 1991, PII) have tried to give an estimate of the importance of this bias and of the possible dependence on the search and galaxy parameters.

In particular it has been claimed that the effect is negligible for CCD and visual searches though, because of the small statistics, this was not a firm conclusion. To test this point we divided our combined galaxy sample in two bins of low and high inclinations[3] and calculated the SN rates separately for the Evans' visual search and for the combined photographic searches. The results are shown in the first two rows of Tab 2, in which in columns 2 and 3 are the SN rates in galaxies of different inclination and in col. 4 their ratio. Since the statistical errors are large no definite conclusion can be drawn, but the indication is that the bias affects also the visual search although to a lesser extent than photographic searches.

Based on the general list of SNe, van den Bergh (1991) argued that the bias due to inclination is most severe for SNII and negligible for SNIa. In PII we found that the bias was roughly a factor 2 larger for type II than for type Ia. Actually, on our present larger sample we find no significant difference between SN Ia and SNII+Ib (Table 2).

We investigated also the dependence on the galaxy morphological types and found that the bias is stronger in late $Sb - Sd$ galaxies than in early $S0 - Sa$ spirals (Table 2).

In previous works (Cappellaro & Turatto, 1988; PII) the inclination bias was corrected adopting a purely empirical approach, that is simply multiplying the SN rate in inclined spirals by a proper factor so that the dependence of SN rate from galaxy inclination disappears.

Here we have adopted a different approach, assuming that the reduced efficiency of SN searches in inclined spirals is due to the large absorption suffered by SNe in these galaxies. In passing we note that since extinction is smaller for longer wavelengths this naturally implies a less severe bias for observations in the visual and red bands than for observations in the B band.

In Sect. 3.2 we already accounted for the internal absorption suffered by SNe in face–on galaxies. In addition, if both dust

[3] The inclination (α) of spirals is derived from the axial ratio R_{25} as reported in RC3 catalog using the relation α = arccos $1/R_{25}$.

Table 2. Comparison of the SN rates in spiral galaxies with small ($\alpha < 45°$) and large ($\alpha \geq 45°$) inclination.

	SN rate [SNu]		
	$\alpha < 45°$	$\alpha \geq 45°$	ratio
Evans' visual	0.85 ± 0.27	0.56 ± 0.16	1.5 ± 0.6
photographic	0.76 ± 0.12	0.39 ± 0.06	1.9 ± 0.4
Ia	0.20 ± 0.04	0.11 ± 0.02	1.8 ± 0.5
II+Ib	0.53 ± 0.11	0.30 ± 0.06	1.8 ± 0.5
S0-Sab	0.34 ± 0.10	0.28 ± 0.07	1.2 ± 0.5
Sb-Sd	0.96 ± 0.16	0.47 ± 0.08	1.9 ± 0.5
All - no corr.	0.73 ± 0.11	0.41 ± 0.06	1.8 ± 0.4
All - sec i	0.76 ± 0.12	0.66 ± 0.09	1.2 ± 0.2
All - adopted	0.87 ± 0.13	0.83 ± 0.11	1.0 ± 0.2

and SNe are distributed in slabs with common median plane, the average SN in a galaxy of inclination α will experience the additional absorption $A_{i,0} \times (\sec\alpha - 1)$, where $A_{i,0} = 1.086 \times \tau_0$ and τ_0 is the half-width optical depth of the dust layer. The value of τ_0 is still debated but typical values are $2\tau_{0,B} \sim 1$ (Bottinelli et al. 1995). A similar value ($2\tau_0^B = 1.28$) was reported by Della Valle & Panagia (1992) based on the direct measurements of the average extinction suffered by SNIa.

These estimates are appropriate for $Sb - Sc$ galaxies whereas extinction is probably smaller in early $S0 - Sa$ spirals (Valentijn 1990). Based on these considerations we adopt $A_{i,0}^B = 0.7$ for $Sb - Sd$ galaxies and $A_{i,0}^B = 0.35$ for the other spirals.

Once the correction for the inclination effect is included in the computation, the ratio of the SN rates in galaxies with inclination $\alpha < 45°$ to that of more inclined galaxies decreases from 1.8 ± 0.4 to 1.2 ± 0.3, consistent with the expected intrinsic value of 1 (Table 2). However a closer examination reveals that the correction is too small for galaxies of intermediate inclination and too large for edge-on spirals. This is better seen in the upper panel of Fig. 4 where we compare the SN rates in spiral galaxies of different inclinations before and after the afore mentioned correction. Changing the value of $A_{i,0}$ would not help in this respect, since the problem is the functional dependence on $\sec\alpha$.

Thus we reversed the problem asking which functional dependence of A_i on inclination would give the appropriate correction. In practice we have determined for each inclination bin the value of the absorption needed to cancel the dependence of the SN rate on inclination. The result of this test is shown in Fig. 4 and can be interpreted in two ways, either the inclination bias is due not only to extinction in the parent galaxies but also to some additional effect (e.g. related to the galaxy surface brightness) or the assumption of a plane parallel geometry for the dust distribution has to be rejected.

The latter interpretation was favored also by van den Bergh (1991) based on the observed distribution of SN in inclined galaxies. He suggested that SNe, in particular SN II in $Sc - Sd$ galaxies, explode at the bottom of chimney-like dust structures. In this scenario the value of A_i as a function of inclination depends on the details of the geometry but, schematically, we expect the extinction to be negligible for small inclination and suddenly to rise when the line of sight to the SNe intersects the wall of the chimney. Actually the empirical extinction law shown in the bottom panel of Fig. 4 could better be explained by a model in which SNe are associated to dust clouds which are elongated along the disk of the galaxy. The fact that the extinction peaks for galaxy inclination around 60° could suggest an axial ratio for the clouds of the order of two. This scenario is consistent with other recent findings. From one side Beckman et al. (1996) showed that dust in spirals has an irregular distribution with an optical depth one order of magnitude larger in the spirals arms than in the inter-arm regions. On the other side SNII, Ib/c and at least half of SN Ia are found to be associated with the spiral arms (Della Valle & Livio 1994, Bartunov et al. 1994), therefore they occur in regions of higher than average extinction.

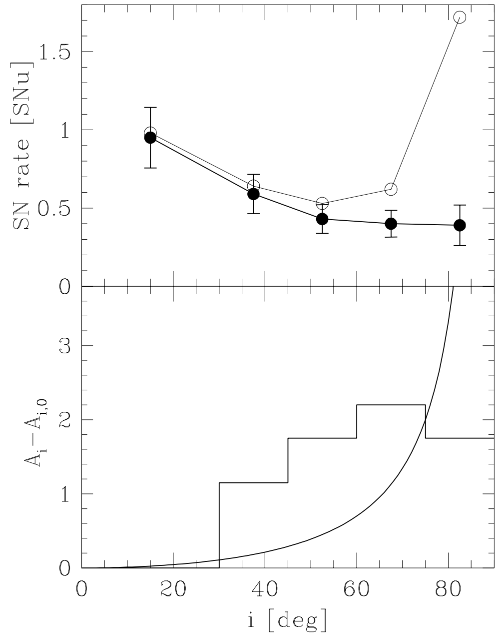

Fig. 4. Upper panel: the SN rates in spirals of different inclination as observed (filled circles) and after the $A_{i,0}(\sec\alpha - 1)$ correction has been applied (open circles). Error-bars relative to the SN statistics are indicated. Bottom panel: comparison between the $A_{i,0}(\sec\alpha - 1)$ extinction law and the extinction law which eliminates the dependence of the SN rates on galaxy inclination.

Conservatively, we will adopt as the functional dependence of the extinction on galaxy inclination the average of the $\sec\alpha$ and of the "empirical" extinction laws reported in Fig. 4. The SN rates for each SN search after this correction are reported in the last column of Table 1. Again the values obtained using alternatively one of the two extinction laws are reported in parenthesis to give an indication of the errors.

We stress that the estimate of the SN rate in spirals, in particular late spirals, are quite sensitive to the adopted value of

Table 3. The rate of SNe in SNu corrected for selection effects.

galaxy	No. of SNe			SN rate [SNu]			
type	Ia	Ib	II	Ia	Ib	II	All
E	7.0			0.13	≤ 0.03	≤ 0.04	0.13
S0	9.0			0.18	≤ 0.04	≤ 0.04	0.18
S0a-Sa	7.1	2.4	1.5	0.25	0.16	0.16	0.57
Sab-Sb	8.7	2.3	11.0	0.17	0.09	0.53	0.78
Sbc-Sc	14.6	5.9	16.5	0.23	0.20	0.75	1.19
Scd-Sd	5.0	0.5	10.5	0.24	0.04	1.22	1.49
Others	3.7	1.8	2.5	0.27	0.22	0.41	0.90
E-S0	16.0	0.0	0.0	0.15	≤ 0.02	≤ 0.02	0.15
S0a-Sb	15.8	4.7	12.5	0.20	0.11	0.40	0.71
Sbc-Sd	19.7	6.3	27.0	0.24	0.16	0.88	1.27

the average extinction in galaxies. For istance if we adopt for $Sb - Sd$ $A^B_{i,0}$ = 0.35 (instead of 0.7) the estimate of the SN rate in these galaxies decreases by about 40%. With this choice however the inclination bias in not corrected, that is the rate of SNe in spirals with $\alpha < 45°$ remains 1.5 larger than in more inclined spirals.

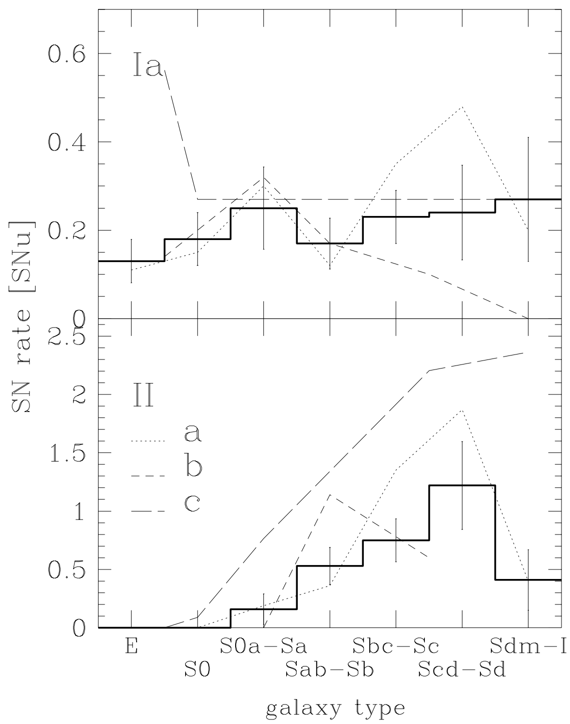

Fig. 5. The rate of SNIa and SNII in the different type of galaxies (upper and lower panel, respectively). SNIb/c are not reported because of the poor statistics (see Table 3). Error-bars due only to SN statistics are indicated. For comparison we report the rates of PII (a), van den Bergh & McClure 1994 (b) and Tammann et al. (1994) (c), scaled to the value of the Hubble constant adopted here (H=75 km s^{-1} Mpc^{-1}).

6. SN rates and uncertainties

With the recipe, the parameters and the correcting factors discussed above we were finally able to compute the SN rates for the combined sample of the five SN searches. The results are reported in Table 3 and the dependence of the SN rates on galaxy morphology is displayed in Fig. 5 for SN Ia and SN II (the statistics for SN Ib are too small). For comparison, recently published estimates are also reported. In general the present results are consistent with the previous estimates, with a few distinctions.

We find that the rate of SNIa increases by about a factor 2 moving from ellipticals to late spirals. This confirms the results of PII but is in contradiction with those of Tammann et al. (1994). Actually, our results are in excellent agreement with those of Tammann et al. for the SNIa rate in spirals (note however their *a priori* assumption of a constant rate from early to late spirals) but our estimate of the rate in ellipticals is a factor 4 smaller. As discussed in Turatto et al. (1994) we believe that the disagreement is due to a bias of SN discoveries in the "fiducial sample" of galaxies.

Traditionally, the discovery of SNIa in ellipticals has been the main argument for placing the progenitors of SNIa among old stellar population. On the other side the fact that the rate in late spirals, which are dominated by stars of population I, is twice that in ellipticals implies that the average age of the progenitors of SNIa in spirals is younger.

It might be expected that the different ages of the progenitor systems lead to observable differences in the outcomes of the explosions. Indeed there are increasing evidences that the variance of SNIa properties in spirals is significantly larger than in ellipticals (Filippenko 1996). Therefore our results support the idea that the progenitors of SN Ia in spirals can have quite different ages.

We also confirm (cf. PII) that SNIb in late spirals are "only" 40% of all type I SNe, ruling out definitively the very high rate suggested by Muller et al. (1992).

The rate of SNe, in particular SNII, is found to peak in the late $Scd - Sd$ spirals instead that in $Sab - Sb$ as indicated by van den Bergh & McClure (1994). The odd result by van den Bergh & McClure is most likely due to the small statistics but differences in the galaxy catalog might also contribute. In fact their reference catalog was the Revised Shapley-Ames Catalogue (Sandage & Tammann, 1981) while we used the RC3.

Finally we point out that the present estimate of the overall SN rate in late spirals is $\sim$ 40% lower than in PII, although the difference is within the errors. This is mainly due to the smaller correction factors adopted here for the galaxy inclination bias. We should stress that because several galaxies have independent observations in different searches, the dependence of the new SN rates on the adopted input parameters and bias corrections is reduced compared with PII.

In most of the previous works, SN rates are presented only with errors due to SN statistics since, because of the poor statistics, these errors dominated over others. In our case, the enlargement of the SN sample strongly reduces the relative importance of statistical errors and other contributions become important, in particular when considering broad bins of galaxy morphological types. In Table 4 we have summarized the different components and the total errors calculated as follows:

1. statistical errors are those due to the statistics of the SN events which are assumed to conform to a Poisson distribution.
2. parameter errors are estimated from the differences of the SN rates computed using extreme values of the input parameters. The cumulative errors due to the parameters are calculated by adding quadratically the individual contributions, assuming that they are uncorrelated. We included an uncertainty of ± 0.25 mag on the SN absolute magnitudes, ± 0.5 mag on the adopted limiting magnitude of each search, and $\pm 50\%$ errors on the absolute magnitude dispersions. For the uncertainties on the shapes of the light curves we adopted for type I the difference between Ia and Ib light curves and for type II the difference between IIP and IIL. We neglected the errors on the galaxy data, that is on radial velocities, luminosities, etc.
3. both the corrections for the nuclear and for the inclination biases have been determined using the average between the correcting factors estimated with two different methods (cf. Sect. 5). As a measure of the propagated uncertainties of each correction we have adopted the difference between the SN rates calculated applying either method.

Finally the total errors are estimated by adding quadratically the three components. As can be seen from Table 4, once the galaxies have been grouped in broad morphological bins the contributions of the three sources of errors are similar. By comparing Table 4 with the similar table of PII it appears that the relative errors of the present estimates are reduced, in the average, by about 20%.

Using the SN rates given in Table 3 we can compute the expected rate of SNe for any given galaxy (or galaxy sample) for which morphological type and luminosity are known. In particular we can estimate the expected rate of SNe in the Galaxy with the assumption that the morphological type is Sb and the luminosity is 2×10^{10} $L_{\odot}$. In a millennium we expect 4 ± 1 SN Ia, 2 ± 1 SN Ib/c and 12 ± 6 SN II, where the uncertainties in the Galaxy morphological type and luminosity are not included.

It is also interesting to compute the expected number of SNe for the "fiducial sample" of galaxies (Tammann et al. 1994) which includes all galaxies from the RSA catalog with recession velocity ≤ 1200 km s^{-1}. By adding the individual contribution of each galaxy we predict 28 SNe per decade. The fraction of SNe actually discovered in these galaxies depends on the intensity of SN search programs and has been growing progressively from 2 SNe per decade in the first half of the century, to 12 SNe in the period 1950-1975, to 20 SNe in most recent years. Given the uncertainties, the last value is consistent with the claims that nowadays nearly all SNe which explode in galaxies of the fiducial sample are discovered.

6.1. Additional types of SNe

Recently a few intrinsically faint SNIa (e.g. SN 1991bg) have been discovered and the question raised if they could represent a separate class or, instead, they are in the tail of a single luminosity distribution for SNIa (the magnitude of SN 1991bg, $B = -16.34$, is about 3σ fainter than the average for SN Ia, Turatto et al. 1996). While the question is still open, it is clear that there can be a severe bias against these faint SNe in magnitude limited SN searches.

By using the control time method we can give an estimate of the rate of this possible class of SNIa assuming that: i) the average absolute magnitude for these SNe is that of SN 1991bg; ii) to this class belong also SNe 1986G, 1992K and SN1991F (Turatto et al. 1996). Note that all these SNe but SN1991F are included in our SN sample.

For faint SN Ia we obtain a rate of 0.05 SNu (averaged through all galaxy types) implying that, although the observed faint Ia events are only 5% of the SNIa in our combined sample, actually they may be about 1/4 of all SNIa explosions (but not the majority of SN Ia events as suggested by Schaefer 1996).

SN II show a wide variety of behaviors. In the present calculation we assumed that SN II are a mixture of SN IIL and IIP with the former being about 0.5 mag brighter than the latter, and both showing wide dispersions in absolute magnitudes. With these assumptions we found that the rates of the two subclasses are almost identical both peaking in late spirals.

In this approach SN 1987A is considered as a normal type II plateau reaching a peak luminosity at the lower end of the luminosity distribution. However it has been argued that SN 1987A could be the representative of a separate class of faint SN II most of which remain undiscovered because of the low intrinsic luminosity (Schmitz & Gaskell 1988). By assuming that this is the correct interpretation, we can give an estimate of the rate of this subclass of SNII, adopting as average parameters for this class those of SN 1987A. The number of 87A-like SNII in our sample ranges from 1 (SN 1987A itself) to 3, if we accept SNe 1973R and 1982F as members of this class (van den Bergh & McClure 1989). These two SNe were both as faint as SN 1987A, but their red colors were most likely due to high extinction (Patat et al. 1994) and they were probably intrinsically "normal" SNII plateau. Conservatively, we estimate that the rate of 87A-like SNe in spirals ranges from 0.06 to 0.18 SNu, that is from 10% to 30% of the rate of "normal" SNII, and are certainly not the preferred outcomes of core collapse.

Another possible separate subclass of SNII is that of type IIn (Schlegel, 1990). SNe assigned to this class show an unusual profile for the Hα emission with a narrow peak sitting on a broad base and no sign of P-Cygni absorption. It is believed that this feature results from the interaction of the SN ejecta with a dense circumstellar material (CSM). This could explain also the bright absolute magnitude and very slow luminosity decline which has been observed in some SN IIn, e.g. SN 1988Z (Turatto et al. 1993a). On the other side if the CSM is not so dense the interaction may be weaker and the light curve could

Table 4. The errors of the rate of SNe (in SNu)

galaxy	statistical			parameters			biases			SN rate [SNu]		
type	Ia	Ib/c	II	Ia	Ib/c	II	Ia	Ib/c	II	Ia	Ib/c	II
E-S0	0.04			0.03			0.02			0.15 ± 0.06		
S0a-Sb	0.05	0.05	0.11	0.04	0.03	0.12	0.04	0.02	0.09	0.20 ± 0.07	0.11 ± 0.06	0.40 ± 0.19
Sbc-Sd	0.05	0.06	0.17	0.05	0.04	0.24	0.05	0.04	0.22	0.24 ± 0.09	0.16 ± 0.08	0.88 ± 0.37

be very similar to that of other SNII, as is the case of SN 1989C (Turatto et al. 1993b)

Two of the SNe included in our sample are classified IIn, namely SNe 1987B and 1987F. SN 1988Z, discovered independently both in the Asiago and OCA searches, is not considered because the parent galaxy is not listed in the RC3. Based on the two observed cases we estimates that the rate of SNIIn in spirals is in the range 0.01-0.03 SNu (the uncertainties result because of the variance in the photometric behaviors). This means that, although in recent years about 15-20% of the observed SN II are classified IIn, actually they are only 2% to 5% of SNII explosions.

7. Conclusions

The major improvement of the present work with respect to previous determinations of the frequency of SNe is in the SN statistics. In fact, our sample counts 110 SNe discovered by the five SN searches in a sample of 7773 RC3 galaxies. Therefore, our SN statistics almost doubles that of previous works (the rates in PII were based on 65 SNe) and is even better than that of the so-called "fiducial sample" which in 1990 counted 96 SNe (Tammann et al. 1994). Also we made an effort to update the algorithm and to estimate the importance of the search biases, using different approaches. Finally, it is important to note that by merging the databases of SN searches with different characteristics we reduced the impact of the assumptions on the input parameters, of the nuclear and inclination bias corrections, and of other possibly hidden, search biases. In this regard, especially important is the inclusion of the Evans' visual search.

The main results of the computation are the following:

1. the rate of SNIa in ellipticals, 0.13 SNu, is confirmed to be lower than that in spirals;
2. the most prolific galaxies are late spirals where 2/3 of the SNe are of type II (0.88 SNu);
3. SN Ib/c are relatively rare, being at most 40% of all SN I (0.16 SNu in late spirals);
4. if SNe 1991bg and 1987A are considered the prototypes of separate classes of faint SNIa and SNII, respectively, the overall SN rates should be raised only by $20 - 30\%$.
5. The average frequency SN IIn in spirals is $\leq$ 0.03 SNu. The fact that they constitute about 20% of all SNII presently discovered is due to their high intrinsic brightness.

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