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Dynamical model of convection in stellar cores

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Abstract. In this work, the classical mixing-length theory is left aside and an alternative picture of convection is considered. This new model assumes that, in stellar conditions, convective transport is mainly achieved by turbulent plumes. This means of energy transport has been examined recently in the context of convective envelopes. Herein, the same approach is applied to the case of convective cores where the physical conditions are somewhat different. We show that turbulent plumes can indeed exist in such domains and may be efficient in the transport of heat. Unfortunately, a free parameter is introduced: the model does not constrain the total number of plumes. However, limits on the value of this parameter are given and the results obtained within these limits do not vary too drastically. Overshooting from convective cores is further evidenced in this model.

Key words: convection – waves – stars: interior

1. Introduction

For decades, our description of stellar convection has relied essentially on the mixing-length theory (MLT). This formalism has been introduced in stellar structure studies to solve the problem of energy transport in convectively unstable zones (Vitense 1953; Böhm-Vitense 1958). The main purpose of this approach is to calculate the mean temperature gradient in these regions and the MLT seems to be quite effective in this task. However, this success is mainly due to the existence of a free parameter whose value is chosen to reproduce the Solar radius. According to the MLT, this parameter, the ratio of the mixing-length to the pressure scale height, defines the characteristic length of convective motions.

Actually, the mixing-length value influences the temperature profile only in the outer layers where the density is very low, making radiative leakage important and convective transport inefficient. For deep enough layers, the temperature gradient remains very close to the adiabatic gradient value (Kippenhahn & Weigert 1991). Thus, in the Solar case, differences in the structure of the convective zone due to different choice of this parameter are noticeable only in the outermost part of the convective envelope (Spruit et al. 1990).

On the other hand, when we are interested in stellar convection from a dynamical point of view, the MLT appears to be not always adequate. This approach describes convective transport as a turbulent process in which turbulent eddies are assumed to mix with the surrounding medium after a distance equal to a mixing-length has been crossed. This way, energy is carried between two points that are a mixing-length apart. This characteristic length is further assumed to represent the mean size of the convective elements. This last assumption raises difficulties when the size of the eddies is larger than the size of the convective domain as it might happen in cores. The MLT picture must obviously fail in these cases and the quantities obtained from this formalism may be questionable.

The knowledge of the magnitude of the velocity is needed when problems such as convective penetration or the production of internal gravity waves are considered. These two processes are known to influence stellar structure and evolution. Overshooting from stellar cores modifies the evolution by increasing the mass of the mixed domain and thus, the quantity of available nuclear fuel. The extent of overshooting appears to be an important parameter of the evolution of stars more massive than the Sun and depends on the velocity field within the convective domain (Roxburgh 1989; Zahn 1991). For their part, internal waves generated by convective motions may contribute in a significant way to the transport of both chemical elements and angular momentum in radiative regions as several studies suggest (Press 1981; García López & Spruit 1991; Schatzman 1993; Montalbán 1994). The efficiency of the transport process also depends on the velocity field at the interface between the unstable and stable zones. A better description of stellar convection is then required to obtain a fair estimate of the influence of these processes.

The idea of introducing turbulent plumes rest on recent numerical simulations of stellar convection. We briefly summarize here the most salient features displayed by these simulations (references may be found in Rieutord & Zahn 1995, in the following, we will refer to this work as RZ). Stellar convection seems to operate essentially through narrow descending flows while the upward motions spread over a bigger volume and are much slower. The downflows present an important contribution

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to the dynamics and the heat transport within the convective region. These flows are lasting structures and may well be laminar plumes. Actually, these plumes should be turbulent: it is a well known fact that numerical simulations are not able to reproduce the highly turbulent nature of convective motions in stars. Relying on these considerations, Rieutord and Zahn (1995) have suggested a simple model of the Solar convective zone (SCZ) in which descending turbulent plumes ensure the totality of the convective transport.

The dynamical quantities afforded by this model have already been exploited in the study of the diffusion of chemical elements in the Solar radiative zone, the transport being induced by internal gravity waves (Montalbán 1994). Lithium depletion in the Sun seems to be reproduced satisfactorily by this approach.

It appeared interesting to extend this picture of convection to conditions prevailing in stellar cores and, thus, provide an alternative to the MLT. The same assumptions as in RZ have been made: principally, turbulent plumes have the same structure in stars and on Earth. The main differences with RZ are that we have to take into account the sphericity of the convective domain, the variation of gravity with depth and the heat sources due to nuclear reactions. In Sect. (2), we recall the properties of turbulent plumes and establish the governing equations. In Sect. (3), the model is applied to a $2M_\odot$ star and the main results are emphasized. Finally, in Sect. (4), we try to outline the limitations of this model.

2. Turbulent plumes in a convective core

A convective core is assumed to reach a statistically mean stationary state in which the motions can be divided into two distinct kind of flows: the ascending motions, represented by an unspecified number (N) of turbulent plumes, and the interstitial background, called interplume medium. The case of descending plumes will not be considered, this choice will be argued in Sect. (4).

We assume the convective transport to be entirely carried out by the plumes. These plumes originate from sources located near the stellar center where energy generation is the largest. Local inhomogeneities are expected to produce large enough density fluctuations to initiate these plumes.

The core will be described as a fully ionized perfect gas in hydrostatic equilibrium, the radiative pressure will not be taken into account. We further simplify the treatment by assuming spherical symmetry, so rotation and magnetic fields are neglected. As mentioned previously, convection in such deep layers is very efficient and leads to a nearly isentropic stratification in the unstable domain. This result will not be put in question, the temperature gradient will be given the adiabatic value throughout the whole core.

To keep as close as possible to RZ, the same notations will be used. However, it is more convenient here to let the vertical coordinate increase in the upward direction.

2.1. Turbulent plumes features

Turbulent plumes are boundary-free shear flows that are driven by buoyancy forces. Among this family of flows, we have to distinguish between suddenly released buoyant elements, called thermals, and flows that are steadily supplied by buoyancy during their motion. Plumes are of the latter kind. However, the motion is not governed by buoyancy forces only. Shear instabilities give rise to turbulence and the flow also evolves under the influence of this turbulence. At the edge of plumes, mesoscale turbulence captures matter from the ambient medium (Turner 1986) so plumes broaden during their motion due to this turbulent entrainment. In a first approximation, the structure of a turbulent plume can be said to result from a balance between buoyancy forces and turbulent entrainment.

When steadily supplied by hot matter at its source, a plume is a long-lived structure and presents the simplifying advantage of reaching a mean stationary state. In terrestrial conditions, a self-similar régime appears in most of the flow (Turner 1969, 1986). This régime is characterized by well established features. If we consider an axisymmetric flow, the mean axial velocity inside the plume is fairly well described by a Gaussian function (List 1982). Then, in spherical coordinates, if the plume axis is set on a radius, the horizontal distribution of the mean radial velocity is given by

$$v_{\rm r}(r,\theta) = V(r)\,{\rm e}^{-(r\theta/b)^2},$$

$b = b(r)$ is a measure of the horizontal extent of the plume and will be called the effective radius; the distance from the axis of the plume is equal to $r\theta$. Because of the symmetry, there is no dependence on the φ-component.

Furthermore, the density contrast is closely related to the axial velocity and its horizontal variation inside the plume may be obtained from the same bell-shaped curve (List 1982):

$$\delta\rho(r,\theta) = \rho(r,\theta) - \rho_0(r) = \Delta\rho(r)\,{\rm e}^{-(r\theta/b)^2},$$

the subscript 0 refers to a physical quantity outside the plume, such functions depend on r only.

The turbulent entrainment is usually taken into account in a form introduced by G.I. Taylor (Morton et al. 1956). It is written as a boundary condition at the edge of a plume; if we set this boundary to be at $\theta_{\rm b}$, the turbulent entrainment is written

$$2\pi r\theta\, v_\theta(r,\theta)\,|_{\theta=\theta_{\rm b}} = -2\pi\, b(r)\,\alpha |v_{\rm r}(r,0)|. \quad (1)$$

According to Taylor's hypothesis, the entrainment rate is proportional to a characteristic velocity that is the mean axial velocity of the plume. The proportionality coefficient α is a parameter whose value is experimentally determined. Afterwards, we will use the commonly assumed constant value $\alpha = 0.083$ (Turner 1986). The above relation states in a simple way that there is a horizontal flow of matter into the plume, whatever the plume is going up or down.

2.2. Equations of motion

Considering a single plume in a steady state, we now derive the equations governing its mean dynamics. The plumes are assumed to be far from each other so they do not have direct interactions. As in RZ, the basic equations are the stationary equations of continuity, conservation of momentum and conservation of energy where viscous terms have been neglected:

$$\nabla\cdot(\rho\boldsymbol{v}) = 0, \tag{2}$$

$$\nabla\cdot(\rho\mathsf{vv}) = \rho\,\boldsymbol{g} - \nabla P, \tag{3}$$

$$\nabla\cdot\left(\frac{1}{2}\rho v^2\boldsymbol{v} + \rho h\boldsymbol{v}\right) = \rho\varepsilon + \rho\boldsymbol{v}\cdot\boldsymbol{g} - \nabla\cdot\boldsymbol{F}_{\rm rad}, \tag{4}$$

ρ is the density, $\boldsymbol{v}$ the mean velocity, vv the tensorial product of the mean velocity, $\boldsymbol{g}$ the gravity acceleration, P the pressure, h the specific enthalpy, ε the energy production rate per unit mass and $\boldsymbol{F}_{\rm rad}$ the radiative flux. All these quantities describe physical conditions inside the plume.

Before proceeding further, we make the following simplifications. The flow is assumed to have no component in the azimuthal direction ($v_\varphi = 0$) and to be in pressure balance with the surrounding medium, pressure fluctuations are thus neglected. Perturbations of the gravitational field, the energy production rate and the radiative flux related to density and temperature fluctuations are not taken into account either. Hence,

$$\nabla P = \nabla P_0 = \rho_0\,\boldsymbol{g},$$

$$\boldsymbol{g} = \boldsymbol{g}_0 = \nabla h_0,$$

$$\rho\varepsilon = \rho_0\varepsilon_0,$$

$$\boldsymbol{F}_{\rm rad} = \boldsymbol{F}_{\rm rad,0}.$$

Dissipative processes are assumed to be small enough so they are neglected: radiative transfer from the plume to the ambient medium is not considered and the motion is adiabatic. These simplifications allow us to rewrite Eq. (2), the radial component of Eq. (3) and Eq. (4) as follows:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\rho v_{\rm r}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\,\rho v_\theta\right) = 0, \tag{5}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\rho v_{\rm r}^2\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\,\rho v_\theta v_{\rm r}\right) = -\delta\rho\,g_0, \tag{6}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\rho e v_{\rm r}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\,\rho e v_\theta\right) = \rho_0\varepsilon_0 - \frac{1}{r^2}\frac{\rm d}{{\rm d}r}\left(r^2F_{\rm rad,0}\right), \tag{7}$$

where $e = \frac{1}{2}v^2 - c_{\rm p}T_0\frac{\delta\rho}{\rho_0}$.

The enthalpy fluctuations have been related to density fluctuations through the perfect gas law. Afterwards, the contribution of the horizontal velocity to the kinetic energy will be omitted.

The next step is to integrate each equation on the horizontal section of the plume. We shall assume that the horizontal extent remains small compared to the distance traveled so that the geometry is locally flat and we can take $\sin\theta \simeq \theta$. Plumes are narrow flows and this hypothesis is amply justified (Tennekes & Lumley 1972). Another difficulty arises when we have to define the outer limit of the turbulent plume. The colatitude θ should vary from 0 to $\theta_{\rm b}$ that is not precisely defined. However, the edge of the flow may be understood as the place where the radial velocity becomes zero and the density contrast vanishes. Then, with the fast decrease of the Gaussian function, the upper limit of the integral may be formally taken equal to infinity.

The energy equation needs more care than the other two equations. If we integrate the right-hand side of Eq. (7) the same way as its left-hand side, we would just take account of the nuclear energy generated inside the plume. In our model, N plumes are said to transport the whole energy. To reproduce this assumption, the r.h.s should be replaced, after integration, by $4\pi r^2\rho_0\varepsilon_0/N$ where $4\pi r^2\rho_0\varepsilon_0$ is the total energy produced on the sphere of radius r. This way, the whole luminosity is equally shared among the N plumes and each plume is treated in the same fashion as the others. Such considerations also apply to the radiative flux. A straightforward integration would forget the contribution of the interplume medium to the radiative transfer. This problem may be solved by using the same expedient: the integrated radiative term is replaced by $-{\rm d}(4\pi r^2F_{\rm rad,0}/N)/{\rm d}r$.

Finally, the integrated equations lead to the following set of ordinary differential equations where the unknowns are ξ, b and V:

$$\frac{\rm d}{\rm dr}\left[b^2\rho_0V\left(\frac{1+\xi}{2}\right)\right] = 2\alpha\,b\,\rho_0|V|, \tag{8}$$

$$\frac{\rm d}{\rm dr}\left[b^2\rho_0V^2\left(\frac{1+2\xi}{3}\right)\right] = 2\rho_0g_0b^2(1-\xi), \tag{9}$$

$$\frac{\rm d}{\rm dr}\left[L_{\rm conv} + L_{\rm kin}\right] = 4\pi r^2\rho_0\varepsilon_0 - \frac{\rm d}{\rm dr}L_{\rm rad,0}, \tag{10}$$

with $\xi = 1 + \frac{\Delta\rho}{\rho_0}$, $\quad L_{\rm conv} = \frac{1}{2}N\pi b^2\rho_0Vc_{\rm p}T_0(1-\xi)$,

$L_{\rm kin} = \frac{1}{6}N\pi b^2\rho_0V^3$ and $L_{\rm rad,0} = 4\pi r^2F_{\rm rad,0}$.

ξ represents the density contrast, $L_{\rm rad,0}$ is the radiative luminosity and $L_{\rm conv}$ and $L_{\rm kin}$ are the convective and kinetic luminosities carried by N turbulent plumes. Equations (8) and (9) are the same as Eqs. (4) and (5) in RZ; Eq. (10) differs from Eq. (7) in RZ because, here, the sum of convective and kinetic luminosities is no more constant with depth. This last equation integrated in the radial direction yields

$$L_{\rm nuc} - L_{\rm rad,0} = L_{\rm conv} + L_{\rm kin}$$

where $L_{\rm nuc} = \int_0^r 4\pi r'^2\rho_0(r')\varepsilon_0(r')\,{\rm d}r'$.

This relation states that the total luminosity, $L_{\rm nuc}$, is transported partly by the radiation, partly by the plumes, in kinetic and enthalpy fluxes form.

From now on, we will neglect the density contrast except in the buoyancy and enthalpy terms: ξ is set equal to unity everywhere else.

2.3. The isentropic static core

Quantities corresponding to physical conditions outside the plume flows appear in Eqs. (8)-(10), the solution of the system requires a specification of the stratification of the static medium surrounding the plumes. This medium may be fairly well modeled by an isentropic perfect gas in hydrostatic equilibrium. This gas is in a fully ionized state due to the high temperatures pertaining to stellar cores. Such a medium is a polytrope of index $n = 3/2$ (Kippenhahn & Weigert 1991). Temperature and density distributions are obtained from the well-known Lane-Emden equation:

$$\frac{1}{x^2}\frac{\mathrm{d}}{\mathrm{dx}}\left(x^2\frac{\mathrm{d}y}{\mathrm{d}x}\right) = -y^{3/2}. \tag{11}$$

Temperature and density distributions are respectively given by

$$T_0 = T_{\rm c}\, y\ , \quad \rho_0 = \rho_{\rm c}\, y^{3/2},$$

where the subscript c denotes the central value of the given quantity; x is the non-dimensional radius defined by

$$x = \frac{r}{r_0} \ \text{ where } \ r_0^2 = \frac{5}{2}\frac{P_{\rm c}}{4\pi G\rho_{\rm c}^2}.$$

For a perfect monoatomic gas, the equation of state and specific heat under constant pressure are

$$P = \frac{\mathscr{R}}{\mu}\rho T,\ c_{\rm p} = \frac{5}{2}\frac{\mathscr{R}}{\mu} \ \text{ with } \ \mu = \frac{1}{1.5X + 0.25Y + 0.5}.$$

Here, μ is the mean molecular weight of a fully ionized medium, X and Y are the respective mass abundances of hydrogen and helium. The chemical composition will be considered homogeneous through the entire core, as a result of the mixing induced by the convective motions. Therefore, the specific heat is a constant.

Once temperature and density variations are known, gravity and radiative flux may be determined. In spherical symmetry, the gravitational acceleration is related to the mass distribution in a simple way:

$$g_0 = \frac{G}{r^2}\int_0^r 4\pi r'^2 \rho_0(r')\,\mathrm{d}r'. \tag{12}$$

As stellar interiors are optically thick, the radiative flux may be expressed in the Eddington approximation:

$$F_{\rm rad,0} = -\frac{16\sigma}{3}\frac{T_0^3}{\kappa_0\rho_0}\frac{\mathrm{d}T_0}{\mathrm{d}r}. \tag{13}$$

Both the mean opacity, κ_0, and thermonuclear reaction rates vary with density, temperature and chemical composition. Whereas approximate analytic power laws are available, we preferred to make use of subroutines written for the stellar evolution code CESAM (Morel 1993; G. Berthomieu et al. 1993), these subroutines rely on tables from Caughlan & Fowler (1988) for the nuclear reactions and on opacities from Iglesias et al. (1992). This choice has been dictated by the fact that if the opacity is underestimated, the radiative flux is overestimated in consequence and the boundary of the convective core is not accurately enough determined.

The physical constants $\mathscr{R}$, G and σ are, respectively, the perfect gas constant, the constant of gravitation and the Stefan-Boltzmann constant.

2.4. Boundary conditions

Near the central singularity, thermodynamical quantities are nearly constant and may be taken equal to their central values. It follows that

$$g_0 \simeq \frac{4\pi}{3}G\rho_{\rm c}r,$$

$$L_{\rm nuc} \simeq \frac{4\pi}{3}\rho_{\rm c}\varepsilon_{\rm c}r^3,$$

$$L_{\rm rad,0} \simeq \frac{4\pi}{3}\frac{16\sigma}{3}\frac{T_{\rm c}^4}{\kappa_{\rm c}\rho_{\rm c}r_0^2}r^3.$$

Near the source of a plume, the kinetic energy may be neglected before the enthalpy term because the velocity is small. Then, Eqs. (8)-(10) imply:

$$b \simeq \frac{2\alpha}{3}r, \tag{14}$$

$$1-\xi \simeq \left[\frac{54}{\pi G\rho_{\rm c}N^2\alpha^4 c_{\rm p}^2 T_{\rm c}^2}\left(\varepsilon_{\rm c} - \frac{16\sigma}{3}\frac{T_{\rm c}^4}{\kappa_{\rm c}\rho_{\rm c}r_0^2}\right)^2\right]^{1/3}, \tag{15}$$

$$V \simeq \left[\frac{2\pi}{3}G\rho_{\rm c}(1-\xi)\right]^{1/2} r. \tag{16}$$

In this region, the plumes present a self-similar régime. These relations define the conditions at the source of a turbulent plume, they will be used as lower boundary conditions.

The upper boundary of the convective core is reached when $L_{\rm nuc} = L_{\rm rad,0}$, following the classical Schwarzschild criterion.

2.5. Introduction of a counter flow

The model above is incomplete. It lacks a description of the dynamics in the interplume medium. In fact, the plumes transport both energy and mass, thus, there is a mass flux in the upward direction. If we require that the whole core should satisfy mass

Table 1. Temperature, density, energy generation rate, mean opacity and mass abundances of hydrogen and helium of the stellar model at the core center and at the Schwarzschild boundary.

r	0	1.232	10^{10} cm
T	2.273	1.742	10^7 K
ρ	72.66	48.75	g cm^{-3}
ε	877.02	12.92	erg g^{-1} s^{-1}
κ	0.542	0.690	cm^2 g^{-1}
X	0.399		
Y	0.579		

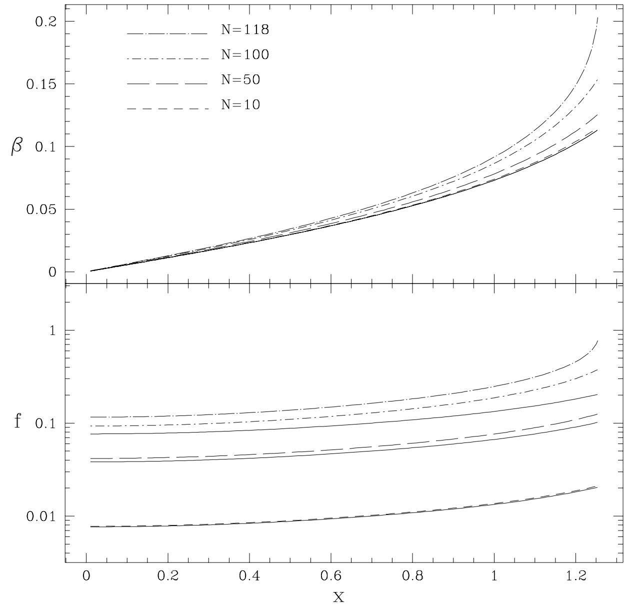

Fig. 1. In the upper panel, the non-dimensional effective radius of a plume is shown as a function of the non-dimensional radius, for different values of N. Solid lines are for case A, dashed lines for case B. This convention will be kept in the following figures. The filling factor is shown in the lower panel. In case A, N is equal to 10, 50, 100 for increasing f.

conservation, we have to introduce motions in the downward direction. This can be easily done if we consider a uniform downflow in the interplume medium. The amplitude of this inverse flow, denoted V_b, is given by the following continuity equation:

$$N\pi b^2\rho_0 V + (4\pi r^2 - N\pi b^2)\rho_0 V_b = 0. \quad (17)$$

Although such an approach may seem crude, this relation gives the correct order of magnitude of the mean downward speed that is needed to keep the mass of the core constant. For simplicity, these motions are assumed not to modify the isentropic stratification, they are strictly adiabatic and do not take part in the energy transport.

The presence of this counter flow modifies slightly the equations governing the dynamics of plumes. The velocity of a plume relative to its environment is changed from V to $(V - V_b)$. Hence, in the mass conservation equation (Eq. (8)), $|V|$ should be replaced by $|V - V_b|$. The momentum equation is also modified by the addition of the quantity $4\alpha b\rho_0|V - V_b|V_b$ in the r.h.s of Eq. (9): now, plumes capture non-zero momentum from the outside. On the other hand, the energy equation remains unchanged as we assumed the energy transport to be unaffected by the inverse current.

3. Application to a $2M_\odot$ star

Above a mass of approximately $1.2M_\odot$, stars develop a convective core during the main sequence evolution. The plume model was applied to a $2M_\odot$ star in an arbitrary stage of evolution during the main sequence phase. The purpose is to illustrate the dynamics of turbulent plumes in convective cores, this choice is based on no particular reasons. Relevant physical quantities that determine conditions in the stellar core are summarized in Table (1).

Two kind of calculations have been conducted during which the number of turbulent plumes has been varied, this number being a free parameter of the model; cases with and without downward counter flow were both examined (these will be called case B and case A respectively). The plumes have been followed from their sources located near the core center (x = 0.01) up to the Schwarzschild boundary ($x_{\rm Sch}$ = 1.253).

There is an upper limit on the number of plumes that may be present at the same time in the convective region, imposed by the fact that plumes cannot take up more space than there is available at a given level. If we define the filling factor to be the ratio of the surface occupied by the plumes to the surface of the sphere at the given radius, this factor has to be less than unity. When flows are narrow enough, so the curvature can be locally neglected, this parameter may be defined as $f = N\pi b^2/4\pi r^2$.

The main features of turbulent plumes dynamics in convective cores are shown in Figs. (1)-(4).

3.1. General behavior of turbulent plumes

Turbulent plumes move across the entire core and broaden during their motion due to turbulent entrainment but they remain fairly narrow flows (Fig. (1)). However, the self-similar régime encountered in terrestrial conditions or in RZ is lost: this is due to the peculiar physics of the core.

From Fig. (2), it appears that the velocity remains small compared to the sound speed (the sound speed is of the order of 5 10^7cm s^{-1}), the Mach number is no where larger than 0.0005. Then, the mean structure of the convective core is not modified by the plumes and the motion is nearly incompressible. A consequence of this highly subsonic flow is the unusually small values of the density contrast and of the related temperature fluctuations (Fig. (3)): a few Kelvins are sufficient to achieve the convective transport. This result amply justifies the simplifications made so far about neglecting the pressure perturbations and the influence of density and temperature fluctuations. This further accounts for the near adiabaticity of the motion.

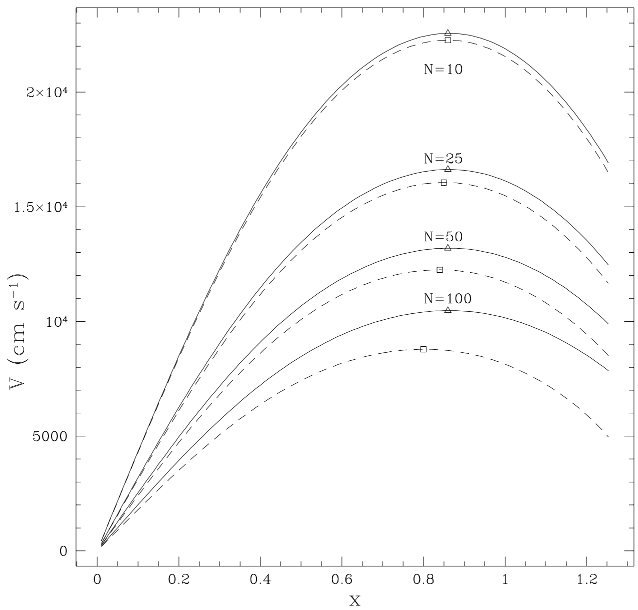

Fig. 2. Radial velocity of a turbulent plume flow as a function of radius, for different values of N. The triangles and squares indicate the highest velocity attained in the flow.

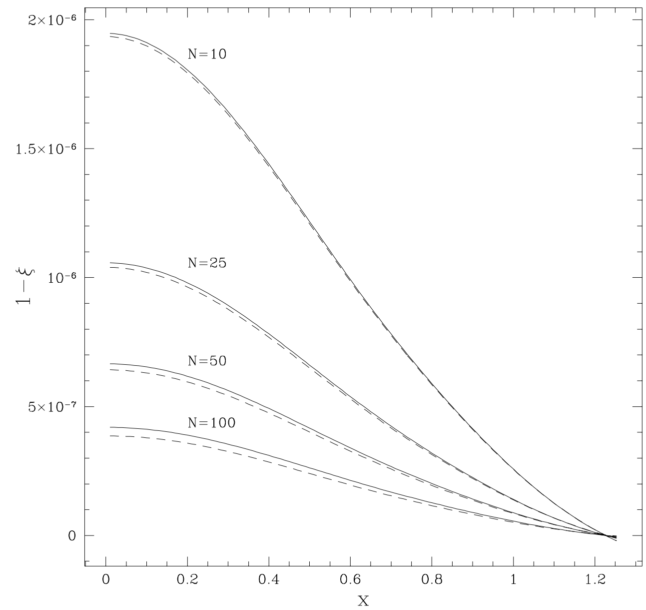

Fig. 3. Density contrast variation for different values of N. The density contrast is positive as we considered ascending plumes corresponding to fluid hotter than the interplume medium. It becomes negative close to the core boundary.

The variation of the velocity is controlled by the distribution of energy inside the core. As shown in Fig. (4), the radiative transport is a significant part of the whole energy transport process, its role becoming more and more important as we approach the outer edge of the core. In the first part of their motion, plumes are accelerated by the energy sources. But, at some level, the available luminosity ($L_{\rm nuc} - L_{\rm rad}$) is no longer able to push their increasing mass up: the convective velocity reaches a maximum value and the plumes begin to decelerate (Fig. (2)).

This may be considered as an individual effect but there is also what we may call a collective effect. The convective speed also depends on the number of plumes and decreases when this number increases: the more plumes there are, the slower they move. This is easily understood by the fact that a lesser velocity is needed to carry up the same luminosity when convective elements are more numerous. In every case, plumes arrive at the Schwarzschild boundary with non-zero velocity and the convective motions overshoot into the neighbouring radiative zone.

The density contrast lessens during the motion as the plumes grow in size and as the need of convective transport lessens when going upwards, radiation becoming more and more efficient.

While the kinetic energy flux is negligible in the bulk of the unstable domain, near the core surface, it becomes of the same order, in absolute value, as the convective flux. Its highest value is reached in the vicinity of the surface while the convective luminosity is maximum somewhere deeper. Before entering the radiative domain, the convective flux changes sign to compensate the kinetic flux so as to satisfy energy conservation. At the same time, the density contrast becomes negative, the upflows are further slowed down as a result.

3.2. Plumes in a static medium (case A)

When the downward current is not taken into account, the plumes grow in size identically, whatever their number. Changing the number of plumes only modifies the radial speed and the density contrast. These quantities adjust themselves so as to reproduce the total luminosity at every depth.

Things happen as if plumes capture matter independently of their speed, contrary to what may be expected from Taylor's entrainment hypothesis (Eq. (1)). This behavior may be understood if we consider that the broadening process depends, after all, only on the conditions encountered in the interplume medium. In the present case, this medium is the same for all the plumes even when their number is changed, in particular, the downflow speed is zero. When a return flow is introduced, the downflow velocity varies with the number of plumes and the effective radius changes as well as we will see. A consequence is that the top velocities are all reached at the same height ($x = 0.85$) when there is no inverse current.

The distribution of energy among the different forms of transport is not modified when the number of plumes is varied (Fig. (4)). It ensues that, in the present case, the velocity and the density contrast of a plume follow simple power laws of N:

$$V(r;N) = \frac{\Pi(r)}{N^{1/3}},$$

$$(1-\xi)(r;N) = \frac{\Xi(r)}{N^{2/3}}.$$

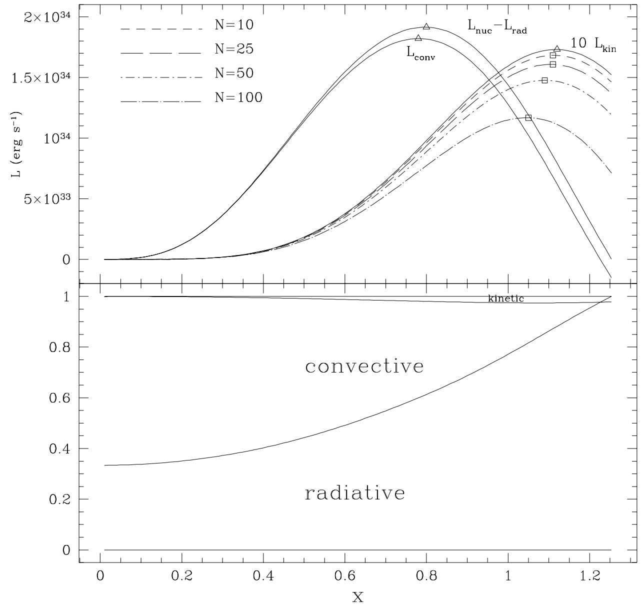

Fig. 4. Distribution of the total luminosity among the radiative, convective and kinetic type of transport. In the lower panel, each contribution is represented as a percentage of the total luminosity, in the case of a static interplume medium. In the upper panel, the variation of luminosity with depth is shown. Triangles and squares indicate the levels where the highest values are reached.

3.3. Plumes with a return flow (case B)

In this case, every dynamical quantities depend on the number of plumes. The more plumes there are, the lesser the convective velocities, as before, and the broader the plumes. The density contrast, although smaller, are not much different from the ones obtained previously.

For a given number of plumes, the effective radius is larger than in case A and the convective speed is smaller as a result. The differences in effective radius and convective speed are small for low values of N, but they become more pronounced as this number is increased. The reason is that, now, the turbulent entrainment rate depends on the number of plumes: the downward speed scales as $V_b = -fV/(1-f)$ (Eq. (17)) and the entrainment rate is proportional to $|V/(1-f)|$. The downward velocity is negative in our sign convention. It is equal, in absolute value, to the upward velocity when the plumes occupy half of the available area. As the number of plumes is increased, the occupation factor grows rapidly and the downflows become much faster than the upflows. In absolute value, the increase rate of the downward velocity with the number of plumes is larger than the decrease rate of the upward velocity. This accounts for a significant enhancement of the turbulent entrainment despite slower plumes and explains the marked differences in effective radius obtained for large values of N.

Due to the high level of turbulent entrainment involved in the present case, the upper limit on the number of plumes is reduced to 118, this number was equal to 491 in the previous case.

Table 2. Velocities and effective radius of turbulent plumes at the Schwarzschild boundary.

	Case A		Case B	
N	V (cm s^{-1})	b (10^9 cm)	V (cm s^{-1})	b (10^9 cm)
10	16904.55	1.111	16486.91	1.130
20	13417.15	1.111	12736.40	1.151
30	11720.96	1.111	10802.96	1.175
40	10649.20	1.111	9501.72	1.202
50	9885.84	1.111	8507.46	1.232
75	8636.08	1.111	6625.60	1.330
100	7846.40	1.111	4965.65	1.507

The highest convective velocities are reached at different levels for different values of the number of plumes because mass distribution in the upflows changes with this number. The largest velocities are reached deeper for larger N. Another reason that accounts for the smaller velocity is the entrainment of negative momentum from the outside (see Sect. (2.5)). However, the energy distribution is not modified significantly.

When the number of plumes is larger than approximately one hundred, the results are less consistent with the assumptions of the model. For these values, plumes are much broader and the narrow flow condition is no longer satisfied accurately enough. Furthermore, the counter flow becomes so important that not taking account of its influence on the convective transport would hardly be justified. Calling $L'_{\rm kin}$ the kinetic luminosity of the inverse current, we have

$$L'_{\rm kin} = -3\left(\frac{f}{1-f}\right)^2 L_{\rm kin}.$$

When $f = 0.366$, the two fluxes are equal in absolute value, $|L'_{\rm kin}|$ becoming more important for higher f.

Finally, given the assumptions that have been made, we argue that this model of convection seems realistic and that core convection may indeed be achieved by means of less than one hundred ascending plumes. Relevant quantities are summarized in Table (2).

4. Discussion

The model presented herein assumes that convective transport in stellar cores is achieved by ascending plumes, contrary to the case of the SCZ considered in RZ where the same assumption was made about descending plumes. In the latter case, the cooling induced by radiative leakage at the surface produce large enough temperature fluctuations that may lead to the formation of descending plumes. The huge density stratification is also responsible for a marked asymmetry between the upflows and the downflows: the motions concentrate in the downward direction while they spread horizontally upwards (Spruit et al. 1990), as a result, the downflows remain narrow while the upflows are not. Thus, in stellar convective envelopes, the existence of turbulent plumes is favored in the downward direction.

The central regions present noticeable differences with the outer layers that may justify our choice to consider ascending plumes only. The variation of the density is far smaller than near the surface by several orders of magnitude. In consequence, the asymmetry of the flows is far less pronounced and descending plumes are not especially favored by the stratification. Actually, we expect the spherical geometry to encourage the propagation of ascending plumes. In the other direction, the plumes will not be allowed to travel across the entire core: because the available area decreases with depth while the plumes get broader during their motion, at some depth, the whole area will unavoidably be filled by plumes. Such a situation is quite unrealistic because mass conservation may no longer be satisfied. Moreover, the cooling at the top of the convective domain is not nearly as drastic as in the surface of the SCZ, so the related temperature fluctuations may hardly form descending plumes that are strong enough to survive and influence the transport of heat throughout the core. On the other hand, nuclear reactions create hot spots from which ascending plumes may develop. Due to the temperature sensitivity of the energy generation, the strongest plumes are more likely to start near the stellar center where the temperature is highest.

Like in RZ, this model suffers from a crude description of the return flow whose influence on the heat transport is assumed to be negligible compared to the luminosity carried by the plumes. We have also transposed to the stellar case the structure of turbulent plumes observed in terrestrial conditions. Bonin & Rieutord (1996) have shown that the entrainment hypothesis is related to the self-similarity of the flow. In stars, the flows cover distances far longer than on Earth and the stratification does not allow to reach this régime. Using Taylor's hypothesis in such conditions is not fully consistent. However, Taylor's condition seems to reproduce the correct scaling of turbulent entrainment and we hope the results obtained in this way to bring correct orders of magnitude.

Despite these weaknesses, this model seems to be, from a dynamical point of view, a good alternative to the classical MLT. Overshooting is a feature of this approach. This phenomenon may be relevant in stellar evolution as it is invoked to reproduce cluster sequences (Maeder 1974, 1975a, 1975b). We have not tried to estimate the extent of overshooting as we expect a drastic change of the entrainment rate in the overshoot domain. When the filling factor becomes large, the downflows are much faster than the plumes. Then, turbulence may be stronger outside plumes than inside them. Taylor's hypothesis relies on the idea that turbulence is much vigorous in a plume than outside of it, this is the reason why the matter inflow is proportional to the mean axial velocity of the plume. When the turbulence is high outside, matter may well be captured from the plume and Taylor's condition does no longer provide the correct scaling.

Until numerical simulations of convection in stellar cores are carried out (such work is in progress, see Dolez et al. 1995), which could validate or invalidate the presence of plumes, this model may be tested in an indirect way by looking at its influence on processes such as chemical elements and angular momentum transport induced by internal gravity waves in radiative zones.

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Appendix A

A.1. Non-dimensional equations

Once written in non-dimensional form, the equations are integrated by a fourth-order Runge-Kutta scheme. The following set of ordinary differential equations has been solved:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = z, \tag{A1}$$

$$\frac{\mathrm{d}z}{\mathrm{d}x} = -2\frac{z}{x} - y^{3/2}, \tag{A2}$$

$$\frac{\mathrm{d}u}{\mathrm{d}x} = -\Gamma z\frac{1-\xi}{u} - 2\alpha\frac{1+f}{(1-f)^2}\frac{u}{\beta}, \tag{A3}$$

$$\frac{\mathrm{d}\beta}{\mathrm{d}x} = \frac{2\alpha}{(1-f)^2} - \frac{3}{4}\frac{z}{y}\beta + \frac{\Gamma}{2}z\frac{(1-\xi)\beta}{u^2}, \tag{A4}$$

$$\Gamma y f u(1-\xi) + \frac{2}{3}fu^3 = \frac{\Lambda}{x^2y^{3/2}}\int_0^x x'^2 y^{3/2} e\,\mathrm{d}x' + \Theta\frac{z}{k}. \tag{A5}$$

The first two equations determine the isentropic stratification while the others describe the dynamics of plumes. The last equation corresponds to energy conservation, it is used to express the density contrast, $(1-\xi)$, in terms of the non-dimensional variables $u = V/V_1$, $\beta = b/r_0$, $e = \varepsilon/\varepsilon_c$ and $k = \kappa/\kappa_c$; V_1 is the initial velocity given by Eq. (16). The constant parameters are:

$$\Gamma = 8\pi G\rho_c\left(\frac{r_0}{V_1}\right)^2, \ \Lambda = \frac{4\varepsilon_c r_0}{V_1^3}$$

and $\Theta = \frac{64\sigma}{3}\frac{T_c^4}{r_0\kappa_c\rho_c^2}$

When the inverse flow is included, f denotes the filling factor introduced in the main text and varies with radius as $N\beta^2/4x^2$. When the downward current is not taken into account, f is zero in Eqs. (A3) and (A4): the equations obtained this way correspond to the static interplume medium case; but in Eq. (A5), f remains the filling factor.

A.2. Adding a return flow

Initial conditions at the source of a plume (see Sect. 2.4.) are modified in the presence of the return flow. In this case, the coefficient B_0 from $b = B_0 r$ satisfies a third degree polynomial:

$$NB_0^3 - 4B_0 + \frac{8\alpha}{3} = 0 \tag{A6}$$

The solutions of a third degree equation of the above type ($x^3 + px + q = 0$) depend on the sign of the discriminant

$$R = (p/3)^3 + (q/2)^2.$$

If $R < 0$, the equation allows three real solutions:

$$x_n = 2\sqrt{-\frac{p}{3}}\cos\left(\frac{\omega}{3} + n\frac{2\pi}{3}\right), \quad n = 0, 1, 2$$

$$\text{with } \cos\omega = -\frac{q}{2}\sqrt{-\left(\frac{3}{p}\right)^3}$$

When $N < 193$, Eq. (A6) admits three real solutions: one of them is negative and is discarded as a result; among the two positive roots, we retain the one closest to the $2\alpha/3$ value obtained in the case without a counter flow. Hence,

$$B_0 = \frac{4}{\sqrt{3N}}\cos\left(\frac{\omega}{3} + \frac{4\pi}{3}\right) \text{ with } \cos\omega = -\frac{\alpha\sqrt{3N}}{2}. \qquad \text{(A7)}$$

This solution is retained because the introduction of the inverse flow is expected to modify initial conditions only slightly as the downward velocity is small near the center. We further have:

$$V = \left[\frac{4\alpha}{9B_0 - 2\alpha}\frac{2\pi}{3}G\rho_c(1-\xi)\right]^{1/2} r. \qquad \text{(A8)}$$

References

Böhm-Vitense E., 1958, Z. Astrophys. 46, 108
Bonin P., Rieutord M., 1996, A&A 310, 221
Berthomieu G., Provost J., Morel P., Lebreton Y., 1993, A&A 268, 775
Caughlan G.R., Fowler W.A., 1988, Atomic Data and Nuclear Tables 40, 284
Dolez N., Glatzmaier G.A., Toomre J., 1995, Spherical Core Convection in Rotating A-type Stars. In: Ulrich R.K., Rhodes Jr. E.J., Däppen W. (eds.) ASP Conf. Ser. vol. 76, GONG'94 : Helio- and Astero-seismology from the Earth and Space. ASP, San Francisco, p.653
García López R.J., Spruit H.C., 1991, ApJ 377, 268
Iglesias C.A., Rogers F.J., Wilson B.J., 1992, ApJ 397, 717
Kippenhahn R., Weigert A., 1994, Stellar Structure and Evolution, Springer-Verlag, Heidelberg
List E.J., 1982, Ann. Rev. Fluid Mech. 14, 189
Maeder A., 1974, A&A 32, 177
Maeder A., 1975a, A&A 40, 303
Maeder A., 1975b, A&A 43, 61
Montalbán J., 1994, A&A 281, 421
Morel P., 1993, The Evolutionary Code CESAM: Numerical Techniques. In: Weiss W.W., Baglin A. (eds.) IAU Coll. 137, ASP Conf. Ser. vol. 40, Inside the stars. ASP, San Francisco, p. 445
Morton B.R., Taylor G.I., Turner J.S., 1956, Proc. Roy. Soc. A 34, 1
Press W.H., 1981, ApJ 245, 286
Rieutord M., Zahn J.-P., 1995, A&A 296, 127
Roxburgh I.W., 1989, A&A 211, 361
Schatzman E., 1993, A&A 279, 431
Spruit H.C., Nordlund Å., Title A.M., 1990, ARA&A 28, 263
Tennekes H., Lumley J.L., 1972, A first course in turbulence, MIT Press, Cambridge, Chap. 4
Turner J., 1969, Ann. Rev. Fluid Mech. 1, 29
Turner J., 1986, J. Fluid Mech. 173, 431
Vitense E., 1953, Z. Astrophys. 32, 135
Zahn J.-P., 1991, A&A 252, 179

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