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Procedure to find $\langle B \rangle$, $\langle R \rangle$ and $\langle I \rangle$ for Cepheids from isolated observations using the complete light curve in V

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Abstract. The successful discovery of Cepheids in nearby galaxies with HST has triggered the development of a both simple and effective method to determine mean magnitudes from a few scattered measurements in combination with the complete light curve observed in V. Based on the empirically derived variation of the scaled difference of magnitude deflections at two passbands with phase, correction values are obtained to transfer the complete light curve in V into light curves in B, R and I, respectively. Applied to isolated observations, the procedure yields a mean magnitude for each phase point observed at that passband. With three to four measurements, the accuracy of the final mean magnitude obtained for Cepheids with normal light curves is comparable to the value obtained from complete light curves.

Key words: methods: data analysis – techniques: photometric – stars: fundamental parameters – Cepheids

1. Introduction and statement of the problem

The $Hubble Space Telescope$ (HST) has now begun to provide the long hoped for Cepheid distances to galaxies as remote as 25 Mpc, [$(m-M)_0 = 32$]. Cepheids have been discovered with HST before repair in IC 4182 and NGC 5253 as the first two galaxies in the calibration program to determine the absolute magnitudes at maximum of type Ia supernovae (Sandage et al. 1992, 1994; Saha et al. 1994, 1995; Sandage & Tammann 1993; Tammann & Sandage 1995). There also have been reports of Cepheid discoveries in M81 (Freedman et al. 1994a), and M101 (Kelson et al. 1996). After repair of the HST, Cepheids have been discovered in M100 (Freedman et al. 1994b, Ferrarese et al. 1996), to be used primarily as a contributor to the recalibration of the Tully-Fisher relation to find its intrinsic dispersion. Large numbers of Cepheids have now also been found in NGC 4536, NGC 4496A, and NGC 4639 (Saha et al. 1996 a, b, 1997; Sandage et al. 1996) as part of an ongoing program of SNe Ia calibration. Furthermore, Cepheids have been found in NGC 3368 (Tanvir et al. 1995), NGC 925 (Silbermann et al. 1996), NGC 3351 (Phelps et al. 1995), and NGC 4414 (Turner et al. 1995) – the latter three as part of the HST Key Project on the Extragalactic Distance Scale.

It is of central importance in all this work to determine the apparent distance modulus of each galaxy in at least two different effective wavelengths so as to estimate the internal reddening, either differential from Cepheid to Cepheid, or general across the Cepheid area. Without such a reddening determination, any individual distance modulus will be in contention, and the primary purposes to which a distance is to be used will be compromised. Hence, Cepheid period-luminosity relations are required in a minimum of two photometric bands, well separated in effective wavelength.

Due to the extreme pressure on HST telescope time, Time Allocation Committees have always taken a minimalist approach in their assignment of the amount of telescope time. The time eventually assigned has always been insufficient to obtain complete Cepheid light curves in two wavelength bands. In view of this, it has become necessary to devise methods to obtain the required mean magnitudes in a particular passband (such as I) from scattered observations in that band when complete light curves, properly phased, exist in some other band such as V.

In our papers on IC 4182, NGC 5253, NGC 4536, NGC 4496A, and NGC 4639 we have used a method devised early in the work on IC 4182 to accomplish this conversion of scattered individual I magnitudes to $\langle I \rangle$. We use the phased V light curve as a template, suitably reduced in amplitude to account for the smaller amplitude in I than in V. The purpose of this note is to present the method, to give templates for conversion of scattered data in the B, R, and I band passes to the mean values $\langle B \rangle$, $\langle R \rangle$, and $\langle I \rangle$, and to assess the accuracy of the method.

Our method differs from the solution by Freedman (1988, her Fig. 5) for the same problem. She derived and adopted fixed amplitude ratios for light curves in $B : V : R : I$ as 1.00 : 0.67 : 0.44 : 0.34 using the multicolor photometry of

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Galactic Cepheids given by Wisniewski & Johnson (1968). In her demonstration she used the B light curve as a template by reducing its amplitude by the appropriate amplitude ratio to match the amplitude of the light curve in any other band pass. In doing that, the well known phase shift as a function of wavelength, discovered by Stebbins (1945) from his six color photometry of Delta Cephei, is encountered over part of the light curves, generally on the falling branch. However, the phase shift is not constant over the cycle, often approaching zero on the rising branch.

Freedman adopts average phase lags of 0.03 at V, 0.07 at R, and 0.10 at I relative to B, and applies these average lags *to all phases*. The precept of her method is that the entire light curve in the fiducial band, after squeezing in amplitude and shifting in phase by the appropriate amount for the bandpass, is a good representation of the unknown light curve at all phases. Freedman states that the resulting mean magnitudes obtained by averaging several individual determinations is accurate to $\pm$ 0.05 mag.

It is to be emphasized that from a physical point of view Freedman's constant phase shift correction is incorrect. The method would be precise if there was no color change with phase. However, it is known that the photospheric temperature of the Cepheids varies with phase, being hottest near maximum light. Because of the observed color change with phase, a squeezed and shifted template will not fit precisely the light curve in another bandpass, shown first by Stebbins' six-color data for Delta Cephei.

It is this lack of automorphism between the squeezed and shifted curves due to the color variation that prompted us to seek the different solution set out in the next section. The detailed correction table and our adopted template curves to convert B, R, and I magnitudes to $\langle B\rangle$, $\langle R\rangle$, and $\langle I\rangle$ are in § 3. Application of the method is in § 4. Comparison of the accuracies of the method presented here with that of Freedman is made in § 5.

2. Basis of the correction procedure

We make no phase shift between the curves but take the effect of the phase shift into account in a correction curve as a function of phase to be determined. It is the determination of the phase variation of these template correction curves that calibrates the method and is the subject of this note.

We postulate, later to be tested using actual data, that we can transfer the information on shape, amplitude, and variable phase shift over the cycle from the complete V light curve to the unknown light curve in B, R, or I by an average correction value C, at every phase φ_V, such that

$$C_{V\rightarrow B}(\varphi) = \frac{[B(\varphi) - <B>] - [V(\varphi) - <V>]}{\Delta V}, \quad (1)$$

$$C_{V\rightarrow R}(\varphi) = \frac{[V(\varphi) - <V>] - [R(\varphi) - <R>]}{\Delta V}, \quad (2)$$

and

$$C_{V\rightarrow I}(\varphi) = \frac{[V(\varphi) - <V>] - [I(\varphi) - <I>]}{\Delta V}. \quad (3)$$

Table 1. Characteristics of the Cepheid sample

Name	Period in days	Amplitude ΔV	Shape of light curve at V
SU Cyg	3.85	0.72	normal, short period
δ Cep	5.37	0.82	normal
W Gem	7.92	0.83	with hump
ζ Gem	10.15	0.51	symmetric
Y Oph	17.12	0.49	normal
T Mon	27.02	1.01	normal

The reason for the postulate is that we expect the individual differences (at any phase) between the observed magnitude and the mean magnitude (intensity mean converted to magnitude units) in wavelength j, relative to the same difference in the V band, will scale as the V amplitude, ΔV. If so, the C functions can be determined empirically by applying the procedure to a set of multicolor light curves such as given by Wisniewski & Johnson (1968). We have done that, and list the C functions in the next section.

Once the correction functions C are known, the required mean magnitude, say $\langle I\rangle$, can be estimated from any single $I(\varphi_V)$ magnitude measured at phase φ_V, by

$$<I> = I(\varphi_V) + [<V> - V(\varphi_V)] + \Delta V\, C_{V\rightarrow I}(\varphi), \quad (4)$$

all data read at the V phase.

Note again that each separate $I(\varphi_V)$ value gives an estimate of $\langle I\rangle$. If there are n measurements of $I(\varphi_V)$ at different phases, each gives an $\langle I\rangle$ value, and the average is taken. An estimate of the error of the resulting $\langle I\rangle$ can then be made from the rms of the average. We have listed such errors of the $\langle I\rangle$ values measured in this way in each of our cited HST papers.

3. Determination of the correction functions

A sample of Cepheids was selected from the atlas of classical Galactic Cepheids published by Wisniewski & Johnson (1968), representing a range in periods, amplitudes and shapes of light curves. The figures of the atlas are shown relative to an arbitrary zero point in magnitude. The mean smooth curves drawn through the $BVRI$ data points of the Cepheids of Table 1 were read off in steps of 0.05 phase units. These measurements provided a reliable estimate of the intensity means expressed in magnitudes. The V amplitude and the phase at V_{max} were also read from the plot. To align the V light curves of the chosen Cepheids in phase, each set of readings m(φ) of the same variable was shifted by the amount $-\varphi(V_{max})$. Correction values $C(\varphi)$ were calculated for the transfers $V \rightarrow B$, $V \rightarrow R$ and $V \rightarrow I$ according to Eqs. (1) to (3).

The result is illustrated in Fig. 1 and corresponds very well with the expected pattern. The four Cepheids with normal light

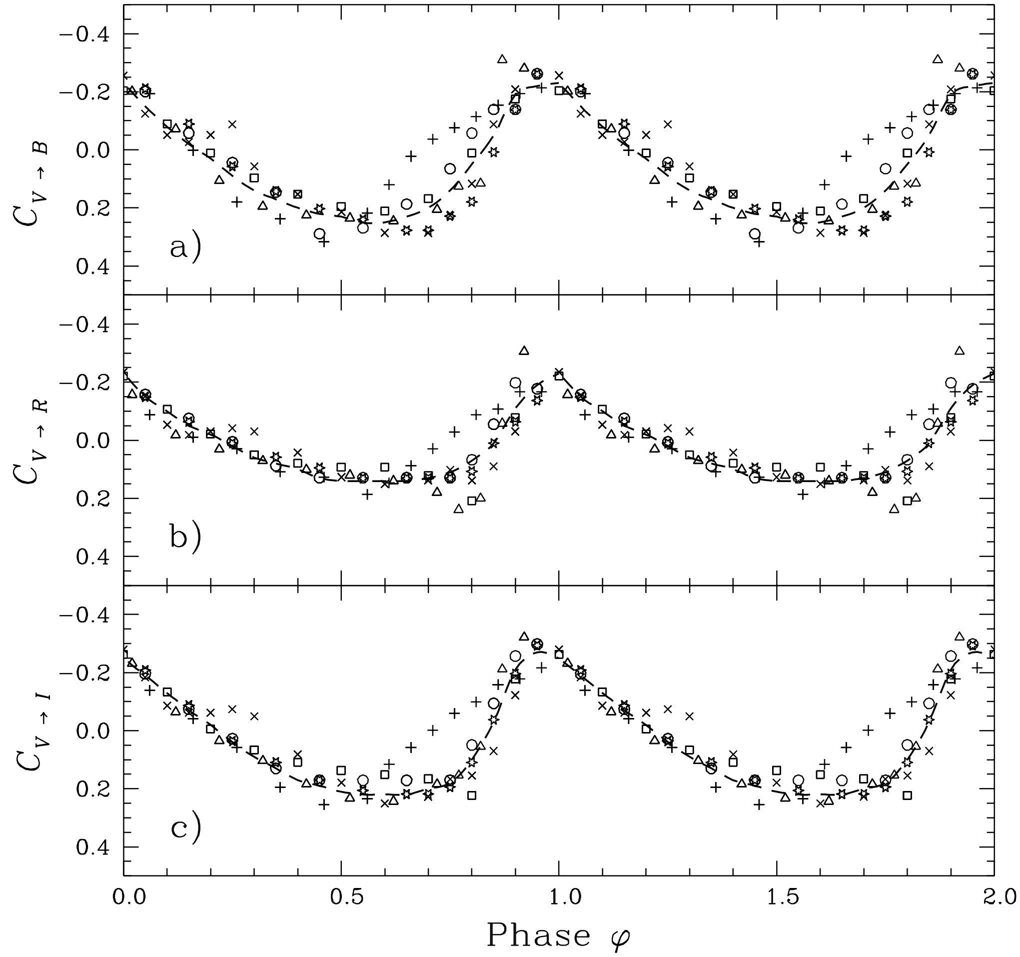

Fig. 1a–c. The correction function for B (panel **a**), R (**b**) and I (**c**) as a function of phase. Different symbols are used to distinguish between the Cepheids: SU Cyg (squares), δ Cep (stars), W Gem (diagonal crosses), ζ Gem (crosses), Y Oph (circles), T Mon (triangles). The mean relation does not take into account the outlying points due to W Gem (secondary bump on the visual light curve) and ζ Gem (highly symmetric light curve).

curves closely follow a mean relation the scatter of which is getting smaller from B to I. Some of the data points obtained from ζ Gem deviate significantly from the average because of the highly symmetrical light curves of that star. The effect is most pronounced in the phase interval $0.6 \leq \varphi \leq 0.8$. The secondary hump in the light curve of W Gem shows up prominently in the range $0.2 \leq \varphi \leq 0.3$. As the majority of Cepheid light curves are asymmetric (e.g., Schaltenbrand & Tammann 1971; Moffett & Barnes 1985) with only minor secondary humps, the adopted correction function (broken line in Fig. 1) disregards the irregular deviations from the mean relation. Table 2 lists values of the adopted correction functions $C_{V\to B}$, $C_{V\to R}$, and $C_{V\to I}$. From these one can reconstruct the individual functions by interpolation. Quantitatively the term $C(\varphi)\cdot\Delta V$ can be as large as $\pm$ 0.25 mag for a Cepheid with an amplitude of 1.0 mag.

4. Application of the correction function

To test the procedure and verify the resulting mean magnitudes, several $BVRI$ data sets of Cepheids were taken from Wisniewski & Johnson (1968) and Freedman et al. (1992) as well as BVI light curves for LMC Cepheids from Walker (1987). The mean magnitudes obtained with our procedure compare very well with those derived from digitizing the smooth curves, converting into intensities, integrating and then converting back to an intensity-averaged magnitude. For three to four isolated measurements per star and a sample of ten different Cepheids the magnitude residuals are $\delta B = 0.02\pm0.06$ (sd), $\delta R = 0.00\pm0.06$ (sd) and $\delta I = 0.00\pm0.05$ (sd). The quality of the final mean magnitude depends strongly on the number of isolated measurements and the actual phase coverage. At optical passbands a minimum of four observations is needed to provide a meaningful check of the consistency of the solution, in the infrared three phase points are sufficient because of the smaller amplitude.

Our relations were derived from observations in the Johnson R and I passbands, whereas modern photometric observations

Table 2. Adopted correction values for B, R, and I as a function of phase

Phase φ_V	$C_{V\rightarrow B}$	$C_{V\rightarrow R}$	$C_{V\rightarrow I}$
0.00	-0.23	-0.23	-0.25
0.05	-0.15	-0.15	-0.19
0.10	-0.08	-0.10	-0.13
0.15	-0.02	-0.05	-0.07
0.20	0.03	-0.02	-0.02
0.25	0.09	0.02	0.04
0.30	0.14	0.06	0.09
0.35	0.17	0.08	0.13
0.40	0.20	0.10	0.17
0.45	0.22	0.13	0.19
0.50	0.23	0.14	0.21
0.55	0.25	0.14	0.22
0.60	0.25	0.14	0.22
0.65	0.23	0.14	0.22
0.70	0.20	0.13	0.20
0.75	0.14	0.11	0.18
0.80	0.05	0.07	0.10
0.85	-0.05	0.01	-0.03
0.90	-0.19	-0.11	-0.21
0.95	-0.22	-0.19	-0.27

are made in the Cousins system. However, this difference is negligible for the present purpose.

5. Comparison with Freedman's solution

Equation (4) differs fundamentally from the Freedman procedure because all phase shift information is contained in the shape of the C correction function. The phase shift in any particular Cepheid is permitted to change as it will over the V phase. Whatever its average run for "normal" Cepheids is reflected in the C function as we have derived it.

Freedman's solution is

$$<I>= I(\varphi') - \frac{\Delta I}{\Delta V}\,[V(\varphi_V) - <V>]\ , \quad (5)$$

where φ' denotes the phase φ_V shifted by +0.07. We have made numerical experiments using both Eqs. (4) and (5) to assess the accuracy of each method. We have carried out Eq. (5) on the six Cepheids in Table 1 at each of the 20 phases, separated by 0.05 phase units, so as to construct correction curves as a function of phase for Eq. (5) as we did for Eqs. (1) to (3), shown as Fig. 1. We found that the Freedman squeeze and phase shift method is excellent, except during the phase interval of the rising branch. This was to be expected because of the way the non-isomorphism of the phase shifted curve manifests itself, as explained in § 1. However, the phase interval is small in which the large deviations occur in the Freedman solution. The problem typically is between phase 0.85 to 1.00. The deviations can produce an error as large as 0.3 mag in this phase interval. This

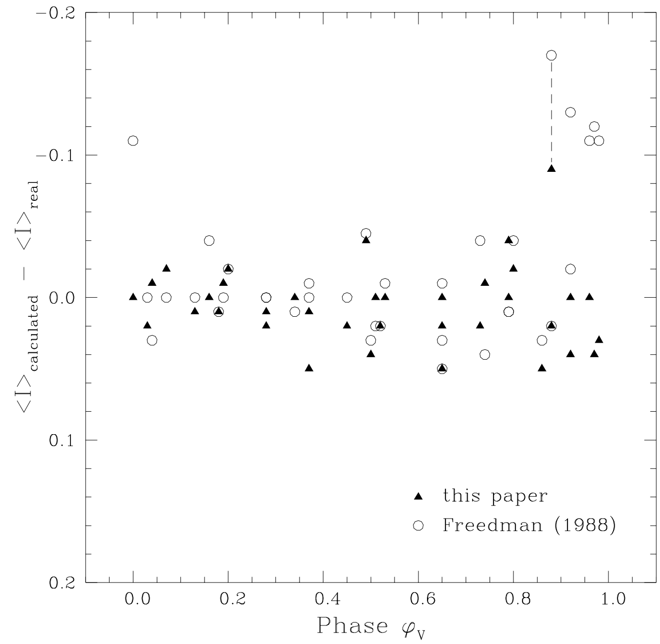

Fig. 2. Comparison of the two procedures to determine $\langle I\rangle$. Plotted are the differences between the calculated and the real $\langle I\rangle$, based on the complete I light curves in the literature, for the four normal Cepheids in Table 1. Filled triangles are based on Eq. (4) while open circles result from Eq. (5). The two extremes at phase 0.88 are due to the short-period Cepheid SU Cyg.

part of the light curve must be avoided when using the Freedman method which does not take into account the color variation with phase.

On the other hand, our method also has a moderately large scatter of the individual points in Fig. 1 about the adopted mean correction curve[1], amounting to an rms of ≈ 0.05 mag for a single measurement for the normal Cepheid curves in Table 1. Hence, if four random points are selected, and their resulting individual $\langle I\rangle$ values determined using Eq. (4), the resulting mean $\langle I\rangle$ found by averaging the four determinations will have an rms of ≈ 0.03 mag. The Freedman solution has similar errors if the bad phase interval is avoided.

To assess the accuracy directly we have selected 8 to 10 random phases for the four normal Cepheids in Table 1 and have applied both Eqs. (4) and (5) to the data to determine the individual $\langle I\rangle$ values at each phase. The result is shown in Fig. 2. Plotted is the difference between the real $\langle I\rangle$ as known for each Cepheid in Table 1 using the complete I light curves in the literature, and the calculated $\langle I\rangle$ from Eqs. (4) and (5) at each random phase. Except for the bad phase interval mentioned above

[1] Cepheid light curves are the result of the combined variations of effective temperature and radius. As pointed out by the referee the next parameter to consider after $T_{\rm eff}$ is the effective surface gravity g. The main dynamical variation of g occurs at the phase of the reversal of the radial velocity, i.e. on the rising branch of the light curve. Very likely this is the physical reason for the relatively large residuals at those phases of the mean light curves in Fig. 1.

Table 3. Quantitative comparison of the two methods

Cepheid	mean residual (this paper)	std dev	mean residual (Freedman 1988)	std dev
SU Cyg	0.01	0.05	-0.02	0.07
δ Cep	0.01	0.01	-0.02	0.06
W Gem	0.03	0.06	-0.00	0.07
Y Oph	0.01	0.02	-0.00	-0.02
T Mon	-0.01	0.02	-0.04	0.06

for the Freedman method, the scatter in both methods is similar. Table 3 gives the difference in $\langle I \rangle$ and its rms value based on these numerical experiments using the 8 to 10 phase points for each star. In the actual HST programs we have had only 2 to 5 individual data points in I, hence the numbers in Table 3 should be increased by about a factor of $\approx$ 1.5 for the HST estimate.

The conclusion is that both methods are competitive, and that each is capable of delivering the important $\langle I \rangle$ data with as few as four random epochs in I at an accuracy level of $\approx$ 0.03 mag due to this problem alone. However, in contrast to Freedman's approach the method presented here makes explicit allowance for the color change with phase as observed in Cepheid light curves.

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