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The effect of dynamical friction on a young stellar cluster prior to the gas removal

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Abstract. Stars are formed preferably in a collective manner in the dense cores of molecular clouds, leading to young stellar clusters, embedded in large amounts of residual interstellar gas. This residual gas will be expelled from the system after some 10^6 yr, mainly due to stellar winds from young protostars. The star formation efficiency, SFE, (ϵ, defined as the ratio of the total stellar mass to the original cloud core mass) is a crucial parameter, which determines, whether the 'naked' stellar system will be a bound one, known as open cluster, or an unbound ever expanding one, known as association. For the stellar cluster to remain bound ϵ = 50% is a lower limit, given the case of a rapid gas removal (gas removal time $\ll$ crossing time), while a SFE of $\sim$ 30% will suffice if the gas is removed slowly (Wilking & Lada 1985).

Following the ideas of Verschueren (1990) we demonstrate by means of a simple model that it is possible to get these lower limits for the star formation efficiency shifted towards smaller values by accounting for the dynamical friction between the stellar system and the interstellar gas. Due to this friction the stars will decelerate within a time scale short compared to the time scale for the onset of stellar winds. In this way we need not to assume the stars being born with a smaller velocity dispersion than that of the gas, as is done by Verschueren. Subsequent contraction of the stellar system and the revirialization of the stars is followed by the gas removal. As a result we find that bound open clusters for $\epsilon \approx 0.1$ as a lower limit are possible.

Key words: clusters: open, and associations – ISM: clouds – ISM: kinetics and dynamics – stars: formation – stellar dynamics

1. Introduction

New observational techniques in the IR- and submm-region have increased our knowledge of the process of star formation enormously in recent years. The respective observations, e.g. of the ρ Ophiuchi, Taurus-Auriga and Orion molecular clouds (Lada 1992; Zinnecker et al. 1993), indicate that stars preferably form in a collective way. Despite the uncertainties regarding the details of protostar formation and pre-main-sequence evolution, the outcome of the star formation process is a young stellar cluster, embedded in a large amount of residual gas. Observations show that, after forming a typical open cluster, the remnant gas will be expelled from the system within $\approx 5 - 10$ Myr, mainly due to the stellar winds (Leisawitz et al. 1989). The fate of the 'naked' stellar cluster, whether it will be a bound system or an unbound ever expanding one, depends crucially on a couple of physical circumstances prior to the gas removal. Among these are the dynamical state of the star-gas-system, the gas removal time, the spatial and velocity distribution of the stars relative to the gas and the star formation efficiency, SFE, ϵ, defined as the ratio of the stellar mass to the original total cloud core mass. This fact has been shown in a number of analytical, as well as numerical studies (Elmegreen 1983, Lada et al. 1984, Elmegreen & Clemens 1985, Verschueren & David 1989, hereafter VD, Verschueren 1990). As for the SFE, a critical value of ϵ=0.50 is derived, given a short gas removal time, compared to the crossing time (see VD for details).

It should be remarked, though, that the SFE used in deriving these results refers to an "effective" value, being present within the volume occupied by the stars at the moment of the gas removal. In fact, based on the ideas introduced for the first time by Lada et al. (1984), Verschueren (1990) showed - for five different cloud models - that in the case of a rapid gas removal the critical SFE, $\epsilon_{\rm crit}$, tends to be lower: when the stars are born with a smaller velocity dispersion than that of the gas, the stellar cluster will collapse and revirialize within a smaller volume. This leads to an increased "effective" SFE, resulting, in turn, in a smaller $\epsilon_{\rm crit}$ for producing a bound open cluster. Summarizing his results, Verschueren gives the general relation $\epsilon_{\rm crit} \approx 0.25 + 0.25\,z$, where z denotes the quadratic ratio of the velocity dispersion of the stars to that of the gas prior to the collapse of the stellar system. Yet, if one assumes that stars are formed with the same velocity dispersion as the gas, one still inferes a critical SFE of $\epsilon_{\rm crit}$=0.50; and, in fact, this result does not depend on whether one consideres a homogeneous or inhomogeneous cloud model.

Our concern in this paper is to show that bound open clusters with a smaller SFE are possible, while neglecting the assump-

Send offprint requests to: A. Saiyadpour tion that stars are born with a smaller velocity dispersion than that of the gas. In doing so we take into account the interaction between the stars and the surrounding gas, namely the dynamical friction (Just et al. 1986). This friction is caused by the fluctuations, which are induced in the coupled star-gas system by the gravitational interaction of the discrete stars with the interstellar medium (Kegel & Völk 1983; Kegel 1987; Deiss et al. 1990). It is the aim of this paper to generalize the physical scenario discussed by Verschueren (1990). For the sake of simplicity, and in order to focus on how dynamical friction affects a young stellar cluster's evolution, we consider a homogeneous cloud core model and compare the results with the respective findings by Verschueren.

The next section contains a description of the underlying physical model and the discussion of the obtained results, followed by a brief discussion in section three.

2. The physical model

We consider a spherical, homogeneous molecular cloud core of radius r_0 and mass M, being in Virial Equilibrium (V.E.). Stars are assumed to form with the velocity dispersion and relativ density distribution equal to that of the gas. There are mainly two physical processes we simulate in our calculations, namely the shrinking of the stellar system and the gas removal: stars will lose kinetic energy due to the dynamical friction with the gas. As a result the stellar system will shrink and revirialize in a smaller volume. Due to this process the stellar system will in total gain kinetic energy. The volume occupied by the stars will shrink further, owing to the continuous influence of dynamical friction. This process will then be stopped after t years by the initiation of the stellar winds, which are assumed to remove the remnant gas out of the system immediately. We calculate the time scale for the stellar system to shrink from an initial radius r_0 down to a critical radius $r_{*,\mathrm{crit}}$. This is the radius, at which the total energy of the stellar system immediately after the gas removal equals zero. As a concrete example we then choose $t = 5 \cdot 10^6$ yr and compute the final radius of the stellar system via energy considerations.

2.1. Analytical approach

For the initial density distribution of our molecular cloud core we assume for simplicity

$$\rho(r) = \begin{cases} \rho_0 & (r \leq r_0) \\ 0 & (r > r_0) \end{cases} . \tag{1}$$

The corresponding mass distribution is

$$M(r) = (4\pi/3) r^3 \rho_0 \, . \tag{2}$$

The gravitational potential is determined by the Poisson equation and is expressed by

$$\phi(r) = 2\pi G \rho_0 (r^2/3 - r_0^2) \, , \tag{3}$$

where the boundry condition $\phi(r_0) = -GM/r_0$ has been applied. G denotes the gravitational constant. We assume that the cloud is initially in hydrostatic equilibrium and therefore also in V.E. The average velocity dispersion V_G, thermal plus turbulent, of the gas is given by

$$V_\mathrm{G}^2 = (4/5)\pi G \rho_0 r_0^2 \, . \tag{4}$$

We further assume that due to the star formation process a fraction $\epsilon = M_*/M$ of the cloud mass is instantaneously turned into stars of total mass M_*, and that they initially have the same velocity dispersion σ_0 as the gas, i.e. $\sigma_0 = V_\mathrm{G}$. The density distribution of the stars is assumed to resemble that of the gas, $\rho_*(r) = \epsilon\rho_0$. The respective distribution of the remaining gas is expressed by $\rho_\mathrm{g}(r) = (1-\epsilon)\rho_0$. The corresponding potentials $\phi_*(r)$ and $\phi_\mathrm{g}(r)$ are given by (3), where ρ_0 is to be replaced by ρ_* and ρ_g, respectively. The total potential and kinetic energy of the stars immediately after their formation, embedded in the external potential of the remaining gas read (Verschueren 1990)

$$U_* = \frac{1}{2}\int_{r_0} \rho_*\phi_* \,\mathrm{d}V + \int_{r_0} \rho_*\phi_\mathrm{g} \,\mathrm{d}V = -\frac{3}{5} G \frac{2-\epsilon}{\epsilon} \frac{M_*^2}{r_0} \, , \tag{5}$$

$$T_* = \frac{1}{2} M_* \sigma_0^2 \, . \tag{6}$$

The cloud was presumed to be in a state of V.E. before star formation, so the same should be valid for the stars immediately after they have been formed. The respective potential energy is expressed by

$$U_*^\mathrm{VE} = -\int_{r_0} \rho_* \boldsymbol{r} \cdot \boldsymbol{\nabla}(\phi_* + \phi_\mathrm{g}) \,\mathrm{d}V = -\frac{3}{5} G \frac{\epsilon M^2}{r_0} \, . \tag{7}$$

The equality $2T_* = -U_*^\mathrm{VE}$ leads to $\frac{3}{5} G \frac{M}{r_0}$ for σ_0^2, which is identical to the rhs of (4). Regarding Eq. (7) and its deviation from Eq. (5) it has to be remarked that when considering the potential energy of the stellar system in V.E., we only need to take into account the variation of the total potential, i.e. potential of the stars plus that of the remnant gas, between the center of the cloud and its boundry.

At this point we assume that the distribution of the gas will not be influenced by the further dynamical evolution of the stars, as long as the stellar winds have not been initiated. This could be achieved by having the original cloud core supported by a magnetic field. We will come back to this in the discussion. In their further evolution the stars will lose kinetic energy due to the dynamical friction with the gas (see below). This causes the stellar system to shrink and occupy a smaller sphere of radius r_*. With the respective friction time scale being large compared to the dynamical time scale, the stellar system will go through a sequence of quasi equilibria and therefore the virial theorem holds approximately at any time, until the stellar winds are initiated. During this evolution the density and potential of the gas remains unchanged, as already presumed. The stars are assumed to retain a uniform density distribution in the collapsed volume, leading to analogous expressions for the density $\rho_{*,\mathrm{c}}(r)$ and potential $\phi_{*,\mathrm{c}}(r)$ as before, only modified by a factor $(r_0/r_*)^3$. The potential energy of the stars embedded in the potential of the gas inside r_* and their kinetic energy are given by (Verschueren 1990)

$$U^{\rm VE}_{*,\rm c} = -\int_{r_*} \rho_{*,\rm c}\boldsymbol{r}\cdot\boldsymbol{\nabla}(\phi_{*,\rm c}+\phi_{\rm g,c})\,{\rm d}V = -\frac{3}{5}G\frac{M_*^2}{\epsilon r_0}\left[\frac{\epsilon r_0}{r_*}+(1-\epsilon)\frac{r_*^2}{r_0^2}\right], \quad (8)$$

$$T_{*,\rm c} = \frac{1}{2}M_*\sigma^2 . \quad (9)$$

$\phi_{\rm g,c}$ is given by the rhs of (3), where ρ_0 and r_0^2 are to be replaced by $\rho_{\rm g}$ and r_*^2, respectively. Applying the virial theorem

$$2T_{*,\rm c} = -U^{\rm VE}_{*,\rm c} \quad (10)$$

to the stellar system in the respective contracted volume yields

$$\frac{\sigma^2}{\sigma_0^2} = \frac{\epsilon r_0}{r_*}+(1-\epsilon)\frac{r_*^2}{r_0^2} . \quad (11)$$

The above relation gives the kinetic energy of the stellar system as a function of its radius.

To gain more insight we consider the state of the stellar system immediately after the remnant gas has been swept away. To simulate a rapid gas removal we apply a zeroth order approximation and just let the gas disappear. Assuming that for the instant r_* and σ will remain unchanged compared to the state immediately before the gas removal, the kinetic energy of the stars will still be given by (9) and their potential energy by

$$\tilde{U}_* = -\frac{3}{5}G\frac{M_*^2}{r_*} . \quad (12)$$

We now ask for the critical radius $r_{*,\rm crit}$, for which the relation $T_{*,\rm c}+\tilde{U}_* = 0$ holds, and obtain

$$\frac{r_{*,\rm crit}}{r_0} = \left(\frac{\epsilon}{1-\epsilon}\right)^{1/3} , \quad (13)$$

where we have made use of Eq. (11), inserting $\sigma_{\rm crit}$ and $r_{*,\rm crit}$ for σ and r_*, respectively. While not being the final radius of the cluster, the above relation does set an upper limit. The stellar cluster will be finally bound, only if its radius immediately after the gas removal satisfies $r_* \leq r_{*,\rm crit}$. Expression (13) together with (11) yield the corresponding expression for the critical velocity dispersion:

$$\frac{\sigma^2_{\rm crit}}{\sigma_0^2} = 2\epsilon^{2/3}(1-\epsilon)^{1/3} . \quad (14)$$

These two quantities, being functions of ϵ only, are depicted in Fig. 1. The emerging question here is the contraction time scale $t_{\rm con}$, in which the stellar system of radius r_0 shrinks to the radius $r_{*,\rm crit}$, and whether this time scale is shorter than the time scale for the initiation of the stellar winds. To address this question we consider again the state of the stellar system in the collapsed volume. The total potential energy of the stars in this volume, embedded in the potential of the remaining gas within the initial radius r_0 is

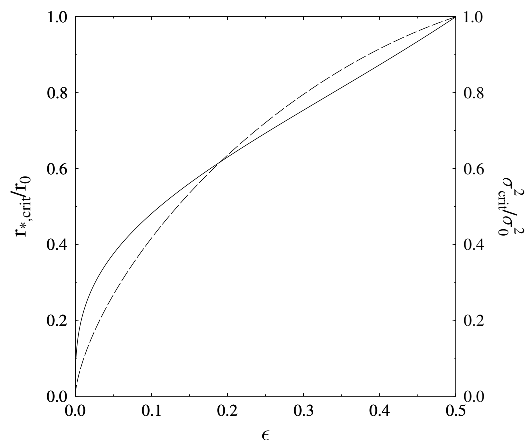

Fig. 1. critical radius (solid curve) and critical velocity dispersion (dashed curve) vs. SFE.

$$U_{*,\rm c} = \frac{1}{2}\int_{r_*}\rho_{*,\rm c}\phi_{*,\rm c}\,{\rm dV} + \int_{r_*}\rho_{*,\rm c}\phi_{\rm g,c}\,{\rm dV} = -\frac{3}{2}G\frac{M_*^2}{\epsilon r_0}\left[\frac{2\epsilon r_0}{5r_*}+(1-\epsilon)\left(1-\frac{r_*^2}{5r_0^2}\right)\right], \quad (15)$$

where $\phi_{\rm g,c}$ is given by the rhs Eq. (3), with ρ_0 being replaced by $\rho_{\rm g}$; r_0^2 is not to be substituted by r_*^2, since we take into account the gas inside, as well as outside r_*. With $U_{*,\rm c}$=$U^{\rm VE}_{*,\rm c}$+$\Delta U_{*,\rm c}$ and $U^{\rm VE}_{*,\rm c}$ given by (8), we obtain

$$\Delta U_{*,\rm c} = -\frac{3}{2}G\frac{M_*^2}{\epsilon r_0}(1-\epsilon)\left(1-\frac{3r_*^2}{5r_0^2}\right) . \quad (16)$$

Hence, together with (10), the temporal variation of the total energy $E_{*,\rm tot}$ of the stars can be written as

$$\frac{{\rm d}E_{*,\rm tot}}{{\rm d}t} = \frac{\rm d}{{\rm d}t}\left(\frac{1}{2}U^{\rm VE}_{*,\rm c}+\Delta U_{*,\rm c}\right) . \quad (17)$$

With the appropriate expressions (8) and (16) we obtain for the rhs of Eq. (17)

$$\frac{\rm d}{{\rm d}t}\left(\frac{1}{2}U^{\rm VE}_{*,\rm c}+\Delta U_{*,\rm c}\right) = \frac{3}{10}G\frac{M_*^2}{\epsilon r_0^2} \times \left[\frac{\epsilon r_0^2}{r_*^2}+4(1-\epsilon)\frac{r_*}{r_0}\right]\frac{{\rm d}r_*}{{\rm d}t} . \quad (18)$$

For sake of completion we now specify the lhs of Eq. (17).

2.1.1. Energy loss rate of the stellar system

As mentioned before, the stars in our model lose kinetic energy due to dynamical friction with the ambient interstellar gas. For an isotropic velocity distribution function $f(v)$ of the stars, the total energy loss rate of the stellar system at a given instant is expressed by

$$\frac{{\rm d}E_{*,\rm tot}}{{\rm d}t} = 4\pi N_*\int_{v_{\rm min}}^{\infty}\frac{{\rm d}E_{\rm s}(v)}{{\rm d}t}f(v)v^2\,{\rm d}v , \quad (19)$$

where N_* denotes the total number of the stars and $\mathrm{d}E_\mathrm{s}/\mathrm{d}t$ the energy loss rate of a single star moving through the gas at a speed v. In a collisional gas, only those stars moving at supersonic speeds are exerted to a gravitational drag force; subsonic stars remain unaffected, since the density perturbations induced in the gas by the stars can adjust themselves symmetrically around each subsonic star on a time scale determined by the propogation speed of pressure perturbations, i.e. by the sound speed c_G (Deiss 1990). However, molecular clouds appear to be in a rather turbulent state, with $V_\mathrm{G} \gg c_\mathrm{G}$, which modifies the above considerations. To account for the turbulence we adopt a lowest order approximation by assuming the pressure perturbations in the gas to propagate at V_G, which implies $v_\mathrm{min} = V_\mathrm{G}$ in Eq. (19). This can be considered as a lower limit, meaning that $v_\mathrm{min} < V_\mathrm{G}$ is identical with an amplification of the dynamical friction.

For stars moving at $v \gtrsim V_\mathrm{G}$, the ambient gaseous medium may be treated as a collisionless system of small particles. According to Rephaeli & Salpeter (1980) the energy loss rate of such a star amounts to

$$\frac{\mathrm{d}E_\mathrm{s}}{\mathrm{d}t} = -\frac{4\pi G^2 M_\mathrm{s}^2 \rho_\mathrm{g}}{v} \ln\Lambda\,, \tag{20}$$

where M_s denotes the mass of the star and $\ln\Lambda$ the Coulomb-logarithm, defined as $\ln\Lambda = \ln(p_\mathrm{max}/p_\mathrm{min})$. p_min and p_max are the smallest and largest impact parameter to be considered, respectively. As a simplification, we adopt $\ln\Lambda$=6 throughout the paper. This value is based upon the implicit presumption that (i) p_min is of the order of the radius of a protostellar nebula (~ 100 AU), and (ii) p_max of the order of the radius r_0 of the cloud core (some tenths of parsec).

Assuming $f(v)$ to be an isotropic Gaussian in the rest frame of the gas

$$f(v) = \frac{1}{(\sqrt{2\pi}\sigma)^3}\,\exp\left(-\frac{v^2}{2\sigma^2}\right)\,, \tag{21}$$

we obtain with (20) for the total energy loss rate of the stellar cluster

$$\frac{\mathrm{d}E_{*,\mathrm{tot}}}{\mathrm{d}t} = -4\sqrt{2\pi}G^2\frac{M_\mathrm{s}^2 N_*}{\sigma}\rho_\mathrm{g}\exp\left(-\frac{V_\mathrm{G}^2}{2\sigma^2}\right)\ln\Lambda\,. \tag{22}$$

In a star forming cloud core, the gas density is generally not smooth but shows considerable substructure. To lowest order, this does not affect the energy loss rate, since dynamical friction is due to the long-range gravitational interaction. One could argue that gaseous substructures can serve as scattering centers for the stars, hence transfering kinetic energy from the gas onto the system of stars. This may be partly true; the main effect of a star passing by a gaseous clump, however, consists in an excitation of internal degrees of freedom within the clump, hence breaking the star's motion; this is just the mechanism of dynamical friction. Thus, Eq. (22) appears to be a reasonable approximation of the total energy loss rate of a young stellar cluster even in a clumpy ambient medium.

2.1.2. Contraction time scale

Inserting (18) and (22) into Eq. (17) yields

$$\begin{aligned}\mathrm{d}t &= -\frac{3N_*\sigma_0}{40\sqrt{2\pi}G\rho_\mathrm{g}\epsilon r_0^2 \ln\Lambda}\frac{\sigma}{\sigma_0}\left[\frac{\epsilon r_0^2}{r_*^2} + 4(1-\epsilon)\frac{r_*}{r_0}\right]\\ &\quad\times\ \exp\left(\frac{\sigma_0^2}{2\sigma^2}\right)\,\mathrm{d}r_*\,,\end{aligned} \tag{23}$$

where we have made use of the fact that $V_\mathrm{G}^2/\sigma_0^2 = 1$. Further, by expressing (σ/σ_0) in terms of (r_*/r_0), see Eq. (11), and substituting $\mathrm{d}r_*$ by $r_0\,\mathrm{d}(r_*/r_0)$ we obtain

$$\frac{3N_*\sigma_0}{40\sqrt{2\pi}G\rho_0\epsilon(1-\epsilon)r_0\ln\Lambda}\int_{r_{*,\mathrm{crit}}/r_0}^{1} I\left(\frac{r_*}{r_0},\epsilon\right)\,\mathrm{d}\left(\frac{r_*}{r_0}\right) = t_\mathrm{con}\,, \tag{24}$$

where the lower integration limit is given by (13) and the function I is defined as

$$\begin{aligned}I\left(\frac{r_*}{r_0},\epsilon\right) &= \left[\frac{\epsilon r_0}{r_*} + (1-\epsilon)\frac{r_*^2}{r_0^2}\right]^{1/2}\left[\frac{\epsilon r_0^2}{r_*^2} + 4(1-\epsilon)\frac{r_*}{r_0}\right]\\ &\quad\times\ \exp\left[\left(2\left(\frac{\epsilon r_0}{r_*} + (1-\epsilon)\frac{r_*^2}{r_0^2}\right)\right)^{-1}\right]\,.\end{aligned} \tag{25}$$

Expressing N_* and σ_0 in (24) in terms of the mass M and the radius r_0 of the original cloud core yields

$$\begin{aligned}\int_{r_{*,\mathrm{crit}}/r_0}^{1}\tilde{I}\left(\frac{r_*}{r_0},\epsilon\right)\,\mathrm{d}\left(\frac{r_*}{r_0}\right) &\equiv g(\epsilon)\\ &= \frac{10\sqrt{10G}\ln\Lambda}{\sqrt{3\pi}}\frac{M_\mathrm{s}\cdot t_\mathrm{con}}{\sqrt{M}r_0^{3/2}}\,,\end{aligned} \tag{26}$$

where $\tilde{I} = I/(1-\epsilon)$ and is depicted in Fig. 2 for eight different ϵ-values. Keeping in mind that the lower integration limit decreases with decreasing ϵ, the figure confirms the expectation regarding the increase of t_con for a given r_0 with decreasing ϵ, the latter being associated with the growth of the surface under the respective curve. Therefore the probability for the stellar system to survive as a bound cluster decreases with decreasing ϵ. As can be seen, with the integral in Eq. (26) being a function of ϵ only, determining the contraction time scale t_con requires values for three parameters, being the mass and radius of the original cloud core and the mass of a single star, yet to be provided.

The energy loss rate of a star [Eq. (20)] is proportional to the density of the ambient gas. Hence, in a inhomogeneous real cloud with a density gradient towards its center, the total energy loss rate of a contracting young stellar cluster increases in the course of its evolution. This implies that expression (24) can be regarded as an upper limit for the contraction time scale of a stellar cluster.

2.1.3. Final radius of the stellar cluster

To compute the final radius of the stellar cluster we once again consider its state immediately after the gas has been removed.

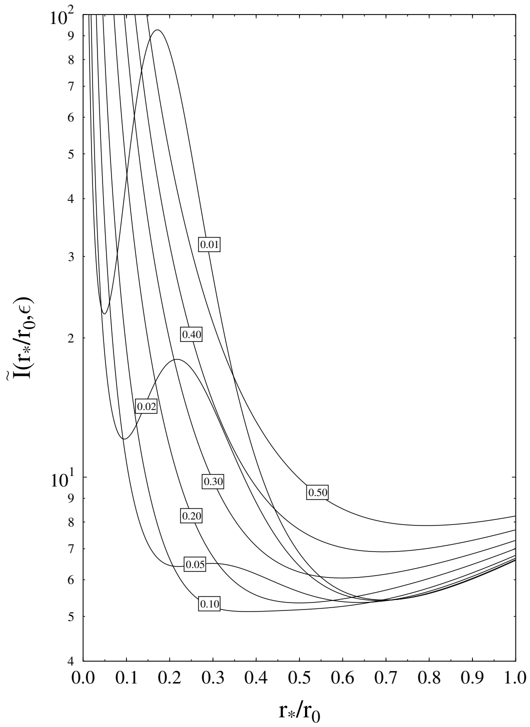

Fig. 2. $\tilde{I}$ plotted against $\frac{r_*}{r_0}$. The label on each curve denotes the respective ϵ-value.

We assume that for the instant the gas removal will not cause any change, regarding the radius r_* of the stellar system and the velocity dispersion σ of the stars. Thus, the kinetic and potential energy of the cluster are given by (9) and (12), respectively, with the velocity dispersion given by (11). The cluster will then come into Virial Equilibrium with the corresponding kinetic $T_{\rm cl}^{\rm VE}$ and potential energy $U_{\rm cl}^{\rm VE}$, which are of the same structure as before, but with the appropriate velocity dispersion $\sigma_{\rm cl}$ of the stars and the Radius $R_{\rm cl}$ of the cluster. Assuming the conservation of the total energy between the two subsequent states

$$T_{*,\rm c} + \tilde{U}_* = T_{\rm cl}^{\rm VE} + U_{\rm cl}^{\rm VE} \tag{27}$$

and applying

$$2T_{\rm cl}^{\rm VE} = -U_{\rm cl}^{\rm VE}\,, \tag{28}$$

yields for the final radius of the cluster

$$R_{\rm cl} = \frac{r_*}{1 - \dfrac{1-\epsilon}{\epsilon}\dfrac{r_*^3}{r_0^3}}\,, \tag{29}$$

where we have inserted the appropriate expressions for the energies. Next, we define

$$\frac{r_*}{r_0} = x \cdot \frac{r_{*,\rm crit}}{r_0}\,, \tag{30}$$

where $x = x(\epsilon, t)$. This, together with (13) yields for the normalized final radius of the cluster

$$\frac{R_{\rm cl}}{r_0} = \frac{x\left(\dfrac{\epsilon}{1-\epsilon}\right)^{1/3}}{1-x^3} \tag{31}$$

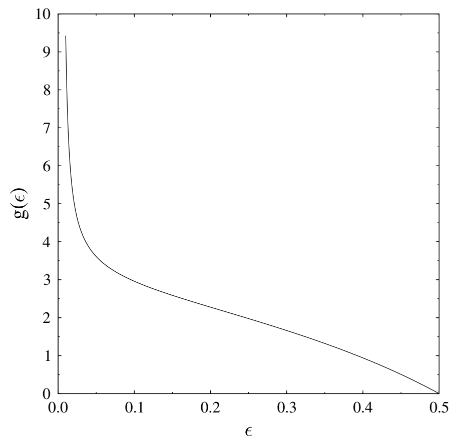

Fig. 3. $g(\epsilon)$ vs. ϵ.

One should keep in mind that the stellar system could by all means shrink down to a radius $r_* < r_{*,\rm crit}$ before the stellar winds set in. Hence, setting the lower integration limit in Eq. (26) equal to $r_{*,\rm crit}/r_0$ can be interpreted as a test. The rhs of the Eq. (31) shows that the cluster will be bound, only if $x < 1$, in agreement with our previous considerations.

2.2. Numerical results

As mentioned before, the integral in Eq. (26) can be evaluated without the knowledge of any other parameter. Fig. 3 shows the corresponding solution as a function of ϵ. Obviously the general tendency for given values of r_0, M and $M_{\rm s}$ is the decrease of the contraction time scale $t_{\rm con}$ with increasing ϵ.

A look at the rhs of Eq. (26) shows that to determine $t_{\rm con}$ with $\ln\Lambda$ already specified, there are two parameters, r_0 and M, yet to be determined. To reduce the number of free parameters we use in our model the density-size relation given by Larson (1981) in the form

$$\rho_0 = \bar{\rho}_0 (r_0/\bar{r}_0)^{-b}\,, \tag{32}$$

where $\bar{\rho}_0 = \bar{n}_0 \cdot m_{\rm H2}$. Parameters $\bar{n}_0$ and $\bar{r}_0$ denote the normalized particle density and radius of the clouds' scaling relation, respectively. $m_{\rm H2}$ is the mass of the hydrogen molecule. We set $\bar{n}_0 = 290\,{\rm cm}^{-3}$, $\bar{r}_0 = 20$ pc and $b = 1.2$ (Dame et al. 1986). Below, we consider cloud radii in the range 0.1 to 0.8 parsec; the dynamical time scale (free-fall time) of such clouds lies in the range $\tau_{\rm ff} = (3\pi/32G\rho_0)^{1/2} \sim (1-3)\times 10^5$ yr. The average mass of the individual stars is chosen to be 1 $M_\odot$. With the parameters chosen, (32) seems to indicate that the column densities are approximately constant in molecular clouds. This, however, may be the result of a selection effect, or may reflect the internal structure of the individual clouds rather than their distribution over physical parameters, as pointed out by Kegel (1989) and Scalo (1990).

Fig. 4 shows the contraction time scale $t_{\rm con}$ for different r_0-values. Two tendencies can be recognized: first, for each ϵ-value there is an increase of $t_{\rm con}$ with increasing radius r_0 of the cloud which is identical with the initial radius of the stellar system.

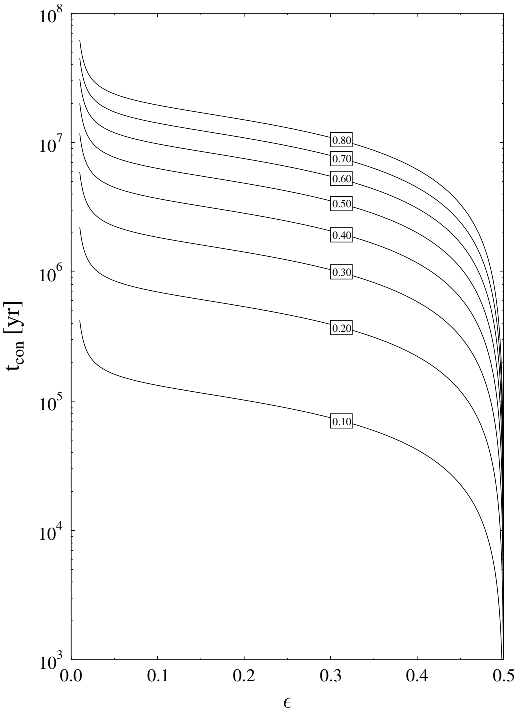

Fig. 4. The time scale for stellar clusters to shrink from an initial radius r_0 down to a radius $r_{*,\rm crit}$, at which the total energy of the cluster immediately after gas removal equals zero. The labels on the curves denote the respective r_0-values in parsec.

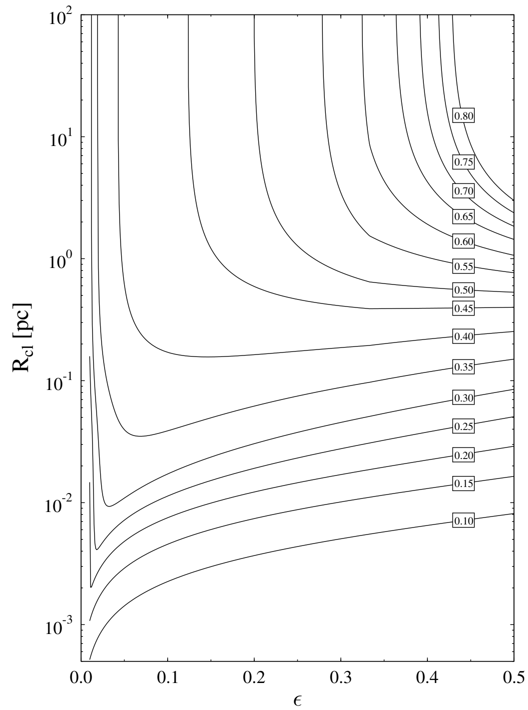

Fig. 5. Radius of the cluster vs. ϵ for different radii r_0 of the molecular cloud core, expressed in parsec by the label of the respective curve. According to Eq. (32) the mass density of the gas is $\approx 5.6 \cdot 10^{-19}$ g cm^{-3} for the smallest core and $\approx 4.6 \cdot 10^{-20}$ g cm^{-3} for the largest one.

This increase of $t_{\rm con}$ is due to the decrease of $\rho_{\rm g}$ (see (32)) and thus of the dynamical friction with increasing r_0. Secondly, for each r_0 the contraction time scale decreases with increasing ϵ, since the latter leads to an increase of the number of the stars, which in turn results in an enhancement of the dynamical friction for $\epsilon < 0.5$. For very small clouds, one infers a contraction time that is smaller than the cloud's free-fall time, especially for ϵ-values near 0.5. The contraction time scale $t_{\rm con}$ is defined as being the time scale for a stellar cluster to shrink from an initial radius r_0 to a radius $r_{*,\rm crit}$ (Fig. 1), at which the total energy of the cluster immediately after gas removal equals zero. However, in a real molecular cloud, the free-fall time gives a lower limit to the time scale of a cluster's contraction. That means, for time scales smaller than the free-fall time Fig. 4 gives formally inconsistent results. However, this is irrelevant to our general results and conclusions derived below, since for a star forming molecular cloud one expects the residual gas to be removed by stellar winds only after a few million years, which is an order of magnitude longer than the free-fall time. Hence, dynamical friction can act over a much longer time span than it is given by $\tau_{\rm ff}$.

Thus, depending on the time scale for the onset of the stellar winds, predictions can be made, regarding the probability for a stellar cluster to survive as a bound cluster. To specify this in more detail we have assumed the stellar winds to initiate after $5 \cdot 10^6$ yr, removing the remnant gas at once. This is based upon observations, due to which the molecular clouds are largely dispersed within 5-10 Myr after forming a typical open cluster (Leisawitz et al. 1989). We have let the stellar cluster shrink during this period and computed the final radius of the stellar system according to Eq. (31). Our results are depicted in Fig. 5, which shows the final cluster radius after gas removal versus ϵ for different radii r_0 of the original cloud core.

Fig. 5 indicates that for each radius r_0 of the initial cloud core there exists a critical SFE, $\epsilon_{\rm crit}$, marked by a steep increase of the cluster radius $R_{\rm cl}$ with decreasing ϵ, below which a bound cluster is no longer possible. This also follows from Fig. 4 and originates in the fact that the radius r_* of the stellar system at the moment of gas removal is larger than $r_{*,\rm crit}$. This in turn is due to a decrease of the dynamical friction with decreasing ϵ: a look at Eq. (22) reveals that dynamical friction is proportional to $N_* \rho_{\rm g}$ with the total number of the stars $N_* \propto \epsilon M$ and the mass density of the remnant gas $\rho_{\rm g} \propto (1-\epsilon)M$. Therefore, we have the maximum of the friction for ϵ=0.5. The limit $\epsilon_{\rm crit}$ is shifted towards higher values with increasing r_0; since $\rho_{\rm g} \propto r_0^{-3}$, a higher N_* corresponding to a higher ϵ is required, in order to make the dynamical friction effective enough to let the stellar system shrink below $r_{*,\rm crit}$ in the proper time. Another point worth noting is the shape of the curves: for $r_0 \gtrsim 0.45$ pc the radius $R_{\rm cl}$ of the cluster decreases monotonically with increasing ϵ, while for $r_0 \lesssim 0.4$ pc the curves have a local minimum at a certain ϵ, which we denote as $\tilde{\epsilon}$ and which moves towards higher values with increasing r_0. In the first case an increase in ϵ leads, as mentioned before, to an amplification of

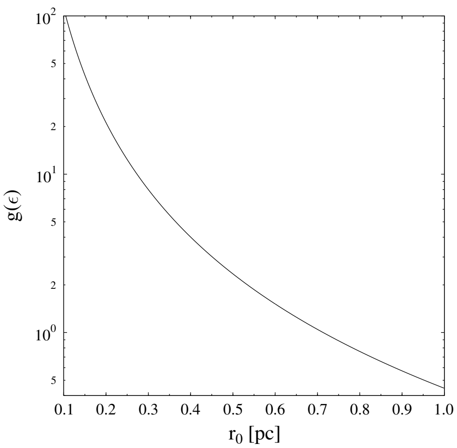

Fig. 6. $g(\epsilon)$ evaluated with $t_{\rm con}$ = $5 \cdot 10^6$ yr as a function of r_0, see the rhs of Eq. (26).

the dynamical friction and in addition, to a decrease of the mass of the gas to be removed. This causes the final radius $R_{\rm cl}$ of the cluster to decrease with increasing ϵ for a given r_0. In the second case the description just given is still valid for $\epsilon < \tilde{\epsilon}$. For $\epsilon > \tilde{\epsilon}$, however, $x(\epsilon)$ increases for a given r_0 with increasing ϵ, since due to the growing number of the stars and small cloud size the stellar system cannot shrink arbitrarily. This can be confirmed as follows: first we determine $g(\epsilon)$ with $t_{\rm con}$ = $5 \cdot 10^6$ yr (see Fig. 6). Next we evaluate $\tilde{g}(\epsilon, x)$, which is defined as the integral in Eq. (26) but with the lower integration limit set equal to $x \cdot (r_{*,\rm crit}/r_0)$, see Eq. (31). The result is depicted in Fig. 7. Now, as an example we read from Fig. 6 the respective $g(\epsilon)$-value ($\sim$ 8) for r_0 = 0.3 pc. It follows from Fig. 7 that for $\tilde{g}(\epsilon, x)$ = 8 the corresponding x-value decreases from ϵ=0.02 to $\epsilon \approx 0.04$ and then increases for $\epsilon \gtrsim 0.05$.

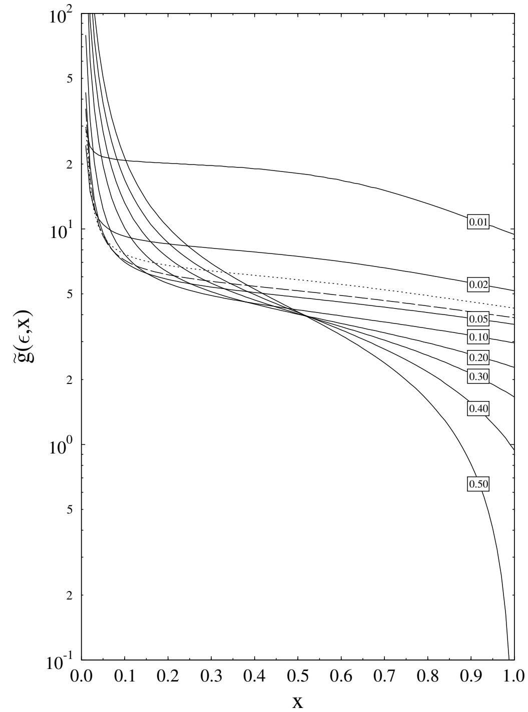

Fig. 7. $\tilde{g}(\epsilon, x)$ vs. x. The label on each curve denotes the respective ϵ-value. The dotted and the dashed curve correspond to ϵ=0.03 and ϵ=0.04, respectively. Note that $\tilde{g}(\epsilon, x = 1) \equiv g(\epsilon)$.

3. Discussion

The majority of stars in our Galaxy is formed in giant molecular clouds as clusters, which will evolve further to become bound open clusters or unbound associations. Recent studies indicate that the probability of formation of bound clusters in the Galaxy is quite low owing to the inefficiency and destructive nature of the star formation process. There are a few parameters, such as gas removal time and initial mass density, which determine the ultimate fate of such clusters. The essential requirement, though, for survival as a bound cluster is a high local star formation efficiency (SFE), defined as $M_{\rm stars}/(M_{\rm stars} + M_{\rm gas})$. Several studies suggest that the formation of a bound cluster requires a SFE of at least 50% in the case of rapid gas removal, while a somewhat lower SFE of $\sim$ 30% will suffice, if the gas is removed slowly (Wilking & Lada 1985; VD). The SFE in the giant molecular clouds in our Galaxy, however, is on average quite low ($\sim$ 5%) (Larson 1990; Pandey et al. 1990; Lada 1992). Thus, the existence of more than 100 bound open clusters within 1-2 kpc of the Sun, with typical life times of about 10^8 yr (Pandey & Mahra 1986; Battinelli et al. 1994) presents an interesting task for star-formation studies.

One should bear in mind, though, that the SFE-values cited above refer to an "effective" value within the volume occupied by the stars at the moment the gas removal sets in. So in order to establish a relation between the effective SFE and the "intrinsic" SFE within the original molecular cloud core, dynamical evolution of the star-gas system before the dispersal of the remnant gas has to be taken into account. Lada et al. (1984) introduced this idea for the first time in their consideration of stars being born with zero velocity dispersion with respect to the gas, which leads to the collapse of the stellar cluster and its subsequent revirialization in a smaller volume. This results in an increased effective SFE and thus in a smaller value of the critical intrinsic SFE ($\epsilon_{\rm crit}$) for producing a bound cluster. The authors derived a value $\epsilon_{\rm crit}$=0.22 for rapid gas removal, pointing out the unlikelihood of any significant gas removal to happen before or during the collapse. This sounds reasonable, since the time scale for the evolution to the main sequence is longer than the dynamical time scales, provided that O stars will not form too early, if at all. Based on this idea, Verschueren (1990) calculated for the case of rapid gas removal $\epsilon_{\rm crit}$ as a function of the velocity dispersion of the stars relative to that of the gas for 5 different cloud models: a homogeneous, an isothermal and 3 Plummer models with 3 different core radii. He derived a typical relation $\epsilon_{\rm crit} \cong 0.25 + 0.25\,z$, where $0 \le z \le 1$ denotes the quadratic ratio of the initial velocity dispersion of the stars to that of the gas. This parameter can also be interpreted as describing the relative strength of the magnetic field. Obviously a SFE of at least 50% ist still inevitable, if the stars are born with the same velocity dispersion as the gas, i.e. (z = 1).

In the present paper we have shown that it is possible to obtain finally bound clusters for substantially lower values of ϵ, if one accounts for the dynamical friction between the stars and the interstellar gas (Just et al. 1986). This friction causes the stars to lose kinetic energy and decelerate, resulting in a shrinking of the stellar system. In our model we have followed the analytical considerations made by Verschueren (1990), modified in two ways: first, we have assumed the stars being born with the same velocity distribution as the gas. The second modification concerns the inclusion of the dynamical friction as an additional process. Our main result is given by Eq. (26). It allows to calculate the contraction time scale $t_{\rm con}$ for the stellar system to shrink from an initial radius r_0 to a critical radius $r_{*,\rm crit}$, at which its total energy immediately after the gas removal equals zero, as a function of ϵ, M, $M_{\rm s}$ and r_0. For practical purposes it is important that this function is a product of four functions, each of which depending on one of the parameters only. Merely the dependence on ϵ, given by $g(\epsilon)$ is somewhat complicated and has to be calculated numerically. Once $g(\epsilon)$ has been determined and ϵ is specified, Eq. (26) constitutes a simple scaling law. Thus, for a given relation between the density and radius of the original cloud core (e.g. (32)), one can construct cores of different sizes and masses, but with the same $t_{\rm con}$ by varying the average mass $M_{\rm s}$ of the stars, which are formed in these cores. Assuming (32) to be valid, we calculated $t_{\rm con}$ for different parameter sets (r_0, ϵ). The general tendency proved to be a decrease of $t_{\rm con}$ with increasing ϵ and decreasing r_0 (see Fig. 4).

In a next step we assumed the stellar winds to initiate after $5 \cdot 10^6$ years, removing the remnant gas in a time short compared to the crossing time and computed the radius of the stellar system in V.E. Thereafter, bound open clusters proved to exist even for ϵ-values as small as 0.1 (Fig. 5). In a stellar cluster without O star formation rapid gas removal may not work. However, one may expect slow gas removal due to stellar winds from low mass stars and pre-main sequence stars. The general tendency of such a scenario is that a young stellar cluster stays longer within its mother molecular cloud, allowing dynamical friction to act over a longer time. This is especially true for systems that are large and massive, but with low star formation efficiency. In that case, according to Fig. 4 rather massive bound stellar clusters could form from molecular clouds with low SFE.

In conclusion we want to stress once again that our main goal in this paper was to generalize the physical scenario discussed by Verschueren (1990). Since his five cloud models revealed no remarkable differences in the final result (his Eq. (60)), we chose the homogeneous cloud model as an example. This appears to be a reasonable assumption, since especially in the late phase of the contraction of a young stellar cluster, the cluster occupies only the inner part of the mother cloud core where the gas density is more or less homogeneous. The large-scale density gradient of the mother molecular cloud has then only a minor effect on the total energy loss rate of the stellar cluster.

For sake of a better comparison, we retained the author's treatment of the gas as being a source of external potential, with its density distribution remaining unchanged, as long as the stellar winds are not being initiated. An alternative would be to let the gas contract with the stellar system in the common gravitational potential. In that case, the velocity dispersion of the gas as well as of the stellar system would increase, due to the additional gain of gravitational binding energy. Hence, the exponential in Eq. (22) would remain more or less unaffected. During contraction the turbulent velocity dispersion, $V_{\rm G}^2$, can not increase arbitrarily since supersonic turbulent motions are heavily damped due to dissipative effects. That means, turbulent kinetic energy is transfered into thermal energy which is eventually radiated away. Assuming that the gas turbulence is continuously generated and excited to a finite amplitude by the motion of the stars (Deiss et al. 1990; Kegel 1987), one expects that at any instant $V_{\rm G}^2$ is comparable to the stars' velocity dispersion σ. Hence, while the gas density would increase during contraction, the turbulent velocity dispersion, $V_{\rm G}^2$, would not necessarily increase. This, would lead to an amplification of the effects discussed further above, since according to Eq. (22) dynamical friction is proportional to the density of the gas.

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References

Battinelli, P., Brandimarti, A., Capuzzo-Dolcetta, R. 1994, A&AS, 104, 379

Dame, T.M., Elmegreen, B.G., Cohen, R.S. 1986, ApJ, 305, 892

Deiss, B.M. 1990, in Reviews in Modern Astronomy 3, ed. Klare, G., p. 350, Springer-Verlag, Berlin, Heidelberg

Deiss, B.M., Just, A., Kegel, W.H. 1990, A&A, 240, 123

Elmegreen, B.G. 1983, MNRAS, 203, 1011

Elmegreen, B.G., Clemens, C. 1985, ApJ, 294, 523

Just, A., Kegel, W.H., Deiss, B.M. 1986, A&A, 164, 337

Kegel, W.H. 1987, in: Physical Processes in Interstellar Clouds, G.E. Morfill, M. Scholer (eds), Reidel, Dordrecht, p. 125

Kegel, W.H. 1989, A&A, 225, 517

Kegel, W.H., Völk, H.J. 1983, A&A 119, 101

Lada, C.J., Margulis, M., Dearborn, D. 1984, ApJ, 285, 141

Lada, E.A. 1992, ApJ, 393, L25

Larson, R.B. 1981, MNRAS, 194, 809

Larson, R.B. 1990, in Physical Processes in Fragmentation and Star Formation, eds. Capuzzo-Dolcetta, R., Chiosi, C., Di Fazio, A., p. 389, Kluwer, Dordrecht

Leisawitz, D., Bash, F.N., Thaddeus, P. 1989, ApJS, 70, 731

Pandey, A.K., Mahra, H.S. 1986, Ap&SS, 126, 167

Pandey, A.K., Paliwal, D.C., Mahra, H.S. 1990, ApJ, 362, 165

Rephaeli, Y., Salpeter, E.E. 1980, ApJ, 240, 20

Scalo, J. 1990, in Physical Processes in Fragmentation and Star Formation, eds. Capuzzo-Dolcetta, R., Chiosi, C., Di Fazio, A., p. 151, Kluwer, Dordrecht

Verschueren, W. 1990, A&A, 234, 156

Verschueren, W., David, M. 1989, A&A, 219, 105

Wilking, B.A., Lada, C.J. 1985, in Prostars and Planets II, eds. D.C. Black and M.S. Mathews, p. 297. University of Arizona Press, Tuscon

Zinnecker, H., McCaughrean, M.J., Wilking, B.A. 1993, in Prostars and Planets III, eds. E.H. Levy, J.I. Lunine, p. 429. University of Arizona Press, Tuscon