

The initial mass function and the dynamical evolution of open clusters

IV. Realistic systems

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Received 19 September 1996 / Accepted 12 December 1996

Abstract. This paper is the fourth one of a series whose chief objective is studying the influence of different mass spectra on the dynamical evolution of open star clusters. Results from several N -body calculations with primordial binaries and mass loss due to stellar evolution are presented. The models show significant differences with those for primordial binaries but no stellar mass loss presented in de la Fuente Marcos (1996b). A differential dynamical behaviour depending on cluster richness is found compared to de la Fuente Marcos (1996a). The evolution of these realistic models is very dependent on the initial mass function. Even for rich clusters, there is a dependence on the binary mass spectrum. The velocity distribution of the escapers is examined and compared with results from previous calculations. The evolution of the primordial binary population is analyzed in detail. The cluster remnant and the final binary population are also studied. Finally, some conclusions about observational properties of Open Cluster Remnants are presented.

Key words: chaos – celestial mechanics, stellar dynamics – Galaxy: open clusters and associations: general – stars: binaries: general – stars: evolution – stars: luminosity function, mass function

1. Introduction

In the last few years, a significant number of pre-main-sequence binaries and multiple systems have been discovered in young open clusters (Brandner et al. 1996; Ghez et al. 1993, 1994; Leinert et al. 1991, 1993; Padgett et al. 1996; Prosser et al. 1994; Richichi et al. 1994; Simon et al. 1992, 1993, 1995). All the surveys carried out find a binary frequency which is greater than or equal to that of the field stars in the range of separations to which they are sensitive. The observational data suggest that the binary cluster population can be interpreted in terms of different formation mechanisms. Wide binaries (with periods greater than 100 years) could originate in capture events

or by a fragmentation process during the collapse of a single rotating protostar. In fact, Hartigan et al. (1994) have shown that about one third of wide pre-main-sequence binary pairs in their sample (projected separations of 400–6000 AU) are not coeval, with the less massive star usually being younger than the more massive star, suggesting that they are formed by capture or exchange (more likely) events. On the other hand, close systems must almost certainly be primordial, although their exact origin is not yet well understood. Several mechanisms have been suggested (fragmentation during the late collapse, gravitational instabilities or even orbital decay) for explaining the formation of these close primordial binaries. Moreover, observations indicate that binary stars are seen since the earliest stages of star formation, which suggests primordial mechanisms for their origin (Harjunpää et al. 1991; Reipurth & Zinnecker 1993; Mathieu 1992, 1994, 1996). In any case, observational results hint that star formation produces mostly binaries. Further information about binaries in clusters can be found in Trimble (1980), Abt (1983), Reipurth (1988), Mathieu (1989), Zinnecker (1989) and an extensive recent review in Bodenheimer et al. (1993).

Binaries in star clusters are of chief importance both in observational and theoretical astrophysics. In some open clusters, a certain number of stars appear above the cluster main-sequence turn-off (blue stragglers); it is currently interpreted (Wheeler 1979) as a result of stellar coalescence or extended main-sequence life-times caused by mixing within the stars (Abt 1985). Another popular explanation for the origin of blue stragglers may be due to binary mass transfer (McCrea 1964). Also, run-away OB stars are interpreted as binaries escaping from star clusters (De Cuyper 1982; Sutantyo 1982; Hills 1983; Gies & Bolton 1986). In addition, binaries in open clusters are used to determine the distance scale (standard candles). Binaries are also the key to explaining X-ray emissions from globular and old open star clusters. From a theoretical point of view, it is clear that primordial binaries (hereafter PBs) can dominate the early stages of the dynamical evolution of star clusters (see Heggie & Aarseth 1992).

Although binaries are heavier than the mean mass in a cluster, their masses do not remain constant throughout their life. Mass loss from stellar evolution, stellar collisions and exchange of mass from one component to another can alter significantly the binary orbit or even disrupt it. Primordial binaries which are too close will be modified or even disrupted by mechanisms such as mass loss or mass transfer during the evolution of the primary from a main sequence star to a giant. Because of their massive nature, binaries tend to occupy the inner regions of star clusters so a change in their physical parameters (mass is the most important) may affect significantly the entire dynamical evolution of the system. Their impact on the cluster evolution depends for the most part on the coupling between the characteristic time-scale for mass segregation in the cluster and that for stellar mass loss. If mass segregation can be reached before significant mass loss has occurred, these primordial binaries act as energy sources more or less continually until they become unstable by stellar evolution or escape. In the case of important stellar mass loss before equipartition, this situation may not be attained and the new evolved binaries may have no tendency to reach the inner regions of the cluster because their masses have decreased in a significant way. However, from a theoretical point of view, a smaller effect is expected than in the case of models without PBs but stellar evolution, because the single stars are also losing mass, so binaries are in any case the most massive objects.

The role of stellar evolution for the dynamics of star clusters has been of interest since the end of the seventies (Angeletti & Giannone 1979, 1980; Angeletti et al. 1980) and there are a number of recent papers (Stodólkiewicz 1982, 1985; Terlevich 1983, 1985, 1987; Applegate 1986; Weinberg & Chernoff 1988; Quinlan & Shapiro 1990; de la Fuente Marcos 1993, 1996a (hereafter Paper II); Aarseth 1996b, c).

The effect of PBs on the dynamical evolution of clusters was studied before stellar evolution. The first paper on the subject was by Aarseth (1975) and it was followed by a number of papers (Aarseth 1980; Spitzer & Mathieu 1980; Giannone & Molteni 1985; McMillan et al. 1990, 1991a, b; Murphy et al. 1990; Gao et al. 1991; Hut et al. 1992; Heggie & Aarseth 1992; McMillan 1993; McMillan & Hut 1994; de la Fuente Marcos 1996b (hereafter Paper III)). In addition, Hills (1975) gave a semi-analytical discussion about this question, Goodman and Hut (1989) pointed out the importance of PBs for the evolution of globular clusters and Leonard and Duncan (1988, 1990) carried out N -body simulations with primordial binaries, although their main emphasis was on escapers. Summaries of most of these studies can be found in the introductory section of McMillan et al. (1990), Gao et al. (1991) and Heggie and Aarseth (1992).

On the contrary, the study of the interplay between stellar mass loss and PBs on the evolution of star clusters has only very recently become of interest. Pols and Marinus (1994) studied the binary stellar evolution in young open clusters using Monte-Carlo simulations, although their chief interest was in pure stellar evolution, and not in the dynamical one. Direct N -body calculations have been performed principally by Aarseth,

but only a few details have been published (Aarseth 1996b, c). He has found that because of stellar evolution the fraction of binaries increases in the central regions of rich star clusters (N up to 10^4). This increase in the central binary fraction is because massive single stars evolve to low-mass stars.

This paper is mainly devoted to study the interplay between the mass spectrum, the PB fraction and the mass loss due to stellar evolution on the dynamical evolution of open star clusters. Moreover, we want to compare our results for the surviving binary fraction with observational data for binaries in open clusters. We are mainly concerned with the binary type for trying to answer the question about the preferential type of surviving binaries in open clusters. It is to be expected that binaries with both components being low-mass stars (late spectral types) will be preferential survivors in rich open star clusters because the time-scale for cluster disruption is larger than their characteristic time-scale for significant mass loss due to stellar evolution. However, for poorly populated open clusters, the disruption time-scale can be significantly smaller than the stellar evolution time even for moderately massive stars, so it should be possible to find binaries with massive components. Also, we are interested in comparing the present results with those from previous papers in this series (de la Fuente Marcos 1995 (hereafter Paper I); Paper II; Paper III) concerning the role of the initial mass function (hereafter IMF) on the dynamical evolution of cluster models.

We have performed five runs each for a total number of stars $N = 100, 250, 500, 750$ with five different IMFs using direct integration methods. As in previous papers, we use the same version of the Aarseth's code NBODY5 (Aarseth 1985; Aarseth 1996a). This code has become a standard in the field of star clusters simulations. Written in FORTRAN, it consists of a fourth-order predictor-corrector integration scheme with individual time steps. It utilizes an Ahmad-Cohen (1973) neighbour scheme to facilitate calculation of the gravitational forces, and handles close encounters via two-, three-, four-, and chain regularization techniques (Kustaanheimo & Stiefel 1965; Aarseth & Zare 1974; Mikkola 1985; Mikkola & Aarseth 1993).

All the calculations have been performed on a VAX 9000/210, running under OpenVMS operating system, at the *Centro de Proceso de Datos (UCM, Moncloa, Madrid)*. This machine has one CPU and its peak performance is about 100 Mflops.

2. Cluster models

In the present section all the physical features of the models we have performed are discussed. This description follows closely that in the previous papers of this series, although in this case all the astrophysical processes discussed above are included simultaneously.

2.1. Initial mass function

The frequency distribution of stellar masses at birth is a fundamental parameter for studying the evolution of star clusters. As

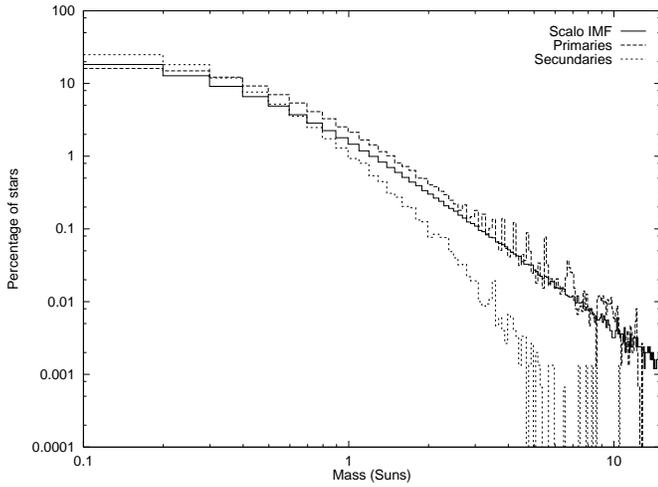


Fig. 1. Distribution function for the modified Scalo IMF with the binary correlation described in Paper III. Note the logarithmic scale for both axes.

Table 1. IMFs used in the calculations

Salpeter IMF	$\alpha = 2.35$
Taff IMF	$\alpha = 2.5$ ($N \leq 100$)
	$\alpha = 2.65$ ($N > 100$)
Miller & Scalo IMF	(Eggleton et al. 1989)*
Kroupa IMF	(Kroupa et al. 1993)
Scalo IMF	(Eggleton 1994) [†]

* This IMF is a fit to the Miller & Scalo's (1979) results.

[†] This IMF is a fit to the Scalo's (1986) results.

in previous papers (Paper I; Paper II; Paper III) several IMFs have been used in the calculations in order to generate an initial distribution of masses. We refer to Paper I for a full discussion of the IMFs used. Models for Kroupa (Kroupa et al. 1993) and Scalo (Scalo 1986; Eggleton 1994) IMFs are used with the binary correlation described in Paper III (Eggleton 1995). In Fig. 1 we see the modified Scalo IMF with the binary correlation used. For models with Salpeter (1955), Taff (1974) and Miller & Scalo (1979) IMFs the two components of the binary have the same mass. The five IMFs used in our calculations are summarized in Table 1.

2.2. Stellar evolution

As in Paper II we use the fitting functions by Eggleton et al. (1989) in order to obtain the stellar diminution of mass as a function of time for Population I stars. These interpolation formulae are explained in the original Eggleton et al's paper and partially in Paper II. All the stars start on a zero-age main sequence (hereafter ZAMS) with a uniform composition of hydrogen, $X = 0.7$, helium, $Y = 0.28$, and metallicity, $Z = 0.02$. For computational convenience, the mass loss is implemented at discrete intervals (accumulated mass diminution of 1 %). Because of very different relative velocities, the actual mass loss is assumed to be instantaneous. The expelled gas leaves the cluster without

any effect on the cluster members. We do not consider mass transfer processes in binaries, so the evolutionary times given by Eggleton's fitting formulae are not changed. Also, there is no presence of accretion disks around stars, so disk accretion can not affect the evolutionary tracks of the cluster stars. There is only one way of changing the evolutionary time-scales for a given star; it can be achieved if two stars collide forming a new object, blue-straggler or Thorne-Żytkow object (Thorne & Żytkow 1977).

2.3. Binary fraction and other binary parameters

The number of primordial binaries in the cluster is conveniently parameterized by the binary fraction f given by:

$$f = \frac{N_b}{N_b + N_s}, \quad (1)$$

where N_b , N_s are, respectively, the number of stars which are binaries and singles. As in Paper III, a binary fraction of $f = 1/3$ is used in the calculations; so the overall multiplicity, defined as the ratio of the number of multiples to the total number of systems, is 0.33 in our present models. Lower bounds from observational surveys in Star Formation Regions are 0.37 (Ophiuchus) and 0.55 (Taurus) (Simon et al. 1995). For a sample of stars in the solar neighbourhood, Duquennoy and Mayor (1991) have found a fraction of 0.57, after correcting for observational bias. For comparing directly with models from previous papers, the number of singles and binaries is chosen in such a way that the total number of stars (not objects) is 100, 250, 500 and 750 respectively. This arbitrary choice has minor effects on the cluster dynamical evolution as described in Paper III.

In order to include a realistic initial binary population there are some other parameters to be determined in addition to the binary fraction. Our initial population of binaries is hard because they are of main dynamical importance for the evolution of star clusters. A binary is defined to be hard or soft (Heggie 1980) depending on whether its binding energy is greater than or less than the local mean kinetic energy per star. As in Paper III, the semi-major axis of the binaries is taken from a uniform distribution:

$$a_b = a_b^0 10^{-q}, \quad (2)$$

where a_b^0 is an input parameter whose value is about $1/N$ in units of R_{vir} (for a hard binary) and q is equal to $X \log \mathcal{R}$. The virial radius, R_{vir} , is defined by

$$R_{\text{vir}} = -G \mathcal{M}^2 / 4E, \quad (3)$$

where E is the total energy of the system, excluding the binding energies of any initial binaries, G is the gravitational constant and \mathcal{M} is the total mass of the cluster; \mathcal{R} is an input parameter, and X is a random number uniformly distributed in the interval $[0, 1]$. The spread in the energy of the binaries is given by the spread in semi-major axis and finally it is given by \mathcal{R} . Small values of \mathcal{R} produce wide binaries, large values give close binaries. The value of \mathcal{R} used in our simulations is the same

that we used in Paper III, $\mathcal{R} = 10$. A typical value for the initial semi-major axis of the binaries in our simulations is about 500 AU (the smaller value is about 70 AU and the maximum is about 3300 AU). The values for the semi-major axis in Table 2 are the upper cut-off for the semi-major axis distribution. Eccentricities are chosen from a random (thermalized) distribution (Jeans 1929) and the same is done for the pericentre, node and inclination. The mass ratio for the PBs in all the models is 1/2, excepting those in which the binary correlation has been used.

2.4. Main features of the models

All the models (for the same N) have the same sequence of random numbers for generating initial conditions; spherical symmetry and constant density are assumed, with the ratio of total kinetic and potential energy fixed at 0.25. Another characteristics common to all models are random and isotropic initial velocities. Also, in all the cases the cluster suffers mass loss due to escape of stars. A star escapes and is removed from the calculation when its distance from the cluster centre is greater than twice the tidal radius. The tidal radius is given by the classical expression $r_t \approx (G \mathcal{M} / T_1)^{1/3}$ where T_1 is defined as a function of A and B Oort constants of galactic rotation: $T_1 = 2\omega[(-A - B) - \omega]$, where ω is the rotational velocity of the star cluster around the galactic nucleus. As in previous papers, the galactic gravitational field is introduced in the models as described in Terlevich (1987). Moreover, we ignore the effect of field stars on the dynamical evolution of the cluster due to their high relative velocities (see discussion in Paper II).

We use a standard and consistent set of units throughout, except where explicitly noted. All lengths are measured in parsecs. Times are measured in terms of the initial half-mass crossing time, defined by

$$T_{cr} = G \mathcal{M}^{5/2} / (-2E)^{3/2}. \quad (4)$$

It represents the time taken for a typical star, moving with velocity $< v^2 >^{1/2}$, to cross the virial diameter ($2 R_{vir}$). As in previous papers of this series we adopt a consistent tidal field for the models which have the same mean stellar density. A typical value for the mean mass density in a real cluster is $1.3 \mathcal{M}_\odot pc^{-3}$ (Lohmann 1971, 1976a, 1976b, 1977a, 1977b) with a range of $0.5-3.2 \mathcal{M}_\odot pc^{-3}$ for his sample of open clusters. These values are typical for evolved clusters, so that a larger mean density of $\simeq 12 \mathcal{M}_\odot pc^{-3}$ is adopted for the whole cluster (Table 2) as considered young and not evolved. All the models do not have the same maximum (\mathcal{M}_{max}) and minimum (\mathcal{M}_{min}) masses for generating the IMF. The Salpeter, Taff and Scalo IMFs have $\mathcal{M}_{max} = 15.0 \mathcal{M}_\odot$ and $\mathcal{M}_{min} = 0.1 \mathcal{M}_\odot$ but the Kroupa and Eggleton IMFs use an algorithm that changes upper and lower limits for masses (see Table 2). Hence models for the Salpeter and Taff IMFs have the value of mean density quoted above but the others have $\simeq 6 \mathcal{M}_\odot pc^{-3}$ (if using the same mean stellar mass, this will give the initial virial radius for different N).

Table 2 gives the disruption time for our present models. If we compare these values with those from Paper III we observe that for $N = 100$ the disruption time is now smaller only for

models with Salpeter, Taff and Miller & Scalo IMFs. However, for models with Kroupa or Scalo IMFs the disruption time is increased even comparing with Paper II. The main difference between the two groups of IMFs is the maximum mass. For the first group, the upper cut-off for the mass spectrum is 15 solar masses but this value is significantly smaller for the second group, so the reason for this behaviour is clearly due to the massive stars. It seems that the supernova events in the core destabilize the cluster in a very efficient way for poorly populated clusters. Two supernova events are enough to provide the energy source which disrupts the cluster. The acceleration of disruption as compared with Paper II is very significant for models with power-law IMF, however for Miller & Scalo IMF the disruption time in the current models is greater than the respective ones for Paper II (due to an upper cut-off in the mass spectrum). For models with $N = 250$, the same trend pointed out above is observed for power-law models with regard to Paper III.

On the contrary, realistic IMFs show greater disruption times than those of Paper II and III. For $N = 500$, the behaviour of the disruption time is very similar to $N = 250$. For $N = 750$, power-law models have disruption times greater than those of Paper III but a bit smaller than those of Paper II. For Miller & Scalo and Kroupa IMFs the disruption times are smaller than those of Paper III and Paper II, however for the Scalo IMF the disruption time is greater than those from Paper II and III. This suggests a complex interplay between the IMF, mass loss from stellar evolution and primordial binaries. For poor clusters the massive stars in PBs dominate the cluster evolution. For rich clusters, the interplay between power-law IMFs, PBs and stellar evolution seems to increase the cluster life-time in relation to models with PBs but no stellar evolution. For intermediate population ($N = 250, 500$), power-law models with all realistic features disrupt earlier than their respective models without mass loss. However, there is no clear interpretation for this behaviour. The increased life-time in some models for $N = 750$ can be explained by the formation of temporarily stable multiple systems (in some cases, hierarchical triple systems) in the cluster remnant; for $N = 500$, a certain number of these systems are also formed in some models. These systems delay the disruption of the evolved cluster because some of them are long-lived. Formation of such systems depends strongly on the fraction of PBs, the cluster membership and the parameter \mathcal{R} . Larger N and PB fraction promote an increased probability of formation of such multiple systems.

3. Characteristic quantities

In this section we compare all of the different runs, concentrating, as in previous papers, on two representative diagnostic parameters. The first one measures the degree of dynamical evolution of the entire cluster. This quantity is called the evolution modulus and is defined by (von Hoerner 1976)

$$W = \log(R_h/R_c), \quad (5)$$

Table 2. Main characteristics of the models

MODEL	IMF*	N^\bullet	N_b	$a_b^0 \times$	$\mathcal{M}_{\max}^\dagger$	$\mathcal{M}_{\min}^\dagger$	$\langle \mathcal{M} \rangle^\dagger$	R_0^\ddagger	$r_{t_0}^*$	$\langle R \rangle_0^\diamond$	T_d^\ominus
I	SA	100	25	0.0160	15.0	0.1	1.0	1.26	5.96	0.78	30.4 (74)
II	TA	100	25	0.0160	15.0	0.1	1.0	1.26	5.95	0.78	47.4 (116)
III	MS	100	25	0.0160	15.0	0.1	0.7	1.26	5.39	1.06	91.9 (261)
IV	KR	100	25	0.0160	5.5 / 1.8	0.2 / 0.1	1.2 / 0.4	1.26	5.16	1.05	111.7 (335)
V	SC	100	25	0.0160	5.8 / 2.0	0.3 / 0.1	1.3 / 0.4	1.26	5.37	1.10	89.9 (256)
VI	SA	250	62	0.0116	15.0	0.1	1.0	1.71	8.11	1.40	118.1 (288)
VII	TA	250	62	0.0116	15.0	0.1	1.0	1.71	8.11	1.32	184.6 (450)
VIII	MS	250	62	0.0116	15.0	0.1	0.6	1.71	7.03	1.37	171.8 (517)
IX	KR	250	62	0.0116	5.5 / 4.8	0.2 / 0.1	0.9 / 0.4	1.71	6.75	1.66	177.5 (568)
X	SC	250	62	0.0116	5.8 / 5.1	0.2 / 0.1	1.0 / 0.5	1.71	7.20	1.62	217.8 (634)
XI	SA	500	125	0.0123	15.0	0.1	1.0	2.15	10.19	2.00	205.1 (498)
XII	TA	500	125	0.0123	15.0	0.1	1.0	2.15	10.19	1.91	280.0 (680)
XIII	MS	500	125	0.0123	15.0	0.1	0.6	2.15	8.75	1.98	204.4 (625)
XIV	KR	500	125	0.0123	5.5 / 5.1	0.2 / 0.1	0.9 / 0.4	2.15	8.51	2.02	270.1 (859)
XV	SC	500	125	0.0123	5.8 / 5.4	0.2 / 0.1	1.1 / 0.5	2.15	9.05	1.98	262.4 (763)
XVI	SA	750	187	0.0081	15.0	0.1	1.0	2.47	11.68	2.30	329.5 (804)
XVII	TA	750	187	0.0081	15.0	0.1	1.0	2.47	11.68	2.30	387.5 (945)
XVIII	MS	750	187	0.0081	15.0	0.1	0.6	2.47	9.98	2.32	162.6 (503)
XIX	KR	750	187	0.0081	5.5 / 5.2	0.2 / 0.1	0.9 / 0.4	2.47	9.73	2.27	227.0 (729)
XX	SC	750	187	0.0081	5.8 / 12.9	0.2 / 0.1	1.0 / 0.6	2.47	10.30	2.27	369.8 (1061)

* SA Salpeter IMF, TA Taff IMF, MS Miller & Scalo IMF, KR Kroupa IMF, SC Scalo IMF.

• Total number of stars ($N_s + 2 N_b$).

× Semi-major axis for PBs in pc.

† In \mathcal{M}_\odot . For KR and SC models (Binary / Single).

‡ Initial virial radius in pc.

* Initial tidal radius in pc.

◊ Initial half-mass radius in pc.

⊙ Disruption time in scaled units (in Mys).

where R_h is the half-mass radius and R_c is the core radius. The core radius defined by Casertano and Hut (1985) is

$$R_{\text{core}}^{(CH)} = \sum_i \rho_i R_i / \sum_i \rho_i. \quad (6)$$

Here, R_i is the distance from star i to the density center, defined as the density-weighted centroid of the system, the density ρ_i is determined by the distance $R_{6,i}$ to star i 's sixth nearest neighbour:

$$\rho_i = M_{6,i} / R_{6,i}^3, \quad (7)$$

where $M_{6,i}$ is the total mass lying within distance $R_{6,i}$ of star i (excluding the mass of star i itself), and the sum is taken over all stars in the system. The second parameter is the escape rate dN/dt which describes the disruption rate of the system.

3.1. Evolution modulus

This quantity has the advantage that it can be obtained directly from observable properties. Figs. 2-6 show the behaviour of W as a function of time for all the models. If we compare the present figures with those from Paper II we observe significant differences.

For Salpeter IMF models, the behaviour of W is very similar for all N in Paper II, however the inclusion of PBs shows great differences, depending on N , for our present models. First, the evolution time-scales (in scaled units) are very different depending on N , accelerating the dynamical evolution in poor clusters. For $N = 100$, the cluster is disrupted even having $W > 0$. For $N = 250$ the evolution is also accelerated but for $N = 500$ the evolutive time-scales are almost the same as without PBs. The evolution is even slower for $N = 750$. As before, this trend can be explained by the formation of temporarily bound multiple systems (triple and quadruple). The comparison between our present results for W and those from Paper III also presents several differences. The evolution of W is slower for greater N but for smaller N it is nearly similar. The mean value of W is now significantly smaller because the halo is less extended than in the case of models with PBs but no stellar evolution.

For Taff IMF models, the behaviour of W is roughly similar to that presented in Paper II except in the case of $N = 100$. In this case, the cluster evolution is accelerated in a very significant way, but the cluster is completely disrupted after reaching $W = 0$. The behaviour is different if comparing with Paper III because

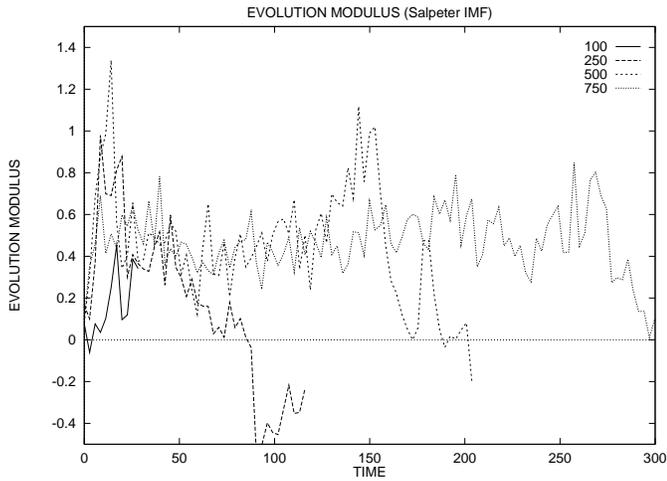


Fig. 2. Evolution modulus for Salpeter's models with $N = 100, 250, 500, 750$. In all figures, time is given in units of the half-mass initial crossing time in the current system.

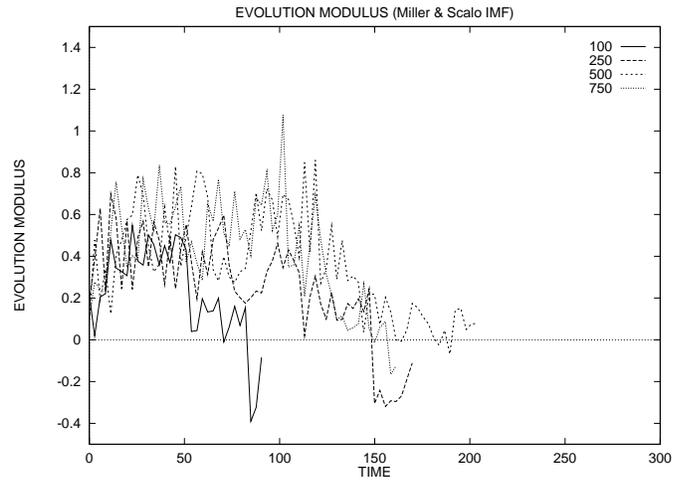


Fig. 4. Evolution modulus for Miller & Scalo's models with $N = 100, 250, 500, 750$.

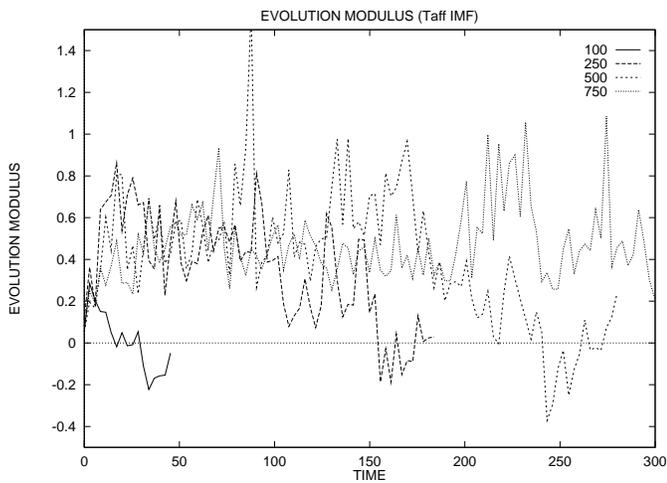


Fig. 3. Evolution modulus for Taff's models with $N = 100, 250, 500, 750$.

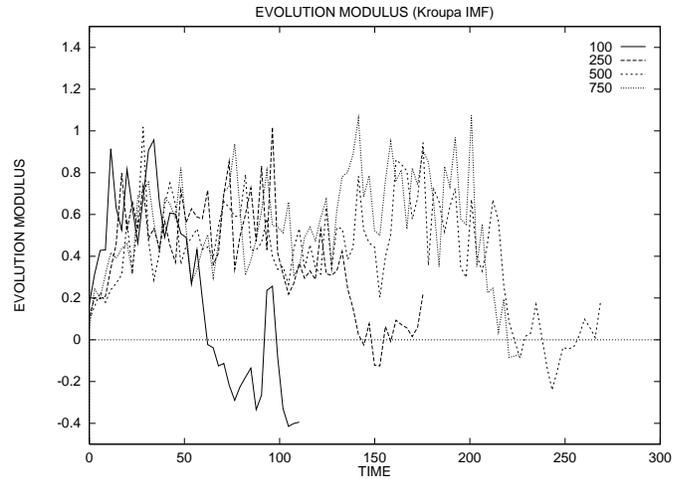


Fig. 5. Evolution modulus for Kroupa's models with $N = 100, 250, 500, 750$.

the core-halo structure seems to be stabilized by stellar mass loss as compared with models with PBs but no mass loss.

Models with Miller & Scalo IMF show practically similar behaviour to those presented in Paper II. The only main differences appear for early stages of the cluster evolution where the mean value of W is greater for models without PBs. This suggests that the haloes formed in models with PBs and stellar evolution are less extended. As regards comparison with models without stellar evolution we also observe features suggesting that the interplay between mass loss and PBs produces less energetic haloes than in the case of models with PBs but no stellar evolution.

For models with Kroupa IMF, we observe that the evolution is very different from that presented in Paper II. For large N the evolution is slowed down but for $N = 100$ the evolution is accelerated although less than for power-law models. The

evolution of W is affected even at early stages of the cluster evolution, when the stellar mass loss is not yet dominant. If comparing with Paper III, the evolution of the cluster seems to be slightly accelerated.

Finally, for Scalo IMF we observe that with regard to Paper II the evolution for models with $N \geq 250$ is significantly slowed down, but speeding up a bit for smaller N . As regards Paper III, the ratio core-halo (in size) seems to be more stable for models with mass loss and the overall evolution is slowed down.

We observe that the inclusion of stellar evolution in models with PBs affects their evolution considerably but in a very uncertain way. However, it is clear that the changes are very IMF dependent. The interpretation of the results for our present models in relation to models with PBs but no stellar evolution is very unclear in comparison with the conclusions we obtained with regard to models without PBs but with or without mass loss due to stellar evolution.

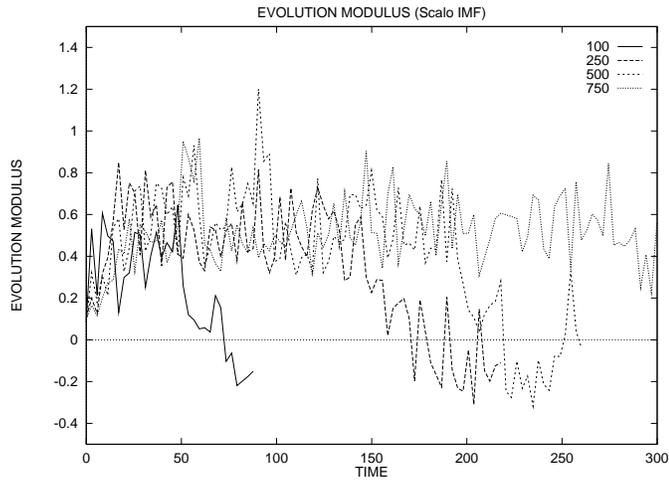


Fig. 6. Evolution modulus for Scalo's models with $N = 100, 250, 500, 750$.

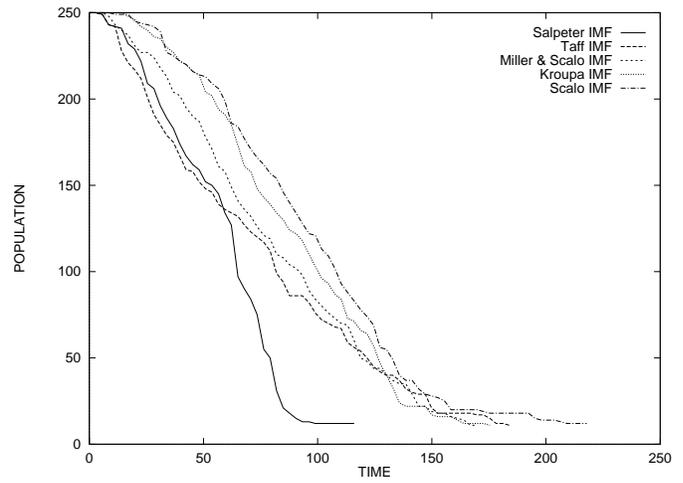


Fig. 8. Evolution of the cluster population with time for models with $N = 250$.

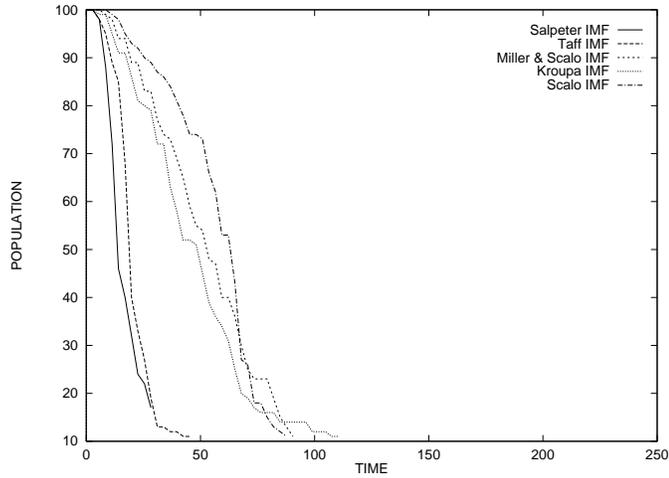


Fig. 7. Evolution of the cluster population with time for models with $N = 100$.

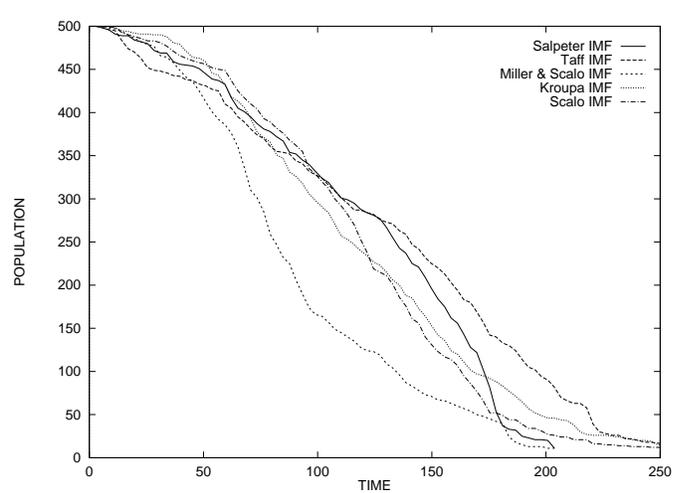


Fig. 9. Evolution of the cluster population with time for models with $N = 500$.

3.2. The escape rate

This is another important quantity for the cluster evolution. Figures 7-10 show the number of cluster stars as a function of time.

For $N = 100$ the inclusion of stellar evolution in models with PBs distinguishes clearly between models with power-law IMFs and realistic ones. In comparison with figures for the escape rate without stellar evolution, the disruption of power-law models is accelerated but in the case of realistic IMFs it is slowed down. Even for Scalo IMF we observe an increase in the duration of the stage in which close encounters are dominant. The almost exponential behaviour that we observe in some models in Paper III disappears because most of the escapers are not due to distant encounters. As regards comparison with Paper II the evolution of the escape rate is very different so we suggest that the escape mechanism induced by binaries with stellar evolution is different from the dominant one in clusters with mass loss due to stellar evolution but no PBs.

The same trend is also observed for $N = 250$ but it is not so clear as in the case of $N = 100$. Models with Taff and Miller & Scalo IMFs show a nearly similar behaviour as in Paper III.

For $N = 500$ the behaviour of the escape rate is very different from that presented in Paper III. Models with massive stars (power-law IMFs) lose their distinctive features and accelerate their disruption rates (in scaled units). For the other IMFs the figures are nearly similar to their respective ones without PBs. With regard to comparison with Paper III, the escape rate is reduced by including stellar evolution.

For $N = 750$ the inclusion of stellar evolution in models with PBs produces a significant change in the escape rate for power-law IMFs. Their escape rate show a slow-down with respect to those from Paper III. However, models with realistic IMFs, except for Scalo IMF, look very similar to those from Paper III. With regard to comparison with Paper II results, the escape rate

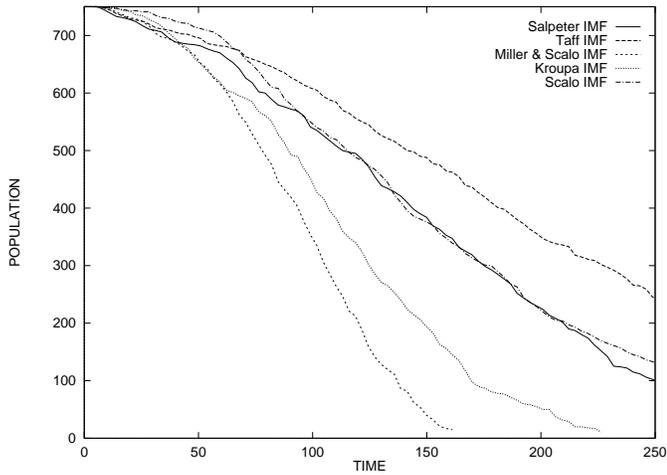


Fig. 10. Evolution of the cluster population with time for models with $N = 750$.

is slowed down for models with Miller & Scalo, Kroupa and power-law IMFs but is accelerated for Scalo IMF.

As we can see, the interpretation of the results is difficult in most of the cases. Only for $N = 100$ can we do this in an easy way. Clearly the differences induced in these models are affected by the massive stars. The stellar evolution in models with small N and PBs is the dominant mechanism for the dynamical evolution of open clusters. For most of the models with small N we found in Paper III almost exponential decay in the escape rate which suggests an evaporative (by distant encounters) dominant escape mechanism, however now we find a change in the shape of the curve. The reason seems to be due to the halo extension. Models including stellar evolution seem to show less extended haloes than models without, so the number of stars available for leaving the system in a smooth way is smaller. This trend almost disappears for models with $N = 250$ but it can still be observed preferentially at early stages of the cluster evolution. For larger N the explanation is not very clear, also we note less extended haloes but the results depend strongly on the IMF. In any case the dominant mechanism for the escape rate at early stages of the star cluster evolution seems to be by close encounters.

4. Evolution of PB population

This section is devoted to study the evolution of the PBs in our present models. We are mainly interested in comparing with results from Paper III. In Figs. 11-14 we see the percentage of primordial binaries in the cluster as a function of time.

From the figures, the first thing to note is the life time of the primordial binary population. For poorly populated models ($N = 100$), we obtain that the life time of the PBs are significantly shorter than for Paper III models. Moreover, the evolution of the surviving fraction of the PB population for models with power-law IMFs seems to be clearly different from that observed for models with realistic IMFs. The diminution in the number of PBs is more rapid because the evolution of the cluster itself is

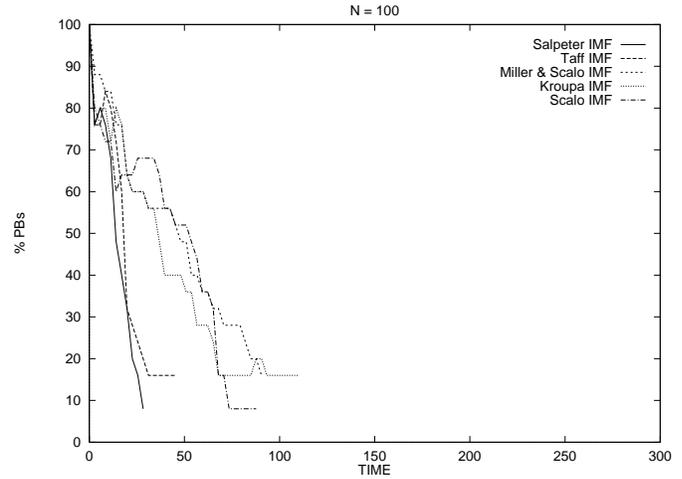


Fig. 11. Evolution of the PB population as a function of time for models with $N = 100$.

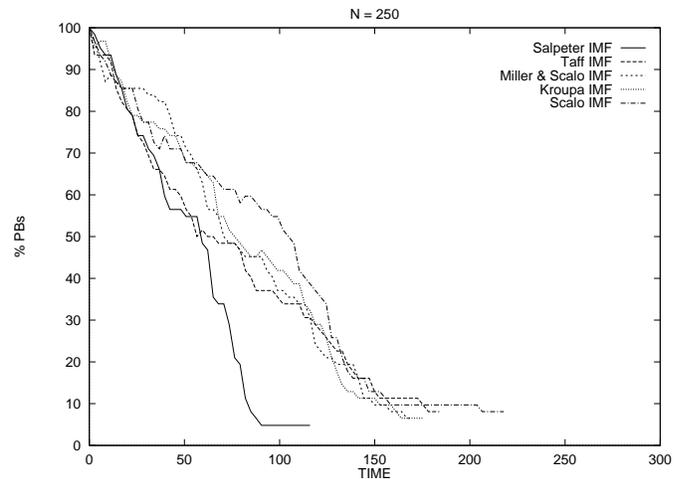


Fig. 12. Evolution of the PB population as a function of time for models with $N = 250$.

quicker. The reason is due to the supernova events. In Salpeter and Taff models two supernova events have occurred; as a result the cluster disintegration and the escape or disruption of PBs are accelerated. For $N = 250$, some differences are observed as regard models from Paper III. Salpeter models show a significant acceleration in the diminution rate of the PB fraction caused, as before, by the supernova explosions (2 for these models). However, the Taff model shows a nearly similar behaviour as in Paper III; the same can be observed for realistic IMFs except for the Scalo model. For $N = 500$ the almost exponential decrease in the percentage of surviving binaries which appears in models from Paper III disappears for the majority of the present models. Initially the same linear diminution is observed, but later the diminution rate slows down by a very significant value. For some models in Paper III the percentage of surviving PBs at the cluster mean life time is smaller than 30 % but now this value reaches almost 50 % for power-law IMF models. Models with $N = 750$

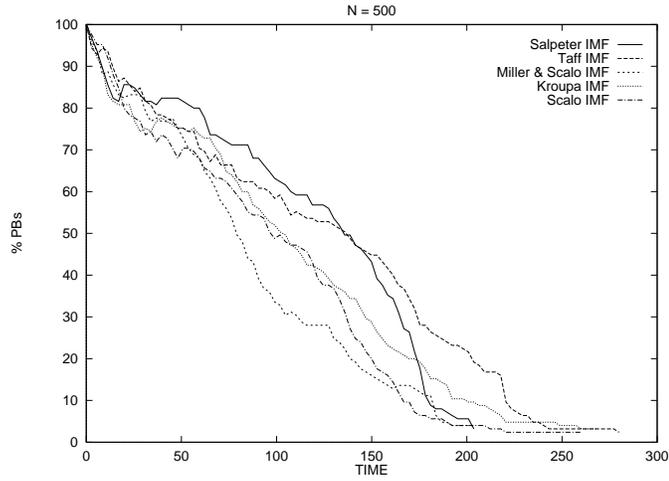


Fig. 13. Evolution of the PB population as a function of time for models with $N = 500$.

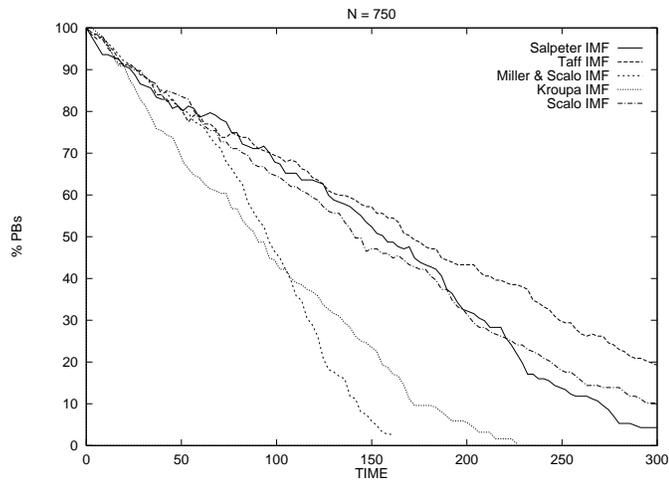


Fig. 14. Evolution of the PB population as a function of time for models with $N = 750$.

also show differences. In Paper III, all the models (excluding the monocomponent one) have a nearly similar behaviour but now models with Salpeter, Taff and Scalo IMFs show almost the same behaviour but the others do not. For the former models the diminution of PBs is almost linear and smaller than for Miller & Scalo and Kroupa IMFs. From Table 2, we see that models for Salpeter, Taff and Scalo IMFs have more massive stars so this may be the reason for this behaviour. In average, our models show that PBs are retained preferentially; this result has also been found by Aarseth (1996d).

Our present considerations suggest that the membership is a main factor for retaining PBs; models with increasing N show smaller diminution rates for the percentage of surviving PBs. Also, the IMF has a main role in the evolution of PBs; for poorly populated clusters massive stars in binaries control the evolution of the entire cluster. For small open clusters the presence of

Table 3. Binary fraction in the core and in the whole cluster

MODEL	f_c^0	f_t^0	f_c^h	f_t^h	f_c^e	f_t^e
I	0.31	0.33	0.00	0.35	0.00	0.13
II	0.33	0.33	0.00	0.28	0.00	0.57
III	0.63	0.33	0.80	0.28	0.36	0.57
IV	0.45	0.33	0.33	0.27	0.18	0.57
V	0.47	0.33	0.27	0.23	0.09	0.22
VI	0.33	0.33	0.32	0.31	0.25	0.50
VII	0.35	0.33	0.00	0.31	0.00	0.83
VIII	0.30	0.33	0.33	0.31	0.27	0.57
IX	0.33	0.33	0.17	0.29	0.09	0.57
X	0.36	0.33	0.36	0.39	0.33	0.71
XI	0.36	0.33	0.75	0.34	0.19	0.57
XII	0.37	0.33	0.42	0.31	0.09	0.38
XIII	0.34	0.33	0.50	0.32	0.09	0.83
XIV	0.36	0.33	0.33	0.28	0.08	0.63
XV	0.27	0.33	0.33	0.28	0.25	0.33
XVI	0.34	0.33	0.00	0.35	0.00	0.63
XVII	0.33	0.33	0.00	0.28	0.00	0.80
XVIII	0.32	0.33	0.47	0.32	0.33	0.50
XIX	0.37	0.33	0.55	0.25	0.13	0.22
XX	0.36	0.33	0.55	0.30	0.29	0.56

massive stars in binaries can accelerate their disruption but for more populated star clusters the effect is to the contrary.

An interesting parameter to study is the binary fraction, f , both in the core and the whole cluster. Table 3 gives the core and total binary fractions for all the models at three selected times in the cluster life: at time equal to zero (f_c^0 , f_t^0), at the time in which the stellar population is a half of the initial one (f_c^h , f_t^h) and at the end of the simulation (f_c^e , f_t^e). In this Table we only consider the primordial binaries because they are the only hard ones; in our present models binaries which are formed dynamically are all soft and short lived. Some quick conclusions arise from the present values of the binary fraction. First, for power-law models there is a strong tendency for binaries to underpopulate the core on a short time scale. Except for $N = 500$ all the power-law models show a smaller binary fraction in the core at the half cluster life. The explanation is due to the supernova events located in the core. They dominate the evolution, driving the binaries outside the core (values for the total binary fraction are nearly similar for all the models at the cluster half life). A typical behaviour of the total binary fraction can be observed in Fig. 15. In all the models an initial decay is observed but the total binary fraction increases in most of the cases when the cluster population has decreased significantly. It suggests a preferential escape of the single stars in clusters with a fraction of PBs. However, the evolution of the core binary fraction with time is extremely irregular. Its behaviour is extremely chaotic although almost in all the models an initial increase is observed as in Aarseth (1996b, c). This effect is greater for rich models and realistic IMFs. Second, the value of the total binary fraction towards the end of our simulations for $N \leq 250$ is 0.57

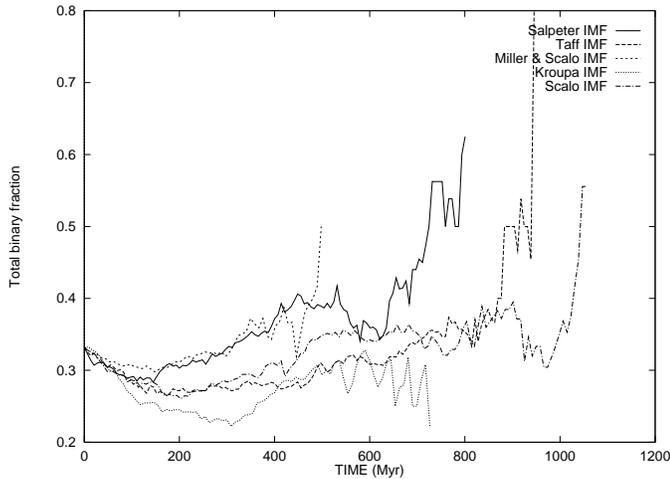


Fig. 15. Evolution of the total binary fraction as a function of time in years for models with $N = 750$. In all the models an initial decay is observed but it increases in most of the models when the cluster population has reached values smaller than one third of the initial.

in many of the models; it is the value currently accepted for the binary fraction in the solar neighbourhood (Duquennoy & Mayor 1991). This value appears preferentially for models with Miller & Scalo and Kroupa IMFs so it is possible to suggest that there probably exists a link between the size of the cluster in which stars are born, their IMF and the binary fraction in a certain region of space.

5. Velocity distribution of the escaping stars

In this section we analyze the velocity distribution of the escaping stars, both singles and binaries. The main emphasis is on comparing present results with previous ones for models without PBs and mass loss, with mass loss but no PBs and with PBs but no mass loss. Most of the results that appear in this section were not discussed previously in the papers of this series.

Stars are considered as escapers and they are removed from the calculations when they reach a distance from the cluster centre greater than twice the tidal radius. At that moment, we note the velocity of the star (if single) or of the centre of mass (if binary). The escape boundary depends on the Galactic parameters so the obtained velocities are higher than those obtained from the theoretical expression ($2 < v >$); many of our velocities have excess escape energy. The values of the mean escape velocity, the standard deviation in the mean, the minimum escape velocity and the maximum escape velocity can be seen in Table 4. We observe two clear tendencies: first, the mean escape velocity increases with N and second, the dispersion of velocities decreases when N increases. This is to some extent connected with the physical scaling in the models. Moreover, the greater mean escape velocities are always for models with Salpeter and Taff IMFs, so it depends on the IMF. This is to be expected because the masses of these models are greater than for the other IMFs. As we can see, when N increases the probabil-

Table 4. Escape velocity for the models*

<i>MODEL</i>	$\langle v_{esc} \rangle$	σ_m	v_{esc}^{min}	v_{esc}^{max}
I	0.935	0.070	0.261	4.450
II	0.866	0.062	0.413	4.426
III	0.822	0.056	0.309	3.511
IV	0.699	0.044	0.000	2.413
V	0.710	0.052	0.223	3.137
VI	1.190	0.065	0.251	7.551
VII	1.270	0.051	0.434	5.992
VIII	0.996	0.044	0.000	5.768
IX	0.858	0.030	0.270	3.994
X	1.049	0.049	0.210	6.915
XII	1.419	0.050	0.000	11.261
XIII	1.456	0.057	0.000	18.427
XIV	1.140	0.031	0.245	5.050
XV	1.051	0.023	0.241	4.726
XVI	1.211	0.036	0.264	7.819
XVII	1.558	0.030	0.345	6.960
XVIII	1.544	0.036	0.345	8.687
XIX	1.141	0.024	0.267	6.386
XX	1.158	0.026	0.000	7.988
XXI	1.246	0.030	0.000	11.080

* All the data in km/s.

ity of high velocity escapers grows; we observe that in all cases the maximum escape velocity is considerably higher than the mean escape velocity. The origin of these high velocity escapers is connected with binary-binary interactions so the high escape velocities must also be connected with binaries of a certain size.

As regards comparison with results from previous models, we observe significant differences. Considering results from models of Paper III, the mean velocity of the escapers is reduced in most of the present models. This tendency is clear for clusters with simple power-law IMFs. However, realistic IMFs show a more complex behaviour. For $N = 100$, the same tendency as for old IMFs is observed (except for Kroupa IMF). For $N = 200$, the mean velocity is larger for the current models. For $N = 500$, this is only observed for Miller & Scalo IMF. For $N = 750$, the same tendency is observed for all the models. Concerning the theoretical implications of these experimental results, it seems that stellar evolution of massive stars is the main process which explains the observed differences. Mass loss in binaries promotes the disruption of a certain number of such systems for models with simple power-law IMFs so the global results suggest the number of initial binaries would have been smaller. On the other hand, present values for the mean velocity of the escapers are larger than those of Paper II, so the presence of PBs increase the velocity of the escapers (due to close encounters between binaries and singles) for models with mass loss.

As regards the relation of velocities of escaping stars with their masses, we observe a bimodal distribution. Most of the escaping particles have velocities not much larger than the mean but a few per cent has velocities greater than three times the

mean. This suggests two processes for the escape of cluster stars. Low velocity escapers are generated by evaporation, i.e. gradual increase of the velocity because of distant encounters and high velocity escapers are produced by close encounters between singles or, more frequently in models with PBs, between singles and binaries or binary-binary encounters.

6. Mass loss in binaries

In order to obtain the change of mass as a function of time, NBODY5 includes the fast fitting functions by Eggleton et al. (1989) for Population I stars. For convenience, mass loss is implemented at discrete intervals when the accumulated contribution reaches 1 percent. The actual mass loss is assumed to be instantaneous, with no further effect on the cluster members or binary companions. Approximate energy conservation can be achieved by performing appropriate corrections to the total potential energy. In the version of NBODY5 used, the binding energy is corrected by the term $\Delta \mathcal{M}/r$ where $\Delta \mathcal{M}$ is the mass loss and r is the binary separation. For close binaries the mass loss is always at apocentre, hence the effect will be smallest. When the mass loss is large, the binary is usually disrupted.

The loss of mass from a binary system is a very complicated dynamical problem. Stellar mass loss causes changes in the orbital elements of the binary. The problem has been mainly studied by Hadjidemetriou (1963, 1966, 1968).

The orbital period can increase or decrease secularly depending upon the mass-flow conditions. The simplest case is where mass is lost isotropically from the system. This situation is assumed in our stellar mass loss events. In our models, the vast majority of massive binaries are disrupted during mass loss events, so the preferential binary survivors are low mass binaries in which the mass loss rate is sufficiently slow to permit a quasi-stable evolution.

By Kepler's third law,

$$\frac{4 \pi^2 a^3}{P^2} = G \mathcal{M} \quad (8)$$

where P is the binary period. It gives, for a constant semi-major axis a , the following relation between the change in period ΔP for a mass loss $\Delta \mathcal{M}$:

$$\frac{\Delta P}{P} = -\frac{\Delta \mathcal{M}}{2 \mathcal{M}}. \quad (9)$$

From Eq. (9) an abrupt change in the period can be achieved by one component losing mass in an eruptive outburst, with the lost material ejected at a high speed. From our simulations, it is found that the evolution of surviving low mass PBs (not exchanged) with the two components of the same mass can be described almost exactly by Eq. (9) with small deviations due to the perturbers; i.e. the semi-major axis remains almost constant during most of the time when the mass loss rate is not too high. The evolution of massive binaries is very complicated because of mass loss. The majority of these binaries are destroyed during the first few crossing times. Their products, mainly white dwarfs, appear sometimes as members of another binary. These

binaries containing collapsed objects have a mass ratio of nearly one.

The above formulae can be used for estimating the changes in the period for an unperturbed binary. The phenomenon is more complex if we consider that the binary has several (in some case many) perturbers. In this case mass loss events near apocentre can easily disrupt the binary because the perturbing force is of the same order as the force between binary components. This depends mainly on the binary semi-major axis; binaries with small semi-major axis are like single stars from a dynamical point of view, so the mass loss events are less destructive for them. On the other hand stellar mass loss in clusters with PBs can favour the survival of certain kinds of binaries; binaries with both low-mass primary and secondary do not suffer a significant mass loss before their possible escape from the cluster. There is observational evidence (Verbunt et al. 1994) that favour the above hypothesis; most X-ray sources in the old open cluster M67 are RS CVn binaries. These are active binary systems in which the primary is a star with a spectral type F or G in the main sequence or subgiant (luminosity class V or IV) and the secondary is a bit cooler and usually more massive and evolved with spectral type K and class IV. Their orbital periods are usually in the range 1–14 days. In models for $N = 750$ we have found several binaries of this spectral type in the remnant, typically with a period of a few hundred years although in other models (not detailed here) with $N = 1500$ and 500 primordial binaries the typical value for the period is about 8 days. In any case, the mean value for the periods of the surviving binaries depends on the initial period distribution.

7. Evolution of the stellar content

As pointed out above all our models start with the stars on the ZAMS. This assumption implies some questionable hypotheses. First, it assumes that the stellar formation in the cluster took place by a single burst. On the other hand, it means there is no preferential stellar mass for the beginning of the star formation. The second of these premises is arguable in the light of some observational discoveries (Herbig 1962; Iben & Talbot 1966; Cohen & Kuhi 1979; Adams et al. 1983; Strom 1985). They suggest that low-mass stars form first and over a long time scale. However, Strom (1985) cautioned that these conclusions rested heavily on the theoretical pre-main-sequence tracks. As pointed out in the excellent review by Zinnecker et al. (1993), low-mass stars acquire their final mass first, but will then take a long time to reach the ZAMS; high-mass stars take longer to accumulate their mass but they evolve rapidly onto the ZAMS. Indeed, the data available are consistent with simultaneous formation of stars of all masses (Stahler 1985; Schroeder & Comins 1988). In spite of these questionable initial hypotheses, the evolution of the stellar content of our models is analyzed in the present section.

We want to study specially the destiny of the collapsed objects in our models. As we see from Table 2, some of our models (preferentially with power-law IMFs) have stars with enough mass to finish their nuclear life in a supernova event. From our

current knowledge of stellar evolution, after such an episode the final product may be a neutron star (called pulsar if its radio emission can be detected from the Earth) or a black hole. The last kind of collapsed object has not been introduced in the stellar evolution routines used in our present models. As regards the formation of neutron stars, it is considered in our models. There are a significant number of neutron stars (pulsars) detected in globular clusters, but at present there is no evidence for these collapsed objects in open clusters. In our models there are no surviving neutron stars. They escape from the cluster shortly after their formation. It is because after the formation of a pulsar, it suffers a kick velocity due to a strong non-symmetric mass loss. Although models with greater N can retain a population of neutron stars, our present models are not able to keep the neutron stars. The time-scale for leaving the cluster for a newly-born neutron star in our models is a few kyr, so it can be considered as an explanation for the null population of neutron stars observed in open clusters. However, it is to be expected from models with greater N ($N = 1500$ in models not presented here) that there is a higher (but low) chance of detecting a neutron star in a highly populated open cluster (maybe M67 is a good candidate to achieve this).

On the contrary, white dwarfs remain in the cluster for many crossing times in most of the models. We have even found a few models in which there are one or two white dwarfs among the members of the cluster remnant. However, there are some differences between the life times for white dwarfs in our models depending both on the IMF and cluster membership.

8. Open cluster remnants

In this section we consider the composition of the final cluster remnant in our models. In Paper III, we noted that the final remnants had a common feature, a high binary richness. This is also observed in the present models but the fraction of purely primordial binaries remaining in the final object resulting from the cluster disruption has decreased. Also we observe that the binaries with two components of the same mass are the preferential survivors. In the majority of the models the number of remaining binaries is three or four (we stop the simulations when the cluster population is $N = 10$). From the results of models for Papers I and II, we also find a certain number of binaries in the cluster remnant (usually 1 or 2). These binaries are formed dynamically, not primordially so one of them is hard and the others (if any) are soft. In our present models all of the surviving binaries are hard.

The final binary population shows distinctive characteristics depending on the cluster richness. For models with $N \leq 250$ the surviving binaries do not have a preferential ratio between the masses of the binary components. We find binaries with both components being low-mass stars ($M \leq 0.8M_{\odot}$) or both massive or one of them massive and the other not. However, for rich models the surviving binaries are always with almost equal-mass components. In a few cases one component is a giant and the other is a low-mass star with a spectral type typically in the range M0-M5. When the two components are on the ZAMS both

the primary and secondary have spectral type in the range K5-M5. In general, the stellar content of the open cluster remnants is mostly ZAMS stars and in a few models there appears a white dwarf or a red giant, especially for rich clusters. In most of the models the binary fraction in the remnant is significantly larger than the initial one; Aarseth (1996d) has also found this effect. In a model with 10,500 stars and 500 binaries, the remnant contains 10 binaries and 39 stars.

Although binaries in cluster remnants are primordial, i.e. at least one of their components was in a PB at the beginning of the simulation, not all the surviving binaries are purely primordial. For rich clusters we find a certain fraction of exchanged binaries (1 or 2 per model). For small clusters it is rare to find an exchanged binary because the cluster disrupts faster so the probability of an exchange during the cluster life is reduced.

The explanation of the above results rests on the time-scales for stellar evolution. For clusters with small N , the cluster disrupts before massive stars have started to leave the ZAMS, so we can find binaries with massive primaries in the cluster remnant. However for rich clusters, the disruption time-scale is greater than the characteristic time-scale for significant mass loss from massive stars, so binaries with massive primaries could be disrupted or ejected during mass loss events. The observational test of our results is very difficult in the case of rich cluster remnants. The detection of a small population (a few tens) of faint stars most of them less luminous than our Sun is a big challenge for the biggest optical telescopes. Moreover, the detection of the binaries in these rich cluster remnants is even more difficult. However, there is some light from this dark picture because stars with such spectral types are expected to be strong radio and/or X-rays sources which can be detected with synthesis techniques or by instruments on satellites. The search of open cluster remnants has been considered by Lodén (1977, 1979, 1980), who analyzed very loose or star-poor clusterings in a large survey of the Southern Milky Way. Thousands of these objects were found and classified into 4 sets. One of them is formed by extremely small and star-poor clusters which he suggested as possibly cluster remnants. The frequency of these objects in his sample is about 20 % but he considers that it can be significantly greater.

9. Conclusions

The main conclusions from this work can be summarized as follows:

- 1.- The inclusion of stellar evolution in cluster models with a fraction of PBs affects the overall dynamical evolution of the cluster in an uncertain way which depends strongly on the cluster richness and the IMF.
- 2.- The stellar evolution in clusters with small population and power-law IMF accelerates their disruption in a very significant way.
- 3.- The escape velocity increases with the cluster richness but the dispersion decreases when N increases. Sometimes, a star can leave the cluster with high velocity.

4.- The final cluster remnant is very rich in binaries, frequently purely primordial but its composition depends strongly on the initial cluster population because of the interplay between the time-scales for cluster disruption and stellar mass loss. Binaries in remnants of poor clusters do not have any special feature in their components but binaries in rich cluster remnants have usually almost the same mass for their components and have late spectral types. Collapsed objects are almost always absent from open cluster remnants.

Acknowledgements. I am very grateful to my Ph. D. thesis supervisor Dr. Sverre J. Aarseth for his generous help, useful comments, for providing his computer code and for sharing some results before publication. The author is also grateful to Dr. P. P. Eggleton for the use of his binary correlation before publication and his IMF fits. I thank the Department of Astrophysics of Universidad Complutense de Madrid for providing excellent computing facilities. All the computations and most of the data analysis were made on the computers of the Universidad Complutense of Madrid, and I thank the computing staff (specially J. Palero and E. Lezcano) for their help during this stage. The author gratefully acknowledges the hospitality of the Institute of Astronomy (Cambridge, U.K.) during several visits when work on the papers of this series was carried out. This research has made use of SIMBAD (operated at CDS, Strasbourg, France), BDA (operated at the Institute of Astronomy, University of Lausanne, Switzerland) and ADS (operated by NASA) databases.

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