

Heavy element diffusion and globular cluster ages

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Abstract. This paper discusses low mass, metal poor stars, presenting new evolutionary computations which take into account the inward diffusion of both He and heavy elements during the H burning phase. The investigation has been extended to He burning models, discussing the luminosity of the Zero Age Horizontal Branch (ZAHB) models originated from progenitors with efficient element diffusion. We find that diffusion of heavy elements plays a non-negligible role in the evolutionary paths of the model. One finds that element diffusion has a sensitive influence on evolutionary models, a less significant effect on the isochrone turn off and, finally, a rather marginal impact on the difference in luminosity between the ZAHB and the isochrone Turn-Off (TO). The consequences of these results on current evaluations of globular cluster ages are discussed in connection with possible sources of errors.

Key words: diffusion – stars: evolution – stars: Population II – globular clusters: general – stars: horizontal-branch

1. Introduction

Over the last years the well known problem of solar neutrinos has stimulated an increasing attention toward theoretical predictions concerning solar structure. Accordingly, several Standard Solar Models have been presented in the literature, following the continuous improvement in the available input physics. However, in the meantime the progress of helioseismologic investigations has produced rather severe constraints on the inner solar structure, bringing to light a non-negligible discrepancy between the actual Sun and all solar models as evaluated according to the traditional evolutionary scenario. As a matter of fact, one finds that all these models underestimate the depth of the subatmospheric convection which affects solar structure as, more in general, it affects the structure of all other low mass stars.

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This troublesome discrepancy has been recently interpreted as an evidence that the diffusion of He and heavy elements plays a not marginal role in solar evolution. The efficiency of element diffusion in stellar structure has been a debated problem. Deliyannis & Demarque (1991) presented arguments for the existence of mixing mechanisms limiting - at least in the surface layers - such efficiency. However the “signature” of element diffusion has become progressively evident in the results of helioseismology, as discussed by Christensen-Dalsgaard (1993), Guzik & Cox (1993) and Bahcall & Pinsonneault (1995). According to Bahcall et al. (1996) one should conclude that solar models that not include diffusion, or with significant internal mixing, are effectively ruled out by helioseismology. When this mechanism is taken into account, Solar Standard Models (SSM) reach a beautiful agreement with helioseismology constraints. According to such an evidence, our FRANEC evolutionary code has been recently implemented to account for element diffusion following the prescriptions given by Bahcall & Pinsonneault (1995). As expected, one finds in this way a SSM which nicely overlaps the results of the two above quoted authors (Ciacio, Degl’Innocenti & Ricci 1996).

Since the evolutionary path of the Sun appears to be deeply affected by element diffusion, it is of obvious interest to extend the investigation to the field of old metal poor Population II stars. One expects in this way to shed new light on the debated problem of globular cluster ages and related constraints on the age of the Universe. As a matter of fact, one cannot disagree with Proffitt & Vandenberg (1991) when they state that similar constraints “can be taken seriously only if ALL of the important physics” is correctly taken into account.

He diffusion in low mass stars has already been discussed in the above quoted exemplary paper by Proffitt & Vandenberg (1991: hereinafter referred to as PVB) who presented and discussed the effect of He diffusion on globular cluster isochrones. In their conclusions, these authors pointed out the possible relevance of significant gravitational settling of elements heavier than He. However, further investigations on the matter (Chaboyer, Sarajedini & Demarque 1992, Chaboyer 1995, Canuto et al. 1996) keep considering the sedimentation of He only. On the other hand, sedimentation of heavy elements

has a non-negligible influence on solar standard models, and we will show in the following that this is also true for metal poor stars of low masses.

In this paper we will study the consequences of introducing He and heavy elements diffusion into theoretical evolutionary structures for low mass, metal poor stars. In the next section we will discuss the evolutionary behaviour of selected stellar models, as computed under different assumptions about the efficiency of diffusion to elucidate the contribution of various mechanisms concerning element sedimentation. The evolutionary computations presented are in all cases based on the most updated input physics used for producing the SSM, as described in Ciaccio et al. (1996). Present stellar models without diffusion can thus be regarded as an update of similar models already presented in the literature (Straniero & Chieffi 1991, Castellani, Chieffi & Pulone 1991).

In Sect. 3 we will present new cluster isochrones, discussing their influence on the present debate about cluster ages. In the same section we will extend the investigation to He burning models, discussing how the luminosity of the Zero Age Horizontal Branch (ZAHB) is affected by element diffusion in the progenitor models. As a result, we will find that element diffusion has a sensitive influence on evolutionary models, a less significant effect on the isochrone turn off and, finally, a rather negligible impact on the difference in luminosity between the ZAHB and the isochrone Turn-Off (TO). The consequences of these results on current evaluations of globular cluster ages will be finally discussed.

2. Element diffusion in low mass metal poor stars

To gain insight on the problem of diffusion let us first discuss the evolutionary behaviour of a stellar model chosen to represent typical metal poor globular cluster stars in some detail, namely adopting a mass $M=0.8 M_{\odot}$ together with original He and metallicity as given by $Y=0.23$ and $Z=0.0004$, respectively. Fig. 1 shows the evolutionary track of such a model, as computed under different assumptions about the efficiency of diffusion. According to labels in that figure one finds: i) the traditional track (ND) where diffusion is not taken into account, ii) a track (dashed) where only He is allowed to diffuse, iii) a track (M1) with diffusion of both He and heavy elements, but without accounting for the effect of sedimentation of metals on matter opacity and, iv) our "best" final track (M2) where all diffusion effects are taken into the right account.

As already known, one finds that He sedimentation makes the evolutionary track redder, with a lower luminosity of the turn off (TO) which is reached earlier. This last occurrence is shown in Table 1 where we report, among other selected quantities, the age of the various models at the TO. From data in Table 1 one recognizes that He sedimentation makes the track turnoff fainter by more than $\delta \log L=0.08$ and younger by more than 1 Gyr. These results can be compared with the results by Proffitt & Michaud (1991), who obtained for a very similar model $\delta \log L=0.05$ with an age decreased by 0.75 Gyr. It appears that

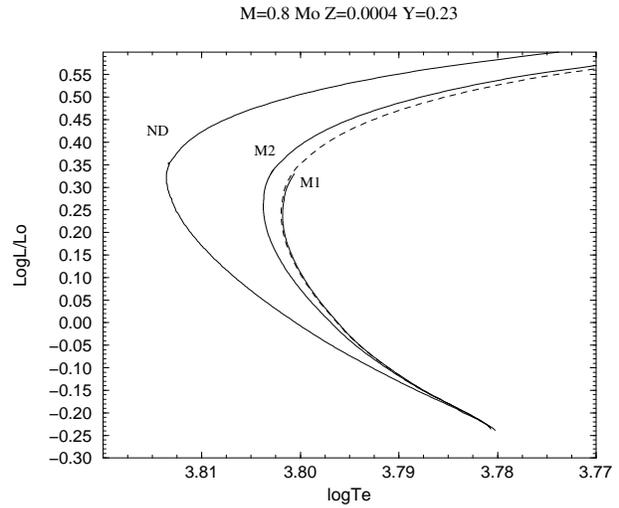


Fig. 1. The HR diagram evolution of the $0.8 M_{\odot}$ as computed under different assumptions about the efficiency of diffusion (see text).

Table 1. Comparison of the TO characteristics of a $0.8 M_{\odot}$ as computed under different assumptions about the efficiency of diffusion (see text).

No Diffusion	$\text{Log}L/L_{\odot}=0.321$	$\text{Log}T_e=3.813$	$t=11.68$ Gyr
He Diffusion	$\text{Log}L/L_{\odot}=0.239$	$\text{Log}T_e=3.802$	$t=10.56$ Gyr
M1	$\text{Log}L/L_{\odot}=0.241$	$\text{Log}T_e=3.802$	$t=10.58$ Gyr
M2	$\text{Log}L/L_{\odot}=0.257$	$\text{Log}T_e=3.804$	$t=10.72$ Gyr

our approach to diffusion gives similar but larger effects on the topology and on the timing of the evolutionary track.

Fig. 1 and Table 1 show that the diffusion of heavy elements plays a further role only through the contribution to stellar opacity, partially counteracting the effect of He depletion. As a whole, when the diffusion of both He and heavy elements is taken into account one finds a track TO fainter (with respect to the case without diffusion) by $\Delta \log L=0.064$, the TO being reached in a time shorter by a bit less than 1 Gyr. It turns out that our results with sedimentation of both He and heavy elements approach the results by Proffitt & Michaud (1991) where only He sedimentation was taken into account. It follows that one can easily foresee that our isochrones will closely approach the diffusion scenario already discussed by PVB.

Fig. 2 shows the effect of diffusion on the distribution of selected elements throughout the structure of our $0.8 M_{\odot}$ model in the phase of exhaustion of central hydrogen, when surface abundances are reduced to $Y_{atm}=0.152$ and $Z_{atm}=0.0003$. However, the surface abundances of these elements will be restored during the first dredge up in the Red Giant phase. Eventually, one finds that our model approaches its He flash with $Y_{atm}=0.228$ in the atmosphere against $Y_{atm}=0.241$ expected if diffusion were not at work. In this respect, one finds that the model behaviour is not far from the evolutionary scenario depicted by PVB, with very similar values for the decrease of surface He. According to the discussion already given by PVB, this decrease implies a decrease in the luminosity of the HB by a small but not completely

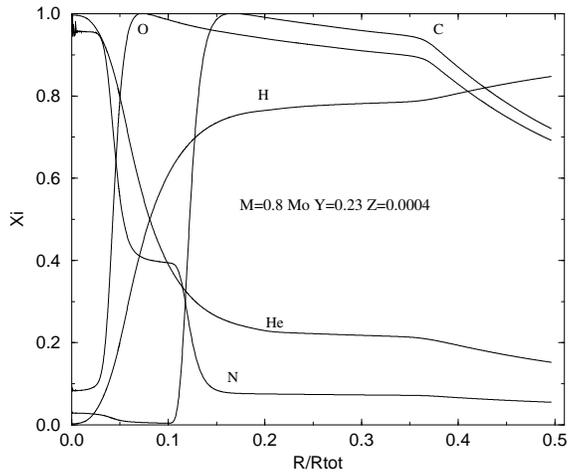


Fig. 2. The distribution of selected elements in the structure of a 0.8 M_{\odot} model with diffusion near the exhaustion of central H.

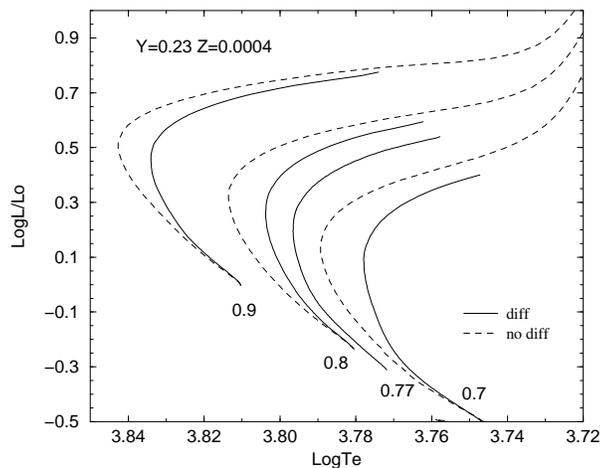


Fig. 3. Evolutionary tracks with or without diffusion and for the labeled assumptions about the mass of the evolving star.

negligible amount. On the contrary, one finds that the temporal depletion of heavy elements in the stellar atmosphere is eventually almost completely smoothed away by the dredge up, the final metallicity of the Red Giant coming back to $Z=0.00038$, with an expected minor influence on the following He burning structures.

Finally, Fig. 3 compares evolutionary tracks with or without diffusion for selected choices of the mass of the evolving stars. Table 2 gives selected quantities for all these evolutionary models taken at their track turnoff. Left to right one finds the mass M of the model (in solar masses), the assumption about the efficiency of diffusion, luminosity, temperature and age of the model at the track turnoff and the variation both in luminosity ($\Delta\log L$) and in age ($\Delta t/t$) at the TO caused by diffusion.

Inspection of these data discloses that the effects of diffusion show a maximum in stars with masses around $M=0.8 M_{\odot}$.

Table 2. Selected quantities at the track turn off for the evolutionary models with and without diffusion, having the masses shown in column 1. The ages t^{TO} are in Gyr.

M/M_{\odot}	diff ?	$\text{Log}L^{TO}$	$\text{Log}Te^{TO}$	t^{TO}	$\Delta\text{Log}L$	$\Delta t/t$
0.70	NO	0.137	3.789	19.28		
0.70	YES	0.090	3.778	18.10	-0.05	-0.06
0.77	YES	0.196	3.797	12.34		
0.80	NO	0.321	3.813	11.68		
0.80	YES	0.257	3.804	10.72	-0.06	-0.09
0.90	NO	0.508	3.843	7.47		
0.90	YES	0.457	3.834	7.05	-0.05	-0.06

The irrelevant differences are within the accuracy of theoretical evolutionary computations. However, one finds that Proffitt & Michaud (1991), who integrated structures with masses equal or below $0.8 M_{\odot}$, already found that the $0.8 M_{\odot}$ model is the most affected by diffusion. An occurrence to be related to the progressive sinking of external convection when going toward less massive cooler stellar structures. Moreover, the lower efficiency of diffusion in the $0.9 M_{\odot}$ model can be understood as a consequence of the shorter evolutionary times, according to the evidence that diffusion is a slow mechanism which needs time. We are thus inclined to regard the quoted maximum as a feature rather than an artifact of the computations.

3. Cluster isochrones and age calibrators

In the previous section we discussed the variation in the luminosity and in the age of the track TO caused by inward diffusion of elements. One may notice that, if isochrone TO were to suffer similar variations, according to current calibrations of TO luminosities in terms of age (as given, e.g. in Castellani, Chieffi & Pulone 1991), the quoted results would imply that diffusion rejuvenates a cluster by no less than 4 Gyr if only He is depleted, or by about 3 Gyr if both He and heavy elements are allowed to diffuse. However, as pointed out by Proffitt & Michaud (1991) and reinforced by PVB, the proper way of discussing ages is through cluster isochrones since consideration of evolutionary tracks on their own exposes one to the risk of misleading results.

According to a well known procedure, evolutionary tracks can be interpolated to eventually produce theoretical isochrones predicting the distribution in the HR diagram of H burning cluster stars for the adopted chemical composition and for various assumptions about the cluster age. Fig. 4 shows a set of similar isochrones as computed with or without diffusion for ages ranging from 10 to 15 billion years, whereas Table 3 gives selected quantities concerning the isochrone turn off.

It appears that over all the explored ranges of ages the effect of diffusion on isochrone TO is about one half the effect found on track TO's. This reinforces that evolutionary tracks alone cannot be used to obtain reliable predictions about the behaviour of isochrones TO's. We note that the difference between track and isochrone TO's is a general feature, but enhanced in

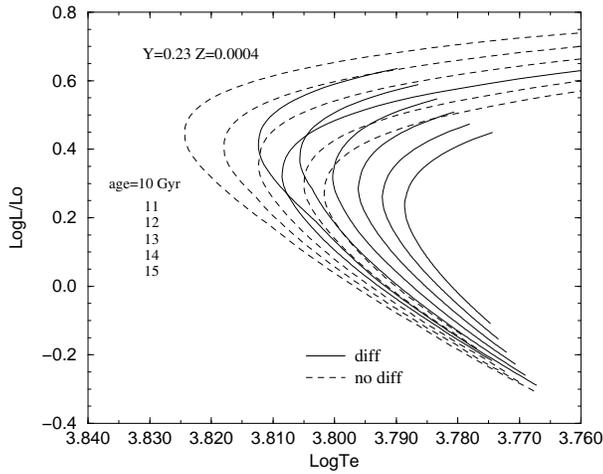


Fig. 4. Selected cluster isochrones with or without diffusion and for the labeled assumptions about the cluster age.

Table 3. Selected quantities at the isochrones turn off for isochrones with and without diffusion.

age (Gyr)	diffusion		no diffusion		ΔLogL
	LogL^{TO}	LogTe^{TO}	LogL^{TO}	LogTe^{TO}	
10	0.414	3.812	0.446	3.824	-0.032
11	0.369	3.806	0.409	3.818	-0.039
12	0.318	3.800	0.349	3.812	-0.030
13	0.287	3.796	0.320	3.809	-0.033
14	0.258	3.792	0.293	3.805	-0.036
15	0.237	3.789	0.268	3.802	-0.031

the diffusive case by the already presented differential effect of diffusion on the evolutionary times of stars of different masses. This is shown in Fig. 5 where we compare the evolutionary track for a $0.8 M_{\odot}$ model, with and without diffusion to the corresponding isochrones as evaluated for the age when the model is just reaching its track TO. As a whole, data in Fig. 4 and Table 3 show that, if diffusion is taken into account, current evaluation of cluster ages based on the luminosity of isochrone TO should be rejuvenated by about 1 Gyr, in agreement with the current evaluation already given accounting for the inward diffusion of He only.

Computational results concerning H burning Red Giant structures can be finally used to evaluate He burning structures as produced by progenitors where diffusion has been active. The $0.8 M_{\odot}$ model with diffusion ignites He within a He core of $M_c=0.4961 M_{\odot}$ against the value of $M_c=0.4993 M_{\odot}$ in the case without diffusion. This is not an unexpected result, since when diffusion is at work a giant reaches He ignition with a lower He abundance in the envelope, and one knows that stars with lower original He have larger He cores (but lower HB luminosity!). However, original He and He in the convective envelope of a

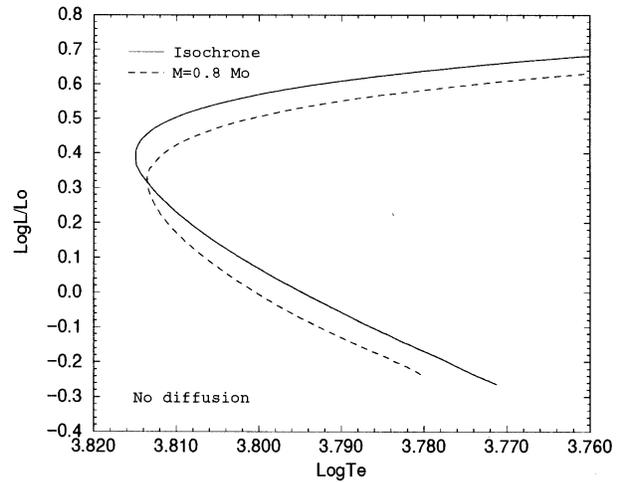
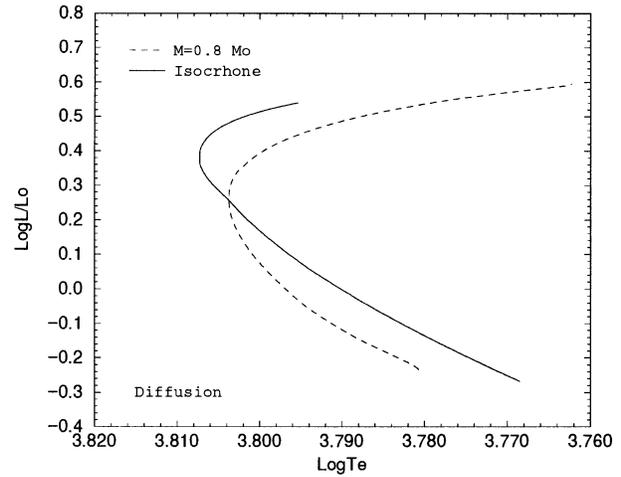


Fig. 5. The evolutionary track (dashed line) of the $0.8 M_{\odot}$ model with or without diffusion (upper and lower panel respectively) compared with the isochrone where the model attains the track TO.

giant are not exactly the same thing, and the above result indicates that the actual mass of the He core has been governed by the amount of He in the stellar envelope, independently of the original values (Y_{or}) of Y.

As a matter of fact, from Bono et al. (1995) in the interval $Y_{or}=0.20\div 0.23$ one derives $dM_c/dY_s=-0.14$ ($dM_c/dY_{or}=-0.12$) and taking $\Delta Y_s=0.013$ from the difference in the atmospheric He after the first dredge up, one finally obtains $\Delta M_c=0.0018$, i.e., exactly the difference found in direct computations with and without diffusion. Note that this is not an obvious result. As a matter of fact, numerical experiments performed artificially decreasing Y_s in the atmosphere of a giant in the upper portion of the RG branch shows that M_c is unaffected by such a variation. This result can be understood by recalling that mass loss in a red giant does not affect the growth of the central He core for the simple reason that the thermodynamic time scale of the core is larger than the evolutionary times in the red

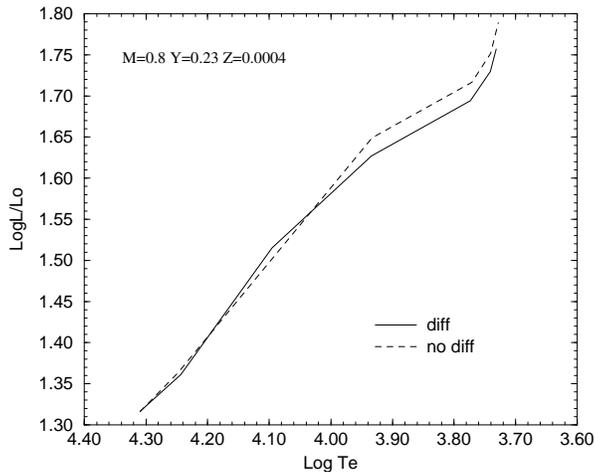


Fig. 6. Zero Age Horizontal Branches for the original chemical composition $Y=0.23$ and $Z=0.0004$, as originated from progenitors with (full line) or without (dashed line) diffusion.

giant phase (Castellani & Castellani 1993). For the same reason, changes of He abundances in the envelope which surrounds a luminous evolving giant do not affect the evolution of the core. On the contrary, the quoted evolutionary result indicates that the depletion of He in the stellar envelope resulting from the combined effects of diffusion and dredge up occurs early enough to allow the stellar core to be sensitive to this variation.

Thus Zero Age Horizontal Branch models with diffusion are expected with surface He smaller by $\Delta Y_s = -0.013$, and with an He core mass appropriate for this atmospheric He abundance. By taking again from Bono et al. (1995) $d \log L_{Z_{AHB}} / dY_s = 1.7$ one can predict that HB luminosity level (at the temperature of the RR Lyrae gap) for stars which experienced diffusion should be fainter by about $\Delta \log L = -0.022$. Fig. 6 shows that such a prediction is fully confirmed by detailed computations of He burning models. As a result, one predicts that diffusion should have a negligible effect on the calibration of the difference in luminosity between HB and TO in terms of the cluster age. As a matter of fact, the expected increase $\Delta \log L \sim 0.01$ for each observed $\Delta \log L_{TO}^{Z_{AHB}}$ implies a decrease in the estimated age of a few 10^8 years.

4. Discussion and conclusions

In this paper we studied the effect of element diffusion on the evolution of old metal poor stars. We found that inward diffusion of heavy elements partially counteracts the effects of He diffusion. As a result, one finds that when diffusion of all elements is taken into account the luminosity of the isochrone TO should decrease of $\Delta \log L \sim -0.03$. In terms of calibration of cluster ages, a similar decrease would imply a rejuvenation of clusters by about 1 Gyr, a bit less but in substantial agreement with the rejuvenation produced by consideration of He diffusion only, as discussed in several papers (see, e.g., Chaboyer 1995 and references therein).

However, as a consequence of diffusion, red giants at the onset of the He flash are expected with lower abundance of He in the envelope, a variation that has to be taken into account and that goes in the direction of decreasing the luminosity of HB stars. Direct computations of He burning HB models confirm that prediction, producing models less luminous by $\Delta \log L \sim -0.02$.

Comparison with Proffitt & Vandenberg (1991) show that we find smaller decreases of turnoff luminosity but similar decreases of the HB luminosity, which now exactly compensates the decrease in luminosity of the turnoff for a given age. This of course reinforces the statement by PVB which already indicated that cluster ages derived from the difference in magnitude between HB and TO are much less sensitive to uncertainties in the diffusion rates than ages derived from the TO luminosity only, adopting some other calibrator of the cluster distance modulus.

According to this theoretical evidence, one should conclude the recent evaluation of about 12 Gyr given for metal poor galactic globulars (Castellani, Brocato & Piersimoni 1996) should safely survive even if element diffusion is taken into account. However, to assess the proper meaning of the above evaluation let us here discuss possible sources of errors. The influence of the input physics and/or of element abundances on the turn-off luminosity has been recently and carefully investigated in a paper by Chaboyer (1995) to which we address the interested reader. Here let us only add some further comments concerning theoretical predictions about the difference in magnitude between HB and TO, bearing in mind – as an order of magnitude – that decreasing the difference by $\Delta \log L_{TO}^{Z_{AHB}} \simeq 0.1$ means decreasing the evaluation of the cluster age by about 4 Gyr.

The influence of cluster metallicity on both TO and HB luminosity has been already evaluated by Chaboyer (1995), by including an empirical dependence of HB luminosity on metals. With our fully theoretical approach we find $d \log L_{TO} / d \log Z \simeq -0.13$ and $d \log L_{HB} / d \log Z \simeq -0.075$. On this basis one finds $d \log L_{TO}^{Z_{AHB}} / d \log Z \simeq 0.06$ which confirms the minor relevance of a precise determination of cluster metallicities as far as the cluster ages is concerned. The influence of the adopted amount of original He is however much more relevant. To introduce this problem let us first notice that the usual procedure to evaluate Y through the number ratio R of HB to Red Giant stars more luminous than the HB luminosity level is open to serious uncertainties, particularly when applied to metal poor clusters with poorly populated instabilities strips. From RG evolutionary times and R calibrations, as given in Bono et al. (1995), one finds that an error of 0.2 mag. in the bottom luminosity of the RG sample, possibly produced by errors in the HB luminosity level AND in the bolometric correction for RG stars, gives Y values ranging from $Y=0.20$ to $Y=0.26$, which is exactly the range explored by Chaboyer (1995).

By taking from Iben & Renzini (1984) $d \log L_{TO} / d \log Y \simeq -0.42$, over the quoted range of Y one finds a variation $\Delta \log L_{TO} \pm 0.013$, which indicates, as reported by Chaboyer (1995), that such a variation in Y scarcely affects the estimated cluster ages. However, theory tells us that at the same time the HB luminosity increases with Y as $d \log L_{HB} / d \log Y \simeq 1.7$. As a whole one finds $\Delta \log L_{TO}^{HB} \simeq \pm 0.06$, which in terms of ages

means an error of the order of 2 Gyr. Thus the estimate of Y could play a relevant role in assessing cluster ages through the observed differences $\Delta \log L_{TO}^{HB}$, the ages decreasing if Y increases.

As a final point, we note that Chaboyer (1995) has already shown that uncertainties in the current evaluations of nuclear reaction cross sections scarcely affect the evaluations of the turn-off luminosity. However, on theoretical grounds one expects that the 3α reactions do govern the He ignition in the RG core, thus determining the mass of the core at the ignition and, in turn, the luminosity of a new born HB star. According to Rolfs & Rodney (1988), one finds that the cross section for this reaction is poorly known, with an estimated error of about 15% (at 1σ). To make this further point clear we performed numerical simulations of RG evolution artificially moving the adopted cross section by $\pm 30\%$. As a result, we found that by increasing (decreasing) the cross section by such an amount the mass of the He core at the flash decreases (increases) by 0.002(0.003). Since $\Delta \log L_{HB}/dM_c \simeq 3.4$ (from Sweigart & Gross, 1978) one may easily see that the quoted cross section scarcely affects the evaluations of ages. As a conclusion, one finds that current evaluations of cluster ages appear generally well grounded in the theory, with the warning that the amount of original He is the main ingredient which appears to be able to move these evaluations by more than 1 Gyr.

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