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ASTRONOMY AND ASTROPHYSICS

The orbital period distribution of novae

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Abstract. A detailed analysis of the orbital period distribution of classical novae is presented. The most important selection effects as well as the nova outburst frequency are evaluated as a function of the binary period in order to determine the intrinsic period distribution of nova progenitors in the galaxy. They cause it to be remarkably different from the observed distribution. The large majority of nova systems have short periods, whereas the observed distribution is much more uniform. A study of the sensitivity of the derived intrinsic distribution on the model parameters and on small number statistics is performed. A comparison of our results with previous work on CV population synthesis reveals a satisfactory overall agreement. Differences appear at very short periods where the population synthesis calculations predict even more systems than can be explained by the observations, and in particular concerning the period gap which does not exist for classical novae. In an appendix the correlation between the orbital period and the outburst amplitude of novae is discussed.

Key words: stars: binaries: close – stars: novae, cataclysmic variables

1. Introduction

Classical novae comprise 35% of all cataclysmic variables (CVs). However, their orbital period distribution is the least well known of all subgroups of CVs. This can be explained by an observational selection effect: While in eruption the light of such systems is emitted by an extended pseudo-photosphere which prevents the direct observation of the binary system, during quiescence most of them are too faint for time-resolved radial velocity measurements. Photometric observations can also be difficult because novae are concentrated in the galactic bulge and thus are usually found in very crowded fields. While some efforts have been made to understand the observed period distribution of the other types of CVs such as polars (e.g. Hameury et al. 1988) and dwarf novae (Shafter 1992), attempts to calculate the intrinsic nova period distribution via population synthesis have been performed by Kolb (1995), but no comparisons with observations are available as yet.

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There are at least three selection effects which influence the observed distribution of nova periods (see Fig. 1 for a schematic representation). The first (S_{-1}) is just related to the possibility to recognize in principle the existence of the object. In other words, it is related to the eruption of a system in the potential novae population independent of the fact whether it is actually observed or not. This "intrinsic" selection effect defines the sample of classical novae. It renders the number of observed systems proportional to the eruption frequency of the prenova binaries and is therefore closely related to the physics of the nova outburst. The second selection effect (S_0) is a "visibility" function that takes into account the magnitude limit of the observed sample. The last effect (S_1) is associated to the probability of detection of a particular orbital period in the sample of known galactic novae. We call this the technical selection function. When time-resolved photometric observations are used to measure periods this effect favours the detection of large modulations, short periods, and high inclinations. Such methods may be specially effective on novae which are observed during the decline to the quiescent state, when the illumination of the secondary by the still very hot and luminous primary may produce large continuum modulations (Patterson 1979). Of course, eclipses also play a prominent role. It is important to point out that more than 75% of the sample of novae with known orbital periods show orbital modulations or eclipses of the primary component. Many periods have been discovered in recent years in time-resolved CCD observations of faint remnants, emphasizing the high potential of this technique. On the other hand, radial velocity measurements are more effective in revealing periods in systems where the inclinations are high (which also favours eclipses!), the period is short (but not too short, see Sect. 5), and the white dwarf mass is low. But such measurements have been less productive in finding periods and are considerably more expensive, requiring larger telescopes than photometric measurements. Moreover, the large majority of the known novae are too faint in quiescence to permit time-resolved spectroscopy even with the most powerful existing instruments.

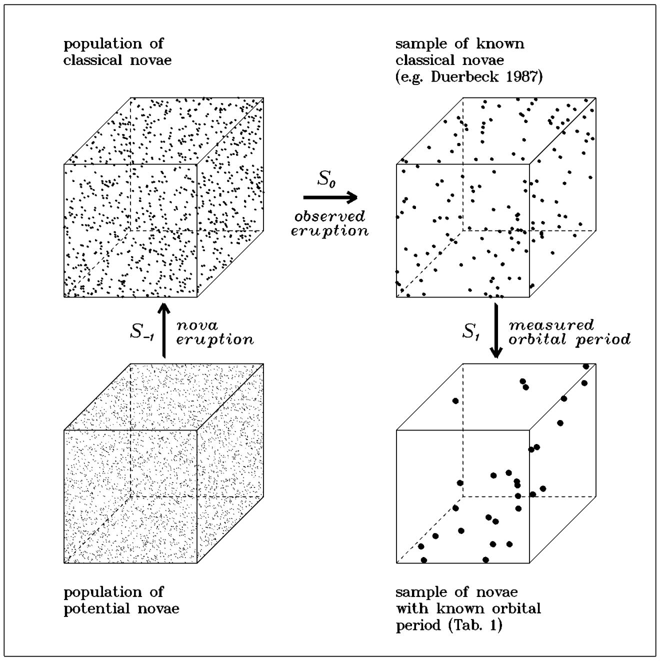

Fig. 1. Schematic representation of selection effects influencing the observed distribution of nova periods

Thus, if $N_{\rm obs}(P)$ is the observed and $N_{\rm P}(P)$ the intrinsic period distribution ("parent" distribution), we may write:

$$N_{\rm obs}(P) = S_{-1} \times S_0 \times S_1 \times N_{\rm P}(P)$$

In this study we aim at the quantification of the selection effects in order to derive the intrinsic distribution of the nova progenitors from the observed period distribution. This may then directly be compared to evolutionary predictions from population synthesis studies (e.g. de Kool 1992; Politano & Webbink 1990, and Kolb 1993).

2. The observed period distribution

The observed orbital periods of novae are listed in Table 1 and their distribution is shown in Fig. 2. Most of the data are extracted from the compilations of Ritter & Kolb (1995), Duerbeck (1987), Warner (1987) and Bruch & Engel (1994). Additionally, the recently determined periods of DO Aql and V849 Oph (Shafter et al. 1993), V368 Aql (Diaz & Bruch 1994) and V909 Sgr and V4077 Sgr are listed. For the last two systems periods are suggested in unpublished photometric data obtained by one of us (MD) at LNA. They still require confirmation. The long periods of DI Lac, V841 Oph and GK Per indicate that their secondary components have already evolved away from the main sequence. In these cases the relations for the mass transfer between the components to be used in Sect. 4 are not valid. Therefore, we cannot regard nova systems with evolved secondaries [and long ($\geq 10^{\rm h}$) orbital periods] here. This leaves us with a sample of 28 novae on which the observed period distribution to be used here is based.

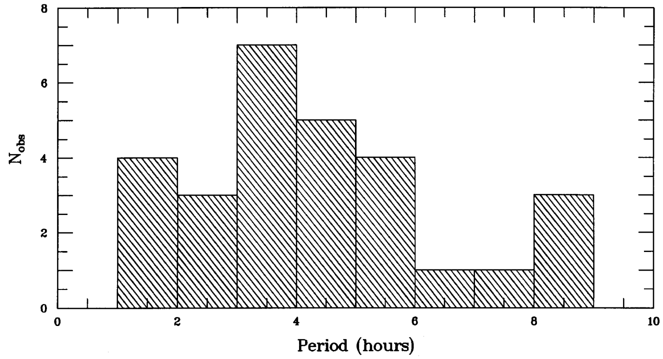

Fig. 2. Observed distribution of nova periods

Apart from the periods, Table 1 contains some further relevant information for each system in the sample. In particular, the t_2- and t_3- times, the outburst amplitude, the quiescent magnitude along with the colour excess, if known, and the observational feature from which the period can be derived are listed. A search for correlations between the period and other directly observable items revealed no dependence of the decay time or the expansion velocity of the shell on the period. But a significant correlation between the nova amplitude and the orbital period exists (see Appendix A).

3. The magnitude-limited sample

The highest probability of detecting a nova is attained during the visual maximum. Around this phase the visual luminosity L_V of the nova is approximately proportional to the bolometric luminosity $L_{\rm bol}$. This is a good first order approximation since the nova reaches the visual maximum when the photospheric radius is largest and the effective temperature permits hydrogen to recombine ($T \approx 7 - 9 \times 10^3$ K) leading to the same bolometric correction for all novae at this phase. It has been shown that many novae undergo a short-lived super-Eddington phase at bolometric maximum [e.g. V1500 Cyg (Wu & Kester 1977), V1668 Cyg (MacDonald 1983)]. For the sake of simplicity it will be considered here that it is short enough to be unimportant for our statistical purposes, neither lasting long enough nor brightening the system enough to considerably increase the probability of detection of the eruption. Therefore, we assume that the magnitude at maximum is determined by the Eddington luminosity. The fact that the slope of the amplitude-period relation for the novae is identical to that of outbursting dwarf novae and can thus be explained by a dependence of intrinsic brightness in quiescence on the period (see Appendix A) indicates that the luminosity at maximum is almost independent of the orbital period and means that the white dwarf mass distribution of novae is - if at all - only a weak function of the orbital period. The Eddington luminosity $L_{\rm Edd}$ is proportional to the white dwarf mass (MacDonald 1983):

$$L_{\rm Edd} = 2.5 \times 10^{38} (1 + X)^{-1} \left(\frac{M_1}{M_\odot} \right) = const \times L_{\rm V} \qquad (1)$$

Table 1. Inventory of classical novae with known orbital periods

System	Year of outburst	Period (hours)	t_2 (days)	t_3 (days)	Range (mag)	$V_{\rm min}$ (mag)	$E(B-V)$ (mag)	Detection method[1]
GQ Mus	1983	1.42476	18	45	$>$14	17.5	0.45	OM/RV
CP Pup	1942	1.4676	5	8	$>$16	15.0	0.25	OM
RW UMi	1956	1.944		140	15	18.7		OM
V1974 Cyg	1992	1.9488						OM
V Per	1887	2.57088			14.5:	18.5		E
QU Vul	1984	2.68224	27	40	13.4	19	0.3	E
V2214 Oph	1988	2.82036	60	100	12.5	17		OM
V603 Aql	1918	3.312	3.5	8	13.1	11.7	0.08	RV
V1668 Cyg	1978	3.3216	12	23	13.3	20	0.36	E
V1500 Cyg	1975	3.35064	2	3.6	14	17.1	0.43	OM/RV
V909 Sgr	1941	3.36:	3.8	7	13.2	20		E
RR Pic	1925	3.48072	80	150	10.9	12.2	0.02	OM/RV
WY Sge	1783	3.687216			13.5	20.7[2]	0.45	E
V4077 Sgr	1982	3.84:	20	100	14	21		OM
DO Aql	1925	4.026288	450	900	7.8	16.5		OM/E
V849 Oph	1919	4.14612	88	175	9.8	17		E
DQ Her	1934	4.64688	67	94	13.2	14.5	0.08	E/RV
CT Ser	1948	4.68	100	$>$100	$>$11	16.6	RV	
T Aur	1891	4.90512	80	100	11	14.9	0.39	E/RV
V533 Her	1963	5.0352	26	44	12	15.6	0.03	OM
PW Vul	1984	5.1288		97	10.6	17		OM
HR Del	1967	5.14008	152	230	8.5	12.1	0.15	RV
V705 Cas	1993	5.472						OM
U Leo	1855	6.4176				17.3		OM
V838 Her	1991	7.14384	1.5	4	15.6	18		E
BT Mon	1939	8.0112	140	190	11.8	15.3		E
V368 Aql	1936	8.16	15	42	11.7	18		E/OM
QZ Aur	1964	8.58	$<$17	26	13	18		E
DI Lac	1910	13.05048	20	43	10.3	14.5	0.41	RV
V841 Oph	1848	14.5008	56	130	11.5	13.4	0.39	RV
GK Per	1982	47.9232	6	13	12.7	13.0	0.29	RV

[1] E = eclipse; OM = orbital modulation; RV = radial velocity
[2] B-magnitude

where M_1 is the white dwarf mass and X is the hydrogen mass fraction. Neglecting the effect of the chemical composition, this equation predicts a bias in the sample of known classical novae (see Fig. 1) towards systems with large white dwarf masses since they can be detected in a larger volume. The number of detectable novae with an orbital period P in a magnitude-limited sample is given by the volume integral:

$$N\left(M_1\right) = \int_0^{2\pi} \int_{-\pi/2}^{\pi/2} \int_0^{r_1} n\left(M_1, r, \Theta, \Psi\right) r^2 \sin\Theta \, dr \, d\Theta \, d\Psi \qquad (2)$$

where r,Ψ and Θ are the distance, the azimutal and the elevation angle, respectively, and n is the space density of novae. r_1=$r_1(M_l)$ is the distance corresponding to the limit in magnitude, m$_l$, imposed by the observations. The systems under consideration in this study are bright enough to permit the period determination at quiescence. If the mean value for the absolute magnitude of novae at minimum (M$_V$=4.6, Bruch 1982b; M$_V$=4.3, Warner 1986) is considered, one finds that the maximum distance in our sample is small enough that Eq. (2) may be formulated in cylindrical coordinates, assuming a constant surface density of progenitors in the galactic disk. This is so because novae in the galactic bulge, where the nova density may be expected to depend on the galactocentric distance, do not contaminate our sample. If $\Sigma(M_1, P)$ is the surface density of novae, Eq. (2) may be written in the approximation of a thin disk ($z \ll r_l$):

$$N(M_1) = \int_0^{2\pi} \int_0^{r_l} \Sigma(M_1)\, r\, dr\, d\Theta \tag{3}$$

with

$$\Sigma(M_1) = \int_{z_{\min}}^{z_{\max}} n(M_1, z)\, dz \tag{4}$$

yielding:

$$N(M_1) = const \times r_l^2(M_1) \times \Sigma(M_1) \tag{5}$$

$r_l(M_l)$ may be related to the bolometric magnitude using the distance modulus and the bolometric correction:

$$\mathrm{m_l} - (\mathrm{M_{bol}} - \mathrm{BC}) = 5 \times \log r_l - 5 + a r_l \tag{6}$$

where a is the mean interstellar absorption per unit of length. After some algebric manipulations, using Eq. (1) and Pogson's equation, one finds:

$$c_1 \mathrm{m_l} + \log L_{\mathrm{Edd}}^{1/2} - c_2 = c_3 a r_l + \log r_l \tag{7}$$

where c_1, c_2 and c_3 are constants. Eq. (7) may be solved numerically for $r_l(M_l)$. In the case of negligible interstellar extinction Eq. (7) reduces to:

$$r_l(a = 0) = const \times L_{\mathrm{Edd}}^{1/2} \tag{8}$$

Of course, in the general case this approximation is not valid. However, Ritter (1986) has shown that for a wide range of interstellar extinction ($0.7 \leq A_V = ar \leq 5$) the limiting distance is well approximated by:

$$r_l(a) = 0.8 \times \left(\frac{r_l(a = 0)}{a}\right)^{1/2} \tag{9}$$

Assuming the interstellar extinction to be homogeneous in the volume occupied by the observed novae, we rewrite Eq. (8) as:

$$r_l = const \times L_{\mathrm{Edd}}^{1/4} \tag{10}$$

Inserting this expression in Eq. (5) and making use of Eq. (1), we finally get:

$$N(M_1, P) = const \times M_1^{1/2} \times \Sigma(M_1, P) \tag{11}$$

This equation constitutes the selection function S_0.

4. The outburst frequency of novae

Obviously, at a given orbital period the number of observed novae should increase in proportion to the total number of outbursts occuring in the nova systems. Thus, the observed distribution of orbital periods $N_{\mathrm{obs}}(P) \sim \nu_{\mathrm{obs}}(P)$ may be written as a function of the intrinsic number of potential novae with this period $N_{\mathrm{P}}(P)$ and corrected for the selection effects discussed in Sect. 1:

$$\nu_{\mathrm{obs}}(P) = \frac{1}{\tau_{\mathrm{obs}}(P)} = \langle \nu_{\mathrm{rec}}(P) \rangle \times N_{\mathrm{P}}(P) \times S_1(P) \tag{12}$$

Here, $\tau_{\mathrm{obs}}(P)$ is the mean time between nova outbursts in a sample of novae with known periods, $\langle \nu_{\mathrm{rec}}(P) \rangle$ is the average theoretical recurrence frequency corrected for the selection effect S_0. S_1 is the technical selection function. If we assume that the average mass transfer rate is a function of the orbital period and equal to the mass accretion rate $\dot{M}$, the theoretical recurrence time of a system may be expressed as a function of the orbital period:

$$\tau_{\mathrm{rec}}(P) = \frac{\Delta M\left(M_1, \dot{M}, L_1, Z_{\mathrm{CEI}}\right)}{\dot{M}(P)} \tag{13}$$

where ΔM is the mass accreted between outbursts which is a function of the white dwarf mass, of the mass accretion rate itself, of the preoutburst white dwarf luminosity L_1 and of the metallicity of the core-envelope interface Z_{CEI} (Starrfield 1989). The mass transfer rate in nova precursors as a function of the orbital period is not well known. We shall adopt here the estimate of Iben et al. (1992), i.e. $\dot{M}=\alpha_{\mathrm{Iben}} \times 10^{-7.28} \times M_2^{3.15}$ for systems subject to magnetic stellar winds (systems above the period gap), where α_{Iben} is a parameter of order unity, and $\dot{M}=10^{-8.80} \times {M_2}^{1.63}$ for systems only subject to gravitational wave radiation (systems below the period gap). Additionally, the empirical $\dot{M}(P)$ relation of Patterson (1984) will be used for comparison. Using the secondary mass-period relation in the form given by Patterson (1984), $\dot{M}$ becomes thus a function of P. One should be aware, however, that this $\dot{M}(P)$ relation is accurate only to the order of magnitude. But note that in the present context only the functional dependence of $\dot{M}$ on M_2 and thus P is of importance, not the absolute value. Following Fujimoto (1982a) we may express ΔM in terms of a critical pressure P_c, for detonation. According to the results of TNR models (e.g. Livio 1988, and Starrfield 1989) and analytical calculations (MacDonald 1983) the white dwarf luminosity has a weak dependence on the critical envelope mass. Neglecting thus the dependence of L_1 on ΔM, P_c can be written as a simple force-pressure relation:

$$P_c\left(\dot{M}, Z_{\mathrm{CEI}}\right) = G \times \frac{\Delta M\left(\dot{M}, Z_{\mathrm{CEI}}\right) \times M_1}{R_1^2} \times \frac{1}{4\pi R_1^2} \tag{14}$$

where R_1 is the white dwarf radius. Neither the numerical value of P_c nor its functional dependence on $\dot{M}$ and Z_{CEI} are well known. Thus, the same holds true for the critical mass ΔM. We adopt here the ansatz of Ritter et al. (1991), assuming that ΔM can be written as:

$$\Delta M(P, M_1) = const \times P_c(Z_{\mathrm{CEI}}) \times \left(\dot{M}(P)\right)^{-\beta} \times \left(\frac{M_1}{R_1^4}\right)^{-\alpha} \tag{15}$$

In principle, the parameters α and β are unknown. However, the ansatz is only sensible if α is close to 1 and β close to 0, implying that the dependence on the white dwarf mass is mainly determined by its effect on the hydrostatic pressure, and that the dependence on $\dot{M}$ is small. Semi-analytical computations by MacDonald (1984) yield $0.1 \lesssim \beta \lesssim 0.5$. Even lower values ($-0.2 \lesssim \beta \lesssim 0.2$) were found in the models of Prialnik et al. (1989) and Starrfield et al. (1986). Concerning the α-parameter, model values for the accreted mass at ignition as a function of the white dwarf mass (Fujimoto 1982b and MacDonald 1984) indicate $\alpha \lesssim 0.7$. We therefore consider $\alpha = 1$ and $\beta = 0$ as a reasonable first approximation. However, the effect of deviations from these values will be explored. In particular, we will permit β to depend on M_1 as suggested by the semi-analytical calculations of MacDonald (1984). We approximate his $\beta - M_1$ relation by a linear fit to the values in Table 1 of Ritter et al. (1991). The dependence of ΔM on chemical composition will be disregarded because there is no reason to assume a relation between the metallicity of the CEI and the orbital period. A possible relation between the mass of the white dwarf (the distribution of which might depend on the orbital period for evolutionary reasons; but see Appendix A) and CEI metallicity will be neglected in the first approximation. Thus, the unknown numerical value of P_c becomes of no consequence in the present connection since, as a constant, it influences the absolute number of outbursts at a given period but not the period *distribution*. Combining Eqs. (13) and (15), we write:

$$\tau_{\rm rec}(P, M_1) = const \times \left(\frac{M_1}{R_1^4}\right)^{-\alpha} \times \left(\dot{M}(P)\right)^{-[1+\beta(M_1)]} = \frac{1}{\nu_{\rm rec}(M_1, P)} \quad (16)$$

This relation defines the theoretical recurrence time for a single system with a given orbital period and white dwarf mass. In the next step we will derive the observed recurrence time of the sample of systems as a function of the parent period frequency distribution $N_{\rm P}(P)$. The white dwarf mass distribution for every period bin will be allowed for by computing the average recurrence frequency over the theoretical mass spectrum of CVs (Politano & Webbink 1990, Kolb 1993). This results in a weighted average frequency of outbursts in an ensemble of systems with a given orbital period. The white dwarf radius may be calculated using the Hamada & Salpeter (1961) relation for cold carbon white dwarfs. In the following treatment it is implicitely assumed that the recurrence time as given by Eq. (13) (typically 10^4 to 10^5 years) is much shorter than the time scale for binary evolution. Under these assumptions it is

$$\langle\nu_{\rm rec}(P)\rangle_{\rm obs} = \frac{\int_{M_1} N(M_1, P) \times \nu_{\rm rec}(M_1, P)\, dM_1}{\int_{M_1} N(M_1, P)\, dM_1} \quad (17)$$

and we can solve Eq. (12) for $N_{\rm P}(P)$ using Eqs. (11) and (16):

$$N_{\rm P}(P) = const \times S_1^{-1} \times \nu_{\rm obs}(P) \times \frac{\int_{M_{1,{\rm l}(P)}}^{M_{1,{\rm u}}} M_1^{1/2}\, \Sigma(M_1, P)\, dM_1}{\int_{M_{1,{\rm l}(P)}}^{M_{1,{\rm u}}} \dot{M}^{[1+\beta(M_1)]} M_1^{1/2}\, \Sigma(M_1, P) \left(\frac{M_1}{R_1^4}\right)^{\alpha} dM_1} \quad (18)$$

The upper limit of the integrals corresponds to the Chandrasekhar mass while the lower limit is set to count only the dynamically and thermally stable systems, defined by the critical mass ratio:

$$q_{\rm crit} \equiv \frac{M_2}{M_{1,{\rm l}}} \iff M_{1,{\rm l}}(P) = \frac{M_2(P)}{q_{\rm crit}} \quad (19)$$

$M_2(P)$ is the secondary mass as a function of the period. The calibration of Patterson (1984) was used as before. The value of $q_{\rm crit}$ depends on the structure of the secondary star. The values of Politano & Webbink (1990) are adopted:

$$q_{\rm crit} = \begin{cases} \frac{2}{3} & \frac{M_2}{M_\odot} < 0.4 \\ \frac{2}{3} + 2.24\left(\frac{M_2}{M_\odot} - 0.4\right)^{1.36} & 0.4 \le \frac{M_2}{M_\odot} \le 0.8 \\ \frac{5}{4} & \frac{M_2}{M_\odot} > 0.8 \end{cases} \quad (20)$$

For computing the integral mean in Eq. (18), a first approximation to the surface density of CVs is required. It is assumed that the white dwarf mass distribution integrated over the orbital period at present is close to the ZACB value. This approximation is justified since most of the accreted mass is ejected in the outbursts. To a good approximation the white dwarf mass can be considered as constant on long time scales, the mass transferred from the secondary between outbursts being removed by the eruption. The secondary mass diminishes as the system evolves by losing angular momentum. Conservative mass-transfer between outbursts ($dM_1/dt = -dM_2/dt$) will be assumed. At a time t_* after the birth of the CV, the secondary mass is simply given by

$$M_2(t_*) = M_2(t = 0) - \dot{M} \times t_* \quad (21)$$

where $\dot{M}$ is the mean mass transfer rate. Therefore, the distribution of zero age masses of Politano & Webbink (1990) should be corrected for evolutionary effects. The approximate treatment of the secondary secular evolution proposed by Ritter et al. (1991) will be adopted here. Since we use the distribution $\Sigma(M_1, P)$ instead of $\Sigma(M_1, M_2)$ the transformation

$$\Sigma(M_1, P) = \Sigma(M_1, M_2(P)) \times J\left(\frac{M_1, M_2}{M_1, P}\right) \quad (22)$$

is applied where

$$J\left(\frac{M_1, M_2}{M_1, P}\right) = \begin{vmatrix} \frac{\partial M_1}{\partial M_1} & \frac{\partial M_2}{\partial M_1} \\ \frac{\partial M_1}{\partial P} & \frac{\partial M_2}{\partial P} \end{vmatrix} = const \times P^{0.22} \quad (23)$$

is the Jacobian of the transformation between (M_1, M_2) and (M_1, P) coordinates. The second equality in Eq. (23) is understood considering the M_2 - P - relation of Patterson (1984) and the fact that M_1 does not depend on P. Hence, the resulting Jacobian determinant is only a function of P and cancels out on the right side of Eq. (18).

To solve Eq. (18) the function $\nu(P)$ must be specified. This can be done by combining the observed periods in bins of arbitrary widths and centres. However, the resulting parent distribution will then depend on the chosen bin parameters. To remain independent of this effect it is preferable to calculate the cumulative distribution function, i.e. the number of novae with a period above a given period P, as a function of P. This is achieved by integrating both sides of Eq. (18) from infinity down to P. Due to the discrete values of the observed nova periods, the integral over the right hand side of Eq. (18) decays into a sum of terms, each corresponding to an observed period. The cumulative parent distribution then becomes a step function with steps at the observed nova periods.

5. The technical selection function

The last unknown quantity in Eq. (18) is S_1, the selection function associated to the probability of measuring orbital periods in novae as a function of P. For obvious reasons eclipses in novae are easier to *detect* if the orbital period is short (while the probability for eclipses to *occur* increases with the mass ratio and thus on the mean with P). The same is true for orbital modulations. In this case another effect enhances the probability to detect short periods (in the optical range): In systems with small orbits the accretion disk is necessarily also small and faint. Asymmetric structures such as a hot spot can thus more easily cause detectable orbital modulations in the light curves than in long period systems with brighter disks.

For periods detected through radial velocity measurements, the situation is not as clear. Using Kepler's third law, a secondary star mass - period relation such as that given by Patterson (1984) and basic geometry, it is possible to derive an equation for the radial velocity amplitude K_1 of the white dwarf which depends only on the orbital period and M_1. At a fixed orbital inclination K_1 increases for a given primary mass roughly by a factor of 3 for periods between $1\overset{\mathrm{h}}{.}5$ and 9^{h}, implying easier detection of longer periods. Moreover at very short periods, the time required to obtain spectra with a sufficiently high S/N ratio may not be short against the orbital period, hampering easy detection. On the other hand, a short period can be detected in shorter, more easily realizable sections of continuous observations.

Currently, we cannot see an objective way to quantify the detection probability for a given period due to these technical selection effects. However, in view of the relatively few orbital periods detected through radial velocity measurements, we expect this probability to decrease with P. Lacking a better prescription, a simple parametrization will be used for the technical selection function S_1. We assume:

$$S_1 = const \times P^{-\gamma} \qquad (24)$$

The parameter γ will probably be a small positive number. We expect $0 \leq \gamma \lesssim 0.5$.

6. Results

6.1. The parent distribution function

Equation (18) was evaluated to derive the cumulative parent distribution function $N_{\rm P}(P)$ of nova progenitors from the observed distribution $\nu_{\rm obs}(P)$ under the quoted assumptions and approximations. It was transformed into the usual (non-cumulative) distribution function by numerical differentiation after having been smoothed by a third order spline fit. This smoothing procedure is required to remove spurious structures caused by accidental groupings of data in the observed distribution. However, it will also remove any real structure on time scales below about an hour. This is the price to be payed for using the cumulative distribution to avoid the uncertainties due to arbitrary binning. The parent distribution is shown on a logarithmic scale Fig. 3 as a solid curve. It was calculated with the parameters listed in Table 2 and normalized in such a way that its integral over all periods is equal to the observed number of nova periods. We will refer to this distribution as the "standard case". The vertical bars indicate the actually observed nova periods.

The observed distribution is based on only 28 novae. It will therefore be sensitive to errors due to small number statistics. In order to assign error margins to the parent distribution due to this effect, a bootstrap method was applied: Taking the observed distribution as representative, 1000 times 28 periods were chosen at random from it. From each of the 1000 resulting distributions the parent distribution was calculated. At each period the "standard deviation" was derived (note that neighbouring periods are not statistically independent and therefore the formally calculated standard deviation must be interpreted with caution) after a $\kappa - \sigma -$clipping with κ = 2.5 in order to reject outliers mainly caused by oscillations of the spline fit in the (unavoidable) cases of some queer accidental period distributions drawn from the observed one by the bootstrap algorithm. The range defined by

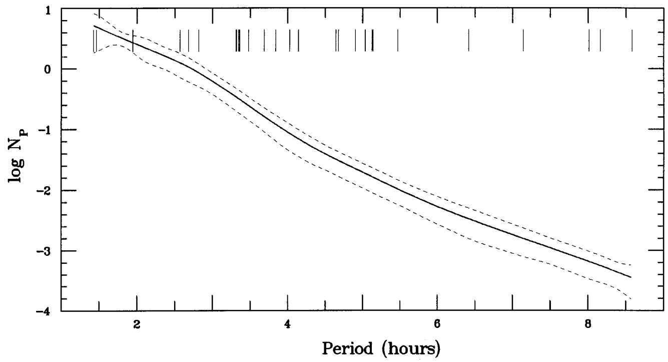

Fig. 3. Parent distribution function N of classical novae (solid line) calculated with the parameters listed in Table 2 (standard case) together with error margins as derived from bootstrap replications (broken line). The vertical bars indicate observed periods.

Table 2. Parameters used for the "standard case"

M_1/R_1^4-exponent α	1.0
$\dot{M}$-exponent β	0.0
technical selection function exponent γ	0.25
$\dot{M} - P -$ relation	Iben et al. (1992)

these standard deviations is limited by the broken lines in Fig. 3.

The parent distribution declines from a maximum at short periods almost exponentially (as indicated by the overall linear trend in log $N_P(P)$) with the period. Intrinsically, there are of the order of ≈ 12 000 times more novae with periods ≈ $1^h\!.5$ than with ≈ $8^h\!.5$, although the observed distribution is about the same at both periods (Fig. 2).

In the first place the shape of the $N_P(P)$ distribution is due to the strong dependence of $\dot{M}$ on P: A much longer time is required for short period systems with a low mass transfer rate until enough mass is transferred to permit a nova outburst. The observed number of outbursts in short period systems being of the same order as that in systems with long periods, the intrinsic number of the former must be drastically higher.

The effect of the $\dot{M} - P$ relation on the parent distribution is enhanced by the influence of the ratio M_1/R_1^4 under the integral in Eq. (18). This can be seen as follows: If P attains a large value, M_2 is also large due to the well-known $M_2 - P$ relation. Then, only high mass white dwarfs contribute to the integrals in Eq. (18) because of the dependence of the integration limits on M_2 via Eqs. (19) and (20). Since high mass white dwarfs have a small radius (which enters with the fourth power here!) the integral mean of M_1/R_1^4 will increase with P, and - since it appears in the denominator of Eq. (18) - leads thus to smaller intrinsic numbers of systems with long periods.

The strong influence of $\dot{M}(P)$ on the parent distribution is visualized in Fig. 4. The solid line is again the distribution according to the standard case. The dashed and dotted lines correspond to the cases $\beta = -0.2$ and $\beta = 0.2$, respectively, i.e. to the limits of β as found by Prialnik et al. (1989) and Starrfield et al. (1986). The dashed-dotted line results from calculations assuming the simplified relation between β and M_1 according to MacDonald (1984) (see Sect. 4). As expected, changing β and thus the influence of $\dot{M}$ modifies the parent distribution considerably. This may cause some concern because even if we were confident that $\beta = 0$ is a good approximation, modifying β is equivalent to modifying the exponent of $\dot{M}$ in the $\dot{M} - P$ relation which is subject to uncertainties. This is also indicated by the thin line resulting from calculations, where the $\dot{M} - P$ law according to Iben et al. (1992) was exchanged by Patterson's (1984) corresponding relation.

Compared with β, the parameters α and γ have a much smaller influence on the $N_P(P)$ distribution. It is shown in Fig. 5 where along with the standard case (solid line) the distributions according to $\alpha = 0.5$ and $\alpha = 1.5$ are shown as thick dashed and dotted lines, respectively. Changing the M_1/R_1^4 dependence by increasing/decreasing α modifies the parent distribution in the sense predicted qualitatively above. The thin dashed and dotted lines refer to the cases $\gamma = 0$ and $\gamma = 0.5$. All graphs remain well within the statistical uncertainties of the standard distribution. Thus, the choice of α and γ is not critical.

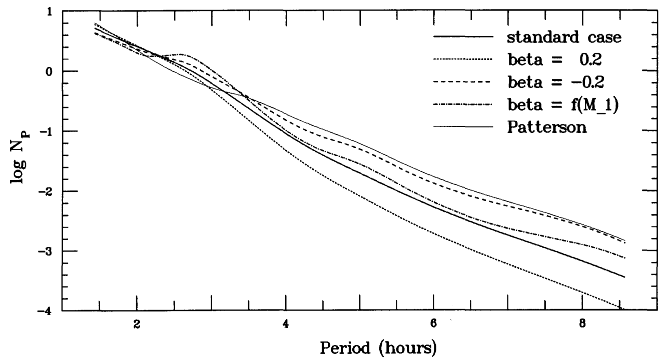

Fig. 4. Parent distribution function N of classical novae calculated using different prescriptions for the mass transfer rate and its influence on novae recurrence times. For details, see text.

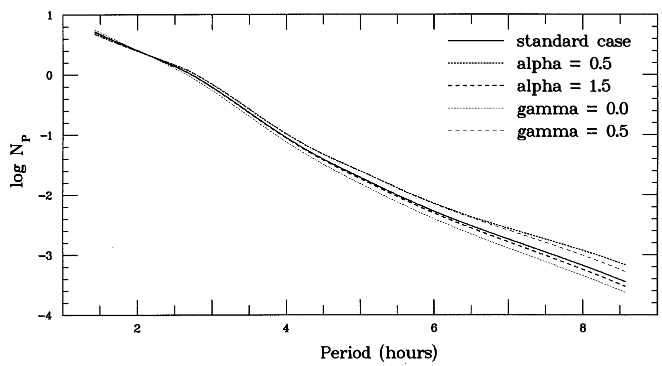

Fig. 5. Parent distribution function N of classical novae calculated using different values of the M_1/R_1^4-exponent α and the technical selection function exponent γ.

6.2. Comparison with population synthesis calculations

The parent distribution derived here can be compared to the distributions derived theoretically from population synthesis calculations. Kolb (1993) presented encompassing calculations for the CV population in general. While both, his and our approaches are not completely independent - in both cases the same theoretical white dwarf mass spectrum for CVs was used - they are sufficiently different to permit a useful comparison: We used the *observed* distribution together with the basic theory of nova outbursts to calculate the intrinsic distribution, taking into account selection effects, while Kolb used the result of calculations of the common envelope phase and evolutionary scenarios of CVs to achieve the same goal.

While the small number of observed nova periods clearly inhibits the comparison of details of the parent distribution with structure apparent in the population synthesis calculations - rendering any decision between the different models considered by Kolb impossible - the general shape of the parent distribution is not grossly different from Kolb's distributions (with the exception of the period gap; see 6.3) in the period range encompassed by the observations. This is shown in Fig. 6, where the standard case (solid line) is shown together with the populations according to models pm1 and pm3 (dotted and dashed thin lines, re-

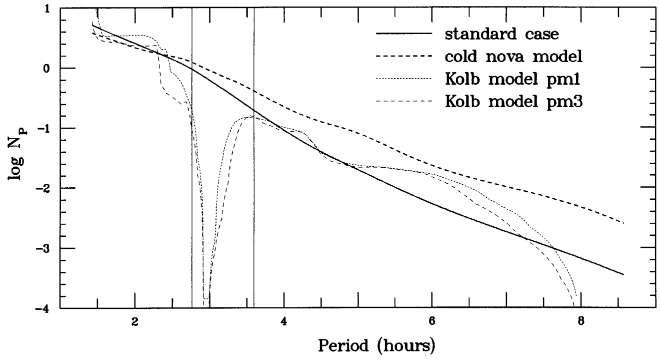

Fig. 6. Comparison of the parent distribution function N of classical (solid line) novae with distributions derived from theoretical population synthesis calculations (taken from Kolb 1993) (dashed and dotted thin lines). The thick dashed line is the distribution calculated under the assumptions of the cold nova model. The thin vertical lines indicate the limits of the period gap as defined by Kolb. For details, see text.

spectively) of Kolb (taken from his Fig. 4). Between $P = 1\overset{h}{.}5$ and $P = 7^{h}$ his distributions exhibit on the mean a decrease in $\log N$ of ≈ 3.2 versus 3.4 in our standard case. In view of the strong dependence of our results on the uncertain $\dot{M} - P$ relation this difference is hardly significant. However, Kolb's distributions predict an excess between $\sim 5^{h}$ and $\sim 7^{h}$ and a quicker drop for $P > 7^{h}$ than our distribution.

In another paper Kolb (1995) used the results of his population synthesis calculations together with an expression for the accreted mass ΔM required to initiate an outburst to predict the distribution of classical novae, defined through their eruptions, under various model assumptions. The resulting distributions should thus correspond to the observed one, were it not for the observational selection effect which Kolb did not consider. In this sense, his calculations are (partly) equivalent to ours, but going in the reverse direction. In fact, the solid line in his Fig. 1 is - within the period range so far observed in nova systems - not unlike the observed period distribution (disregarding the period gap) shown in Fig. 2: The density of systems at low periods is somewhat less than at intermediate ($3^{h} < P < 5^{h}$) periods and drops off towards higher values. The strong peak at very small periods in Kolb's (1995) distribution and also in his populations synthesis calculations (Kolb 1993) is beyond the observed short limit. The implications of this fact will be discussed in Sect. 6.4.

Kolb (1995) used the same prescription for the accreted mass ΔM required to ignite a nova outburst as we did in the standard model (i.e. β=0). However, he also considered a modified model, the cold nova model, where ΔM depends on $\dot{M}$ in a different way. This results in a nova distribution with a significant deficit of outbursts at small periods and an enhancement at intermediate ones (dotted line in his Fig. 1). We calculated the parent distribution function using the same law for ΔM and show it as a dashed thick line in Fig. 6. While the overall inclination at intermediate periods is not unlike that of Kolb's (1993) synthetic populations, it would predict a much too small relative number of short period novae. Thus, considering the period distribution from Kolb's population synthesis, the observed nova period distribution does not support the cold nova model.

6.3. The period gap

While we thus find an overall agreement of the intrinsic period distribution of novae and the theoretically predicted general CV distribution, the observed nova distribution and in consequence the calculated parent distribution does not show the period gap which evolutionary scenarios for CVs predict and which is so obvious in the *overall* observed period distribution. This lead Baptista et al. (1993) to the conclusion that no such gap exists for classical novae. From his theoretical calculations Kolb (1993) finds the period gap to be defined by the limits $2\overset{h}{.}34 \leq P \leq 3\overset{h}{.}24$. This is in reasonable agreement with the observed gap for the overall CV population: A period distribution function constructed from data in the Ritter catalogue (Ritter & Kolb 1993) clearly allows the empirical gap to be defined as $2\overset{h}{.}11 \leq P \leq 3\overset{h}{.}20$. Three novae (V Per, QU Vul and V2214 Oph) are observed to lie in this range.

Due to CVs born in the gap and young enough not yet to have evolved out of it, the gap is not expected to be totally devoid of novae. The calculations of the relative number of nova events by Kolb (1995), based on his synthetic CV population models (including the gap) can be used to find the probability to observe a given number of systems in the gap. Since Kolb disregarded observational selection effects a comparison with the observed distribution is not strictly possible. However, the overall similarity of the observed and calculated distribution (see Sect. 6.2) secures that the error will remain moderate.

Taking Kolb's (1995) distribution and the limits of the period gap (his Fig. 1, solid line) which is only marginally smaller than found in his previous study, the probability that out of a randomly chosen sample of 28 periods exactly 3 (3 or more) fall into the gap is only 1.1% (1.3%). So small probabilities justify concern and the question if classical novae have a way to avoid detaching and thus to stop mass transfer when passing through the classical period gap.

6.4. The low-period cutoff

Kolb's (1993) population synthesis calculations predict of the order of two times more CVs below the observed low period limit of CVs in general and classical novae in particular than above it. The predicted minimum period lying below the observed one can be explained by the insufficient knowledge of stellar opacities at low temperatures, rendering the calculated period cutoff highly uncertain. This issue is discussed by Kolb (1993) as well as by Rappaport et al. (1982).

Thus, the predicted presence of a large number of CVs below the observed period limit may just be due to an uncomplete understanding of CV evolution. However, this does not resolve the problem but only shifts it: These systems should then populate the range just above the empirical minimum period and thus the number of novae should be raised above the observed number. Therefore, compared to the overall synthetic CV population, classical novae appear to be deficient in the range between minimum period and period gap. However, as a caveat it must be mentioned, that on the observational side we are always dealing with small number statistics.

7. Conclusions

Using the observed orbital period distribution of classical novae we have calculated the intrinsic distribution of systems which are able to undergo nova outbursts, considering the selection effects related to (1) the frequency of nova eruptions in a given system in dependence of the principal parameters which according to current theories determine the outburst rate as a function of orbital period (mainly primary star mass and mass accretion rate), (2) the magnitude limitation of the sample for which orbital periods are measured, and (3) the observational techniques by which nova periods are determined.

In view of the strong dependence of the intrinsic distribution on the mass transfer rate, the functional dependence of which on the orbital period is neither theoretically nor observationally well established, and considering the small number of observed nova periods introducing considerable statistical uncertainties, its overall shape is remarkably similar to distributions from population synthesis calculations for catalcysmic variables in general.

However, there are also differences. The observed period distribution and thus in consequence the calculated intrinsic one does not show any indication of the 2-3 hour period gap which is such a prominent feature in the distribution functions of other types of CVs (in particular dwarf novae). The probability that this is due to small number statistics is very small. Thus, there seems to be a way for classical novae to avoid entering a detached phase when passing through the corresponding period range in the cause of their orbital evolution. Since dwarf novae behave differently in this respect both kinds of systems might not just be different manifestations of identical objects passing through different phases of a cyclic evolution. Within current theories the existence of the period gap is a natural consequence of the reaction of the secondary star to the orbital period evolution. Its absence in classical novae therefore forces a reconsideration of these theories.

Another difference concerning the period distribution of novae and the predicted one for CVs in general appears in the low period regime between the classical gap and the short period cutoff. However, the theoretical understanding of the period evolution in this range being far from complete, and the number of observed novae being so small that statistical uncertainties are large, the apparent deficiency of novae at very short periods might not yet be significant. To confirm if the observations are really at odds with the theories in this respect, a larger number of nova periods and improved theories are required.

Obviously much more measurements of nova periods are desirable in order to inspire more confidence in the results of the present study. With modern CCD techniques observational projects dedicated to this aim can be realized even at comparatively small telescopes. Currently, such a project is carried out by us at the LNA, and we urge other observers to do the same.

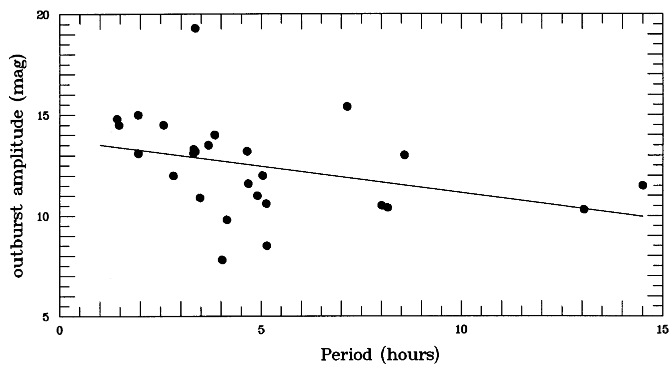

Fig. 7. Orbital period - outburst amplitude relation for classical novae together with the best fit straight line

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Appendix A: the period-amplitude relation of novae

In order to study the period-amplitude relation of novae, the amplitudes of all novae with known period were calculated from the maximum and minimum magnitudes quoted in the catalogue of Duerbeck (1987). The only system excluded from this investigation is GK Per because due to its evolved secondary its period is much longer than that of any other nova.

The outburst amplitudes are plotted against orbital period P in Fig. 7. It is obvious that there is a definite trend in the sense that the amplitude is lower if the period is longer. The most deviating points are V1500 Cyg which has an unusually high amplitude, and DO Aql and HR Del which have low amplitudes. Note that the latter is intrinsically extraordinarily bright in quiescence (Bruch 1982a) and that the observed outburst amplitude is therefore lower than that of nova with mean quiescent brightness and the same intrinsic strength of the outburst.

A linear least squares fit to the data yields a formal period-amplitude relation (shown as a solid line in Fig. 7) with an inclination of $a = -0.26 \pm 0.14$. The correlation coefficient is $r = -0.35$. The significance level at which the null hypothesis of zero correlation is disproved (see e.g. Press et al. 1986) is $s = 0.063$. (Excluding V1500 Cyg, DO Aql and HR Del leads to $a = -0.24 \pm 0.05$, $r = -0.48$ and $s = 0.013$.)

Since the observed amplitude of the outburst depends on the intrinsic strength of the eruption *and* the quiescent absolute magnitude of the system, i.e. on the background before which the outburst is seen, any relation between period and absolute magnitude of novae in quiescence will influence the period-amplitude relation. No such relation is known for nova, but Warner (1987) found that the absolute magnitude of dwarf novae in outburst depends linearily on the orbital period (if a correction is made to reduce all systems to the same orbital inclination). The accretion disks of outbursting dwarf novae are expected to behave similar to steady state disks, and so are the disks of novae in quiescence. Therefore, it may be assumed that the period-magnitude relation found by Warner is also valid for classical novae.

Warner's relation indicates that the absolute magnitude increases (i.e. its numerical value decreases!) with the period by 0.259 ± 0.024 mag/hour. Within the error limits this is identical to the decrease of the observed outburst amplitude of novae found above.

In conclusion we can state that the intrinsic strength of a nova outburst does not depend on the orbital period of the system.

Nova theory predicts that the outburst amplitude should increase with the white dwarf mass. The independence of intrinsic outburst amplitude and period therefore also indicates that the mass distribution of the white dwarfs in novae does not depend noticeably on the period.

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