

Properties of theoretical RRab light curves^{*}

M.U. Feuchtinger and E.A. Dorfi

Institut für Astronomie der Universität Wien, Türkenschanzstraße 17, A-1180 Wien, Austria

Received 30 September 1996 / Accepted 13 December 1996

Abstract. Using an adaptive radiation hydrodynamics code we have computed a grid of fundamental mode RR Lyrae models in order to give a detailed comparison to observed RR Lyrae light curves and to investigate the influence of several numerical parameters inherent to the computational method. The results can be summarized as follows: Concerning the comparison to observations the proposed solution of the well known RR Lyrae phase discrepancy in Feuchtinger & Dorfi (1996) is corroborated on the basis of more than 50 models. It is shown how the Fourier parameters vary during the crossings of the instability strip. Based on the large number of theoretical models we can study the properties of theoretical RRab light curves and e.g. state that no clear relation exists which links the fundamental stellar parameters to the low order Fourier coefficients. Investigating the long term evolution and the influence of several critical parameters like numerical viscosity, number of grid points or switching temperature between Lagrangian and adaptive grid, it turns out that the Fourier parameters are essentially independent on the special choice of our numerical parameters.

Key words: stars: variables – stars: interiors – stars: oscillations – methods: numerical

1. Introduction

In the past years great effort has been made to improve the nonlinear stellar pulsation calculations (e.g. Dorfi & Feuchtinger 1991, Cox et al. 1992, Gehmeyr 1992ab,1993, Fokin 1992, Feuchtinger & Dorfi 1994,1996, Buchler et al. 1996). These studies have been motivated by several persisting problems concerning the former generation of Lagrangian pulsation models (cf. the proceedings edited by Buchler 1990) which led to a deep insight into many aspects of the nonlinear dynamics of pulsating stars. However, the ‘standard’ Lagrangian codes suffer both from an inadequate numerical resolution and a lack of relevant

physical processes like time-dependent radiative transfer, influence of the stellar atmosphere, appropriate outer boundary conditions and simple fits to the opacity tables (e.g. Bono & Stellingwerf 1994). As a consequence, in particular RR Lyrae light curves computed with Lagrangian codes show conspicuous numerical perturbations and a detailed comparison to observations by Fourier decomposition reveals significant differences in the low order Fourier phases (Simon 1985, Simon & Aikawa 1986, Kovács 1990). This so-called *RR Lyrae phase discrepancy* being attributed to the absence of convective energy transport and an insufficient treatment of the radiative transfer in the outer layers (Kovács 1993), has been resolved by adaptive pulsation calculations which solve the full time-dependent radiation hydrodynamical equations (Feuchtinger & Dorfi 1996). The reliability of such computations enables us now to undertake a more systematic investigation than the aforementioned exploratory study.

Beginning with the work of Walraven (1955) it has been argued that a Fourier decomposition of light curves can link the observed shape of the luminosity changes to the stellar parameters. Simon & Lee (1981) have shown that for Cepheids the change of the Fourier parameter provides a quantitative description of the Hertzsprung progression. However, the corresponding work of Simon & Teays (1982) on RR Lyrae stars gives only ranges for the Fourier parameters and no clear progression can be extracted from the observational data. Using linear pulsation theory Simon (1979) has shown how the Fourier coefficients are related to the perturbations in luminosity and radius but a large number of uncertainties inherent to this method allows only rough estimates on the fundamental stellar parameters. Adopting our nonlinear models of RR Lyrae stars we will examine this conjecture.

In Sect. 2 we briefly describe the physical input and the Fourier decomposition. Sect. 3 is devoted to a comparison between the theoretical results and observations of RR Lyrae stars. In Sect. 4 we discuss the relation between the Fourier parameters and stellar quantities and describe the changes of the Fourier parameter during the crossings of the instability strip. The paper is closed by conclusions in Sect. 5. and an appendix A where we discuss the influence of the numerical parameters on the results.

Send offprint requests to: M.U. Feuchtinger
(FM@AMOK.AST.UNIVIE.AC.AT)

* Table 1 is only available in electronic form at CDS via anonymous ftp cdsarc.strasbg.fr (130.79.128.5) or via <http://cdsweb.u-strasbg.fr/Abstract.html>

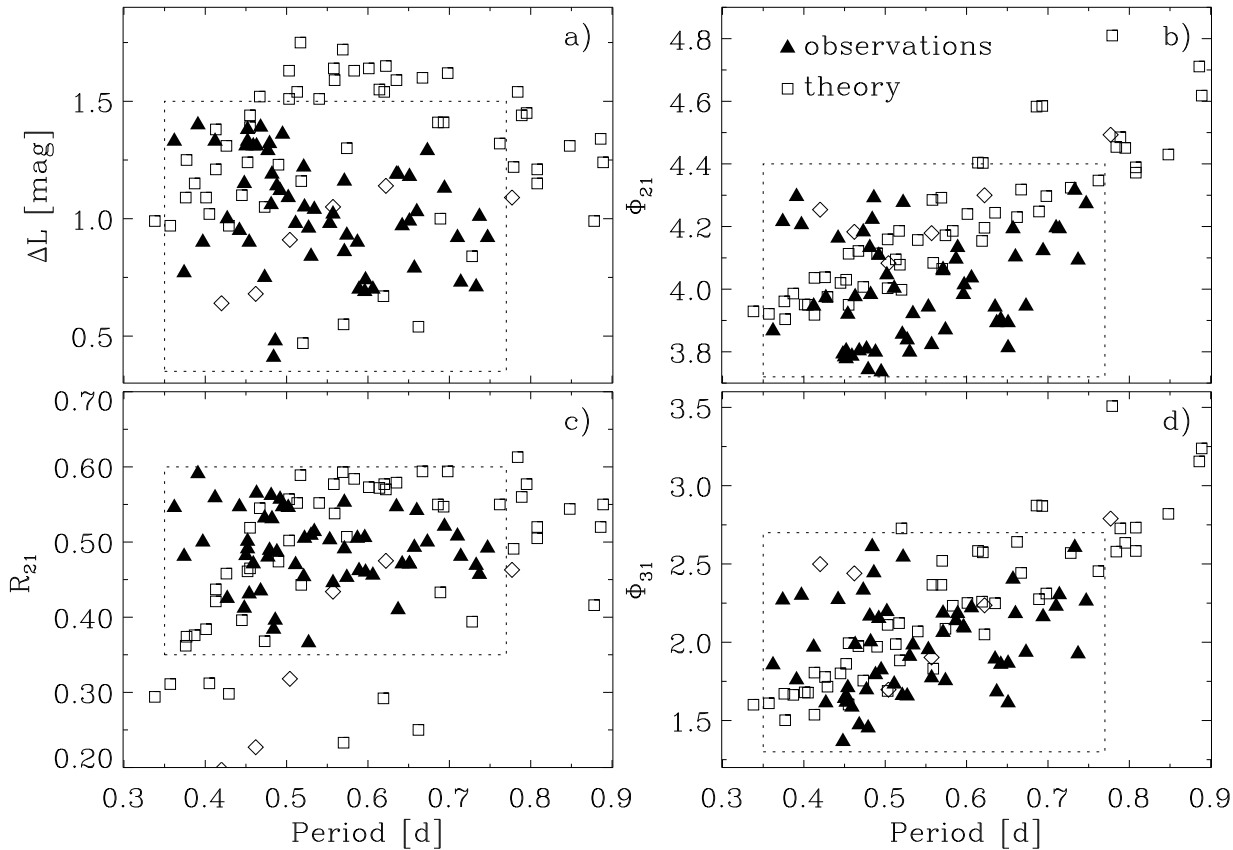


Fig. 1a–d. The observations of RR Lyrae field stars (filled triangles, Simon & Teays 1982) and theoretical RR Lyrae computations, open squares ($Z = 0.001$) and diamonds ($Z = 0.0001$). The dotted lines enclose the observational data. **a** Luminosity variation ΔL in magnitudes, **b** the Fourier amplitude R_{21} , **c** the Fourier phases Φ_{21} and **d** Φ_{31} .

2. Physical input and model parameters

The radiative RR Lyrae models are calculated by solving the full system of the nonlinear radiation hydrodynamical equations (e.g. Dorfi & Feuchtinger 1995) together with the Livermore opacities (OPAL, Iglesias & Rogers 1991) and a realistic equation of state provided by Wuchterl (1990). At temperatures below 6000 K the OPAL-tables are extended by molecular opacities from Alexander et al. (1989). Time-dependent radiative transfer is treated in the grey approximation and since it is not necessary to take into account the stellar atmosphere, the radiative moment equations are closed by the Eddington approximation (e.g. Feuchtinger & Dorfi 1994). It has turned out that a time-dependent treatment of the radiation field is essential for the temporal behaviour of the radiation field which determines the shape of the light curve and correspondingly the Fourier parameters. All physical equations are reformulated for an adaptive grid (Dorfi & Drury 1987) which determines the position of the individual grid points. A detailed description of the numerical method together with applications on nonlinear stellar pulsations can be found in Dorfi & Feuchtinger (1995).

Convective energy transport is not included in our present study which could alter some results on the long period side. Gehmeyr (1992a, 1993) has taken into account convective en-

ergy transport. He has extended the equations of radiation hydrodynamics by the Stellingwerf–Castor model (Stellingwerf 1982) for time-dependent turbulent convection and investigated the red end of the instability strip. However, for fixed stellar mass, luminosity and effective temperature convection decreases the temperature gradient and therefore increases the stellar radius and decreases the mean stellar density. As seen in all our computations the extended RR Lyrae models exhibit more pronounced nonlinearities in the light curves than the bluer radiative models. Consequently, the Fourier phases Φ_{21} and Φ_{31} increase even more than depicted in Fig. 2 towards the lower mean densities whereas the convective energy transport will reduce the pulsation amplitudes in accordance with a decrease of the Fourier ratio R_{21} . A detailed discussion on these effects employing the convective transport scheme of Kuhfuß (1986) will be given in a forthcoming paper.

The Fourier decomposition of RR-Lyrae light curves has been done by using a standard nonlinear least square fit on a cos-series with 8 terms as described e.g. in Simon & Teays (1982). We note in passing that Jurcsik & Kovács (1996) have applied a sin-series to the observed light curves when they derive their period–phase–[Fe/H] relation. Due to the lack of observational material on the velocity variations of RR Lyrae stars we can-

not compare our theoretical velocity changes with such data and we are restricted to the decomposition of our bolometric light curves. From the Fourier amplitudes and phases the following low order combinations are then derived $R_{21} = A_2/A_1$, $\Phi_{21} = \Phi_2 - 2\Phi_1$ and $\Phi_{31} = \Phi_3 - 3\Phi_1$ (e.g. Feuchtinger & Dorfi 1996). We also want to emphasize that the absolute bolometric magnitude M_{bol} is used for the Fourier decomposition of our theoretical light curves and compared to the observed V magnitudes.

3. Comparison with observations

To get a quantitative comparison between theoretical and observational RR Lyrae light and radial velocity curves a number of authors have carried out Fourier decompositions (e.g. Simon & Teays 1982, Simon 1985). From this Fourier decomposition the low order combinations R_{21} , Φ_{21} and Φ_{31} are utilized to characterize the amplitude as well as the shape and phase information and recently also to obtain the $[\text{Fe}/\text{H}]$ from RR Lyrae light curves (e.g. Jurcsik & Kovács 1996). After resolving the so-called phase discrepancy (Feuchtinger & Dorfi 1996) between observations and theoretical models of earlier fully Lagrangian computations we can take the observational data base of (Simon & Teays 1982) for comparison with a large number of our theoretical RR Lyrae models as summarized in Table 1. The Fourier coefficients of a large sample of observed RRab field star light curves have been taken directly from the tables in Simon & Teays (1982).

In Fig. 1 we have plotted their observational data (filled triangles) of RR Lyrae field stars together with the results of our computations (open squares and diamonds) compiled in Table 1. This table is also available in electronic form from the authors. The dotted lines indicate the observational range of the periods and Fourier parameter. Since we have computed only radiative models the cooler models with larger periods can be less realistic because the convective energy transport is not included in these pulsation calculations. All models presented are fundamental mode pulsators and populate well the observed parameter ranges. The agreement between observations and our theoretical model is quite satisfactory although for Φ_{21} a small but systematic difference remains. Several reasons can be responsible for such a small offset in Φ_{21} , i.e. uncertainties in the opacities, differences between bolometric and visual light curves as well as the lack of convective energy transport leading to slightly different stellar structures across the entire instability strip. In order to check the influence of the metallicity on the Fourier parameters one sequence corresponding to $M = 0.85 M_{\odot}$ and $L = 65 L_{\odot}$ has been computed both with $Z = 0.001$ and $Z = 0.0001$. From the open diamonds in Fig. 1 representing the models with lower metallicity and the direct comparison in Table 1 no trend can be drawn towards lower Φ_{21} . Hence, at least for the models calculated up to now the metallicity effects are probably not the reason for this offset in Φ_{21} .

For RR Lyrae stars with periods longer than 0.8 days only few Fourier decompositions exist which indicate an increase

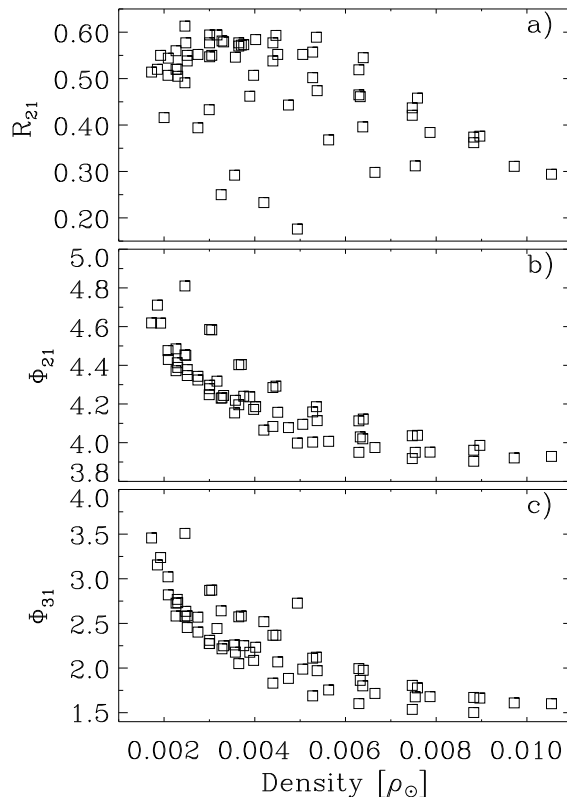


Fig. 2a–c. Fourier coefficients as a function of the mean stellar density in units of the mean solar density ρ_{\odot} . **a** The Fourier amplitude R_{21} , **b** the Fourier phases Φ_{21} and **c** Φ_{31} .

of both phases (Pel 1976, Kwee & Diethelm 1984) but up to now no trend can be extracted from these observations. On the one hand our radiative calculations predict a clear increase of the Fourier phases towards longer periods. However, we are not able to estimate the influence of convection on the Fourier phases without convective pulsation models and hence we must await further computations in order to see if this increase is real or only an artefact due to the lack of convection.

As seen in both data sets no clear correlation can be extracted from the observations as well as from the theoretical models. However, as mentioned in the next section the Fourier parameter change due to evolutionary effects but all fundamental stellar parameters seem to influence the shape of the light curve in a complicated and non-unique way.

4. Evolutionary effects

Based on the large number of theoretical RR Lyrae models of high numerical quality as summarized in Table 1 we investigate the influence of stellar parameters on the Fourier coefficients R_{21} , Φ_{21} and Φ_{31} .

Fig. 2 shows the dependence of these Fourier coefficients on the mean stellar density ρ in units of the mean solar density defined by $\rho_{\odot} = M_{\odot}/R_{\odot}^3$. Since $P \propto \rho^{-1/2}$ the overall increase of the Fourier phases clearly reveals the dependence

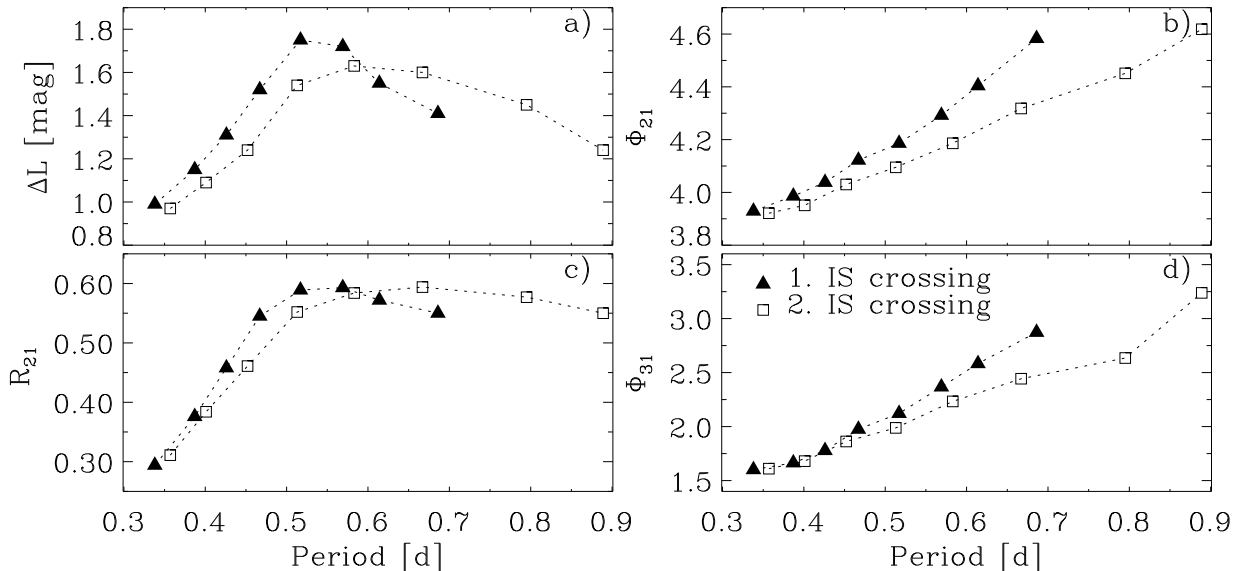


Fig. 3a–d. Evolution of amplitude and Fourier coefficients as a function of the effective temperature T_{eff} during two crossings of the instability strip for a RR Lyrae model with $M = 0.66 M_{\odot}$ and $Z = 0.001$. **a** the luminosity variation ΔL in magnitudes, **b** the Fourier phase Φ_{21} , **c** the Fourier amplitude R_{21} and **d** the Fourier phase Φ_{31} .

on the period and the amplitude ratio R_{21} basically shows the instability strip with a broad maximum around $0.004 \rho_{\odot}$. However, in contrast to Fig. 1 we chose this plot to show how at a given mean density the range of Fourier phases increases with decreasing ρ . Consequently, for low mean densities the Fourier phases do not only depend on the pulsation period alone as also seen in Fig. 3.

Without plotting the correlations between the variables we have to state for the models calculated that we do not find a clear connection between the Fourier parameters and fundamental stellar parameters like mass, luminosity or radius. Beside a general characteristics of the shape of the light curve it seems therefore rather difficult to relate a particular combination of the Fourier parameter to other stellar parameters. However, as explored in the following we find a notable variation of the Fourier parameters as a star evolves across the instability strip and the lowest values for R_{21} , Φ_{21} and Φ_{31} are located at the bluest RR Lyrae pulsators.

Taking standard RR Lyrae evolutionary calculations using the same OPAL opacities (e.g. Koopmann et al. 1994) we can follow the evolution of the Fourier parameters as the star moves across the instability strip in the HR–diagram (cf. the models with $M = 0.66 M_{\odot}$ in Table 1). It is clear that the period should be decreasing as the star evolves from red to blue. After the turnaround in the HR–diagram we expect an increasing period for the RR Lyrae becoming redder again.

In Fig. 3 we plot the evolution of the luminosity change together with the Fourier coefficients during such crossings of the instability strip for a RR Lyrae model with $M = 0.66 M_{\odot}$ and $Z = 0.001$. The first sequence (filled triangles) comes from the almost horizontal path with $L \simeq 45 L_{\odot}$ and the effective temperature varying between 6100 K and 7500 K. The second

crossing (open squares) leads to an increasing luminosity from $47.5 L_{\odot}$ at $T_{\text{eff}} = 7500$ K to $61.2 L_{\odot}$ at $T_{\text{eff}} = 6100$ K with values given in Table 1. In both cases we see clear trends in all Fourier coefficients R_{21} , Φ_{21} and Φ_{31} . First, the luminosity variation is clearly correlated with the Fourier amplitude ratio R_{21} (cf. Fig. 3a,c) and both maxima occur at the same temperatures. Secondly, the Fourier phases Φ_{21} and Φ_{31} depicted in Figs. 3b,d increase both with the period or the decreasing effective temperature. Correspondingly, the nature of these light curves changes from a more sinusoidal shape to a more triangular form characterized by the rapid luminosity increase and followed by the slow descending part. From this figure and the fact that the luminosity is larger during the second crossing we expect in general lower Fourier coefficients as the stars evolves towards the red edge. Note that during the first crossing the Fourier phases Φ_{21} and Φ_{31} are always decreasing whereas the second crossing is accompanied by monotonically increasing Fourier phases. For periods lower than 0.6 days the Fourier phases seem to depend only on the mean stellar density irrespective of their evolutionary status and the difference between first and second crossing remains rather small. At larger periods these differences become more significant but it is not clear whether convective energy transport will alter this behaviour.

From the large sample of stellar evolution calculations of RR Lyraes (e.g. Lee & Demarque 1990) we can estimate the typical crossing times of the instability strip. For our RR Lyrae model with $M = 0.66 M_{\odot}$, $Z = 0.001$ and $Y_{\text{He}} = 0.23$ presented in Fig. 3 we find about $20 \cdot 10^6$ years for the first crossing and about $4 \cdot 10^6$ years for the second crossing. Hence, it will be rather impossible to observe the changes in the corresponding Fourier phase Φ_{21} and Φ_{31} since according to the evolutionary time scale they will be of the order of 10^{-6} yr^{-1} . The situation can

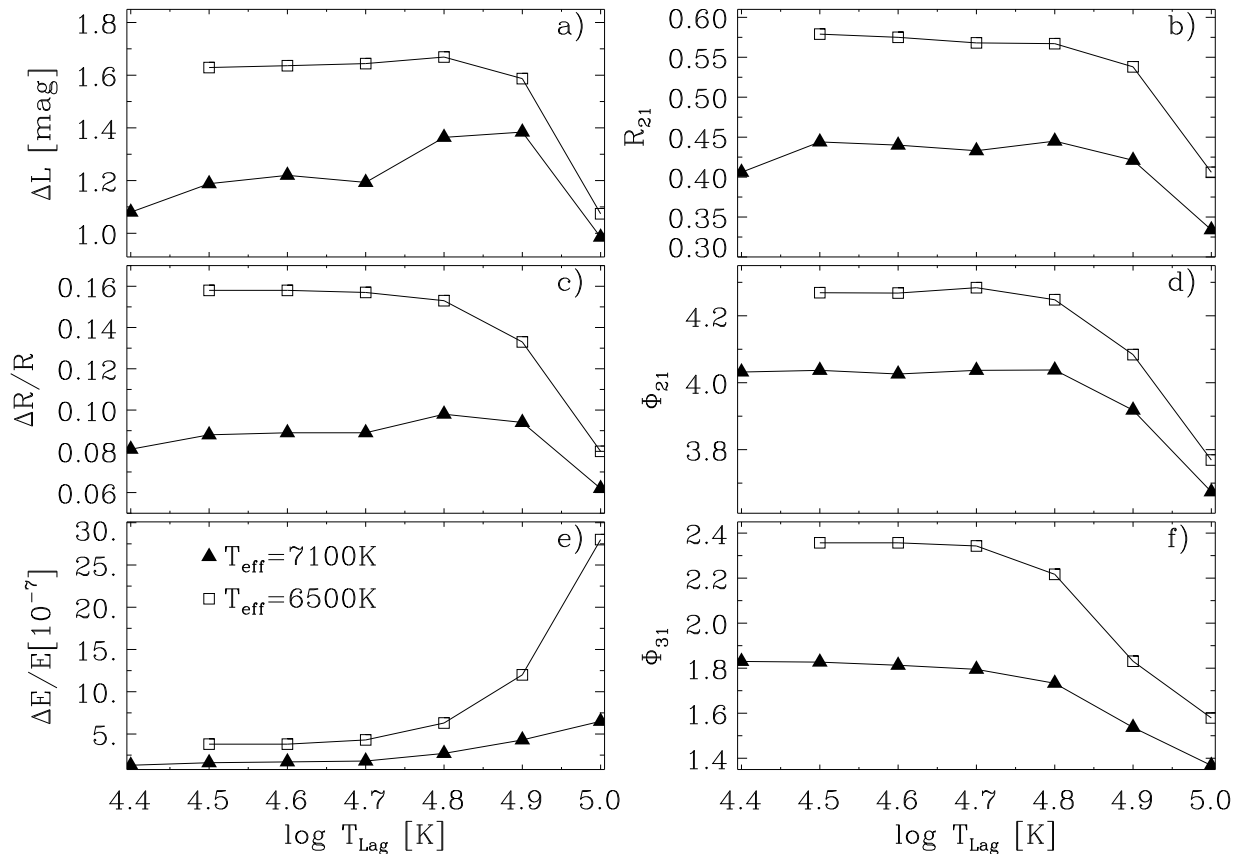


Fig. 4a–f. The dependence of the pulsation characteristics on the location of the Lagrangian switch point fixed by the temperature T_{Lag} for two different RR Lyrae models (see text). **a** Luminosity variation ΔL during a pulsation in magnitudes, **b** the Fourier amplitude R_{21} , **c** relative radius variation $\Delta R/R$, **d** the Fourier phase Φ_{21} , **e** the total energy error $\Delta E/E$ in units of 10^{-7} , **f** the Fourier phase Φ_{31} .

be somewhat better if we can observe stars near the red upper part of the instability strip where the evolution proceeds faster.

5. Conclusions

Since the observed shapes of the light curves have been analyzed by low order Fourier parameters R_{21} , Φ_{21} and Φ_{31} , we have undertaken the same effort on newly calculated theoretical RRab pulsations. Due to the numerical improvements by adaptive radiation hydrodynamical calculations a direct comparison becomes feasible between observational light curves and theoretical pulsation calculations. From the computations we clearly see monotonic changes of the Fourier parameters as the stars evolve along their paths in the HR–diagram. These secular changes are rather small but we can in principle distinguish between the two crossings of the instability strip. However, it seems that a number of stellar parameters determines the Fourier parameters (i.e. the shape of the light curve) and hence it is impossible to extract directly the stellar properties from comparing our numerical results with observations. From the correlation between the mean density and the Fourier parameters we see how the decreasing mean density increases the range of the Fourier phases as the star moves towards the red

edge of the instability strip. In such a mean density diagram the Fourier amplitude ratio also depicts the pulsation amplitude changes across the instability strip.

All our computed models predict a clear trend of increasing Fourier phases with increasing period which is also indicated by the observations of RR Lyrae stars with periods longer than one day. However, computations for cool RR Lyrae stars have to include convective energy transport in order to make reliable predictions on the details of the light curves.

In the appendix of this paper we study the influence of basic numerical parameters and the long term evolution of our models. From this investigation three important points can be drawn. First, the long term evolution of our models clearly shows constancy of the pulsation properties over the calculated 5000 pulsation cycles. Although the total energy error is increasing in time all characteristic pulsation quantities remain unchanged. Hence we conclude that the conservation of total energy is an unsuitable criterion for the quality of nonlinear stellar pulsation models. Secondly, the pulsation properties are independent of the location of the Lagrangian switch point if it is chosen in the range of $4.5 \leq \log T_{\text{Lag}} \leq 4.7$. Thirdly, the nonlinear period and the Fourier phases do not essentially depend on the value of the numerical viscosity parameter.

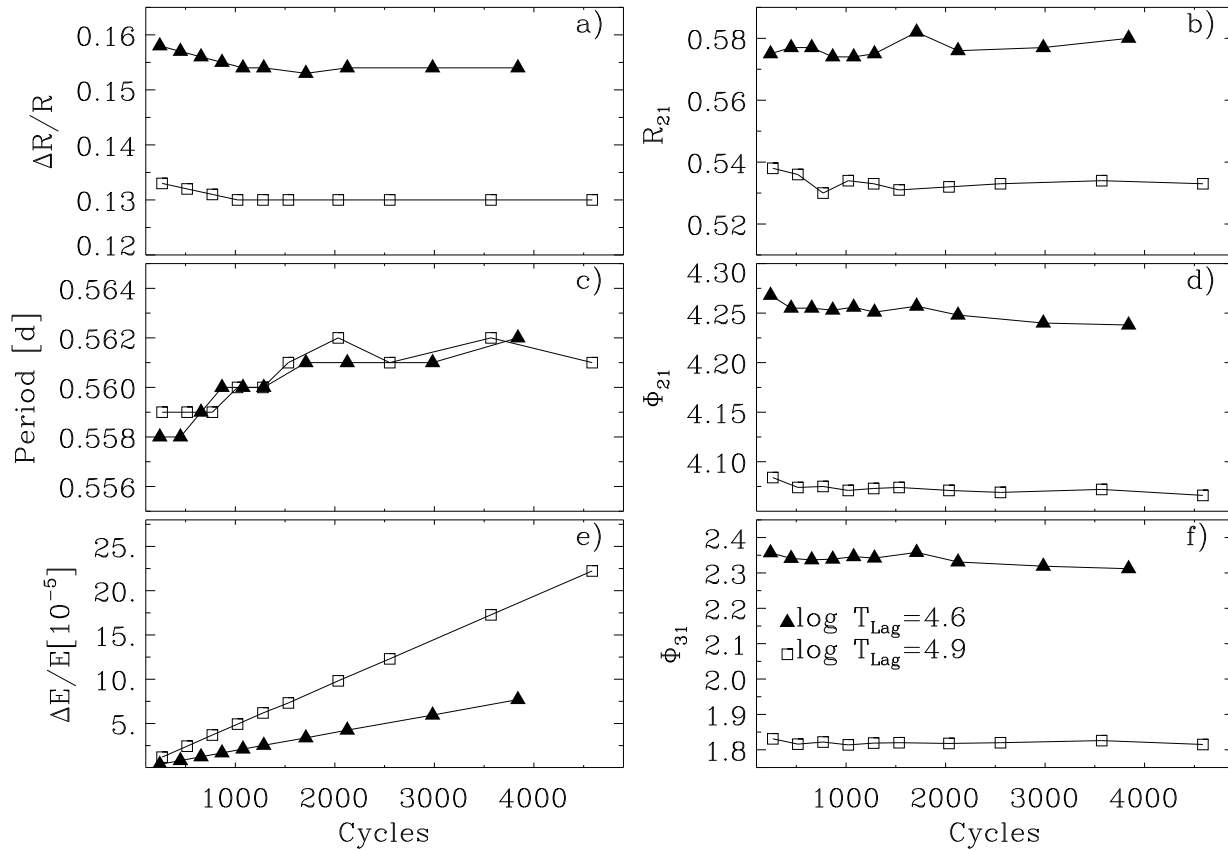


Fig. 5a–f. Long term evolution of the Fourier coefficients as a function of the number of periods for one model with different Lagrangian switch points. **a** The relative radius variation $\Delta R/R$, **b** the Fourier amplitude R_{21} , **c** the pulsation period, **d** the Fourier phases Φ_{21} and **e** the accumulated total energy error in units 10^{-5} of the initial total energy, **f** the Fourier phases Φ_{31} .

By enlarging the set of computational models through calculations with different elemental abundances it would also be possible e.g. to test the observational period–phase–[Fe/H] relation as discussed by Jurcsik & Kovács (1996). Since so far we have calculated only one sequence (cf. Table 1) with a lower metallicity of $Z = 0.0001$ we cannot verify their relation on the basis of our RR Lyrae computations.

Acknowledgements. This work is supported by the *Fonds zur Förderung der wissenschaftlichen Forschung (FWF)* under project number S7305–AST.

Appendix A

A.1. Numerical influence

As discussed in previous papers (e.g. Kovács 1993, Feuchtinger & Dorfi 1996) all nonlinear pulsation calculations by adaptive radiation hydrodynamical codes depend basically on two numerical parameters. First, the location where an inner Lagrangian grid joins the adaptive grid and secondly the amount of artificial viscosity used to spread shock waves over a finite length. Note that the numerical value of the viscosity can be reduced by at least two orders of magnitude if an adaptive code is used. The following three short sections are devoted to a brief

discussion of these numerical parameters demonstrating how accurate the physical results can be controlled.

.1. Lagrangian switch point

Starting with the initial hydrostatic model we fix the Lagrangian switch point at a certain grid index located at the minimum of ∇_{ad} within the second Helium ionization zone i.e. T_{Lag} around 30000 K. Inside this index the grid points move Lagrangian whereas outside we employ the adaptive grid. In this way the errors induced by advection across the computational cells remain small and it is possible to follow the pulsation over many thousands cycles (see also Sect. .1).

In Fig. 4 we plot the dependence of the pulsation characteristics on the Lagrangian switch point as a function of the temperature $\log T_{\text{Lag}}$ for two different RR Lyrae models with $T_{\text{eff}} = 7100$ K (filled triangles) and $T_{\text{eff}} = 6500$ K (open squares). It is evident that all pulsation variables are almost independent on the exact value of T_{Lag} as long as the Lagrangian switch point is not located too deep in the envelope ($\log T_{\text{Lag}} \leq 4.7$) causing an increase of the advection errors during a pulsation cycle (Fig. 4e). On the other hand the adaptive grid is needed to provide the resolution within the ionization zones ruling out fully Lagrangian computations. However,

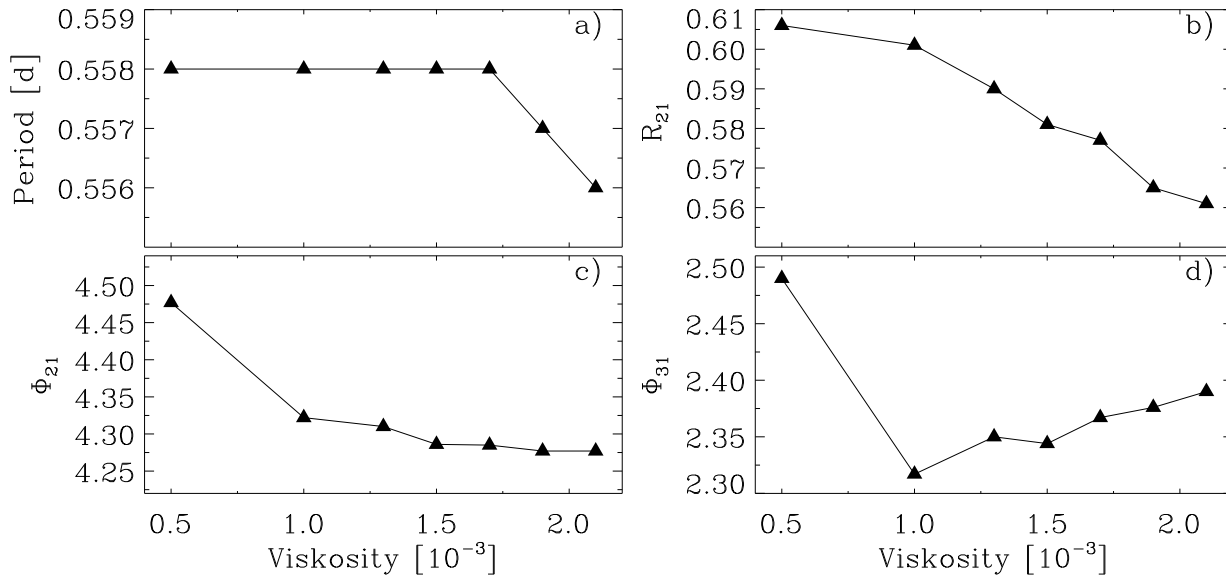


Fig. 6a–d. The influence of the artificial viscosity in units of the local radius on the pulsation properties. **a** The pulsation period in days, **b** the Fourier amplitude R_{21} , **c** the Fourier phases Φ_{21} and **d** Φ_{31} .

over the entire instability strip (characterized by the two effective temperatures depicted) this behaviour of constant pulsation characteristics for $4.5 \leq \log T_{\text{Lag}} \leq 4.7$ allows calculations with the same set of numerical parameters. Hence, the location of the Lagrangian switch point does not influence the pulsation properties if it is chosen within the quoted temperature range. For all calculations presented in this study we take the minimum of the adiabatic temperature gradient ∇_{ad} in the second Helium ionization zone which is close to $\log T_{\text{Lag}} = 4.6$. Note further that an increase of the advection error leads to decreasing Fourier phases Φ_{21} and Φ_{31} .

A.2. Long term evolution

As already discussed in Feuchtinger & Dorfi (1994) we solve an equation for the internal energy instead of the total energy equation. As a consequence we ensure total energy conservation only up to a known iteration accuracy by the adopted numerical scheme. In the literature it is often argued that exact total energy conservation is in particular necessary when calculating nonlinear stellar pulsations (e.g. Buchler 1990, Buchler et al. 1996). In order to demonstrate that our RR Lyrae models are not affected by this total energy error we present the long term evolution of one model for 2 different locations of the Lagrangian switch point leading to different values of the accumulated total energy error. This long term evolution over almost 5000 pulsation cycles is depicted in Fig. 5. Concerning the total energy error (Fig. 5e) some points need to be emphasized. The overall decrease of the total energy is due to a different resolution during the expansion and contraction phases (Feuchtinger & Dorfi 1994). If this energy loss would affect the long term evolution we would expect a decrease of the pulsation period with time. However, as seen in Fig. 5c we observe the oppo-

site behaviour of the period which we explain by higher modes being damped out as the pulsation evolves (cf. Fig. 5d,f). This slight increase of the period occurs for both Lagrangian switch points of $\log T_{\text{Lag}} = 4.6$ (filled triangles) and $\log T_{\text{Lag}} = 4.9$ (open squares). In the case of $\log T_{\text{Lag}} = 4.9$ the relative total energy error at the end of this sequence is about $2.5 \cdot 10^{-4}$ which is already about 1000 times the typical kinetic energy of the pulsating star. Showing the constancy over the entire computational run without any clear trend we quote the relative errors of about 1 percent in the radial changes and of about 0.5 percent in the other pulsation characteristics like period P and Fourier parameters R_{21} , Φ_{21} and Φ_{31} . Keeping in mind our numerical results on various pulsating stars we think that the error in conserving the total energy cannot be used as a suitable criterion for the quality of the nonlinear computations.

A.3. Numerical viscosity

The problems of stellar pulsations involving numerical viscosity have been discussed in several papers (e.g. Kovács 1990, Dorfi & Feuchtinger 1995). This viscosity to broaden shock waves is constructed according to the geometrically correct tensor formulation of Tscharnuter & Winkler (1979). Hence, in the present study we emphasize only two points. First, the non-adaptive codes have to spread the shock fronts over a fixed number of grid points causing that the shock fronts vary by their thickness depending on the local resolution. Secondly, within an adaptive code we are able to specify a physical length scale over which a shock front is broadened and the grid points are clustered around the wave to resolve it properly. Throughout the results discussed in the previous sections we use a relative length of $1.5 \cdot 10^{-3}$ of the local radius for the numerical viscosity.

Fig. 6 plots the influence of the numerical viscosity as a function of the relative length over which a shock front is broadened. Since the viscosity decreases the amplitude of the pulsation increases as seen in the corresponding Fourier amplitude ratio R_{21} (Fig. 6b). After reducing the viscosity below $1.7 \cdot 10^{-3}$ the period (Fig. 6a) remains constant and we adopt a value of $1.5 \cdot 10^{-3}$ for our computations since the observed properties of RR Lyrae stars impose limits of the amplitude of the luminosity variations through the instability strip. A tinier viscosity leads also to additional small features in the light curves due to the finite radial resolution of computations.

References

- Alexander, D.R., Augason, G.C., Johnson, H.R. 1989, ApJ 345, 1014
 Bono G., Stellingwerf R.F. 1994, ApJS 93, 233
 Buchler J.R. (Ed.) 1990, The Numerical Modelling of Non-linear Stellar Pulsations, NATO Workshop, Les Arcs, D. Reidel, Dordrecht
 Buchler J.R., Kolláth Z., Marom A. 1996, ApSS, in press
 Cox A.N., Deupree R.G., Gehmeyr M. 1992, Physica D 60, 139
 Dorfi E.A., Drury L.O'C. 1987, J. Comp. Phys. 69, 175
 Dorfi E.A., Feuchtinger M.U. 1991, A&A 249, 417
 Dorfi E.A., Feuchtinger M.U. 1995, Comp. Phys. Comm. 89, 69
 Feuchtinger M., Dorfi E.A. 1994, A&A 291, 225
 Feuchtinger M., Dorfi E.A. 1996, A&A 306, 837
 Fokin A.B. 1992, MNRAS 256, 26
 Gehmeyr M. 1992a, ApJ 399, 265
 Gehmeyr M. 1992b, ApJ 399, 272
 Iglesias, C.A., Rogers, F.J. 1991, ApJ 371, 408
 Gehmeyr M. 1993, ApJ 412, 341
 Jurcsik J., Kovács G. 1996, A&A, 312, 111
 Koopmann R.A., Lee Y.W., Demarque P., Howard J.M. 1994, ApJ 423, 380
 Kovács G., 1990, in: The Numerical Modelling of Non-linear Stellar Pulsations, Ed. J.R. Buchler, NATO Workshop, Les Arcs, D. Reidel, Dordrecht
 Kovács G. 1993, in: New Perspectives on Stellar Pulsation and Pulsating Variable Stars, Eds. J.M. Nemeč and J.M. Matthews, IAU Colloquium 139, Victoria, Cambridge Univ. Press
 Kuhfuß R. 1986, A&A 160, 116
 Kwee K.K., Diethelm R. 1984, A&AS 55, 77
 Lee Y.W., Demarque P. 1990, ApJS 73, 709
 Pel J.W. 1976, A&AS 24, 413
 Simon N.R. 1979, A&A 74, 30
 Simon N.R. 1985, ApJ 299, 723
 Simon N.R., Aikawa T. 1986, ApJ 304, 249
 Simon N.R., Lee A.S. 1981, ApJ 248, 291
 Simon N.R., Teays T.J. 1982, ApJ 261, 586
 Stellingwerf R.F. 1982, ApJ 262, 330
 Tscharnuter W.M., Winkler K.-H.A. 1979, Comp. Phys. Comm. 18, 171
 Walraven Th. 1955, Bull. Astron. Inst. Neth. 12, 233
 Wuchterl G. 1990, A&A 238, 83